Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.12-e-x-^m-a+b-sin-c+d-x^n-^p

Nasser M. Abbasi

May 18, 2020 Compiled on May 18, 2020 at 5:28am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [357]. This is test number [69].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (357)	% 0. (0)
Mathematica	% 96.64 (345)	% 3.36 (12)
Maple	% 68.63 (245)	% 31.37 (112)
Maxima	% 62.46 (223)	% 37.54 (134)
Fricas	% 85.43 (305)	% 14.57 (52)
Sympy	% 26.05 (93)	% 73.95 (264)
Giac	% 43.7 (156)	% 56.3 (201)

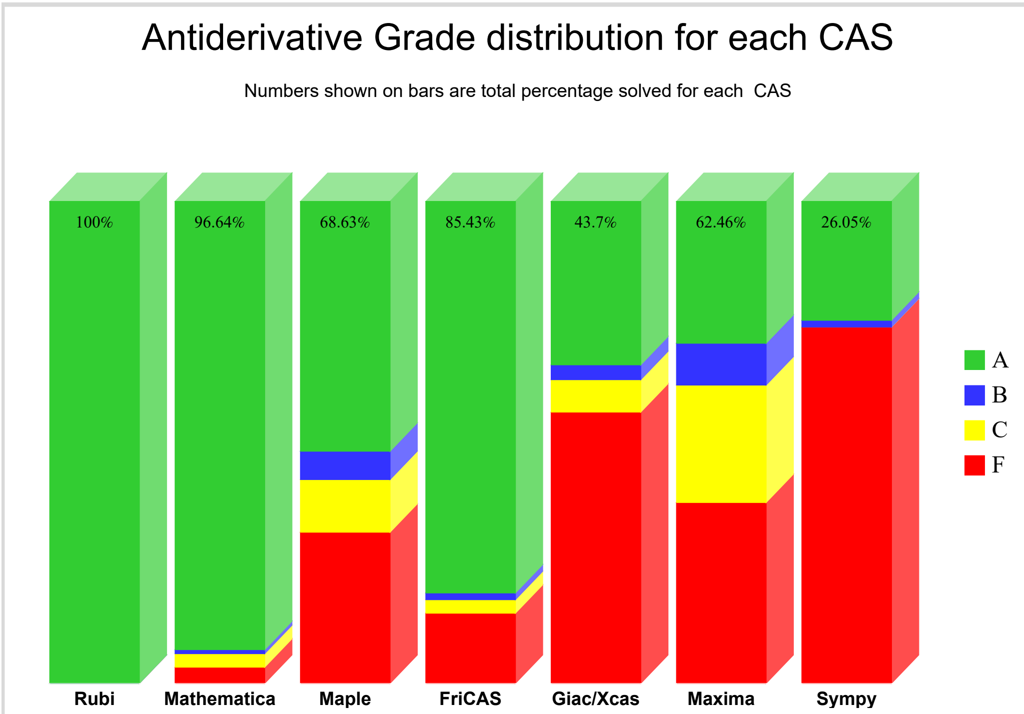
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

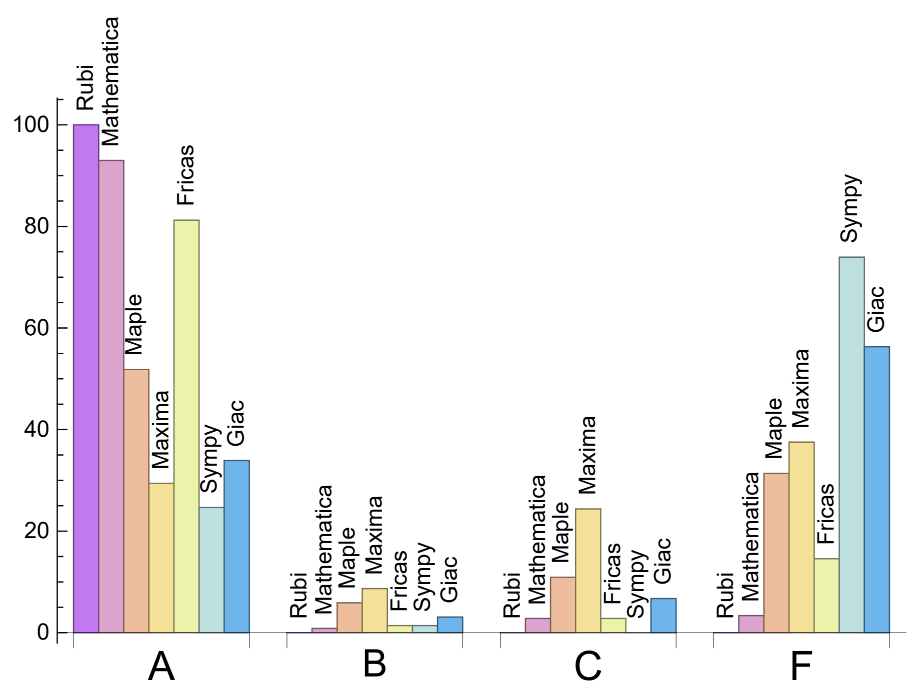
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	93.	0.84	2.8	3.36
Maple	51.82	5.88	10.92	31.37
Maxima	29.41	8.68	24.37	37.54
Fricas	81.23	1.4	2.8	14.57
Sympy	24.65	1.4	0.	73.95
Giac	33.89	3.08	6.72	56.3

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	131.66	0.77	96.	1.
Mathematica	3.01	120.64	0.73	81.	0.82
Maple	0.16	160.21	1.01	62.	0.87
Maxima	2.4	448.49	4.03	78.	1.38
Fricas	1.57	349.06	2.21	204.	2.1
Sympy	3.21	83.75	1.15	20.	0.89
Giac	0.57	147.97	1.02	0.	0.

1.4 list of integrals that has no closed form antiderivative

{38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 55, 56, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 101, 102, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 195, 196, 205, 206, 215, 216, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {133, 182, 272, 273, 274, 291, 296}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

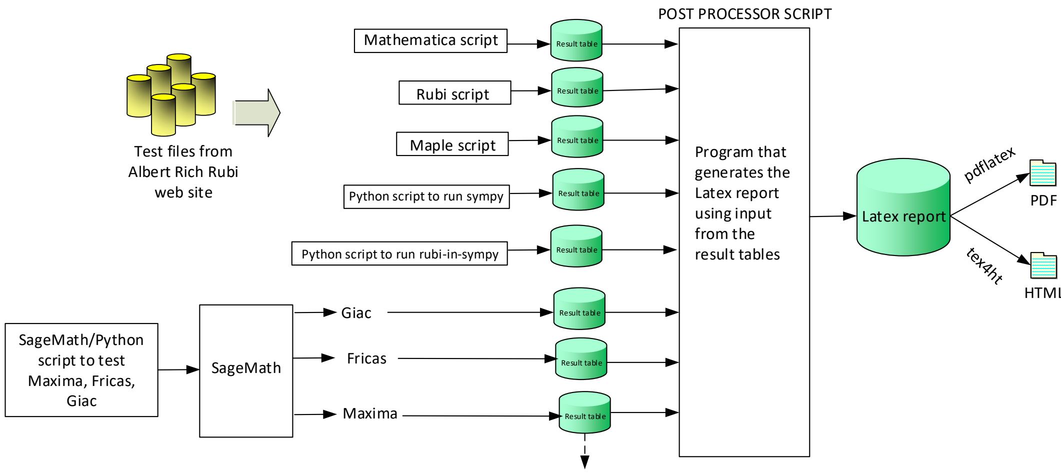
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 192, 194, 195, 196, 198, 199, 202, 204, 205, 206, 207, 208, 209, 213, 214, 215, 216, 218, 219, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 284, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade: { 183, 193, 203 }

C grade: { 190, 191, 197, 210, 211, 212, 217, 220, 221, 222 }

F grade: { 171, 172, 173, 200, 201, 226, 260, 261, 282, 283, 285, 286}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 69, 70, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 101, 102, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 162, 163, 164, 169, 170, 175, 176, 177, 178, 179, 180, 181, 185, 186, 189, 195, 196, 197, 198, 199, 200, 205, 206, 209, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

B grade: { 109, 110, 116, 117, 118, 153, 154, 155, 165, 166, 167, 168, 187, 188, 190, 191, 201, 207, 208, 293, 298 }

C grade: { 16, 17, 27, 139, 142, 210, 211, 220, 221, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 331, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 355 }

F grade: { 35, 36, 43, 44, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 131, 133, 140, 141, 143, 144, 171, 172, 173, 174, 182, 183, 184, 192, 193, 194, 202, 203, 204, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 310, 318, 326, 327, 328, 329, 330, 332, 333, 334, 342, 350, 351, 352, 353, 354, 356, 357 }

2.1.4 Maxima

A grade: { 1, 2, 3, 12, 13, 14, 23, 24, 25, 33, 38, 39, 40, 41, 42, 51, 55, 57, 58, 69, 70, 83, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97, 101, 107, 115, 122, 124, 125, 126, 127, 128, 129, 130, 132, 134, 145, 146, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 189, 195, 196, 205, 206, 209, 215, 216, 225, 226, 231, 238, 245, 255, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 284, 287, 299, 300, 301, 302, 303, 305, 306, 307, 309, 311, 312, 313, 314, 319, 321, 336, 343, 345

B grade: { 37, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 82, 187, 188, 192, 193, 194, 202, 203, 204, 207, 208, 335, 337, 338 }

C grade: { 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 34, 59, 60, 71, 72, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 123, 153, 154, 155, 156, 165, 166, 167, 168, 197, 198, 199, 212, 213, 214, 217, 218, 219, 222, 223, 224, 288, 289, 290, 294, 295, 315, 316, 317, 320, 322, 323, 324, 325, 339, 340, 341, 344, 346, 347, 348, 349 }

F grade: { 35, 36, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 56, 81, 89, 90, 91, 92, 98, 99, 100, 102, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 200, 201, 210, 211, 220, 221, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 282, 283, 285, 286, 291, 292, 293, 296, 297, 298, 304, 308, 310, 318, 326, 327, 328, 329, 330, 331, 332, 333, 334, 342, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 192, 193, 194, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 237, 238, 245, 246, 247, 248, 255, 256, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 331, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 355 }

B grade: { 36, 44, 81, 89, 184 }

C grade: { 35, 43, 190, 191, 200, 201, 210, 211, 220, 221 }

F grade: { 131, 133, 139, 140, 141, 142, 143, 144, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 254, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 326, 327, 328, 329, 330, 332, 333, 350, 351, 352, 353, 354, 356, 357 }

2.1.6 Sympy

A grade: { 1, 2, 3, 9, 12, 13, 14, 23, 24, 25, 29, 31, 33, 34, 37, 38, 39, 40, 41, 42, 46, 48, 49, 51, 55, 56, 57, 58, 69, 70, 82, 83, 84, 85, 86, 87, 88, 91, 93, 94, 95, 97, 101, 102, 106, 107, 108, 109, 110, 114, 115, 116, 117, 118, 122, 124, 125, 126, 128, 129, 157, 169, 170, 175, 176, 187, 188, 189, 209, 215, 216, 259, 264, 265, 270, 271, 275, 299, 300, 301, 302, 303, 311, 312, 313, 314, 319, 321 }

B grade: { 7, 8, 28, 32, 127 }

C grade: { }

F grade: { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 30, 35, 36, 43, 44, 45, 47, 50, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 90, 92, 96, 98, 99, 100, 103, 104, 105, 111, 112, 113, 119, 120, 121, 123, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 304, 305, 306, 307, 308, 309, 310, 315, 316, 317, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 12, 13, 14, 15, 23, 24, 25, 26, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 69, 70, 71, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 101, 102, 107, 115, 124, 125, 126, 127, 128, 129, 130, 132, 134, 157, 158, 163, 164, 169, 170, 175, 176, 180, 181, 185, 186, 195, 196, 205, 206, 208, 209, 215, 216, 225, 226, 227, 228, 229, 230, 231, 237, 238, 259, 264, 265, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

B grade: { 5, 6, 16, 17, 27, 60, 72, 187, 188, 189, 207 }

C grade: { 7, 8, 9, 18, 19, 20, 28, 29, 31, 32, 153, 154, 155, 156, 165, 166, 167, 168, 212, 213, 214, 235, 236, 239 }

F grade: { 10, 11, 21, 22, 30, 35, 36, 43, 44, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 89, 98, 99, 100, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 131, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 159, 160, 161, 162, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 232, 233, 234, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	62	63	115	65	93
normalized size	1	1.	0.89	1.09	1.11	2.02	1.14	1.63
time (sec)	N/A	0.073	0.086	0.01	0.981	1.991	3.952	1.177

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	40	50	93	49	82
normalized size	1	1.	1.	0.91	1.14	2.11	1.11	1.86
time (sec)	N/A	0.043	0.004	0.007	0.976	1.923	0.987	1.176

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	41	27	28	49	31	35
normalized size	1	1.	1.64	1.08	1.12	1.96	1.24	1.4
time (sec)	N/A	0.021	0.014	0.005	0.961	1.835	0.231	1.147

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	28	68	144	0	43
normalized size	1	1.	0.94	0.9	2.19	4.65	0.	1.39
time (sec)	N/A	0.034	0.049	0.009	1.155	2.012	0.	1.112

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	47	77	197	0	134
normalized size	1	1.	0.91	0.89	1.45	3.72	0.	2.53
time (sec)	N/A	0.091	0.081	0.009	1.148	1.928	0.	1.149

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	86	65	78	242	0	275
normalized size	1	1.	1.16	0.88	1.05	3.27	0.	3.72
time (sec)	N/A	0.125	0.087	0.009	1.156	1.928	0.	1.117

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	125	89	387	297	488	223
normalized size	1	1.	1.03	0.74	3.2	2.45	4.03	1.84
time (sec)	N/A	0.134	0.251	0.009	1.627	2.078	4.247	1.15

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	104	68	350	252	223	196
normalized size	1	1.	1.02	0.67	3.43	2.47	2.19	1.92
time (sec)	N/A	0.068	0.195	0.009	1.646	2.089	2.931	1.158

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	61	48	315	204	66	138
normalized size	1	1.	0.82	0.65	4.26	2.76	0.89	1.86
time (sec)	N/A	0.043	0.142	0.003	1.634	1.989	0.526	1.12

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	91	66	366	221	0	0
normalized size	1	1.	1.03	0.75	4.16	2.51	0.	0.
time (sec)	N/A	0.074	0.182	0.008	1.163	1.936	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	83	369	284	0	0
normalized size	1	1.	1.04	0.73	3.24	2.49	0.	0.
time (sec)	N/A	0.091	0.216	0.009	1.178	2.061	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	122	140	143	265	209	244
normalized size	1	1.	0.75	0.86	0.88	1.63	1.28	1.5
time (sec)	N/A	0.247	0.39	0.016	1.034	2.054	7.3	1.119

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	92	93	117	188	136	166
normalized size	1	1.	0.9	0.91	1.15	1.84	1.33	1.63
time (sec)	N/A	0.134	0.221	0.016	1.019	1.978	2.255	1.16

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	62	70	119	95	77
normalized size	1	1.	0.9	1.07	1.21	2.05	1.64	1.33
time (sec)	N/A	0.049	0.123	0.013	0.984	2.043	0.607	1.12

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	69	146	321	0	104
normalized size	1	1.	0.96	0.93	1.97	4.34	0.	1.41
time (sec)	N/A	0.105	0.166	0.033	1.214	2.045	0.	1.152

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	116	203	167	425	0	305
normalized size	1	1.	1.01	1.77	1.45	3.7	0.	2.65
time (sec)	N/A	0.221	0.254	0.247	1.242	2.067	0.	1.123

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	158	262	174	510	0	605
normalized size	1	1.	0.93	1.55	1.03	3.02	0.	3.58
time (sec)	N/A	0.29	0.465	0.227	1.221	2.166	0.	1.496

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	234	189	807	595	0	444
normalized size	1	1.	0.95	0.77	3.27	2.41	0.	1.8
time (sec)	N/A	0.242	0.573	0.016	1.853	2.253	0.	1.427

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	191	142	743	498	0	382
normalized size	1	1.	0.96	0.72	3.75	2.52	0.	1.93
time (sec)	N/A	0.157	0.532	0.016	1.818	2.303	0.	1.527

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	147	99	660	385	0	263
normalized size	1	1.	0.96	0.65	4.31	2.52	0.	1.72
time (sec)	N/A	0.109	0.323	0.015	1.782	2.029	0.	1.448

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	184	137	743	439	0	0
normalized size	1	1.	0.98	0.73	3.97	2.35	0.	0.
time (sec)	N/A	0.162	0.514	0.014	1.258	2.162	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	226	175	756	556	0	0
normalized size	1	1.	0.95	0.73	3.16	2.33	0.	0.
time (sec)	N/A	0.197	0.663	0.014	1.252	2.446	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	113	107	182	143	165
normalized size	1	1.	0.64	0.97	0.91	1.56	1.22	1.41
time (sec)	N/A	0.13	0.266	0.013	0.99	2.231	11.958	1.125

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	66	81	138	92	81
normalized size	1	1.	0.73	0.84	1.03	1.75	1.16	1.03
time (sec)	N/A	0.074	0.153	0.011	0.985	2.208	4.041	1.123

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	36	61	46	35
normalized size	1	1.	1.	0.79	1.09	1.85	1.39	1.06
time (sec)	N/A	0.031	0.028	0.006	0.964	2.166	1.037	1.109

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	48	120	262	0	63
normalized size	1	1.	0.93	0.87	2.18	4.76	0.	1.15
time (sec)	N/A	0.095	0.074	0.027	1.221	2.304	0.	1.122

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	162	135	367	0	251
normalized size	1	1.	0.99	1.78	1.48	4.03	0.	2.76
time (sec)	N/A	0.22	0.13	0.194	1.215	2.295	0.	1.107

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	159	132	695	447	439	350
normalized size	1	1.	0.85	0.7	3.7	2.38	2.34	1.86
time (sec)	N/A	0.225	0.433	0.013	2.245	2.464	6.389	1.144

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	117	99	651	374	129	250
normalized size	1	1.	0.76	0.65	4.25	2.44	0.84	1.63
time (sec)	N/A	0.082	0.23	0.01	1.793	2.201	2.457	1.123

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	167	130	725	440	0	0
normalized size	1	1.	0.99	0.77	4.32	2.62	0.	0.
time (sec)	N/A	0.146	0.433	0.01	1.261	2.482	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	58	131	200	116	131
normalized size	1	1.	0.89	0.82	1.85	2.82	1.63	1.85
time (sec)	N/A	0.053	0.067	0.011	1.527	2.189	4.989	1.104

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	78	158	254	291	169
normalized size	1	1.	0.89	0.93	1.88	3.02	3.46	2.01
time (sec)	N/A	0.078	0.149	0.019	1.538	2.193	6.099	1.131

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	50	74	126	95	70
normalized size	1	1.	1.	0.75	1.1	1.88	1.42	1.04
time (sec)	N/A	0.045	0.043	0.005	0.984	2.301	12.665	1.108

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	73	154	51	53
normalized size	1	1.	0.93	0.89	1.66	3.5	1.16	1.2
time (sec)	N/A	0.1	0.101	0.019	1.119	2.323	5.821	1.097

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	289	0	0	3376	0	0
normalized size	1	1.	0.8	0.	0.	9.33	0.	0.
time (sec)	N/A	0.879	0.21	0.047	0.	3.321	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	188	0	0	2481	0	0
normalized size	1	1.	0.77	0.	0.	10.13	0.	0.
time (sec)	N/A	0.514	0.065	0.037	0.	3.447	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	48	10905	458	192	85
normalized size	1	1.	1.	1.	227.19	9.54	4.	1.77
time (sec)	N/A	0.069	0.081	0.017	155.146	2.107	16.044	1.114

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.395	0.042	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.351	0.04	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.365	0.035	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.023	0.032	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.268	0.036	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	513	0	0	5577	0	0
normalized size	1	1.	0.77	0.	0.	8.41	0.	0.
time (sec)	N/A	1.302	2.309	0.669	0.	4.061	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	302	0	0	3438	0	0
normalized size	1	1.	0.93	0.	0.	10.61	0.	0.
time (sec)	N/A	0.596	0.949	0.586	0.	3.975	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	164	0	798	0	194
normalized size	1	1.	1.	1.8	0.	8.77	0.	2.13
time (sec)	N/A	0.102	0.205	0.041	0.	2.107	0.	1.117

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	5.923	0.598	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	8.169	0.921	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	4.192	0.371	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	4.807	0.509	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	7.192	0.696	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.846	0.723	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	373	0	0	894	0	0
normalized size	1	1.	0.84	0.	0.	2.01	0.	0.
time (sec)	N/A	0.48	8.562	0.415	0.	2.356	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	551	0	0	555	0	0
normalized size	1	1.	1.97	0.	0.	1.99	0.	0.
time (sec)	N/A	0.263	6.513	0.374	0.	2.033	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	263	0	0
normalized size	1	1.	1.11	0.	0.	1.96	0.	0.
time (sec)	N/A	0.119	1.571	0.125	0.	1.694	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.412	0.08	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.659	0.224	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	73	50	93	49	82
normalized size	1	1.	1.	1.66	1.14	2.11	1.11	1.86
time (sec)	N/A	0.052	0.008	0.019	0.979	1.753	4.557	1.095

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	41	27	28	49	31	35
normalized size	1	1.	1.64	1.08	1.12	1.96	1.24	1.4
time (sec)	N/A	0.027	0.022	0.001	0.966	1.666	0.567	1.093

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	0	68	144	0	43
normalized size	1	1.	0.94	0.	2.19	4.65	0.	1.39
time (sec)	N/A	0.039	0.051	0.045	1.513	1.661	0.	1.143

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	0	77	197	0	134
normalized size	1	1.	0.91	0.	1.45	3.72	0.	2.53
time (sec)	N/A	0.102	0.087	0.072	1.137	1.709	0.	1.113

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	124	0	402	204	0	0
normalized size	1	1.	1.11	0.	3.59	1.82	0.	0.
time (sec)	N/A	0.085	0.228	0.063	1.177	1.66	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	108	0	375	149	0	0
normalized size	1	1.	1.19	0.	4.12	1.64	0.	0.
time (sec)	N/A	0.064	0.126	0.05	1.154	1.738	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	366	180	0	0
normalized size	1	1.	1.19	0.	3.62	1.78	0.	0.
time (sec)	N/A	0.078	0.211	0.068	1.156	1.76	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	143	0	369	230	0	0
normalized size	1	1.	1.1	0.	2.84	1.77	0.	0.
time (sec)	N/A	0.102	0.373	0.082	1.163	1.776	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	124	0	396	201	0	0
normalized size	1	1.	1.17	0.	3.74	1.9	0.	0.
time (sec)	N/A	0.072	0.191	0.06	1.137	1.887	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	138	0	363	147	0	0
normalized size	1	1.	1.68	0.	4.43	1.79	0.	0.
time (sec)	N/A	0.029	0.1	0.057	1.129	1.623	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	120	0	366	188	0	0
normalized size	1	1.	1.19	0.	3.62	1.86	0.	0.
time (sec)	N/A	0.055	0.193	0.067	1.178	1.735	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	146	0	369	231	0	0
normalized size	1	1.	1.16	0.	2.93	1.83	0.	0.
time (sec)	N/A	0.071	0.447	0.091	1.164	1.718	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	92	137	117	189	143	166
normalized size	1	1.	0.86	1.28	1.09	1.77	1.34	1.55
time (sec)	N/A	0.133	0.281	0.095	1.016	1.762	8.309	1.105

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	62	70	119	99	77
normalized size	1	1.	0.87	1.03	1.17	1.98	1.65	1.28
time (sec)	N/A	0.057	0.146	0.014	0.986	1.662	1.248	1.096

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	146	328	0	107
normalized size	1	1.	0.89	0.	1.82	4.1	0.	1.34
time (sec)	N/A	0.093	0.181	0.17	1.221	1.704	0.	1.106

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	116	0	167	425	0	305
normalized size	1	1.	0.95	0.	1.37	3.48	0.	2.5
time (sec)	N/A	0.219	0.267	0.225	1.244	1.788	0.	1.124

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	339	0	830	446	0	0
normalized size	1	1.	1.36	0.	3.33	1.79	0.	0.
time (sec)	N/A	0.206	0.63	0.211	1.287	1.815	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	283	0	770	328	0	0
normalized size	1	1.	1.47	0.	3.99	1.7	0.	0.
time (sec)	N/A	0.136	0.366	0.173	1.258	1.696	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	332	0	743	392	0	0
normalized size	1	1.	1.45	0.	3.24	1.71	0.	0.
time (sec)	N/A	0.189	0.584	0.197	1.28	1.832	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	292	0	756	502	0	0
normalized size	1	1.	1.03	0.	2.67	1.77	0.	0.
time (sec)	N/A	0.236	2.505	0.243	1.264	1.855	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	339	0	818	433	0	0
normalized size	1	1.	1.43	0.	3.45	1.83	0.	0.
time (sec)	N/A	0.15	0.578	0.202	1.244	1.867	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	281	0	755	327	0	0
normalized size	1	1.	1.54	0.	4.13	1.79	0.	0.
time (sec)	N/A	0.075	0.275	0.13	1.227	1.834	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	332	0	743	405	0	0
normalized size	1	1.	1.48	0.	3.3	1.8	0.	0.
time (sec)	N/A	0.134	0.562	0.208	1.279	1.923	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	294	0	756	506	0	0
normalized size	1	1.	1.07	0.	2.75	1.84	0.	0.
time (sec)	N/A	0.183	2.516	0.22	1.274	1.913	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	188	0	0	2483	0	0
normalized size	1	1.	0.77	0.	0.	10.13	0.	0.
time (sec)	N/A	0.503	0.165	0.043	0.	2.954	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	49	10905	458	202	86
normalized size	1	1.	1.	0.96	213.82	8.98	3.96	1.69
time (sec)	N/A	0.078	0.095	0.016	154.506	1.776	24.677	1.101

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.444	0.035	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.46	0.041	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.606	0.033	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.318	0.038	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.027	0.033	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.364	0.039	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	302	0	0	3438	0	0
normalized size	1	1.	0.93	0.	0.	10.61	0.	0.
time (sec)	N/A	0.593	0.993	0.773	0.	3.284	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	91	167	0	798	0	197
normalized size	1	1.	0.97	1.78	0.	8.49	0.	2.1
time (sec)	N/A	0.109	0.205	0.015	0.	1.889	0.	1.124

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	10.025	0.647	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	12.269	0.727	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	6.846	0.501	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	10.659	0.884	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	8.222	0.593	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	11.617	0.671	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.864	0.678	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	373	0	0	926	0	0
normalized size	1	1.	0.84	0.	0.	2.1	0.	0.
time (sec)	N/A	0.414	12.47	0.389	0.	2.002	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	556	0	0	578	0	0
normalized size	1	1.	1.95	0.	0.	2.03	0.	0.
time (sec)	N/A	0.232	6.819	0.381	0.	1.906	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	149	0	0	274	0	0
normalized size	1	1.	1.11	0.	0.	2.04	0.	0.
time (sec)	N/A	0.1	1.601	0.124	0.	1.742	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.407	0.082	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.721	0.223	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	73	116	224	0	0
normalized size	1	1.	0.9	0.94	1.49	2.87	0.	0.
time (sec)	N/A	0.131	0.073	0.012	1.146	2.043	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	57	103	203	0	0
normalized size	1	1.	0.87	0.95	1.72	3.38	0.	0.
time (sec)	N/A	0.098	0.052	0.01	1.141	2.002	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	38	78	144	0	0
normalized size	1	1.	1.	1.19	2.44	4.5	0.	0.
time (sec)	N/A	0.072	0.023	0.011	1.143	1.937	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	58	109	17	0
normalized size	1	1.	1.	1.05	2.76	5.19	0.81	0.
time (sec)	N/A	0.028	0.047	0.009	1.144	1.922	1.339	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	27	14	16
normalized size	1	1.	1.	1.08	1.33	2.25	1.17	1.33
time (sec)	N/A	0.014	0.013	0.004	0.956	1.691	1.261	1.105

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	42	68	69	29	0
normalized size	1	1.	1.	1.45	2.34	2.38	1.	0.
time (sec)	N/A	0.025	0.004	0.007	1.134	1.56	2.427	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	95	69	95	46	0
normalized size	1	1.	0.84	2.11	1.53	2.11	1.02	0.
time (sec)	N/A	0.046	0.052	0.009	1.381	1.675	4.456	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	165	68	112	61	0
normalized size	1	1.	1.	2.7	1.11	1.84	1.	0.
time (sec)	N/A	0.068	0.005	0.008	1.148	1.538	7.662	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	96	134	290	0	0
normalized size	1	1.	0.89	0.99	1.38	2.99	0.	0.
time (sec)	N/A	0.169	0.17	0.014	1.155	1.446	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	76	120	246	0	0
normalized size	1	1.	1.	1.17	1.85	3.78	0.	0.
time (sec)	N/A	0.104	0.161	0.015	1.15	1.54	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	52	89	167	0	0
normalized size	1	1.	1.	1.27	2.17	4.07	0.	0.
time (sec)	N/A	0.091	0.091	0.014	1.136	1.452	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	36	69	144	31	0
normalized size	1	1.	0.86	0.97	1.86	3.89	0.84	0.
time (sec)	N/A	0.05	0.059	0.011	1.127	1.435	4.679	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	34	34	72	262	30
normalized size	1	1.	1.03	1.1	1.1	2.32	8.45	0.97
time (sec)	N/A	0.027	0.056	0.007	0.963	1.213	4.391	1.099

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	97	92	132	445	0
normalized size	1	1.	0.84	1.9	1.8	2.59	8.73	0.
time (sec)	N/A	0.04	0.079	0.011	1.128	1.329	6.388	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	54	197	92	158	654	0
normalized size	1	1.	0.62	2.26	1.06	1.82	7.52	0.
time (sec)	N/A	0.065	0.127	0.013	1.132	1.285	9.598	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	334	92	194	726	0
normalized size	1	1.	0.61	3.12	0.86	1.81	6.79	0.
time (sec)	N/A	0.081	0.18	0.012	1.133	1.563	16.805	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	59	539	209	0	0
normalized size	1	1.	1.01	0.74	6.74	2.61	0.	0.
time (sec)	N/A	0.058	0.134	0.01	1.411	1.728	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	58	123	0	0
normalized size	1	1.	1.	0.88	2.32	4.92	0.	0.
time (sec)	N/A	0.029	0.049	0.011	1.123	1.556	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	47	495	186	0	0
normalized size	1	1.	0.81	0.63	6.6	2.48	0.	0.
time (sec)	N/A	0.031	0.102	0.009	1.174	1.706	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	38	20	0
normalized size	1	1.	1.	0.93	1.2	2.53	1.33	0.
time (sec)	N/A	0.016	0.014	0.003	0.942	1.673	4.299	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	89	65	359	236	0	0
normalized size	1	1.	0.92	0.67	3.7	2.43	0.	0.
time (sec)	N/A	0.059	0.156	0.009	1.163	1.795	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	23	8	8
normalized size	1	1.	1.	0.88	1.	2.88	1.	1.
time (sec)	N/A	0.009	0.01	0.005	0.938	1.625	0.306	1.1

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	15	20	50	29	20
normalized size	1	1.	1.1	0.71	0.95	2.38	1.38	0.95
time (sec)	N/A	0.02	0.023	0.009	0.941	1.658	0.903	1.099

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	57	20	22
normalized size	1	1.	1.	0.77	1.	2.59	0.91	1.
time (sec)	N/A	0.012	0.015	0.006	0.949	1.628	0.303	1.099

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	41	52	41	134	379	41
normalized size	1	1.	0.59	0.75	0.59	1.94	5.49	0.59
time (sec)	N/A	0.043	0.049	0.01	0.95	1.623	1.513	1.1

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	62	59	63	173	80	63
normalized size	1	1.	0.71	0.68	0.72	1.99	0.92	0.72
time (sec)	N/A	0.064	0.055	0.01	0.973	1.675	8.298	1.091

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	1.019	0.985	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	1.392	0.85	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	88	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.164	1.123	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.929	0.948	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	148	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.449	0.967	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.269	0.765	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	0	131	0	0
normalized size	1	1.	0.92	0.96	0.	5.24	0.	0.
time (sec)	N/A	0.038	0.061	0.006	0.	1.763	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	45	0	166	0	0
normalized size	1	1.	0.86	1.05	0.	3.86	0.	0.
time (sec)	N/A	0.061	0.077	0.007	0.	1.712	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	0	275	0	0
normalized size	1	1.	0.81	0.78	0.	4.1	0.	0.
time (sec)	N/A	0.092	0.106	0.012	0.	1.723	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	77	0	309	0	0
normalized size	1	1.	0.84	0.97	0.	3.91	0.	0.
time (sec)	N/A	0.101	0.105	0.013	0.	1.793	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	74	0	0	0	0
normalized size	1	1.	1.09	0.85	0.	0.	0.	0.
time (sec)	N/A	0.027	0.084	0.077	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	94	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.224	0.102	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	177	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.281	0.287	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	118	110	0	0	0	0
normalized size	1	1.	1.08	1.01	0.	0.	0.	0.
time (sec)	N/A	0.079	0.202	0.111	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	129	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.519	0.1	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	225	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.579	0.217	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	44	43	68	0	0
normalized size	1	1.	0.86	1.26	1.23	1.94	0.	0.
time (sec)	N/A	0.032	0.071	0.006	1.013	1.727	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	44	39	66	0	0
normalized size	1	1.	0.85	1.29	1.15	1.94	0.	0.
time (sec)	N/A	0.03	0.063	0.013	1.007	1.843	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	44	0	189	0	0
normalized size	1	1.	1.02	0.96	0.	4.11	0.	0.
time (sec)	N/A	0.089	0.074	0.006	0.	2.069	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	66	0	213	0	0
normalized size	1	1.	0.87	0.99	0.	3.18	0.	0.
time (sec)	N/A	0.12	0.131	0.021	0.	2.148	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	99	0	394	0	0
normalized size	1	1.	0.84	0.88	0.	3.49	0.	0.
time (sec)	N/A	0.215	0.19	0.018	0.	2.167	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	65	0	254	0	0
normalized size	1	1.	0.87	0.83	0.	3.26	0.	0.
time (sec)	N/A	0.111	0.123	0.007	0.	1.993	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	82	89	0	294	0	0
normalized size	1	1.	0.86	0.94	0.	3.09	0.	0.
time (sec)	N/A	0.151	0.172	0.019	0.	1.933	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	141	144	0	522	0	0
normalized size	1	1.	0.85	0.87	0.	3.16	0.	0.
time (sec)	N/A	0.264	0.285	0.014	0.	1.999	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	173	586	3780	582	0	1381
normalized size	1	1.	0.78	2.63	16.95	2.61	0.	6.19
time (sec)	N/A	0.312	1.027	0.014	4.5	1.671	0.	1.242

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	117	291	2245	392	0	903
normalized size	1	1.	0.78	1.94	14.97	2.61	0.	6.02
time (sec)	N/A	0.164	0.618	0.005	3.696	1.61	0.	1.197

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	120	829	192	0	495
normalized size	1	1.	0.96	1.74	12.01	2.78	0.	7.17
time (sec)	N/A	0.073	0.174	0.006	2.482	1.654	0.	1.177

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	42	193	117	0	193
normalized size	1	1.	1.	1.08	4.95	3.	0.	4.95
time (sec)	N/A	0.008	0.015	0.007	1.836	1.592	0.	1.108

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	5.198	0.131	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	9.835	0.196	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	440	365	0	1023	0	0
normalized size	1	1.	1.31	1.08	0.	3.04	0.	0.
time (sec)	N/A	0.42	0.87	0.013	0.	2.062	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	265	225	0	713	0	0
normalized size	1	1.	1.14	0.97	0.	3.06	0.	0.
time (sec)	N/A	0.249	0.492	0.01	0.	1.859	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	95	101	0	383	0	0
normalized size	1	1.	0.79	0.84	0.	3.19	0.	0.
time (sec)	N/A	0.128	0.284	0.012	0.	1.736	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	52	0	177	0	0
normalized size	1	1.	1.	0.87	0.	2.95	0.	0.
time (sec)	N/A	0.034	0.033	0.007	0.	1.691	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.508	0.237	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	18.378	0.425	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	218	1248	6666	760	0	1449
normalized size	1	1.	0.64	3.66	19.55	2.23	0.	4.25
time (sec)	N/A	0.572	2.947	0.013	4.861	1.959	0.	1.218

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	151	669	3950	513	0	952
normalized size	1	1.	0.59	2.61	15.43	2.	0.	3.72
time (sec)	N/A	0.34	1.825	0.008	3.634	1.839	0.	1.237

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	309	1435	335	0	525
normalized size	1	1.	0.93	2.53	11.76	2.75	0.	4.3
time (sec)	N/A	0.178	0.601	0.01	2.53	1.82	0.	1.197

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	136	335	242	0	204
normalized size	1	1.	0.81	1.64	4.04	2.92	0.	2.46
time (sec)	N/A	0.042	0.071	0.009	1.798	1.643	0.	1.132

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	15.25	0.105	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	25.994	0.178	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	434	434	0	0	0	1007	0	0
normalized size	1	1.	0.	0.	0.	2.32	0.	0.
time (sec)	N/A	0.444	102.679	0.157	0.	1.971	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	0	0	0	774	0	0
normalized size	1	1.	0.	0.	0.	2.76	0.	0.
time (sec)	N/A	0.276	40.177	0.106	0.	1.874	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	235	0	0	0	578	0	0
normalized size	1	1.	0.	0.	0.	2.46	0.	0.
time (sec)	N/A	0.192	76.397	0.083	0.	1.77	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	115	0	0	277	0	0
normalized size	1	1.	1.07	0.	0.	2.59	0.	0.
time (sec)	N/A	0.029	0.017	0.052	0.	1.8	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	57.888	0.105	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	121.895	0.164	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	467	302	0	1040	0	0
normalized size	1	1.	1.26	0.81	0.	2.8	0.	0.
time (sec)	N/A	0.479	1.464	0.016	0.	2.075	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	242	150	0	670	0	0
normalized size	1	1.	1.22	0.76	0.	3.38	0.	0.
time (sec)	N/A	0.259	0.772	0.013	0.	1.982	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	100	80	0	352	0	0
normalized size	1	1.	0.95	0.76	0.	3.35	0.	0.
time (sec)	N/A	0.068	0.17	0.009	0.	1.728	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	4.556	0.221	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	21.707	0.309	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	405	0	0	1116	0	0
normalized size	1	1.	1.23	0.	0.	3.38	0.	0.
time (sec)	N/A	0.3	2.524	0.289	0.	2.082	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	700	0	0	761	0	0
normalized size	1	1.	2.98	0.	0.	3.24	0.	0.
time (sec)	N/A	0.146	2.272	0.21	0.	2.014	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	203	0	0	408	0	0
normalized size	1	1.	1.9	0.	0.	3.81	0.	0.
time (sec)	N/A	0.027	0.487	0.101	0.	1.867	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	5.15	0.273	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	26.665	0.379	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	138	1246	1486	431	549	1848
normalized size	1	1.	0.34	3.04	3.62	1.05	1.34	4.51
time (sec)	N/A	0.399	1.769	0.013	1.185	1.751	2.702	1.6

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	85	366	470	208	231	714
normalized size	1	1.	0.46	1.98	2.54	1.12	1.25	3.86
time (sec)	N/A	0.159	0.416	0.008	1.022	1.689	0.742	1.299

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	61	84	111	66	225
normalized size	1	1.	0.93	1.13	1.56	2.06	1.22	4.17
time (sec)	N/A	0.028	0.074	0.007	0.95	1.642	0.491	1.202

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	238	785	0	524	0	0
normalized size	1	1.	1.	3.3	0.	2.2	0.	0.
time (sec)	N/A	0.749	1.47	0.021	0.	1.836	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	397	1817	0	873	0	0
normalized size	1	1.	1.17	5.36	0.	2.58	0.	0.
time (sec)	N/A	0.983	3.577	0.048	0.	1.968	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	419	0	2504	740	0	0
normalized size	1	1.	1.1	0.	6.55	1.94	0.	0.
time (sec)	N/A	0.306	3.146	0.036	2.572	2.315	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	705	0	1449	524	0	0
normalized size	1	1.	2.42	0.	4.98	1.8	0.	0.
time (sec)	N/A	0.2	2.61	0.02	1.983	2.333	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	0	466	198	0	0
normalized size	1	1.	1.07	0.	4.05	1.72	0.	0.
time (sec)	N/A	0.081	0.149	0.004	1.371	2.149	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	10.778	0.036	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	13.406	0.039	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	611	557	696	1184	1119	0	0
normalized size	1	1.	0.91	1.14	1.94	1.83	0.	0.
time (sec)	N/A	0.791	2.2	0.059	2.048	2.386	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	367	295	549	621	0	0
normalized size	1	1.	1.22	0.98	1.82	2.06	0.	0.
time (sec)	N/A	0.393	0.62	0.028	1.552	2.284	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	99	84	167	360	0	0
normalized size	1	1.	1.05	0.89	1.78	3.83	0.	0.
time (sec)	N/A	0.118	0.076	0.015	1.206	2.167	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	C	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	276	0	438	0	741	0	0
normalized size	1	1.	0.	1.59	0.	2.68	0.	0.
time (sec)	N/A	1.203	15.316	0.029	0.	2.359	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	C	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	350	350	0	2724	0	1003	0	0
normalized size	1	1.	0.	7.78	0.	2.87	0.	0.
time (sec)	N/A	0.929	180.033	0.052	0.	2.431	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	463	0	2916	1211	0	0
normalized size	1	1.	1.19	0.	7.48	3.11	0.	0.
time (sec)	N/A	0.428	2.252	0.056	2.929	2.799	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	835	0	1623	825	0	0
normalized size	1	1.	3.33	0.	6.47	3.29	0.	0.
time (sec)	N/A	0.226	2.653	0.043	2.078	2.041	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	166	0	516	359	0	0
normalized size	1	1.	1.44	0.	4.49	3.12	0.	0.
time (sec)	N/A	0.082	0.44	0.012	1.383	1.944	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	13.111	0.056	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	15.474	0.055	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	633	256	2704	2904	776	0	2103
normalized size	1	1.	0.4	4.27	4.59	1.23	0.	3.32
time (sec)	N/A	0.647	2.509	0.013	1.428	1.763	0.	1.621

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	147	801	919	362	0	613
normalized size	1	1.	0.51	2.78	3.19	1.26	0.	2.13
time (sec)	N/A	0.269	0.604	0.009	1.093	1.67	0.	1.397

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	134	162	155	95	111
normalized size	1	1.	0.76	1.58	1.91	1.82	1.12	1.31
time (sec)	N/A	0.057	0.108	0.009	0.965	1.704	1.519	1.222

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	118	327	0	1119	0	0
normalized size	1	1.	0.3	0.83	0.	2.83	0.	0.
time (sec)	N/A	1.39	1.756	0.019	0.	1.931	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	555	180	1175	0	1789	0	0
normalized size	1	1.	0.32	2.12	0.	3.22	0.	0.
time (sec)	N/A	2.124	1.11	0.067	0.	2.124	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	432	395	1769	782	0	1049
normalized size	1	1.	0.84	0.77	3.45	1.52	0.	2.04
time (sec)	N/A	0.535	2.313	0.01	2.389	2.071	0.	1.332

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	213	175	832	450	0	549
normalized size	1	1.	0.88	0.72	3.42	1.85	0.	2.26
time (sec)	N/A	0.264	0.824	0.009	1.884	1.879	0.	1.26

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	86	373	297	0	230
normalized size	1	1.	0.88	0.66	2.87	2.28	0.	1.77
time (sec)	N/A	0.074	0.148	0.006	1.623	1.791	0.	1.193

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	21.876	0.035	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	20.727	0.036	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	855	855	929	936	1354	1670	0	0
normalized size	1	1.	1.09	1.09	1.58	1.95	0.	0.
time (sec)	N/A	1.051	4.581	0.078	2.101	2.176	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	540	391	618	814	0	0
normalized size	1	1.	1.29	0.93	1.47	1.94	0.	0.
time (sec)	N/A	0.504	0.856	0.03	1.572	2.039	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	108	186	412	0	0
normalized size	1	1.	0.98	0.79	1.37	3.03	0.	0.
time (sec)	N/A	0.162	0.102	0.016	1.226	1.821	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	170	156	0	1359	0	0
normalized size	1	1.	0.39	0.36	0.	3.13	0.	0.
time (sec)	N/A	1.92	2.743	0.027	0.	2.34	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	566	566	313	1556	0	1908	0	0
normalized size	1	1.	0.55	2.75	0.	3.37	0.	0.
time (sec)	N/A	2.63	1.229	0.083	0.	2.733	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	630	613	452	3386	1303	0	0
normalized size	1	1.	0.97	0.72	5.37	2.07	0.	0.
time (sec)	N/A	0.748	2.931	0.017	2.911	2.437	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	378	225	1628	771	0	0
normalized size	1	1.	1.19	0.71	5.12	2.42	0.	0.
time (sec)	N/A	0.385	1.173	0.017	1.998	2.072	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	146	105	716	406	0	0
normalized size	1	1.	1.04	0.74	5.08	2.88	0.	0.
time (sec)	N/A	0.112	0.151	0.013	1.441	2.019	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	29.668	0.058	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	180.033	0.056	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	226	0	0	568	0	594
normalized size	1	1.	0.78	0.	0.	1.97	0.	2.06
time (sec)	N/A	0.269	0.547	0.04	0.	7.63	0.	1.254

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	111	0	0	356	0	198
normalized size	1	1.	0.55	0.	0.	1.76	0.	0.98
time (sec)	N/A	0.178	0.284	0.032	0.	7.76	0.	1.173

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	97	0	0	317	0	173
normalized size	1	1.	0.61	0.	0.	1.98	0.	1.08
time (sec)	N/A	0.138	0.211	0.033	0.	7.271	0.	1.19

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	70	0	0	219	0	112
normalized size	1	1.	0.82	0.	0.	2.58	0.	1.32
time (sec)	N/A	0.07	0.076	0.034	0.	7.241	0.	1.197

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	31	120	0	47
normalized size	1	1.	1.	0.	0.74	2.86	0.	1.12
time (sec)	N/A	0.052	0.068	0.033	1.049	1.665	0.	1.178

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.145	0.034	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	115	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	0.181	0.035	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	184	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.263	0.355	0.035	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	0	0	0	639
normalized size	1	1.	0.66	0.	0.	0.	0.	2.39
time (sec)	N/A	0.273	0.738	0.032	0.	0.	0.	1.339

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	160	0	0	0	0	284
normalized size	1	1.	0.7	0.	0.	0.	0.	1.25
time (sec)	N/A	0.198	0.455	0.033	0.	0.	0.	1.223

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	0	0	232	0	171
normalized size	1	1.	0.81	0.	0.	2.61	0.	1.92
time (sec)	N/A	0.086	0.06	0.034	0.	7.919	0.	1.201

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	31	123	0	70
normalized size	1	1.	1.	0.	0.7	2.8	0.	1.59
time (sec)	N/A	0.063	0.075	0.033	1.049	1.712	0.	1.221

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	96	0	0	0	0	113
normalized size	1	1.	0.72	0.	0.	0.	0.	0.85
time (sec)	N/A	0.125	0.147	0.034	0.	0.	0.	1.139

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	133	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	0.242	0.033	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	87	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.156	0.033	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	208	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	0.34	0.042	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	131	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	0.183	0.04	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	88	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.181	0.04	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	42	154	0	0
normalized size	1	1.	0.93	0.	0.93	3.42	0.	0.
time (sec)	N/A	0.047	0.066	0.04	1.04	2.077	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	72	0	0	282	0	0
normalized size	1	1.	0.79	0.	0.	3.1	0.	0.
time (sec)	N/A	0.076	0.088	0.042	0.	7.58	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	107	0	0	382	0	0
normalized size	1	1.	0.62	0.	0.	2.22	0.	0.
time (sec)	N/A	0.148	0.166	0.042	0.	8.887	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	112	0	0	437	0	0
normalized size	1	1.	0.52	0.	0.	2.01	0.	0.
time (sec)	N/A	0.189	0.276	0.041	0.	8.789	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	237	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	0.906	0.041	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	228	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	0.364	0.041	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	113	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.303	0.04	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	90	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.213	0.043	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	136	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.327	0.04	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	96	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.171	0.042	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	42	157	0	0
normalized size	1	1.	0.94	0.	0.89	3.34	0.	0.
time (sec)	N/A	0.066	0.082	0.042	1.033	1.463	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	72	0	0	317	0	0
normalized size	1	1.	0.76	0.	0.	3.34	0.	0.
time (sec)	N/A	0.096	0.109	0.042	0.	6.577	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	165	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.907	0.043	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	192	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	1.254	0.043	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	6.093	0.187	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	503	503	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.391	11.53	0.039	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	369	369	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	7.58	0.087	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	192	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.829	0.086	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.096	0.06	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	1.688	0.036	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	1.606	0.09	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	539	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.534	14.637	0.07	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	403	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.345	12.585	0.118	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	215	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.493	0.151	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.246	0.142	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	2.789	0.072	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	2.675	0.118	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	856	856	786	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.966	21.681	0.33	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	556	556	552	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.457	4.532	0.403	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	277	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	1.969	0.206	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.651	0.335	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	3.167	0.293	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	1.375	0.055	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	1.296	0.073	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.267	0.066	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.66	0.075	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.813	0.045	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	180.165	0.4	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	180.133	2.198	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	11.19	1.366	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	180.108	0.362	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	180.128	0.359	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	2.097	0.994	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	150	209	348	571	0	0
normalized size	1	1.	0.67	0.93	1.55	2.55	0.	0.
time (sec)	N/A	0.457	0.597	0.037	1.342	1.402	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	79	115	207	387	0	0
normalized size	1	1.	0.67	0.97	1.75	3.28	0.	0.
time (sec)	N/A	0.239	0.213	0.022	1.221	1.325	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	43	88	163	0	0
normalized size	1	1.	1.32	1.13	2.32	4.29	0.	0.
time (sec)	N/A	0.078	0.03	0.01	1.129	1.308	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	83	142	0	366	0	0
normalized size	1	1.	0.81	1.38	0.	3.55	0.	0.
time (sec)	N/A	0.279	0.202	0.023	0.	1.537	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	85	144	0	382	0	0
normalized size	1	1.	0.9	1.53	0.	4.06	0.	0.
time (sec)	N/A	0.222	0.741	0.019	0.	1.334	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	151	527	0	959	0	0
normalized size	1	1.	0.65	2.26	0.	4.12	0.	0.
time (sec)	N/A	0.487	1.857	0.02	0.	1.594	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	252	265	435	828	0	0
normalized size	1	1.	0.99	1.04	1.71	3.26	0.	0.
time (sec)	N/A	0.616	0.561	0.039	1.418	1.634	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	105	110	185	373	0	0
normalized size	1	1.	1.12	1.17	1.97	3.97	0.	0.
time (sec)	N/A	0.226	0.145	0.024	1.221	1.553	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	195	321	0	776	0	0
normalized size	1	1.	0.76	1.26	0.	3.04	0.	0.
time (sec)	N/A	0.663	0.401	0.034	0.	1.477	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	263	308	0	805	0	0
normalized size	1	1.	1.35	1.58	0.	4.13	0.	0.
time (sec)	N/A	0.391	1.451	0.029	0.	1.693	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	740	1124	0	2053	0	0
normalized size	1	1.	1.57	2.39	0.	4.37	0.	0.
time (sec)	N/A	0.957	3.466	0.038	0.	2.069	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	1.046	1.049	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.57	0.94	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.037	0.062	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.078	0.	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.135	0.	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	112.861	3.425	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	19.451	2.381	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	3.275	1.77	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	2.947	0.002	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	99.551	0.	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	1.399	0.513	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	0	0	213	0	0
normalized size	1	1.	0.82	0.	0.	1.85	0.	0.
time (sec)	N/A	0.287	0.125	0.154	0.	1.745	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	47	151	197	176	143	0
normalized size	1	1.	0.49	1.57	2.05	1.83	1.49	0.
time (sec)	N/A	0.208	0.198	0.079	1.606	1.68	29.811	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	40	133	134	155	117	0
normalized size	1	1.	0.54	1.8	1.81	2.09	1.58	0.
time (sec)	N/A	0.182	0.224	0.075	1.549	1.755	11.83	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	117	81	135	76	0
normalized size	1	1.	0.67	2.6	1.8	3.	1.69	0.
time (sec)	N/A	0.127	0.132	0.072	1.517	1.695	5.147	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	105	42	104	53	0
normalized size	1	1.	1.	4.2	1.68	4.16	2.12	0.
time (sec)	N/A	0.018	0.063	0.111	1.46	1.695	1.943	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	228	57	265	0	0
normalized size	1	1.	0.65	4.15	1.04	4.82	0.	0.
time (sec)	N/A	0.166	0.057	0.078	1.643	1.694	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	155	328	339	0	0
normalized size	1	1.	0.66	2.01	4.26	4.4	0.	0.
time (sec)	N/A	0.177	0.179	0.083	1.661	1.791	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	69	183	365	401	0	0
normalized size	1	1.	0.59	1.58	3.15	3.46	0.	0.
time (sec)	N/A	0.206	0.145	0.082	1.674	1.695	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	138	0	0	270	0	0
normalized size	1	1.	0.9	0.	0.	1.76	0.	0.
time (sec)	N/A	0.301	0.3	0.162	0.	1.805	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	38	135	43	157	92	0
normalized size	1	1.	0.66	2.33	0.74	2.71	1.59	0.
time (sec)	N/A	0.181	0.091	0.08	1.54	1.607	30.286	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	105	240	481	425	0	0
normalized size	1	1.	0.68	1.55	3.1	2.74	0.	0.
time (sec)	N/A	0.214	0.271	0.123	1.72	1.708	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	119	22	120	66	0
normalized size	1	1.	1.	3.84	0.71	3.87	2.13	0.
time (sec)	N/A	0.104	0.052	0.071	1.536	1.61	5.097	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	80	157	444	356	0	0
normalized size	1	1.	0.68	1.34	3.79	3.04	0.	0.
time (sec)	N/A	0.059	0.121	0.083	1.713	1.819	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	47	268	63	285	0	0
normalized size	1	1.	0.64	3.67	0.86	3.9	0.	0.
time (sec)	N/A	0.121	0.058	0.083	1.697	1.741	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	105	232	489	412	0	0
normalized size	1	1.	0.78	1.72	3.62	3.05	0.	0.
time (sec)	N/A	0.156	0.277	0.092	1.739	1.581	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	214	70	375	0	0
normalized size	1	1.	0.68	2.18	0.71	3.83	0.	0.
time (sec)	N/A	0.203	0.128	0.087	1.691	1.664	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	142	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	0.325	0.295	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.278	0.189	0.125	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	0.187	0.182	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	129	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.185	0.154	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	119	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.137	0.128	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	47	280	0	293	0	0
normalized size	1	1.	0.64	3.84	0.	4.01	0.	0.
time (sec)	N/A	0.151	0.074	0.144	0.	1.778	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.205	0.16	0.185	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	114	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.174	0.112	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	142	0	0	301	0	0
normalized size	1	1.	0.84	0.	0.	1.78	0.	0.
time (sec)	N/A	0.301	0.537	0.134	0.	1.864	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	79	208	386	259	0	0
normalized size	1	1.	0.48	1.26	2.34	1.57	0.	0.
time (sec)	N/A	0.188	0.296	0.08	1.634	1.635	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	69	190	296	227	0	0
normalized size	1	1.	0.5	1.37	2.13	1.63	0.	0.
time (sec)	N/A	0.164	0.281	0.078	1.581	1.71	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	174	219	198	0	0
normalized size	1	1.	0.7	2.2	2.77	2.51	0.	0.
time (sec)	N/A	0.102	0.174	0.073	1.516	1.788	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	47	158	157	146	0	0
normalized size	1	1.	0.85	2.87	2.85	2.65	0.	0.
time (sec)	N/A	0.023	0.094	0.113	1.471	1.629	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	50	283	70	288	0	0
normalized size	1	1.	0.51	2.86	0.71	2.91	0.	0.
time (sec)	N/A	0.209	0.087	0.079	1.576	1.75	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	65	211	378	332	0	0
normalized size	1	1.	0.76	2.45	4.4	3.86	0.	0.
time (sec)	N/A	0.184	0.148	0.089	1.648	1.795	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	85	238	420	404	0	0
normalized size	1	1.	0.71	2.	3.53	3.39	0.	0.
time (sec)	N/A	0.23	0.203	0.088	1.703	1.842	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	189	0	0	359	0	0
normalized size	1	1.	0.9	0.	0.	1.72	0.	0.
time (sec)	N/A	0.277	0.895	0.132	0.	1.803	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	200	63	219	0	0
normalized size	1	1.	0.74	2.2	0.69	2.41	0.	0.
time (sec)	N/A	0.183	0.234	0.086	1.551	1.599	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	113	309	539	387	0	0
normalized size	1	1.	0.58	1.58	2.76	1.98	0.	0.
time (sec)	N/A	0.231	0.287	0.112	1.761	1.852	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	182	38	162	0	0
normalized size	1	1.	0.85	2.8	0.58	2.49	0.	0.
time (sec)	N/A	0.1	0.118	0.075	1.546	1.588	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	93	224	483	305	0	0
normalized size	1	1.	0.63	1.51	3.26	2.06	0.	0.
time (sec)	N/A	0.058	0.102	0.087	1.726	1.702	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	60	331	74	304	0	0
normalized size	1	1.	0.52	2.88	0.64	2.64	0.	0.
time (sec)	N/A	0.133	0.096	0.085	1.689	1.614	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	107	301	510	339	0	0
normalized size	1	1.	0.81	2.28	3.86	2.57	0.	0.
time (sec)	N/A	0.148	0.202	0.096	1.751	1.751	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	79	277	86	366	0	0
normalized size	1	1.	0.49	1.72	0.53	2.27	0.	0.
time (sec)	N/A	0.215	0.145	0.095	1.691	1.693	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.365	0.832	0.211	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	161	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.341	0.543	0.124	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	168	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.315	0.558	0.157	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	160	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	0.549	0.23	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	149	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.267	0.173	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	63	343	0	317	0	0
normalized size	1	1.	0.52	2.83	0.	2.62	0.	0.
time (sec)	N/A	0.179	0.138	0.144	0.	1.868	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	125	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.275	0.344	0.158	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	129	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	0.352	0.115	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [177] had the largest ratio of [0.7]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	16	0.25
2	A	5	4	1.	16	0.25
3	A	4	3	1.	14	0.214
4	A	5	4	1.	16	0.25
5	A	7	6	1.	16	0.375
6	A	8	6	1.	16	0.375
7	A	7	6	1.	16	0.375
8	A	6	5	1.	16	0.312
9	A	4	3	1.	12	0.25
10	A	6	5	1.	16	0.312
11	A	7	6	1.	16	0.375
12	A	10	8	1.	18	0.444
13	A	7	6	1.	18	0.333
14	A	2	2	1.	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	9	6	1.	18	0.333
16	A	13	8	1.	18	0.444
17	A	15	8	1.	18	0.444
18	A	13	8	1.	18	0.444
19	A	11	8	1.	18	0.444
20	A	8	5	1.	14	0.357
21	A	11	8	1.	18	0.444
22	A	13	8	1.	18	0.444
23	A	7	5	1.	14	0.357
24	A	4	4	1.	14	0.286
25	A	3	2	1.	12	0.167
26	A	8	4	1.	14	0.286
27	A	12	6	1.	14	0.429
28	A	10	5	1.	14	0.357
29	A	8	4	1.	10	0.4
30	A	9	5	1.	14	0.357
31	A	6	3	1.	10	0.3
32	A	7	4	1.	14	0.286
33	A	3	2	1.	12	0.167
34	A	8	6	1.	12	0.5
35	A	11	7	1.	18	0.389
36	A	9	6	1.	18	0.333
37	A	4	4	1.	16	0.25
38	A	0	0	0.	0	0.
39	A	0	0	0.	0	0.
40	A	0	0	0.	0	0.
41	A	0	0	0.	0	0.
42	A	0	0	0.	0	0.
43	A	19	11	1.	18	0.611
44	A	12	9	1.	18	0.5
45	A	6	6	1.	16	0.375
46	A	0	0	0.	0	0.
47	A	0	0	0.	0	0.
48	A	0	0	0.	0	0.
49	A	0	0	0.	0	0.
50	A	0	0	0.	0	0.
51	A	0	0	0.	0	0.
52	A	13	5	1.	20	0.25
53	A	9	5	1.	20	0.25
54	A	5	3	1.	18	0.167
55	A	0	0	0.	0	0.
56	A	0	0	0.	0	0.
57	A	5	4	1.	16	0.25
58	A	4	3	1.	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	5	4	1.	16	0.25
60	A	7	6	1.	16	0.375
61	A	6	4	1.	16	0.25
62	A	5	3	1.	14	0.214
63	A	6	4	1.	16	0.25
64	A	7	5	1.	16	0.312
65	A	6	4	1.	16	0.25
66	A	4	2	1.	12	0.167
67	A	6	4	1.	16	0.25
68	A	7	5	1.	16	0.312
69	A	7	6	1.	18	0.333
70	A	2	2	1.	18	0.111
71	A	9	6	1.	18	0.333
72	A	13	8	1.	18	0.444
73	A	11	7	1.	18	0.389
74	A	9	5	1.	16	0.312
75	A	11	7	1.	18	0.389
76	A	13	7	1.	18	0.389
77	A	11	7	1.	18	0.389
78	A	8	4	1.	14	0.286
79	A	11	7	1.	18	0.389
80	A	13	7	1.	18	0.389
81	A	9	6	1.	18	0.333
82	A	4	4	1.	18	0.222
83	A	0	0	0.	0	0.
84	A	0	0	0.	0	0.
85	A	0	0	0.	0	0.
86	A	0	0	0.	0	0.
87	A	0	0	0.	0	0.
88	A	0	0	0.	0	0.
89	A	12	9	1.	18	0.5
90	A	6	6	1.	18	0.333
91	A	0	0	0.	0	0.
92	A	0	0	0.	0	0.
93	A	0	0	0.	0	0.
94	A	0	0	0.	0	0.
95	A	0	0	0.	0	0.
96	A	0	0	0.	0	0.
97	A	0	0	0.	0	0.
98	A	13	5	1.	20	0.25
99	A	9	5	1.	20	0.25
100	A	5	3	1.	18	0.167
101	A	0	0	0.	0	0.
102	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
103	A	7	5	1.	12	0.417
104	A	6	5	1.	10	0.5
105	A	5	5	1.	8	0.625
106	A	3	3	1.	12	0.25
107	A	2	2	1.	12	0.167
108	A	3	3	1.	12	0.25
109	A	4	3	1.	12	0.25
110	A	5	3	1.	12	0.25
111	A	9	6	1.	14	0.429
112	A	8	8	1.	12	0.667
113	A	6	6	1.	10	0.6
114	A	5	4	1.	14	0.286
115	A	3	3	1.	14	0.214
116	A	3	3	1.	14	0.214
117	A	5	5	1.	14	0.357
118	A	5	4	1.	14	0.286
119	A	5	5	1.	8	0.625
120	A	3	3	1.	12	0.25
121	A	4	4	1.	12	0.333
122	A	2	2	1.	12	0.167
123	A	5	5	1.	12	0.417
124	A	2	2	1.	12	0.167
125	A	3	2	1.	14	0.143
126	A	3	3	1.	6	0.5
127	A	5	5	1.	8	0.625
128	A	7	5	1.	8	0.625
129	A	0	0	0.	0	0.
130	A	0	0	0.	0	0.
131	A	3	3	1.	20	0.15
132	A	0	0	0.	0	0.
133	A	5	5	1.	22	0.227
134	A	0	0	0.	0	0.
135	A	3	3	1.	12	0.25
136	A	5	4	1.	14	0.286
137	A	8	4	1.	14	0.286
138	A	8	4	1.	14	0.286
139	A	3	2	1.	8	0.25
140	A	5	3	1.	10	0.3
141	A	8	3	1.	10	0.3
142	A	3	2	1.	12	0.167
143	A	5	3	1.	14	0.214
144	A	8	3	1.	14	0.214
145	A	3	3	1.	16	0.188
146	A	3	3	1.	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	5	5	1.	16	0.312
148	A	7	6	1.	18	0.333
149	A	12	6	1.	18	0.333
150	A	6	5	1.	16	0.312
151	A	8	6	1.	18	0.333
152	A	14	6	1.	18	0.333
153	A	10	8	1.	18	0.444
154	A	7	6	1.	18	0.333
155	A	5	4	1.	16	0.25
156	A	1	1	1.	10	0.1
157	A	0	0	0.	0	0.
158	A	0	0	0.	0	0.
159	A	16	11	1.	18	0.611
160	A	12	10	1.	18	0.556
161	A	8	7	1.	16	0.438
162	A	3	3	1.	10	0.3
163	A	0	0	0.	0	0.
164	A	0	0	0.	0	0.
165	A	14	10	1.	20	0.5
166	A	11	8	1.	20	0.4
167	A	7	6	1.	18	0.333
168	A	3	3	1.	12	0.25
169	A	0	0	0.	0	0.
170	A	0	0	0.	0	0.
171	A	14	9	1.	20	0.45
172	A	10	7	1.	20	0.35
173	A	8	5	1.	18	0.278
174	A	3	2	1.	12	0.167
175	A	0	0	0.	0	0.
176	A	0	0	0.	0	0.
177	A	18	14	1.	20	0.7
178	A	12	11	1.	18	0.611
179	A	5	5	1.	12	0.417
180	A	0	0	0.	0	0.
181	A	0	0	0.	0	0.
182	A	13	10	1.	20	0.5
183	A	8	5	1.	18	0.278
184	A	3	2	1.	12	0.167
185	A	0	0	0.	0	0.
186	A	0	0	0.	0	0.
187	A	14	3	1.	22	0.136
188	A	8	3	1.	20	0.15
189	A	3	3	1.	14	0.214
190	A	8	4	1.	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
191	A	10	6	1.	22	0.273
192	A	12	9	1.	22	0.409
193	A	9	6	1.	20	0.3
194	A	4	3	1.	14	0.214
195	A	0	0	0.	0	0.
196	A	0	0	0.	0	0.
197	A	23	5	1.	22	0.227
198	A	14	5	1.	20	0.25
199	A	6	5	1.	14	0.357
200	A	13	5	1.	22	0.227
201	A	10	6	1.	22	0.273
202	A	14	8	1.	22	0.364
203	A	8	3	1.	20	0.15
204	A	4	3	1.	14	0.214
205	A	0	0	0.	0	0.
206	A	0	0	0.	0	0.
207	A	20	4	1.	22	0.182
208	A	11	4	1.	20	0.2
209	A	4	3	1.	14	0.214
210	A	11	4	1.	22	0.182
211	A	13	6	1.	22	0.273
212	A	17	10	1.	22	0.454
213	A	10	8	1.	20	0.4
214	A	5	5	1.	14	0.357
215	A	0	0	0.	0	0.
216	A	0	0	0.	0	0.
217	A	29	5	1.	22	0.227
218	A	17	5	1.	20	0.25
219	A	7	5	1.	14	0.357
220	A	16	5	1.	22	0.227
221	A	13	6	1.	22	0.273
222	A	24	13	1.	22	0.591
223	A	15	12	1.	20	0.6
224	A	7	7	1.	14	0.5
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	9	4	1.	27	0.148
228	A	7	4	1.	27	0.148
229	A	6	4	1.	27	0.148
230	A	4	4	1.	27	0.148
231	A	3	3	1.	27	0.111
232	A	6	6	1.	27	0.222
233	A	7	6	1.	27	0.222
234	A	9	6	1.	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	9	8	1.	27	0.296
236	A	8	8	1.	27	0.296
237	A	5	5	1.	27	0.185
238	A	4	4	1.	27	0.148
239	A	6	6	1.	27	0.222
240	A	7	7	1.	27	0.259
241	A	7	7	1.	27	0.259
242	A	9	6	1.	27	0.222
243	A	7	6	1.	27	0.222
244	A	6	6	1.	27	0.222
245	A	3	3	1.	27	0.111
246	A	4	4	1.	27	0.148
247	A	6	4	1.	27	0.148
248	A	7	4	1.	27	0.148
249	A	11	9	1.	27	0.333
250	A	10	9	1.	27	0.333
251	A	8	7	1.	27	0.259
252	A	7	7	1.	27	0.259
253	A	8	8	1.	27	0.296
254	A	6	6	1.	27	0.222
255	A	4	4	1.	27	0.148
256	A	5	5	1.	27	0.185
257	A	9	9	1.	27	0.333
258	A	10	9	1.	27	0.333
259	A	0	0	0.	0	0.
260	A	14	5	1.	16	0.312
261	A	11	5	1.	16	0.312
262	A	8	5	1.	14	0.357
263	A	3	2	1.	12	0.167
264	A	0	0	0.	0	0.
265	A	0	0	0.	0	0.
266	A	16	6	1.	20	0.3
267	A	13	6	1.	20	0.3
268	A	10	6	1.	18	0.333
269	A	4	2	1.	16	0.125
270	A	0	0	0.	0	0.
271	A	0	0	0.	0	0.
272	A	28	10	1.	22	0.454
273	A	19	10	1.	20	0.5
274	A	8	4	1.	18	0.222
275	A	0	0	0.	0	0.
276	A	0	0	0.	0	0.
277	A	0	0	0.	0	0.
278	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
279	A	0	0	0.	0	0.
280	A	0	0	0.	0	0.
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	0	0	0.	0	0.
284	A	0	0	0.	0	0.
285	A	0	0	0.	0	0.
286	A	0	0	0.	0	0.
287	A	0	0	0.	0	0.
288	A	23	6	1.	20	0.3
289	A	15	6	1.	18	0.333
290	A	6	5	1.	12	0.417
291	A	12	6	1.	20	0.3
292	A	7	6	1.	20	0.3
293	A	15	6	1.	20	0.3
294	A	27	11	1.	20	0.55
295	A	12	8	1.	14	0.571
296	A	22	6	1.	22	0.273
297	A	12	8	1.	22	0.364
298	A	27	11	1.	22	0.5
299	A	0	0	0.	0	0.
300	A	0	0	0.	0	0.
301	A	0	0	0.	0	0.
302	A	0	0	0.	0	0.
303	A	0	0	0.	0	0.
304	A	0	0	0.	0	0.
305	A	0	0	0.	0	0.
306	A	0	0	0.	0	0.
307	A	0	0	0.	0	0.
308	A	0	0	0.	0	0.
309	A	0	0	0.	0	0.
310	A	4	3	1.	18	0.167
311	A	5	3	1.	18	0.167
312	A	4	3	1.	18	0.167
313	A	3	3	1.	16	0.188
314	A	2	2	1.	14	0.143
315	A	4	4	1.	18	0.222
316	A	5	5	1.	18	0.278
317	A	6	5	1.	18	0.278
318	A	4	3	1.	20	0.15
319	A	4	4	1.	20	0.2
320	A	5	5	1.	20	0.25
321	A	3	3	1.	18	0.167
322	A	4	4	1.	16	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	4	4	1.	20	0.2
324	A	5	5	1.	20	0.25
325	A	6	6	1.	20	0.3
326	A	4	3	1.	20	0.15
327	A	4	3	1.	20	0.15
328	A	4	3	1.	20	0.15
329	A	4	3	1.	18	0.167
330	A	4	3	1.	16	0.188
331	A	4	4	1.	20	0.2
332	A	4	3	1.	20	0.15
333	A	4	3	1.	20	0.15
334	A	6	4	1.	18	0.222
335	A	5	4	1.	18	0.222
336	A	5	5	1.	18	0.278
337	A	3	3	1.	16	0.188
338	A	3	3	1.	14	0.214
339	A	6	5	1.	18	0.278
340	A	6	6	1.	18	0.333
341	A	8	7	1.	18	0.389
342	A	6	4	1.	20	0.2
343	A	4	4	1.	20	0.2
344	A	7	6	1.	20	0.3
345	A	4	4	1.	18	0.222
346	A	6	5	1.	16	0.312
347	A	6	5	1.	20	0.25
348	A	7	7	1.	20	0.35
349	A	8	7	1.	20	0.35
350	A	6	4	1.	20	0.2
351	A	6	4	1.	20	0.2
352	A	6	4	1.	20	0.2
353	A	6	4	1.	18	0.222
354	A	6	4	1.	16	0.25
355	A	6	5	1.	20	0.25
356	A	6	4	1.	20	0.2
357	A	6	4	1.	20	0.2

Chapter 3

Listing of integrals

3.1 $\int x^5 \left(a+b\sin\left(c+dx^2\right)\right)\,dx$

Optimal. Leaf size=57

$$\frac{ax^6}{6}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b\cos\left(c+dx^2\right)}{d^3}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}$$

```
[Out] (a*x^6)/6 + (b*Cos[c + d*x^2])/d^3 - (b*x^4*Cos[c + d*x^2])/(2*d) + (b*x^2*
Sin[c + d*x^2])/d^2
```

Rubi [A] time = 0.0730884, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3379, 3296, 2638}

$$\frac{ax^6}{6}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b\cos\left(c+dx^2\right)}{d^3}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 + (b*Cos[c + d*x^2])/d^3 - (b*x^4*Cos[c + d*x^2])/(2*d) + (b*x^2*
Sin[c + d*x^2])/d^2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^5\left(a+b\sin\left(c+dx^2\right)\right)\,dx &= \int\left(ax^5+bx^5\sin\left(c+dx^2\right)\right)\,dx\\
&= \frac{ax^6}{6}+b\int x^5\sin\left(c+dx^2\right)\,dx\\
&= \frac{ax^6}{6}+\frac{1}{2}b\,\text{Subst}\left(\int x^2\sin(c+dx)\,dx,x,x^2\right)\\
&= \frac{ax^6}{6}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{b\,\text{Subst}\left(\int x\cos(c+dx)\,dx,x,x^2\right)}{d}\\
&= \frac{ax^6}{6}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}-\frac{b\,\text{Subst}\left(\int\sin(c+dx)\,dx,x,x^2\right)}{d^2}\\
&= \frac{ax^6}{6}+\frac{b\cos\left(c+dx^2\right)}{d^3}-\frac{bx^4\cos\left(c+dx^2\right)}{2d}+\frac{bx^2\sin\left(c+dx^2\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.0859434, size = 51, normalized size = 0.89

$$\frac{ad^3x^6-3b\left(d^2x^4-2\right)\cos\left(c+dx^2\right)+6bdx^2\sin\left(c+dx^2\right)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*d^3*x^6 - 3*b*(-2 + d^2*x^4)*Cos[c + d*x^2] + 6*b*d*x^2*Sin[c + d*x^2])/
(6*d^3)
```

Maple [A] time = 0.01, size = 62, normalized size = 1.1

$$\frac{ax^6}{6}+b\left(-\frac{x^4\cos\left(dx^2+c\right)}{2d}+2\frac{1}{d}\left(1/2\frac{x^2\sin\left(dx^2+c\right)}{d}+1/2\frac{\cos\left(dx^2+c\right)}{d^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^2+c)),x)
```

```
[Out] 1/6*a*x^6+b*(-1/2/d*x^4*cos(d*x^2+c)+2/d*(1/2/d*x^2*sin(d*x^2+c)+1/2/d^2*co
s(d*x^2+c)))
```

Maxima [A] time = 0.980933, size = 63, normalized size = 1.11

$$\frac{1}{6}ax^6+\frac{\left(2\,dx^2\sin\left(dx^2+c\right)-\left(d^2x^4-2\right)\cos\left(dx^2+c\right)\right)b}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/2*(2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*b/d
^3
```

Fricas [A] time = 1.99113, size = 115, normalized size = 2.02

$$\frac{ad^3x^6 + 6\,bdx^2\sin\left(dx^2 + c\right) - 3\left(bd^2x^4 - 2\,b\right)\cos\left(dx^2 + c\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^3*x^6 + 6*b*d*x^2*sin(d*x^2 + c) - 3*(b*d^2*x^4 - 2*b)*cos(d*x^2 +
 c))/d^3
```

Sympy [A] time = 3.95245, size = 65, normalized size = 1.14

$$\begin{cases} \frac{ax^6}{6} - \frac{bx^4\cos\left(c+dx^2\right)}{2d} + \frac{bx^2\sin\left(c+dx^2\right)}{d^2} + \frac{b\cos\left(c+dx^2\right)}{d^3} & \text{for } d \neq 0 \\ \frac{x^6(a+b\sin(c))}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**6/6 - b*x**4*cos(c + d*x**2)/(2*d) + b*x**2*sin(c + d*x**2)
/d**2 + b*cos(c + d*x**2)/d**3, Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Giac [A] time = 1.17725, size = 93, normalized size = 1.63

$$\frac{adx^6 + 3\left(\frac{2\,x^2\sin\left(dx^2+c\right)}{d} - \frac{\left(\left(dx^2+c\right)^2 - 2\left(dx^2+c\right)c + c^2 - 2\right)\cos\left(dx^2+c\right)}{d^2}\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/6*(a*d*x^6 + 3*(2*x^2*sin(d*x^2 + c)/d - ((d*x^2 + c)^2 - 2*(d*x^2 + c)*c
 + c^2 - 2)*cos(d*x^2 + c)/d^2)*b)/d
```

3.2 $\int x^3 \left(a+b \sin \left(c+d x^2\right)\right) \, dx$

Optimal. Leaf size=44

$$\frac{ax^4}{4}+\frac{b\sin\left(c+dx^2\right)}{2d^2}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}$$

```
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Rubi [A] time = 0.0425759, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3379, 3296, 2637}

$$\frac{ax^4}{4}+\frac{b\sin\left(c+dx^2\right)}{2d^2}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\sin\left(c+dx^2\right)\right)dx &= \int\left(ax^3+bx^3\sin\left(c+dx^2\right)\right)dx\\ &= \frac{ax^4}{4}+b\int x^3\sin\left(c+dx^2\right)dx\\ &= \frac{ax^4}{4}+\frac{1}{2}b\,\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^2\right)\\ &= \frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\,\text{Subst}\left(\int\cos(c+dx)\,dx,x,x^2\right)}{2d}\\ &= \frac{ax^4}{4}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}+\frac{b\sin\left(c+dx^2\right)}{2d^2}\end{aligned}$$

Mathematica [A] time = 0.0042806, size = 44, normalized size = 1.

$$\frac{ax^4}{4}+\frac{b\sin\left(c+dx^2\right)}{2d^2}-\frac{bx^2\cos\left(c+dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 - (b*x^2*Cos[c + d*x^2])/(2*d) + (b*Sin[c + d*x^2])/(2*d^2)
```

Maple [A] time = 0.007, size = 40, normalized size = 0.9

$$\frac{ax^4}{4}+b\left(-\frac{x^2\cos\left(dx^2+c\right)}{2\,d}+\frac{\sin\left(dx^2+c\right)}{2\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^2+c)),x)
```

```
[Out] 1/4*a*x^4+b*(-1/2/d*x^2*cos(d*x^2+c)+1/2/d^2*sin(d*x^2+c))
```

Maxima [A] time = 0.976055, size = 50, normalized size = 1.14

$$\frac{1}{4}ax^4-\frac{\left(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right)\right)b}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 - 1/2*(d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*b/d^2
```

Fricas [A] time = 1.92282, size = 93, normalized size = 2.11

$$\frac{ad^2x^4-2\,bdx^2\cos\left(dx^2+c\right)+2\,b\sin\left(dx^2+c\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^2*x^4 - 2*b*d*x^2*cos(d*x^2 + c) + 2*b*sin(d*x^2 + c))/d^2
```

Sympy [A] time = 0.986867, size = 49, normalized size = 1.11

$$\begin{cases} \frac{ax^4}{4} - \frac{bx^2\cos\left(c+dx^2\right)}{2d} + \frac{b\sin\left(c+dx^2\right)}{2d^2} & \text{for } d \neq 0 \\ \frac{x^4(a+b\sin(c))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**4/4 - b*x**2*cos(c + d*x**2)/(2*d) + b*sin(c + d*x**2)/(2*d
**2), Ne(d, 0)), (x**4*(a + b*sin(c))/4, True))
```

Giac [A] time = 1.17592, size = 82, normalized size = 1.86

$$\frac{\frac{\left(\left(dx^2+c\right)^2-2\left(dx^2+c\right)c\right)a}{d} - \frac{2\left(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right)\right)b}{d}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/4*(((d*x^2 + c)^2 - 2*(d*x^2 + c)*c)*a/d - 2*(d*x^2*cos(d*x^2 + c) - sin(
d*x^2 + c))*b/d)/d
```

3.3 $\int x\left(a+b\sin\left(c+dx^2\right)\right)dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}$$

```
[Out] (a*x^2)/2 - (b*Cos[c + d*x^2])/(2*d)
```

Rubi [A] time = 0.0212534, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 3379, 2638}

$$\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 - (b*Cos[c + d*x^2])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\sin\left(c+dx^2\right)\right)dx&=\int\left(ax+bx\sin\left(c+dx^2\right)\right)dx\\&=\frac{ax^2}{2}+b\int x\sin\left(c+dx^2\right)dx\\&=\frac{ax^2}{2}+\frac{1}{2}b\,\text{Subst}\left(\int\sin(c+dx)\,dx,x,x^2\right)\\&=\frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0135029, size = 41, normalized size = 1.64

$$\frac{ax^2}{2}+\frac{b\sin(c)\sin\left(dx^2\right)}{2d}-\frac{b\cos(c)\cos\left(dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 - (b*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sin[c]*Sin[d*x^2])/(2*d)
```

Maple [A] time = 0.005, size = 27, normalized size = 1.1

$$\frac{a\left(dx^2+c\right)-b\cos\left(dx^2+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^2+c)),x)
```

```
[Out] 1/2/d*(a*(d*x^2+c)-b*cos(d*x^2+c))
```

Maxima [A] time = 0.961183, size = 28, normalized size = 1.12

$$\frac{1}{2}ax^2-\frac{b\cos\left(dx^2+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 - 1/2*b*cos(d*x^2 + c)/d
```

Fricas [A] time = 1.8349, size = 49, normalized size = 1.96

$$\frac{adx^2-b\cos\left(dx^2+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x^2 - b*cos(d*x^2 + c))/d
```

Sympy [A] time = 0.23124, size = 31, normalized size = 1.24

$$\begin{cases} \frac{ax^2}{2}-\frac{b\cos\left(c+dx^2\right)}{2d} & \text{for } d\neq 0 \\ \frac{x^2(a+b\sin(c))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((a*x**2/2 - b*cos(c + d*x**2)/(2*d), Ne(d, 0)), (x**2*(a + b*sin(
c))/2, True))
```

Giac [A] time = 1.14744, size = 35, normalized size = 1.4

$$\frac{\left(dx^2+c\right)a-b\cos\left(dx^2+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x^2 + c)*a - b*cos(d*x^2 + c))/d
```

3.4 $\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx$

Optimal. Leaf size=31

$$a\log(x)+\frac{1}{2}b\sin(c)\text{CosIntegral}\left(dx^2\right)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)$$

```
[Out] a*Log[x] + (b*CosIntegral[d*x^2]*Sin[c])/2 + (b*Cos[c]*SinIntegral[d*x^2])/
2
```

Rubi [A] time = 0.0342408, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3377, 3376, 3375}

$$a\log(x)+\frac{1}{2}b\sin(c)\text{CosIntegral}\left(dx^2\right)+\frac{1}{2}b\cos(c)\text{Si}\left(dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x,x]
```

```
[Out] a*Log[x] + (b*CosIntegral[d*x^2]*Sin[c])/2 + (b*Cos[c]*SinIntegral[d*x^2])/
2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin\left(c+dx^2\right)}{x}\,dx &= \int \left(\frac{a}{x}+\frac{b\sin\left(c+dx^2\right)}{x}\right)dx\\ &= a\log(x)+b\int \frac{\sin\left(c+dx^2\right)}{x}\,dx\\ &= a\log(x)+(b\cos(c))\int \frac{\sin\left(dx^2\right)}{x}\,dx+(b\sin(c))\int \frac{\cos\left(dx^2\right)}{x}\,dx\\ &= a\log(x)+\frac{1}{2}b\mathbf{Ci}\left(dx^2\right)\sin(c)+\frac{1}{2}b\cos(c)\mathbf{Si}\left(dx^2\right)\end{aligned}$$

Mathematica [A] time = 0.0490486, size = 29, normalized size = 0.94

$$a\log(x)+\frac{1}{2}b\left(\sin(c)\mathbf{CosIntegral}\left(dx^2\right)+\cos(c)\mathbf{Si}\left(dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x,x]
```

```
[Out] a*Log[x] + (b*(CosIntegral[d*x^2]*Sin[c] + Cos[c]*SinIntegral[d*x^2]))/2
```

Maple [A] time = 0.009, size = 28, normalized size = 0.9

$$a\ln(x)+\frac{b\cos(c)\,\mathrm{Si}\left(dx^2\right)}{2}+\frac{b\mathrm{Ci}\left(dx^2\right)\sin(c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x,x)
```

```
[Out] a*ln(x)+1/2*b*cos(c)*Si(d*x^2)+1/2*b*Ci(d*x^2)*sin(c)
```

Maxima [C] time = 1.15545, size = 68, normalized size = 2.19

$$-\frac{1}{4}\left(\left(i\,\mathrm{Ei}\left(i\,dx^2\right)-i\,\mathrm{Ei}\left(-i\,dx^2\right)\right)\cos(c)-\left(\mathrm{Ei}\left(i\,dx^2\right)+\mathrm{Ei}\left(-i\,dx^2\right)\right)\sin(c)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="maxima")
```

```
[Out] -1/4*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2)
)*sin(c))*b + a*log(x)
```

Fricas [A] time = 2.01222, size = 144, normalized size = 4.65

$$\frac{1}{2}\,b\cos(c)\,\mathrm{Si}\left(dx^2\right)+a\log(x)+\frac{1}{4}\left(b\,\mathrm{Ci}\left(dx^2\right)+b\,\mathrm{Ci}\left(-dx^2\right)\right)\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*cos(c)*sin_integral(d*x^2) + a*log(x) + 1/4*(b*cos_integral(d*x^2) +
b*cos_integral(-d*x^2))*sin(c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \sin \left(c + dx^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x, x)
```

Giac [A] time = 1.11189, size = 43, normalized size = 1.39

$$\frac{1}{2}\, b \operatorname{Ci}\left(dx^2\right) \sin\left(c\right) + \frac{1}{2}\, b \cos\left(c\right) \operatorname{Si}\left(dx^2\right) + \frac{1}{2}\, a \log\left(dx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x,x, algorithm="giac")
```

```
[Out] 1/2*b*cos_integral(d*x^2)*sin(c) + 1/2*b*cos(c)*sin_integral(d*x^2) + 1/2*a
*log(d*x^2)
```

3.5 $\int \frac{a+b\sin(c+dx^2)}{x^3}\,dx$

Optimal. Leaf size=53

$$-\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{2x^2}$$

```
[Out] -a/(2*x^2) + (b*d*Cos[c]*CosIntegral[d*x^2])/2 - (b*Sin[c + d*x^2])/(2*x^2)
 - (b*d*Sin[c]*SinIntegral[d*x^2])/2
```

Rubi [A] time = 0.0911441, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3379, 3297, 3303, 3299, 3302}

$$-\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*d*Cos[c]*CosIntegral[d*x^2])/2 - (b*Sin[c + d*x^2])/(2*x^2)
 - (b*d*Sin[c]*SinIntegral[d*x^2])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+dx^2\right)}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b\sin\left(c+dx^2\right)}{x^3}\right)dx \\
&= -\frac{a}{2x^2}+b\int \frac{\sin\left(c+dx^2\right)}{x^3}\,dx \\
&= -\frac{a}{2x^2}+\frac{1}{2}b\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{a}{2x^2}-\frac{b\sin\left(c+dx^2\right)}{2x^2}+\frac{1}{2}(bd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,x^2\right) \\
&= -\frac{a}{2x^2}-\frac{b\sin\left(c+dx^2\right)}{2x^2}+\frac{1}{2}(bd\cos(c))\,\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,x^2\right)-\frac{1}{2}(bd\sin(c))\,\text{Subst}\left(\int\right. \\
&= -\frac{a}{2x^2}+\frac{1}{2}bd\cos(c)\text{Ci}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{2x^2}-\frac{1}{2}bd\sin(c)\text{Si}\left(dx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0812176, size = 48, normalized size = 0.91

$$
-\frac{a-bdx^2\cos(c)\text{CosIntegral}\left(dx^2\right)+bdx^2\sin(c)\text{Si}\left(dx^2\right)+b\sin\left(c+dx^2\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^3,x]
```

```
[Out] -(a - b*d*x^2*Cos[c]*CosIntegral[d*x^2] + b*Sin[c + d*x^2] + b*d*x^2*Sin[c]
*SinIntegral[d*x^2])/(2*x^2)
```

Maple [A] time = 0.009, size = 47, normalized size = 0.9

$$
-\frac{a}{2\,x^2}+b\left(-\frac{\sin\left(dx^2+c\right)}{2\,x^2}+d\left(\frac{\cos(c)\,\text{Ci}\left(dx^2\right)}{2}-\frac{\sin(c)\,\text{Si}\left(dx^2\right)}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^3,x)
```

```
[Out] -1/2*a/x^2+b*(-1/2/x^2*sin(d*x^2+c)+d*(1/2*cos(c)*Ci(d*x^2)-1/2*sin(c)*Si(d
*x^2)))
```

Maxima [C] time = 1.14813, size = 77, normalized size = 1.45

$$
\frac{1}{4}\left(\left(\Gamma\left(-1,i\,dx^2\right)+\Gamma\left(-1,-i\,dx^2\right)\right)\cos(c)-\left(i\,\Gamma\left(-1,i\,dx^2\right)-i\,\Gamma\left(-1,-i\,dx^2\right)\right)\sin(c)\right)bd-\frac{a}{2\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*x
^2) - I*gamma(-1, -I*d*x^2))*sin(c))*b*d - 1/2*a/x^2
```

Fricas [A] time = 1.9283, size = 197, normalized size = 3.72

$$-\frac{2\,bdx^2\sin\left(c\right)\mathrm{Si}\left(dx^2\right)-\left(bdx^2\,\mathrm{Ci}\left(dx^2\right)+bdx^2\,\mathrm{Ci}\left(-dx^2\right)\right)\cos\left(c\right)+2\,b\sin\left(dx^2+c\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d*x^2*sin(c)*sin_integral(d*x^2) - (b*d*x^2*cos_integral(d*x^2) +
 b*d*x^2*cos_integral(-d*x^2))*cos(c) + 2*b*sin(d*x^2 + c) + 2*a)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sin\left(c+dx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**3, x)
```

Giac [B] time = 1.14906, size = 134, normalized size = 2.53

$$\frac{\left(dx^2+c\right)bd^2\cos\left(c\right)\mathrm{Ci}\left(dx^2\right)-bcd^2\cos\left(c\right)\mathrm{Ci}\left(dx^2\right)-\left(dx^2+c\right)bd^2\sin\left(c\right)\mathrm{Si}\left(dx^2\right)+bcd^2\sin\left(c\right)\mathrm{Si}\left(dx^2\right)-bd^2\sin\left(d\right.}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((d*x^2 + c)*b*d^2*cos(c)*cos_integral(d*x^2) - b*c*d^2*cos(c)*cos_inte
gral(d*x^2) - (d*x^2 + c)*b*d^2*sin(c)*sin_integral(d*x^2) + b*c*d^2*sin(c)
*sin_integral(d*x^2) - b*d^2*sin(d*x^2 + c) - a*d^2)/(d^2*x^2)
```

3.6 $\int \frac{a+b\sin(c+dx^2)}{x^5}\,dx$

Optimal. Leaf size=74

$$-\frac{a}{4x^4}-\frac{1}{4}bd^2\sin(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}$$

```
[Out] -a/(4*x^4) - (b*d*Cos[c + d*x^2])/(4*x^2) - (b*d^2*CosIntegral[d*x^2]*Sin[c
])/4 - (b*Sin[c + d*x^2])/(4*x^4) - (b*d^2*Cos[c]*SinIntegral[d*x^2])/4
```

Rubi [A] time = 0.125384, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3379, 3297, 3303, 3299, 3302}

$$-\frac{a}{4x^4}-\frac{1}{4}bd^2\sin(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^5,x]
```


```
[Out] -a/(4*x^4) - (b*d*Cos[c + d*x^2])/(4*x^2) - (b*d^2*CosIntegral[d*x^2]*Sin[c
])/4 - (b*Sin[c + d*x^2])/(4*x^4) - (b*d^2*Cos[c]*SinIntegral[d*x^2])/4
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^2\right)}{x^5}\right)dx\\
&= -\frac{a}{4x^4}+b\int \frac{\sin\left(c+dx^2\right)}{x^5}\,dx\\
&= -\frac{a}{4x^4}+\frac{1}{2}b\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{a}{4x^4}-\frac{b\sin\left(c+dx^2\right)}{4x^4}+\frac{1}{4}(bd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{\sin(c+dx)}{x}\,dx,x,x^2\right)\\
&= -\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}\left(bd^2\cos(c)\right)\text{Subst}\left(\int \frac{\sin(dx)}{x}\,dx,x,x^2\right)-\\
&= -\frac{a}{4x^4}-\frac{bd\cos\left(c+dx^2\right)}{4x^2}-\frac{1}{4}bd^2\text{Ci}\left(dx^2\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{4x^4}-\frac{1}{4}bd^2\cos(c)\text{Si}\left(dx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0866247, size = 86, normalized size = 1.16

$$
-\frac{a}{4x^4}-\frac{1}{4}bd^2\left(\sin(c)\text{CosIntegral}\left(dx^2\right)+\cos(c)\text{Si}\left(dx^2\right)\right)-\frac{b\cos\left(dx^2\right)\left(dx^2\cos(c)+\sin(c)\right)}{4x^4}+\frac{b\sin\left(dx^2\right)\left(dx^2\sin(c\right.}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^5,x]
```

```
[Out] -a/(4*x^4) - (b*Cos[d*x^2]*(d*x^2*Cos[c] + Sin[c]))/(4*x^4) + (b*(-Cos[c] +
 d*x^2*Sin[c])*Sin[d*x^2])/(4*x^4) - (b*d^2*(CosIntegral[d*x^2]*Sin[c] + Co
s[c]*SinIntegral[d*x^2]))/4
```

Maple [A] time = 0.009, size = 65, normalized size = 0.9

$$
-\frac{a}{4\,x^4}+b\left(-\frac{\sin\left(dx^2+c\right)}{4\,x^4}+\frac{d}{2}\left(-\frac{\cos\left(dx^2+c\right)}{2\,x^2}-d\left(\frac{\cos(c)\,\text{Si}\left(dx^2\right)}{2}+\frac{\sin(c)\,\text{Ci}\left(dx^2\right)}{2}\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^5,x)
```

```
[Out] -1/4*a/x^4+b*(-1/4/x^4*sin(d*x^2+c)+1/2*d*(-1/2/x^2*cos(d*x^2+c)-d*(1/2*cos
(c)*Si(d*x^2)+1/2*sin(c)*Ci(d*x^2))))
```

Maxima [C] time = 1.15642, size = 78, normalized size = 1.05

$$\frac{1}{4}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos\left(c\right)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin\left(c\right)\right)bd^2-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*d
*x^2) + gamma(-2, -I*d*x^2))*sin(c))*b*d^2 - 1/4*a/x^4
```

Fricas [A] time = 1.928, size = 242, normalized size = 3.27

$$-\frac{2\,bd^2x^4\cos\left(c\right)\operatorname{Si}\left(dx^2\right)+2\,bdx^2\cos\left(dx^2+c\right)+2\,b\sin\left(dx^2+c\right)+\left(bd^2x^4\operatorname{Ci}\left(dx^2\right)+bd^2x^4\operatorname{Ci}\left(-dx^2\right)\right)\sin\left(c\right)+2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*d^2*x^4*cos(c)*sin_integral(d*x^2) + 2*b*d*x^2*cos(d*x^2 + c) + 2
*b*sin(d*x^2 + c) + (b*d^2*x^4*cos_integral(d*x^2) + b*d^2*x^4*cos_integral
(-d*x^2))*sin(c) + 2*a)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sin\left(c+dx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**5, x)
```

Giac [B] time = 1.11717, size = 275, normalized size = 3.72

$$-\frac{\left(dx^2+c\right)^2bd^3\operatorname{Ci}\left(dx^2\right)\sin\left(c\right)-2\left(dx^2+c\right)bcd^3\operatorname{Ci}\left(dx^2\right)\sin\left(c\right)+bc^2d^3\operatorname{Ci}\left(dx^2\right)\sin\left(c\right)+\left(dx^2+c\right)^2bd^3\cos\left(c\right)\operatorname{Si}\left(dx^2\right)}{4\left(\left(dx^2+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((d*x^2 + c)^2*b*d^3*cos_integral(d*x^2)*sin(c) - 2*(d*x^2 + c)*b*c*d^
3*cos_integral(d*x^2)*sin(c) + b*c^2*d^3*cos_integral(d*x^2)*sin(c) + (d*x^
2 + c)^2*b*d^3*cos(c)*sin_integral(d*x^2) - 2*(d*x^2 + c)*b*c*d^3*cos(c)*si
n_integral(d*x^2) + b*c^2*d^3*cos(c)*sin_integral(d*x^2) + (d*x^2 + c)*b*d^
3*cos(d*x^2 + c) - b*c*d^3*cos(d*x^2 + c) + b*d^3*sin(d*x^2 + c) + a*d^3)/(
((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

3.7 $\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)\,dx$

Optimal. Leaf size=121

$$\frac{ax^5}{5}-\frac{3\sqrt{\frac{\pi}{2}}\,b\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{4d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,b\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}$$

```
[Out] (a*x^5)/5 - (b*x^3*Cos[c + d*x^2])/(2*d) - (3*b*Sqrt[Pi/2]*Cos[c]*FresnelS[
Sqrt[d]*Sqrt[2/Pi]*x])/(4*d^(5/2)) - (3*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[
2/Pi]*x]*Sin[c])/(4*d^(5/2)) + (3*b*x*Sin[c + d*x^2])/(4*d^2)
```

Rubi [A] time = 0.134059, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3385, 3386, 3353, 3352, 3351}

$$\frac{ax^5}{5}-\frac{3\sqrt{\frac{\pi}{2}}\,b\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{4d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,b\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 - (b*x^3*Cos[c + d*x^2])/(2*d) - (3*b*Sqrt[Pi/2]*Cos[c]*FresnelS[
Sqrt[d]*Sqrt[2/Pi]*x])/(4*d^(5/2)) - (3*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[
2/Pi]*x]*Sin[c])/(4*d^(5/2)) + (3*b*x*Sin[c + d*x^2])/(4*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\sin\left(c+dx^2\right)\right)dx &= \int\left(ax^4+bx^4\sin\left(c+dx^2\right)\right)dx\\ &= \frac{ax^5}{5}+b\int x^4\sin\left(c+dx^2\right)dx\\ &= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}+\frac{(3b)\int x^2\cos\left(c+dx^2\right)dx}{2d}\\ &= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{(3b)\int\sin\left(c+dx^2\right)dx}{4d^2}\\ &= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}+\frac{3bx\sin\left(c+dx^2\right)}{4d^2}-\frac{(3b\cos(c))\int\sin\left(dx^2\right)dx}{4d^2}-\frac{(3b\sin(c))\int}{4}\\ &= \frac{ax^5}{5}-\frac{bx^3\cos\left(c+dx^2\right)}{2d}-\frac{3b\sqrt{\frac{\pi}{2}}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{4d^{5/2}}-\frac{3b\sqrt{\frac{\pi}{2}}C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{4d^{5/2}}+\frac{3bx\,\mathrm{s}}{}\end{aligned}$$

Mathematica [A] time = 0.25147, size = 125, normalized size = 1.03

$$\frac{ax^5}{5}-\frac{3\sqrt{\frac{\pi}{2}}b\left(\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)+\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right)}{4d^{5/2}}-\frac{bx\cos\left(dx^2\right)\left(2dx^2\cos(c)-3\sin(c)\right)}{4d^2}+\frac{bx\sin\left(dx^2\right)\left(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 - (b*x*Cos[d*x^2]*(2*d*x^2*Cos[c] - 3*Sin[c]))/(4*d^2) - (3*b*Sqr
t[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt[2/Pi
]*x]*Sin[c]))/(4*d^(5/2)) + (b*x*(3*Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(4
*d^2)
```

Maple [A] time = 0.009, size = 89, normalized size = 0.7

$$\frac{ax^5}{5}+b\left(-\frac{x^3\cos\left(dx^2+c\right)}{2d}+\frac{3}{2d}\left(\frac{x\sin\left(dx^2+c\right)}{2d}-\frac{\sqrt{2}\sqrt{\pi}}{4}\left(\cos(c)\,\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)+\sin(c)\,\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^2+c)),x)
```

```
[Out] 1/5*a*x^5+b*(-1/2/d*x^3*cos(d*x^2+c)+3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2
)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*Fres
nelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Maxima [C] time = 1.62742, size = 387, normalized size = 3.2

$$\frac{1}{5}\,ax^5 - \frac{\left(16\,dx^3|d|\cos\left(dx^2+c\right) - 24\,x|d|\sin\left(dx^2+c\right) - \sqrt{\pi}\left(\left(\left(-3i\,\cos\left(\frac{1}{4}\,\pi + \frac{1}{2}\,\arctan\left(0,d\right)\right) - 3i\,\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,\mathrm{ar}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 - 1/32*(16*d*x^3*abs(d)*cos(d*x^2 + c) - 24*x*abs(d)*sin(d*x^2 +
c) - sqrt(pi)*(((-3*I*cos(1/4*pi + 1/2*arctan2(0, d)) - 3*I*cos(-1/4*pi + 1
/2*arctan2(0, d)) - 3*sin(1/4*pi + 1/2*arctan2(0, d)) + 3*sin(-1/4*pi + 1/2
*arctan2(0, d)))*cos(c) - (3*cos(1/4*pi + 1/2*arctan2(0, d)) + 3*cos(-1/4*p
i + 1/2*arctan2(0, d)) - 3*I*sin(1/4*pi + 1/2*arctan2(0, d)) + 3*I*sin(-1/4
*pi + 1/2*arctan2(0, d)))*sin(c))*erf(sqrt(I*d)*x) + ((3*I*cos(1/4*pi + 1/2
*arctan2(0, d)) + 3*I*cos(-1/4*pi + 1/2*arctan2(0, d)) - 3*sin(1/4*pi + 1/2
*arctan2(0, d)) + 3*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) - (3*cos(1/4*p
i + 1/2*arctan2(0, d)) + 3*cos(-1/4*pi + 1/2*arctan2(0, d)) + 3*I*sin(1/4*p
i + 1/2*arctan2(0, d)) - 3*I*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(c))*erf(
sqrt(-I*d)*x))*sqrt(abs(d)))*b/(d^2*abs(d))
```

Fricas [A] time = 2.07763, size = 297, normalized size = 2.45

$$\frac{8\,ad^3x^5 - 20\,bd^2x^3\cos\left(dx^2+c\right) - 15\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right) - 15\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right) + 30\,bdx\sin\left(d\right.}{40\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*d^3*x^5 - 20*b*d^2*x^3*cos(d*x^2 + c) - 15*sqrt(2)*pi*b*sqrt(d/pi
)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) - 15*sqrt(2)*pi*b*sqrt(d/pi)*fre
snel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 30*b*d*x*sin(d*x^2 + c))/d^3
```

Sympy [B] time = 4.24711, size = 488, normalized size = 4.03

$$\frac{ax^5}{5} - \frac{5\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\sin\left(c\right)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\sin\left(c\right)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)}{2} - \frac{21\sqrt{2}\sqrt{\pi}bx^4\sqrt{\frac{1}{d}}\cos\left(c\right)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\Gamma\left(\right.}{32\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**2+c)),x)
```

```
[Out] a*x**5/5 - 5*sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt
(d)*x/sqrt(pi))*gamma(1/4)/(32*gamma(9/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1
/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 - 21*sqrt(2)*sqrt(pi)*b*x
**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(32*ga
mma(11/4)) + sqrt(2)*sqrt(pi)*b*x**4*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt
(d)*x/sqrt(pi))/2 - 15*sqrt(2)*sqrt(pi)*b*sqrt(1/d)*sin(c)*fresnelc(sqrt(2)
*sqrt(d)*x/sqrt(pi))*gamma(1/4)/(128*d**2*gamma(9/4)) - 63*sqrt(2)*sqrt(pi)
*b*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))*gamma(3/4)/(128*d*
```

```
*2*gamma(11/4)) + 5*b*x**3*sqrt(1/d)*sin(c)*sin(d*x**2)*gamma(1/4)/(32*sqrt
(d)*gamma(9/4)) - 21*b*x**3*sqrt(1/d)*cos(c)*cos(d*x**2)*gamma(3/4)/(32*sqr
t(d)*gamma(11/4)) + 15*b*x*sqrt(1/d)*sin(c)*cos(d*x**2)*gamma(1/4)/(64*d**(
3/2)*gamma(9/4)) + 63*b*x*sqrt(1/d)*sin(d*x**2)*cos(c)*gamma(3/4)/(64*d**(3
/2)*gamma(11/4))
```

Giac [C] time = 1.14956, size = 223, normalized size = 1.84

$$\frac{1}{5}ax^5 - \frac{3i\sqrt{2}\sqrt{\pi}b\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{16d^2\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}} + \frac{3i\sqrt{2}\sqrt{\pi}b\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{16d^2\left(\frac{id}{|d|}+1\right)\sqrt{|d|}} + \frac{i\left(2ibdx^3-3bx\right)e^{\left(idx^2\right.}}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/5*a*x^5 - 3/16*I*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*
sqrt(abs(d)))*e^(I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d))) + 3/16*I*sqrt(2)
*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d^2
*(I*d/abs(d) + 1)*sqrt(abs(d))) + 1/8*I*(2*I*b*d*x^3 - 3*b*x)*e^(I*d*x^2 +
I*c)/d^2 + 1/8*I*(2*I*b*d*x^3 + 3*b*x)*e^(-I*d*x^2 - I*c)/d^2
```

3.8 $\int x^2 \left(a+b\sin\left(c+dx^2\right)\right)\,dx$

Optimal. Leaf size=102

$$\frac{ax^3}{3}+\frac{\sqrt{\frac{\pi}{2}}\,b\cos(c)\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{2d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,b\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{bx\cos\left(c+dx^2\right)}{2d}$$

```
[Out] (a*x^3)/3 - (b*x*Cos[c + d*x^2])/(2*d) + (b*Sqrt[Pi/2]*Cos[c]*FresnelC[Sqrt
[d]*Sqrt[2/Pi]*x])/(2*d^(3/2)) - (b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x]*Sin[c])/(2*d^(3/2))
```

Rubi [A] time = 0.068195, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3385, 3354, 3352, 3351}

$$\frac{ax^3}{3}+\frac{\sqrt{\frac{\pi}{2}}\,b\cos(c)\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{2d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,b\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{bx\cos\left(c+dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 - (b*x*Cos[c + d*x^2])/(2*d) + (b*Sqrt[Pi/2]*Cos[c]*FresnelC[Sqrt
[d]*Sqrt[2/Pi]*x])/(2*d^(3/2)) - (b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x]*Sin[c])/(2*d^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^2\left(a+b\sin\left(c+dx^2\right)\right)dx=\int\left(ax^2+bx^2\sin\left(c+dx^2\right)\right)dx$$
$$=\frac{ax^3}{3}+b\int x^2\sin\left(c+dx^2\right)dx$$
$$=\frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{b\int\cos\left(c+dx^2\right)dx}{2d}$$
$$=\frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{(b\cos(c))\int\cos\left(dx^2\right)dx}{2d}-\frac{(b\sin(c))\int\sin\left(dx^2\right)dx}{2d}$$
$$=\frac{ax^3}{3}-\frac{bx\cos\left(c+dx^2\right)}{2d}+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{3/2}}-\frac{b\sqrt{\frac{\pi}{2}}S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{2d^{3/2}}$$

Mathematica [A] time = 0.194966, size = 104, normalized size = 1.02

$$\frac{ax^3}{3}+\frac{\sqrt{\frac{\pi}{2}}b\left(\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right)}{2d^{3/2}}+\frac{bx\sin(c)\sin\left(dx^2\right)}{2d}-\frac{bx\cos(c)\cos\left(dx^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 - (b*x*Cos[c]*Cos[d*x^2])/(2*d) + (b*Sqrt[Pi/2]*(Cos[c]*FresnelC[
Sqrt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]))/(2*d^(3/2))
 + (b*x*Sin[c]*Sin[d*x^2])/(2*d)
```

Maple [A] time = 0.009, size = 68, normalized size = 0.7

$$\frac{ax^3}{3}+b\left(-\frac{x\cos\left(dx^2+c\right)}{2d}+\frac{\sqrt{2}\sqrt{\pi}}{4}\left(\cos(c)\,\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)-\sin(c)\,\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)\right)d^{-\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^2+c)),x)
```

```
[Out] 1/3*a*x^3+b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fre
snelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2
))))
```

Maxima [C] time = 1.64559, size = 350, normalized size = 3.43

$$\frac{1}{3}ax^3-\frac{\left(8x|d|\cos\left(dx^2+c\right)-\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,d)\right)+\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,d)\right)-i\sin\left(\frac{1}{4}\pi+\frac{1}{2}\text{arctar}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 - 1/16*(8*x*abs(d)*cos(d*x^2 + c) - sqrt(pi)*(((cos(1/4*pi + 1/2*
arctan2(0, d)) + cos(-1/4*pi + 1/2*arctan2(0, d)) - I*sin(1/4*pi + 1/2*arct
```

```
an2(0, d)) + I*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) - (I*cos(1/4*pi + 1
/2*arctan2(0, d)) + I*cos(-1/4*pi + 1/2*arctan2(0, d)) + sin(1/4*pi + 1/2*a
rctan2(0, d)) - sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(c))*erf(sqrt(I*d)*x)
+ ((cos(1/4*pi + 1/2*arctan2(0, d)) + cos(-1/4*pi + 1/2*arctan2(0, d)) + I*
sin(1/4*pi + 1/2*arctan2(0, d)) - I*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c
) - (-I*cos(1/4*pi + 1/2*arctan2(0, d)) - I*cos(-1/4*pi + 1/2*arctan2(0, d)
) + sin(1/4*pi + 1/2*arctan2(0, d)) - sin(-1/4*pi + 1/2*arctan2(0, d)))*sin
(c))*erf(sqrt(-I*d)*x))*sqrt(abs(d)))*b/(d*abs(d))
```

Fricas [A] time = 2.08909, size = 252, normalized size = 2.47

$$\frac{4\,ad^2x^3+3\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-3\,\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-6\,bdx\cos\left(dx^2+c\right)}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*x^3 + 3*sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*
sqrt(d/pi)) - 3*sqrt(2)*pi*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*s
in(c) - 6*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [B] time = 2.9315, size = 223, normalized size = 2.19

$$\frac{ax^3}{3}-\frac{bd^{\frac{3}{2}}x^5\sqrt{\frac{1}{d}}\cos\left(c\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{b\sqrt{d}x^3\sqrt{\frac{1}{d}}\sin\left(c\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{d^2x^4}{4}\right)}{8\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{2}\sqrt{\pi}bx^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**2+c)),x)
```

```
[Out] a*x**3/3 - b*d**(3/2)*x**5*sqrt(1/d)*cos(c)*gamma(3/4)*gamma(5/4)*hyper((3/
4, 5/4), (3/2, 7/4, 9/4), -d**2*x**4/4)/(8*gamma(7/4)*gamma(9/4)) - b*sqrt(
d)*x**3*sqrt(1/d)*sin(c)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4,
 7/4), -d**2*x**4/4)/(8*gamma(5/4)*gamma(7/4)) + sqrt(2)*sqrt(pi)*b*x**2*sq
rt(1/d)*sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi))/2 + sqrt(2)*sqrt(pi)*b*
x**2*sqrt(1/d)*cos(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi))/2
```

Giac [C] time = 1.15799, size = 196, normalized size = 1.92

$$\frac{1}{3}\,ax^3-\frac{bxe^{\left(i\,dx^2+i\,c\right)}}{4\,d}-\frac{bxe^{\left(-i\,dx^2-i\,c\right)}}{4\,d}-\frac{\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{\left(i\,c\right)}}{8\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}-\frac{\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-}}{8\,d\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 - 1/4*b*x*e^(I*d*x^2 + I*c)/d - 1/4*b*x*e^(-I*d*x^2 - I*c)/d - 1/
8*sqrt(2)*sqrt(pi)*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(
I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/8*sqrt(2)*sqrt(pi)*b*erf(-1/2*s
qrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/(d*(I*d/abs(d) + 1)*sqrt(a
bs(d)))
```

3.9 $\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx$

Optimal. Leaf size=74

$$ax+\frac{\sqrt{\frac{\pi}{2}}b\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}b\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}$$

```
[Out] a*x + (b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sqrt[d] + (b*Sqr
t[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d]
```

Rubi [A] time = 0.042957, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3353, 3352, 3351}

$$ax+\frac{\sqrt{\frac{\pi}{2}}b\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}b\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x^2],x]
```

```
[Out] a*x + (b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sqrt[d] + (b*Sqr
t[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sin\left(c+dx^2\right)\right)\,dx &= ax+b\int \sin\left(c+dx^2\right)\,dx\\
&= ax+(b\cos(c))\int \sin\left(dx^2\right)\,dx+(b\sin(c))\int \cos\left(dx^2\right)\,dx\\
&= ax+\frac{b\sqrt{\frac{\pi}{2}}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{b\sqrt{\frac{\pi}{2}}C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)}{\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.142439, size = 61, normalized size = 0.82

$$ax+\frac{\sqrt{\frac{\pi}{2}}b\left(\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)+\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x^2],x]
```

```
[Out] a*x + (b*Sqrt[Pi/2]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c]))/Sqrt[d]
```

Maple [A] time = 0.003, size = 48, normalized size = 0.7

$$ax+\frac{b\sqrt{2}\sqrt{\pi}}{2}\left(\cos(c)\,\mathrm{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)+\sin(c)\,\mathrm{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(d*x^2+c),x)
```

```
[Out] a*x+1/2*b*2^(1/2)*Pi^(1/2)/d^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1
/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))
```

Maxima [C] time = 1.63441, size = 315, normalized size = 4.26

$$ax-\frac{\sqrt{\pi}\left(\left(\left(-i\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,d)\right)-i\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,d)\right)-\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,d)\right)+\sin\left(-\frac{1}{4}\pi+\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="maxima")
```

```
[Out] a*x - 1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, d)) - I*cos(-1/4*pi +
1/2*arctan2(0, d)) - sin(1/4*pi + 1/2*arctan2(0, d)) + sin(-1/4*pi + 1/2*ar
ctan2(0, d)))*cos(c) - (cos(1/4*pi + 1/2*arctan2(0, d)) + cos(-1/4*pi + 1/2
*arctan2(0, d)) - I*sin(1/4*pi + 1/2*arctan2(0, d)) + I*sin(-1/4*pi + 1/2*a
rctan2(0, d)))*sin(c))*erf(sqrt(I*d)*x) + ((I*cos(1/4*pi + 1/2*arctan2(0, d
)) + I*cos(-1/4*pi + 1/2*arctan2(0, d)) - sin(1/4*pi + 1/2*arctan2(0, d)) +
 sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) - (cos(1/4*pi + 1/2*arctan2(0, d)
) + cos(-1/4*pi + 1/2*arctan2(0, d)) + I*sin(1/4*pi + 1/2*arctan2(0, d)) -
I*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(c))*erf(sqrt(-I*d)*x))*b/sqrt(abs(d
))
```

Fricas [A] time = 1.98925, size = 204, normalized size = 2.76

$$\frac{\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+\sqrt{2}\pi b\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)+2\,adx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) + sqr
t(2)*pi*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) + 2*a*d*x)/d
```

Sympy [A] time = 0.526034, size = 66, normalized size = 0.89

$$ax + \frac{\sqrt{2}\sqrt{\pi}b\left(\sin\left(c\right)C\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right) + \cos\left(c\right)S\left(\frac{\sqrt{2}\sqrt{d}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{d}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x**2+c),x)
```

```
[Out] a*x + sqrt(2)*sqrt(pi)*b*(sin(c)*fresnelc(sqrt(2)*sqrt(d)*x/sqrt(pi)) + cos
(c)*fresnels(sqrt(2)*sqrt(d)*x/sqrt(pi)))*sqrt(1/d)/2
```

Giac [C] time = 1.11975, size = 138, normalized size = 1.86

$$-\frac{1}{4}\left(-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}} + \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{\left(\frac{id}{|d|}+1\right)\sqrt{|d|}}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d))
)*e^(I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))) + I*sqrt(2)*sqrt(pi)*erf(-1/2*sq
rt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((I*d/abs(d) + 1)*sqrt(abs(
d))))*b + a*x
```

3.10 $\int \frac{a+b\sin(c+dx^2)}{x^2}\,dx$

Optimal. Leaf size=88

$$-\frac{a}{x}+\sqrt{2\pi}b\sqrt{d}\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\sqrt{2\pi}b\sqrt{d}\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{b\sin(c+dx^2)}{x}$$

```
[Out] -(a/x) + b*Sqrt[d]*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] - b*Sqr
t[d]*Sqrt[2*Pi]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - (b*Sin[c + d*x^2])/
x
```

Rubi [A] time = 0.0744913, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3387, 3354, 3352, 3351}

$$-\frac{a}{x}+\sqrt{2\pi}b\sqrt{d}\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\sqrt{2\pi}b\sqrt{d}\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{b\sin(c+dx^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^2,x]
```


```
[Out] -(a/x) + b*Sqrt[d]*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] - b*Sqr
t[d]*Sqrt[2*Pi]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - (b*Sin[c + d*x^2])/
x
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{a+b\sin\left(c+dx^2\right)}{x^2}\,dx = \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^2\right)}{x^2}\right)dx$$
$$= -\frac{a}{x}+b\int \frac{\sin\left(c+dx^2\right)}{x^2}\,dx$$
$$= -\frac{a}{x}-\frac{b\sin\left(c+dx^2\right)}{x}+(2bd)\int \cos\left(c+dx^2\right)dx$$
$$= -\frac{a}{x}-\frac{b\sin\left(c+dx^2\right)}{x}+(2bd\cos(c))\int \cos\left(dx^2\right)dx-(2bd\sin(c))\int \sin\left(dx^2\right)dx$$
$$= -\frac{a}{x}+b\sqrt{d}\sqrt{2\pi}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-b\sqrt{d}\sqrt{2\pi}S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)-\frac{b\sin\left(c+dx^2\right)}{x}$$

Mathematica [A] time = 0.182091, size = 91, normalized size = 1.03

$$-\frac{a}{x}+\sqrt{2\pi}b\sqrt{d}\left(\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right)-\frac{b\sin(c)\cos\left(dx^2\right)}{x}-\frac{b\cos(c)\sin\left(dx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*Cos[d*x^2]*Sin[c])/x + b*Sqrt[d]*Sqrt[2*Pi]*(Cos[c]*FresnelC[Sq
rt[d]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c]) - (b*Cos[c]*Si
n[d*x^2])/x
```

Maple [A] time = 0.008, size = 66, normalized size = 0.8

$$-\frac{a}{x}+b\left(-\frac{\sin\left(dx^2+c\right)}{x}+\sqrt{d}\sqrt{2}\sqrt{\pi}\left(\cos(c)\,\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)-\sin(c)\,\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^2,x)
```

```
[Out] -a/x+b*(-1/x*sin(d*x^2+c)+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(x*d^(1/
2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] time = 1.16251, size = 366, normalized size = 4.16

$$-\frac{\sqrt{x^2|d|}\left(\left(\left(i\Gamma\left(-\frac{1}{2},idx^2\right)-i\Gamma\left(-\frac{1}{2},-idx^2\right)\right)\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,d)\right)+\left(i\Gamma\left(-\frac{1}{2},idx^2\right)-i\Gamma\left(-\frac{1}{2},-idx^2\right)\right)\cos\left(-\frac{1}{4}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(x^2*abs(d))*(((I*gamma(-1/2, I*d*x^2) - I*gamma(-1/2, -I*d*x^2))*
cos(1/4*pi + 1/2*arctan2(0, d)) + (I*gamma(-1/2, I*d*x^2) - I*gamma(-1/2, -
```

```
I*d*x^2))*cos(-1/4*pi + 1/2*arctan2(0, d)) - (gamma(-1/2, I*d*x^2) + gamma(
-1/2, -I*d*x^2))*sin(1/4*pi + 1/2*arctan2(0, d)) + (gamma(-1/2, I*d*x^2) +
gamma(-1/2, -I*d*x^2))*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) + ((gamma(-
1/2, I*d*x^2) + gamma(-1/2, -I*d*x^2))*cos(1/4*pi + 1/2*arctan2(0, d)) + (g
amma(-1/2, I*d*x^2) + gamma(-1/2, -I*d*x^2))*cos(-1/4*pi + 1/2*arctan2(0, d
)) + (I*gamma(-1/2, I*d*x^2) - I*gamma(-1/2, -I*d*x^2))*sin(1/4*pi + 1/2*ar
ctan2(0, d)) + (-I*gamma(-1/2, I*d*x^2) + I*gamma(-1/2, -I*d*x^2))*sin(-1/4
*pi + 1/2*arctan2(0, d)))*sin(c))*b/x - a/x
```

Fricas [A] time = 1.93645, size = 221, normalized size = 2.51

$$\frac{\sqrt{2}\pi bx\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right) - \sqrt{2}\pi bx\sqrt{\frac{d}{\pi}}\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right) - b\sin\left(dx^2 + c\right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(2)*pi*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - sqrt(
2)*pi*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) - b*sin(d*x^2
 + c) - a)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sin\left(c + dx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(dx^2 + c\right) + a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)/x^2, x)
```

3.11 $\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{a}{3x^3}-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{b\sin\left(c+dx^2\right)}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}$$

```
[Out] -a/(3*x^3) - (2*b*d*Cos[c + d*x^2])/(3*x) - (2*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*
FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/3 - (2*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d
]*Sqrt[2/Pi]*x]*Sin[c])/3 - (b*Sin[c + d*x^2])/(3*x^3)
```

Rubi [A] time = 0.0907686, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3387, 3388, 3353, 3352, 3351}

$$-\frac{a}{3x^3}-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{b\sin\left(c+dx^2\right)}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])/x^4,x]
```

```
[Out] -a/(3*x^3) - (2*b*d*Cos[c + d*x^2])/(3*x) - (2*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*
FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/3 - (2*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d
]*Sqrt[2/Pi]*x]*Sin[c])/3 - (b*Sin[c + d*x^2])/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b\sin\left(c+dx^2\right)}{x^4}\right)dx\\
&= -\frac{a}{3x^3}+b\int \frac{\sin\left(c+dx^2\right)}{x^4}\,dx\\
&= -\frac{a}{3x^3}-\frac{b\sin\left(c+dx^2\right)}{3x^3}+\frac{1}{3}(2bd)\int \frac{\cos\left(c+dx^2\right)}{x^2}\,dx\\
&= -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{b\sin\left(c+dx^2\right)}{3x^3}-\frac{1}{3}\left(4bd^2\right)\int \sin\left(c+dx^2\right)dx\\
&= -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{b\sin\left(c+dx^2\right)}{3x^3}-\frac{1}{3}\left(4bd^2\cos(c)\right)\int \sin\left(dx^2\right)dx-\frac{1}{3}\left(4bd^2\sin(c\right.\\
&= -\frac{a}{3x^3}-\frac{2bd\cos\left(c+dx^2\right)}{3x}-\frac{2}{3}bd^{3/2}\sqrt{2\pi}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2}{3}bd^{3/2}\sqrt{2\pi}C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\sin(c)
\end{aligned}$$

Mathematica [A] time = 0.215891, size = 119, normalized size = 1.04

$$-\frac{a}{3x^3}-\frac{2}{3}\sqrt{2\pi}bd^{3/2}\left(\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)+\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)\right)-\frac{b\cos\left(dx^2\right)\left(2dx^2\cos(c)+\sin(c)\right)}{3x^3}+\frac{b\sin\left(dx^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*Cos[d*x^2]*(2*d*x^2*Cos[c] + Sin[c]))/(3*x^3) - (2*b*d^(3/2
)*Sqrt[2*Pi]*(Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + FresnelC[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c]))/3 + (b*(-Cos[c] + 2*d*x^2*Sin[c])*Sin[d*x^2])/(3*x^3)
```

Maple [A] time = 0.009, size = 83, normalized size = 0.7

$$-\frac{a}{3\,x^3}+b\left(-\frac{\sin\left(dx^2+c\right)}{3\,x^3}+\frac{2\,d}{3}\left(-\frac{\cos\left(dx^2+c\right)}{x}-\sqrt{d}\sqrt{2}\sqrt{\pi}\left(\cos\left(c\right)\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)+\sin\left(c\right)\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{d}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))/x^4,x)
```

```
[Out] -1/3*a/x^3+b*(-1/3/x^3*sin(d*x^2+c)+2/3*d*(-1/x*cos(d*x^2+c)-d^(1/2)*2^(1/2
)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d
^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Maxima [C] time = 1.17759, size = 369, normalized size = 3.24

$$-\frac{\sqrt{x^2|d|}\left(\left(\left(i\Gamma\left(-\frac{3}{2},i\,dx^2\right)-i\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\cos\left(\frac{3}{4}\pi+\frac{3}{2}\arctan\left(0,d\right)\right)+\left(i\Gamma\left(-\frac{3}{2},i\,dx^2\right)-i\Gamma\left(-\frac{3}{2},-i\,dx^2\right)\right)\cos\left(-\frac{3}{4}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(x^2*abs(d))*(((I*gamma(-3/2, I*d*x^2) - I*gamma(-3/2, -I*d*x^2))*
cos(3/4*pi + 3/2*arctan2(0, d)) + (I*gamma(-3/2, I*d*x^2) - I*gamma(-3/2, -
I*d*x^2))*cos(-3/4*pi + 3/2*arctan2(0, d)) - (gamma(-3/2, I*d*x^2) + gamma(
-3/2, -I*d*x^2))*sin(3/4*pi + 3/2*arctan2(0, d)) + (gamma(-3/2, I*d*x^2) +
gamma(-3/2, -I*d*x^2))*sin(-3/4*pi + 3/2*arctan2(0, d)))*cos(c) + ((gamma(-
3/2, I*d*x^2) + gamma(-3/2, -I*d*x^2))*cos(3/4*pi + 3/2*arctan2(0, d)) + (g
amma(-3/2, I*d*x^2) + gamma(-3/2, -I*d*x^2))*cos(-3/4*pi + 3/2*arctan2(0, d
)) + (I*gamma(-3/2, I*d*x^2) - I*gamma(-3/2, -I*d*x^2))*sin(3/4*pi + 3/2*ar
ctan2(0, d)) + (-I*gamma(-3/2, I*d*x^2) + I*gamma(-3/2, -I*d*x^2))*sin(-3/4
*pi + 3/2*arctan2(0, d)))*sin(c))*b*abs(d)/x - 1/3*a/x^3
```

Fricas [A] time = 2.06092, size = 284, normalized size = 2.49

$$-\frac{2\sqrt{2}\pi bdx^3\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+2\sqrt{2}\pi bdx^3\sqrt{\frac{d}{\pi}}\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+2\,bdx^2\cos\left(dx^2+c\right)+b\sin\left(dx^2+c\right)+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/p
i)) + 2*sqrt(2)*pi*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin
(c) + 2*b*d*x^2*cos(d*x^2 + c) + b*sin(d*x^2 + c) + a)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sin\left(c+dx^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\sin\left(dx^2+c\right)+a}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)/x^4, x)
```

3.12 $\int x^5 \left(a + b\sin\left(c + dx^2\right)\right)^2 dx$

Optimal. Leaf size=163

$$\frac{a^2x^6}{6} + \frac{2abx^2\sin\left(c+dx^2\right)}{d^2} + \frac{2ab\cos\left(c+dx^2\right)}{d^3} - \frac{abx^4\cos\left(c+dx^2\right)}{d} + \frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2} + \frac{b^2\sin\left(c+dx^2\right)\cos\left(c-\right.}{8d^3}$$

```
[Out] -(b^2*x^2)/(8*d^2) + (a^2*x^6)/6 + (b^2*x^6)/12 + (2*a*b*Cos[c + d*x^2])/d^
3 - (a*b*x^4*Cos[c + d*x^2])/d + (2*a*b*x^2*Sin[c + d*x^2])/d^2 + (b^2*Cos[
c + d*x^2]*Sin[c + d*x^2])/(8*d^3) - (b^2*x^4*Cos[c + d*x^2]*Sin[c + d*x^2]
)/(4*d) + (b^2*x^2*Sin[c + d*x^2]^2)/(4*d^2)
```

Rubi [A] time = 0.246997, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3379, 3317, 3296, 2638, 3311, 30, 2635, 8}

$$\frac{a^2x^6}{6} + \frac{2abx^2\sin\left(c+dx^2\right)}{d^2} + \frac{2ab\cos\left(c+dx^2\right)}{d^3} - \frac{abx^4\cos\left(c+dx^2\right)}{d} + \frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2} + \frac{b^2\sin\left(c+dx^2\right)\cos\left(c-\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^2])^2,x]
```


```
[Out] -(b^2*x^2)/(8*d^2) + (a^2*x^6)/6 + (b^2*x^6)/12 + (2*a*b*Cos[c + d*x^2])/d^
3 - (a*b*x^4*Cos[c + d*x^2])/d + (2*a*b*x^2*Sin[c + d*x^2])/d^2 + (b^2*Cos[
c + d*x^2]*Sin[c + d*x^2])/(8*d^3) - (b^2*x^4*Cos[c + d*x^2]*Sin[c + d*x^2]
)/(4*d) + (b^2*x^2*Sin[c + d*x^2]^2)/(4*d^2)
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\sin\left(c+dx^2\right)\right)^2 dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+b\sin(c+dx))^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^2x^2+2abx^2\sin(c+dx)+b^2x^2\sin^2(c+dx)\right)dx,x,x^2\right)\\ &= \frac{a^2x^6}{6}+(ab)\operatorname{Subst}\left(\int x^2\sin(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\operatorname{Subst}\left(\int x^2\sin^2(c+dx)\,dx,x,x^2\right.\\ &= \frac{a^2x^6}{6}-\frac{abx^4\cos\left(c+dx^2\right)}{d}-\frac{b^2x^4\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4d^2}+\frac{1}{4}\\ &= \frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}+\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right.}{8d^3}\\ &= -\frac{b^2x^2}{8d^2}+\frac{a^2x^6}{6}+\frac{b^2x^6}{12}+\frac{2ab\cos\left(c+dx^2\right)}{d^3}-\frac{abx^4\cos\left(c+dx^2\right)}{d}+\frac{2abx^2\sin\left(c+dx^2\right)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.389886, size = 122, normalized size = 0.75

$$\frac{8a^2d^3x^6-48ab\left(d^2x^4-2\right)\cos\left(c+dx^2\right)+96abdx^2\sin\left(c+dx^2\right)-6b^2d^2x^4\sin\left(2\left(c+dx^2\right)\right)+3b^2\sin\left(2\left(c+dx^2\right)\right)-6b^2}{48d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (8*a^2*d^3*x^6 + 4*b^2*d^3*x^6 - 48*a*b*(-2 + d^2*x^4)*Cos[c + d*x^2] - 6*b
^2*d*x^2*Cos[2*(c + d*x^2)] + 96*a*b*d*x^2*Sin[c + d*x^2] + 3*b^2*Sin[2*(c
+ d*x^2)] - 6*b^2*d^2*x^4*Sin[2*(c + d*x^2)])/(48*d^3)
```

Maple [A] time = 0.016, size = 140, normalized size = 0.9

$$\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{b^2}{2}\left(\frac{x^4\sin\left(2\,dx^2+2\,c\right)}{4\,d}-\frac{1}{d}\left(-\frac{x^2\cos\left(2\,dx^2+2\,c\right)}{4\,d}+\frac{\sin\left(2\,dx^2+2\,c\right)}{8\,d^2}\right)\right)+2\,ab\left(-1/2\,\frac{x^4\cos\left(dx^2+c\right)}{d}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] 1/6*a^2*x^6+1/12*b^2*x^6-1/2*b^2*(1/4/d*x^4*sin(2*d*x^2+2*c)-1/d*(-1/4/d*x^
2*cos(2*d*x^2+2*c)+1/8/d^2*sin(2*d*x^2+2*c)))+2*a*b*(-1/2/d*x^4*cos(d*x^2+c
)+2/d*(1/2/d*x^2*sin(d*x^2+c)+1/2/d^2*cos(d*x^2+c)))
```

Maxima [A] time = 1.03411, size = 143, normalized size = 0.88

$$\frac{1}{6}a^2x^6 + \frac{\left(2\,dx^2\sin\left(dx^2+c\right) - \left(d^2x^4-2\right)\cos\left(dx^2+c\right)\right)ab}{d^3} + \frac{\left(4\,d^3x^6 - 6\,dx^2\cos\left(2\,dx^2+2\,c\right) - 3\left(2\,d^2x^4-1\right)\sin\left(2\,c\right.\right.}{48\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + (2*d*x^2*sin(d*x^2 + c) - (d^2*x^4 - 2)*cos(d*x^2 + c))*a*b/d
^3 + 1/48*(4*d^3*x^6 - 6*d*x^2*cos(2*d*x^2 + 2*c) - 3*(2*d^2*x^4 - 1)*sin(2
*d*x^2 + 2*c))*b^2/d^3
```

Fricas [A] time = 2.05386, size = 265, normalized size = 1.63

$$\frac{2\left(2\,a^2+b^2\right)d^3x^6 - 6\,b^2dx^2\cos\left(dx^2+c\right)^2 + 3\,b^2dx^2 - 24\left(abd^2x^4 - 2\,ab\right)\cos\left(dx^2+c\right) + 3\left(16\,abdx^2 - \left(2\,b^2d^2x^4 - b^2\right.\right.}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(2*(2*a^2 + b^2)*d^3*x^6 - 6*b^2*d*x^2*cos(d*x^2 + c)^2 + 3*b^2*d*x^2
- 24*(a*b*d^2*x^4 - 2*a*b)*cos(d*x^2 + c) + 3*(16*a*b*d*x^2 - (2*b^2*d^2*x^
4 - b^2)*cos(d*x^2 + c))*sin(d*x^2 + c))/d^3
```

Sympy [A] time = 7.29998, size = 209, normalized size = 1.28

$$\begin{cases} \frac{a^2x^6}{6} - \frac{abx^4\cos\left(c+dx^2\right)}{d} + \frac{2abx^2\sin\left(c+dx^2\right)}{d^2} + \frac{2ab\cos\left(c+dx^2\right)}{d^3} + \frac{b^2x^6\sin^2\left(c+dx^2\right)}{12} + \frac{b^2x^6\cos^2\left(c+dx^2\right)}{12} - \frac{b^2x^4\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d} + \frac{b^2x}{} \\ \frac{x^6\left(a+b\sin\left(c\right)\right)^2}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 - a*b*x**4*cos(c + d*x**2)/d + 2*a*b*x**2*sin(c + d*
x**2)/d**2 + 2*a*b*cos(c + d*x**2)/d**3 + b**2*x**6*sin(c + d*x**2)**2/12 +
 b**2*x**6*cos(c + d*x**2)**2/12 - b**2*x**4*sin(c + d*x**2)*cos(c + d*x**2
)/(4*d) + b**2*x**2*sin(c + d*x**2)**2/(8*d**2) - b**2*x**2*cos(c + d*x**2)
**2/(8*d**2) + b**2*sin(c + d*x**2)*cos(c + d*x**2)/(8*d**3), Ne(d, 0)), (x
**6*(a + b*sin(c))**2/6, True))
```

Giac [A] time = 1.11926, size = 244, normalized size = 1.5

$$\frac{8\,a^2dx^6+48\left(\frac{2\,x^2\sin\left(dx^2+c\right)}{d}-\frac{\left(\left(dx^2+c\right)^2-2\left(dx^2+c\right)c+c^2-2\right)\cos\left(dx^2+c\right)}{d^2}\right)ab-\left(\frac{6\,x^2\cos\left(2\,dx^2+2\,c\right)}{d}+\frac{3\left(2\left(dx^2+c\right)^2-4\left(dx^2+c\right)c+2\,c^2-1\right)\sin\left(2\,dx^2+\cdots\right.}{d^2}\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(8*a^2*d*x^6 + 48*(2*x^2*sin(d*x^2 + c)/d - ((d*x^2 + c)^2 - 2*(d*x^2
+ c)*c + c^2 - 2)*cos(d*x^2 + c)/d^2)*a*b - (6*x^2*cos(2*d*x^2 + 2*c)/d + 3
*(2*(d*x^2 + c)^2 - 4*(d*x^2 + c)*c + 2*c^2 - 1)*sin(2*d*x^2 + 2*c)/d^2 - 4
*((d*x^2 + c)^3 - 3*(d*x^2 + c)^2*c + 3*(d*x^2 + c)*c^2)/d^2)*b^2)/d
```

3.13 $\int x^3 \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=102

$$\frac{a^2x^4}{4}+\frac{ab\sin\left(c+dx^2\right)}{d^2}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{b^2\sin^2\left(c+dx^2\right)}{8d^2}-\frac{b^2x^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}+\frac{b^2x^4}{8}$$

```
[Out] (a^2*x^4)/4 + (b^2*x^4)/8 - (a*b*x^2*Cos[c + d*x^2])/d + (a*b*Sin[c + d*x^2
])/d^2 - (b^2*x^2*Cos[c + d*x^2]*Sin[c + d*x^2])/(4*d) + (b^2*Sin[c + d*x^2
]^2)/(8*d^2)
```

Rubi [A] time = 0.133626, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 3317, 3296, 2637, 3310, 30}

$$\frac{a^2x^4}{4}+\frac{ab\sin\left(c+dx^2\right)}{d^2}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{b^2\sin^2\left(c+dx^2\right)}{8d^2}-\frac{b^2x^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}+\frac{b^2x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4)/8 - (a*b*x^2*Cos[c + d*x^2])/d + (a*b*Sin[c + d*x^2
])/d^2 - (b^2*x^2*Cos[c + d*x^2]*Sin[c + d*x^2])/(4*d) + (b^2*Sin[c + d*x^2
]^2)/(8*d^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\sin\left(c+dx^2\right)\right)^2dx &= \frac{1}{2}\text{Subst}\left(\int x(a+b\sin(c+dx))^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(a^2x+2abx\sin(c+dx)+b^2x\sin^2(c+dx)\right)dx,x,x^2\right)\\ &= \frac{a^2x^4}{4}+(ab)\,\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\,\text{Subst}\left(\int x\sin^2(c+dx)\,dx,x,x^2\right)\\ &= \frac{a^2x^4}{4}-\frac{abx^2\cos\left(c+dx^2\right)}{d}-\frac{b^2x^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}+\frac{b^2\sin^2\left(c+dx^2\right)}{8d^2}+\frac{1}{4}b^2\\ &= \frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{abx^2\cos\left(c+dx^2\right)}{d}+\frac{ab\sin\left(c+dx^2\right)}{d^2}-\frac{b^2x^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}\end{aligned}$$

Mathematica [A] time = 0.220874, size = 92, normalized size = 0.9

$$\frac{4a^2d^2x^4+16ab\sin\left(c+dx^2\right)-16abdx^2\cos\left(c+dx^2\right)-2b^2dx^2\sin\left(2\left(c+dx^2\right)\right)-b^2\cos\left(2\left(c+dx^2\right)\right)+2b^2d^2x^4}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (4*a^2*d^2*x^4 + 2*b^2*d^2*x^4 - 16*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c
 + d*x^2)] + 16*a*b*Sin[c + d*x^2] - 2*b^2*d*x^2*Sin[2*(c + d*x^2)])/(16*d^
2)
```

Maple [A] time = 0.016, size = 93, normalized size = 0.9

$$\frac{a^2x^4}{4}+\frac{b^2x^4}{8}-\frac{b^2}{2}\left(\frac{x^2\sin\left(2\,dx^2+2\,c\right)}{4\,d}+\frac{\cos\left(2\,dx^2+2\,c\right)}{8\,d^2}\right)+2\,ab\left(-1/2\,\frac{x^2\cos\left(dx^2+c\right)}{d}+1/2\,\frac{\sin\left(dx^2+c\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/8*b^2*x^4-1/2*b^2*(1/4/d*x^2*sin(2*d*x^2+2*c)+1/8/d^2*cos(2*d
*x^2+2*c))+2*a*b*(-1/2/d*x^2*cos(d*x^2+c)+1/2/d^2*sin(d*x^2+c))
```

Maxima [A] time = 1.01859, size = 117, normalized size = 1.15

$$\frac{1}{4}a^2x^4-\frac{\left(dx^2\cos\left(dx^2+c\right)-\sin\left(dx^2+c\right)\right)ab}{d^2}+\frac{\left(2\,d^2x^4-2\,dx^2\sin\left(2\,dx^2+2\,c\right)-\cos\left(2\,dx^2+2\,c\right)\right)b^2}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 - (d*x^2*cos(d*x^2 + c) - sin(d*x^2 + c))*a*b/d^2 + 1/16*(2*d^2
*x^4 - 2*d*x^2*sin(2*d*x^2 + 2*c) - cos(2*d*x^2 + 2*c))*b^2/d^2
```

Fricas [A] time = 1.97841, size = 188, normalized size = 1.84

$$\frac{(2a^2+b^2)d^2x^4 - 8abdx^2\cos(dx^2+c) - b^2\cos(dx^2+c)^2 - 2(b^2dx^2\cos(dx^2+c) - 4ab)\sin(dx^2+c)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^2 + b^2)*d^2*x^4 - 8*a*b*d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c
)^2 - 2*(b^2*d*x^2*cos(d*x^2 + c) - 4*a*b)*sin(d*x^2 + c))/d^2
```

Sympy [A] time = 2.25549, size = 136, normalized size = 1.33

$$\begin{cases} \frac{a^2x^4}{4} - \frac{abx^2\cos(c+dx^2)}{d} + \frac{ab\sin(c+dx^2)}{d^2} + \frac{b^2x^4\sin^2(c+dx^2)}{8} + \frac{b^2x^4\cos^2(c+dx^2)}{8} - \frac{b^2x^2\sin(c+dx^2)\cos(c+dx^2)}{4d} + \frac{b^2\sin^2(c+dx^2)}{8d^2} & \text{for } d \\ \frac{x^4(a+b\sin(c))^2}{4} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 - a*b*x**2*cos(c + d*x**2)/d + a*b*sin(c + d*x**2)/d
**2 + b**2*x**4*sin(c + d*x**2)**2/8 + b**2*x**4*cos(c + d*x**2)**2/8 - b**
2*x**2*sin(c + d*x**2)*cos(c + d*x**2)/(4*d) + b**2*sin(c + d*x**2)**2/(8*d
**2), Ne(d, 0)), (x**4*(a + b*sin(c))**2/4, True))
```

Giac [A] time = 1.16025, size = 166, normalized size = 1.63

$$\frac{\frac{4\left((dx^2+c)^2 - 2(dx^2+c)c\right)a^2}{d} - \frac{16\left(dx^2\cos(dx^2+c) - \sin(dx^2+c)\right)ab}{d} - \frac{\left(2dx^2\sin(2dx^2+2c) - 2(dx^2+c)^2 + 4(dx^2+c)c + \cos(2dx^2+2c)\right)b^2}{d}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*((d*x^2 + c)^2 - 2*(d*x^2 + c)*c)*a^2/d - 16*(d*x^2*cos(d*x^2 + c)
- sin(d*x^2 + c))*a*b/d - (2*d*x^2*sin(2*d*x^2 + 2*c) - 2*(d*x^2 + c)^2 + 4
*(d*x^2 + c)*c + cos(2*d*x^2 + 2*c))*b^2/d)/d
```

3.14 $\int x \left(a+b\sin\left(c+dx^2\right)\right)^2 \, dx$

Optimal. Leaf size=58

$$\frac{1}{4}x^2\left(2a^2+b^2\right)-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}$$

```
[Out] ((2*a^2 + b^2)*x^2)/4 - (a*b*Cos[c + d*x^2])/d - (b^2*Cos[c + d*x^2]*Sin[c
+ d*x^2])/(4*d)
```

Rubi [A] time = 0.0485753, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3379, 2644}

$$\frac{1}{4}x^2\left(2a^2+b^2\right)-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^2)/4 - (a*b*Cos[c + d*x^2])/d - (b^2*Cos[c + d*x^2]*Sin[c
+ d*x^2])/(4*d)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+b\sin(c+dx))^2\,dx, x, x^2\right)\\ &= \frac{1}{4}\left(2a^2+b^2\right)x^2-\frac{ab\cos\left(c+dx^2\right)}{d}-\frac{b^2\cos\left(c+dx^2\right)\sin\left(c+dx^2\right)}{4d}\end{aligned}$$

Mathematica [A] time = 0.123084, size = 52, normalized size = 0.9

$$-\frac{-2\left(2a^2+b^2\right)\left(c+dx^2\right)+8ab\cos\left(c+dx^2\right)+b^2\sin\left(2\left(c+dx^2\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] -(-2*(2*a^2 + b^2)*(c + d*x^2) + 8*a*b*Cos[c + d*x^2] + b^2*Sin[2*(c + d*x^
2)])/(8*d)
```

Maple [A] time = 0.013, size = 62, normalized size = 1.1

$$\frac{1}{2d}\left(b^2\left(-\frac{\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)}{2}+\frac{dx^2}{2}+\frac{c}{2}\right)-2\,ab\cos\left(dx^2+c\right)+a^2\left(dx^2+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] 1/2/d*(b^2*(-1/2*cos(d*x^2+c)*sin(d*x^2+c)+1/2*d*x^2+1/2*c)-2*a*b*cos(d*x^2
+c)+a^2*(d*x^2+c))
```

Maxima [A] time = 0.984389, size = 70, normalized size = 1.21

$$\frac{1}{2}\,a^2x^2+\frac{\left(2\,dx^2-\sin\left(2\,dx^2+2\,c\right)\right)b^2}{8\,d}-\frac{ab\cos\left(dx^2+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(2*d*x^2 - sin(2*d*x^2 + 2*c))*b^2/d - a*b*cos(d*x^2 + c)
/d
```

Fricas [A] time = 2.04341, size = 119, normalized size = 2.05

$$\frac{\left(2\,a^2+b^2\right)dx^2-b^2\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)-4\,ab\cos\left(dx^2+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d*x^2 - b^2*cos(d*x^2 + c)*sin(d*x^2 + c) - 4*a*b*cos(d*
x^2 + c))/d
```

Sympy [A] time = 0.607337, size = 95, normalized size = 1.64

$$\begin{cases}\frac{a^2x^2}{2}-\frac{ab\cos\left(c+dx^2\right)}{d}+\frac{b^2x^2\sin^2\left(c+dx^2\right)}{4}+\frac{b^2x^2\cos^2\left(c+dx^2\right)}{4}-\frac{b^2\sin\left(c+dx^2\right)\cos\left(c+dx^2\right)}{4d} & \text{for } d\neq 0\\ \frac{x^2\left(a+b\sin\left(c\right)\right)^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 - a*b*cos(c + d*x**2)/d + b**2*x**2*sin(c + d*x**2)*
*2/4 + b**2*x**2*cos(c + d*x**2)**2/4 - b**2*sin(c + d*x**2)*cos(c + d*x**2
)/(4*d), Ne(d, 0)), (x**2*(a + b*sin(c))**2/2, True))
```

Giac [A] time = 1.11968, size = 77, normalized size = 1.33

$$\frac{4\left(dx^2+c\right)a^2+\left(2\,dx^2+2\,c-\sin\left(2\,dx^2+2\,c\right)\right)b^2-8\,ab\cos\left(dx^2+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x^2 + c)*a^2 + (2*d*x^2 + 2*c - sin(2*d*x^2 + 2*c))*b^2 - 8*a*b*c
os(d*x^2 + c))/d
```

3.15 $\int \frac{(a+b\sin(c+dx^2))^2}{x} dx$

Optimal. Leaf size=74

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\sin(c)\text{CosIntegral}\left(dx^2\right)+ab\cos(c)\text{Si}\left(dx^2\right)-\frac{1}{4}b^2\cos(2c)\text{CosIntegral}\left(2dx^2\right)+\frac{1}{4}b^2\sin(2c$$

```
[Out] -(b^2*Cos[2*c]*CosIntegral[2*d*x^2])/4 + ((2*a^2 + b^2)*Log[x])/2 + a*b*Cos
Integral[d*x^2]*Sin[c] + a*b*Cos[c]*SinIntegral[d*x^2] + (b^2*Sin[2*c]*SinI
ntegral[2*d*x^2])/4
```

Rubi [A] time = 0.105475, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3403, 6, 3378, 3376, 3375, 3377}

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\sin(c)\text{CosIntegral}\left(dx^2\right)+ab\cos(c)\text{Si}\left(dx^2\right)-\frac{1}{4}b^2\cos(2c)\text{CosIntegral}\left(2dx^2\right)+\frac{1}{4}b^2\sin(2c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x,x]
```

```
[Out] -(b^2*Cos[2*c]*CosIntegral[2*d*x^2])/4 + ((2*a^2 + b^2)*Log[x])/2 + a*b*Cos
Integral[d*x^2]*Sin[c] + a*b*Cos[c]*SinIntegral[d*x^2] + (b^2*Sin[2*c]*SinI
ntegral[2*d*x^2])/4
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
```

]

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}\,dx = \int \left(\frac{a^2}{x}+\frac{b^2}{2x}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+\frac{2ab\sin\left(c+dx^2\right)}{x}\right)dx$$
$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+\frac{2ab\sin\left(c+dx^2\right)}{x}\right)dx$$
$$= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x}\,dx$$
$$= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab\cos(c))\int \frac{\sin\left(dx^2\right)}{x}\,dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \frac{\cos\left(2dx^2\right)}{x}\,dx+(2a$$
$$= -\frac{1}{4}b^2\cos(2c)\text{Ci}\left(2dx^2\right)+\frac{1}{2}\left(2a^2+b^2\right)\log(x)+ab\text{Ci}\left(dx^2\right)\sin(c)+ab\cos(c)\text{Si}\left(dx^2\right)+\frac{1}{4}b$$

Mathematica [A] time = 0.1655, size = 71, normalized size = 0.96

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)-\frac{1}{4}b\left(-4a\sin(c)\text{CosIntegral}\left(dx^2\right)-4a\cos(c)\text{Si}\left(dx^2\right)+b\cos(2c)\text{CosIntegral}\left(2dx^2\right)-b\sin(2c)\text{Si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x,x]
```

```
[Out] ((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^2] - 4*a*CosInt
egral[d*x^2]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^2] - b*Sin[2*c]*SinIntegra
l[2*d*x^2]))/4
```

Maple [A] time = 0.033, size = 69, normalized size = 0.9

$$\ln(x)\,a^2+\frac{\ln(x)\,b^2}{2}+\frac{b^2\text{Si}\left(2\,dx^2\right)\sin(2\,c)}{4}-\frac{b^2\text{Ci}\left(2\,dx^2\right)\cos(2\,c)}{4}+ab\cos(c)\,\text{Si}\left(dx^2\right)+ab\text{Ci}\left(dx^2\right)\sin(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x,x)
```

```
[Out] ln(x)*a^2+1/2*ln(x)*b^2+1/4*b^2*Si(2*d*x^2)*sin(2*c)-1/4*b^2*Ci(2*d*x^2)*co
s(2*c)+a*b*cos(c)*Si(d*x^2)+a*b*Ci(d*x^2)*sin(c)
```

Maxima [C] time = 1.21379, size = 146, normalized size = 1.97

$$-\frac{1}{2}\left(\left(i\,\text{Ei}\left(i\,dx^2\right)-i\,\text{Ei}\left(-i\,dx^2\right)\right)\cos(c)-\left(\text{Ei}\left(i\,dx^2\right)+\text{Ei}\left(-i\,dx^2\right)\right)\sin(c)\right)ab-\frac{1}{8}\left(\left(\text{Ei}\left(2i\,dx^2\right)+\text{Ei}\left(-2i\,dx^2\right)\right)\cos(2\,c)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*((I*Ei(I*d*x^2) - I*Ei(-I*d*x^2))*cos(c) - (Ei(I*d*x^2) + Ei(-I*d*x^2)
)*sin(c))*a*b - 1/8*((Ei(2*I*d*x^2) + Ei(-2*I*d*x^2))*cos(2*c) - (-I*Ei(2*I
*d*x^2) + I*Ei(-2*I*d*x^2))*sin(2*c) - 4*log(x))*b^2 + a^2*log(x)
```

Fricas [A] time = 2.04536, size = 321, normalized size = 4.34

$$\frac{1}{4}b^2\sin(2c)\operatorname{Si}\left(2dx^2\right)+ab\cos(c)\operatorname{Si}\left(dx^2\right)-\frac{1}{8}\left(b^2\operatorname{Ci}\left(2dx^2\right)+b^2\operatorname{Ci}\left(-2dx^2\right)\right)\cos(2c)+\frac{1}{2}\left(2a^2+b^2\right)\log(x)+\frac{1}{2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*sin(2*c)*sin_integral(2*d*x^2) + a*b*cos(c)*sin_integral(d*x^2) - 1
/8*(b^2*cos_integral(2*d*x^2) + b^2*cos_integral(-2*d*x^2))*cos(2*c) + 1/2*
(2*a^2 + b^2)*log(x) + 1/2*(a*b*cos_integral(d*x^2) + a*b*cos_integral(-d*x
^2))*sin(c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x, x)
```

Giac [A] time = 1.15248, size = 104, normalized size = 1.41

$$-\frac{1}{4}b^2\cos(2c)\operatorname{Ci}\left(2dx^2\right)+ab\operatorname{Ci}\left(dx^2\right)\sin(c)+ab\cos(c)\operatorname{Si}\left(dx^2\right)-\frac{1}{4}b^2\sin(2c)\operatorname{Si}\left(-2dx^2\right)+\frac{1}{2}a^2\log\left(dx^2\right)+\frac{1}{4}b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] -1/4*b^2*cos(2*c)*cos_integral(2*d*x^2) + a*b*cos_integral(d*x^2)*sin(c) +
a*b*cos(c)*sin_integral(d*x^2) - 1/4*b^2*sin(2*c)*sin_integral(-2*d*x^2) +
1/2*a^2*log(d*x^2) + 1/4*b^2*log(d*x^2)
```

3.16 $\int \frac{(a+b\sin(c+dx^2))^2}{x^3}\,dx$

Optimal. Leaf size=115

$$-\frac{2a^2+b^2}{4x^2}+abd\cos(c)\text{CosIntegral}\left(dx^2\right)-abd\sin(c)\text{Si}\left(dx^2\right)-\frac{ab\sin\left(c+dx^2\right)}{x^2}+\frac{1}{2}b^2d\sin(2c)\text{CosIntegral}\left(2dx^2\right)+$$

```
[Out] -(2*a^2 + b^2)/(4*x^2) + (b^2*Cos[2*(c + d*x^2)])/(4*x^2) + a*b*d*Cos[c]*Co
sIntegral[d*x^2] + (b^2*d*CosIntegral[2*d*x^2]*Sin[2*c])/2 - (a*b*Sin[c + d
*x^2])/x^2 - a*b*d*Sin[c]*SinIntegral[d*x^2] + (b^2*d*Cos[2*c]*SinIntegral[
2*d*x^2])/2
```

Rubi [A] time = 0.221334, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3380, 3297, 3303, 3299, 3302, 3379}

$$-\frac{2a^2+b^2}{4x^2}+abd\cos(c)\text{CosIntegral}\left(dx^2\right)-abd\sin(c)\text{Si}\left(dx^2\right)-\frac{ab\sin\left(c+dx^2\right)}{x^2}+\frac{1}{2}b^2d\sin(2c)\text{CosIntegral}\left(2dx^2\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

```
[Out] -(2*a^2 + b^2)/(4*x^2) + (b^2*Cos[2*(c + d*x^2)])/(4*x^2) + a*b*d*Cos[c]*Co
sIntegral[d*x^2] + (b^2*d*CosIntegral[2*d*x^2]*Sin[2*c])/2 - (a*b*Sin[c + d
*x^2])/x^2 - a*b*d*Sin[c]*SinIntegral[d*x^2] + (b^2*d*Cos[2*c]*SinIntegral[
2*d*x^2])/2
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{b^2}{2x^3}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^3}+\frac{2ab\sin\left(c+dx^2\right)}{x^3}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^3}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^3}+\frac{2ab\sin\left(c+dx^2\right)}{x^3}\right)dx \\
&= -\frac{2a^2+b^2}{4x^2}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^3}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^3}\,dx \\
&= -\frac{2a^2+b^2}{4x^2}+(ab)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^2\right)-\frac{1}{4}b^2\,\text{Subst}\left(\int \frac{\cos(2c+2dx)}{x^2}\,dx,x \right. \\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}-\frac{ab\sin\left(c+dx^2\right)}{x^2}+(abd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, \right. \\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}-\frac{ab\sin\left(c+dx^2\right)}{x^2}+(abd\cos(c))\,\text{Subst}\left(\int \frac{\cos(dx)}{x} \right. \\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{4x^2}+abd\cos(c)\text{Ci}\left(dx^2\right)+\frac{1}{2}b^2d\text{Ci}\left(2dx^2\right)\sin(2c)-\frac{ab\,\text{s}}{}
\end{aligned}
$$

Mathematica [A] time = 0.254218, size = 116, normalized size = 1.01

$$\frac{-2a^2+4abdx^2\cos(c)\text{CosIntegral}\left(dx^2\right)-4abdx^2\sin(c)\text{Si}\left(dx^2\right)-4ab\sin\left(c+dx^2\right)+2b^2dx^2\sin(2c)\text{CosIntegral}\left(2\right.}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^3,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*d*x^2*Cos[c]*CosIntegral[d*x
^2] + 2*b^2*d*x^2*CosIntegral[2*d*x^2]*Sin[2*c] - 4*a*b*Sin[c + d*x^2] - 4*
```

```
a*b*d*x^2*Sin[c]*SinIntegral[d*x^2] + 2*b^2*d*x^2*Cos[2*c]*SinIntegral[2*d*
x^2])/(4*x^2)
```

Maple [C] time = 0.247, size = 203, normalized size = 1.8

$$-\frac{\operatorname{csgn}\left(dx^2\right)\mathrm{e}^{-2\,ic}\pi\,b^2d}{4}+\frac{\operatorname{Si}\left(2\,dx^2\right)\mathrm{e}^{-2\,ic}b^2d}{2}-\frac{i}{4}\operatorname{Ei}\left(1,-2\,idx^2\right)\mathrm{e}^{-2\,ic}b^2d+\frac{i}{4}b^2d\operatorname{Ei}\left(1,-2\,idx^2\right)\mathrm{e}^{2\,ic}-\frac{abd\operatorname{Ei}\left(1,-idx^2\right)\mathrm{e}^{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^3,x)
```

```
[Out] -1/4*csgn(d*x^2)*exp(-2*I*c)*Pi*b^2*d+1/2*Si(2*d*x^2)*exp(-2*I*c)*b^2*d-1/4
*I*Ei(1,-2*I*d*x^2)*exp(-2*I*c)*b^2*d+1/4*I*b^2*d*Ei(1,-2*I*d*x^2)*exp(2*I*
c)-1/2*a*b*d*Ei(1,-I*d*x^2)*exp(I*c)+1/2*I*csgn(d*x^2)*exp(-I*c)*Pi*a*b*d-I
*Si(d*x^2)*exp(-I*c)*a*b*d-1/2*Ei(1,-I*d*x^2)*exp(-I*c)*a*b*d-1/2/x^2*a^2-1
/4*b^2/x^2-a*b*sin(d*x^2+c)/x^2+1/4*b^2*cos(2*d*x^2+2*c)/x^2
```

Maxima [C] time = 1.2421, size = 167, normalized size = 1.45

$$\frac{1}{2}\left(\left(\Gamma\left(-1,i\,dx^2\right)+\Gamma\left(-1,-i\,dx^2\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1,i\,dx^2\right)-i\,\Gamma\left(-1,-i\,dx^2\right)\right)\sin\left(c\right)\right)abd+\frac{\left(\left(\left(i\,\Gamma\left(-1,2i\,dx^2\right)-i\,\Gamma\left(-1,-2i\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((gamma(-1, I*d*x^2) + gamma(-1, -I*d*x^2))*cos(c) - (I*gamma(-1, I*d*x
^2) - I*gamma(-1, -I*d*x^2))*sin(c))*a*b*d + 1/4*(((I*gamma(-1, 2*I*d*x^2)
- I*gamma(-1, -2*I*d*x^2))*cos(2*c) + (gamma(-1, 2*I*d*x^2) + gamma(-1, -2*
I*d*x^2))*sin(2*c))*d*x^2 - 1)*b^2/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 2.06748, size = 425, normalized size = 3.7

$$\frac{2\,b^2dx^2\cos\left(2\,c\right)\operatorname{Si}\left(2\,dx^2\right)-4\,abdx^2\sin\left(c\right)\operatorname{Si}\left(dx^2\right)+2\,b^2\cos\left(dx^2+c\right)^2-4\,ab\sin\left(dx^2+c\right)-2\,a^2-2\,b^2+2\left(abdx^2\operatorname{Ci}\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d*x^2*cos(2*c)*sin_integral(2*d*x^2) - 4*a*b*d*x^2*sin(c)*sin_in
tegral(d*x^2) + 2*b^2*cos(d*x^2 + c)^2 - 4*a*b*sin(d*x^2 + c) - 2*a^2 - 2*b
^2 + 2*(a*b*d*x^2*cos_integral(d*x^2) + a*b*d*x^2*cos_integral(-d*x^2))*cos
(c) + (b^2*d*x^2*cos_integral(2*d*x^2) + b^2*d*x^2*cos_integral(-2*d*x^2))*
sin(2*c))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**3, x)
```

Giac [B] time = 1.12347, size = 305, normalized size = 2.65

$$\frac{4\left(dx^2+c\right)abd^2\cos\left(c\right)\operatorname{Ci}\left(dx^2\right)-4\,abcd^2\cos\left(c\right)\operatorname{Ci}\left(dx^2\right)+2\left(dx^2+c\right)b^2d^2\operatorname{Ci}\left(2\,dx^2\right)\sin\left(2\,c\right)-2\,b^2cd^2\operatorname{Ci}\left(2\,dx^2\right)\sin\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x^2 + c)*a*b*d^2*cos(c)*cos_integral(d*x^2) - 4*a*b*c*d^2*cos(c)*
cos_integral(d*x^2) + 2*(d*x^2 + c)*b^2*d^2*cos_integral(2*d*x^2)*sin(2*c)
- 2*b^2*c*d^2*cos_integral(2*d*x^2)*sin(2*c) - 4*(d*x^2 + c)*a*b*d^2*sin(c)
*sin_integral(d*x^2) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^2) - 2*(d*x^2 +
c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^2) + 2*b^2*c*d^2*cos(2*c)*sin_integ
ral(-2*d*x^2) + b^2*d^2*cos(2*d*x^2 + 2*c) - 4*a*b*d^2*sin(d*x^2 + c) - 2*a
^2*d^2 - b^2*d^2)/(d^2*x^2)
```

3.17 $\int \frac{(a+b\sin(c+dx^2))^2}{x^5}\,dx$

Optimal. Leaf size=169

$$-\frac{2a^2+b^2}{8x^4}-\frac{1}{2}abd^2\sin(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{2}abd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{ab\sin\left(c+dx^2\right)}{2x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{1}{2}b^2d^2\cos$$

```
[Out] -(2*a^2 + b^2)/(8*x^4) - (a*b*d*Cos[c + d*x^2])/(2*x^2) + (b^2*Cos[2*(c + d
*x^2)])/(8*x^4) + (b^2*d^2*Cos[2*c]*CosIntegral[2*d*x^2])/2 - (a*b*d^2*CosI
ntegral[d*x^2]*Sin[c])/2 - (a*b*Sin[c + d*x^2])/(2*x^4) - (b^2*d*Sin[2*(c +
 d*x^2)])/(4*x^2) - (a*b*d^2*Cos[c]*SinIntegral[d*x^2])/2 - (b^2*d^2*Sin[2*
c]*SinIntegral[2*d*x^2])/2
```

Rubi [A] time = 0.290192, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3380, 3297, 3303, 3299, 3302, 3379}

$$-\frac{2a^2+b^2}{8x^4}-\frac{1}{2}abd^2\sin(c)\text{CosIntegral}\left(dx^2\right)-\frac{1}{2}abd^2\cos(c)\text{Si}\left(dx^2\right)-\frac{ab\sin\left(c+dx^2\right)}{2x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{1}{2}b^2d^2\cos$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

```
[Out] -(2*a^2 + b^2)/(8*x^4) - (a*b*d*Cos[c + d*x^2])/(2*x^2) + (b^2*Cos[2*(c + d
*x^2)])/(8*x^4) + (b^2*d^2*Cos[2*c]*CosIntegral[2*d*x^2])/2 - (a*b*d^2*CosI
ntegral[d*x^2]*Sin[c])/2 - (a*b*Sin[c + d*x^2])/(2*x^4) - (b^2*d*Sin[2*(c +
 d*x^2)])/(4*x^2) - (a*b*d^2*Cos[c]*SinIntegral[d*x^2])/2 - (b^2*d^2*Sin[2*
c]*SinIntegral[2*d*x^2])/2
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{b^2}{2x^5}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^5}+\frac{2ab\sin\left(c+dx^2\right)}{x^5}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^5}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^5}+\frac{2ab\sin\left(c+dx^2\right)}{x^5}\right)dx\\
&= -\frac{2a^2+b^2}{8x^4}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^5}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^5}\,dx\\
&= -\frac{2a^2+b^2}{8x^4}+(ab)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^3}\,dx,x,x^2\right)-\frac{1}{4}b^2\,\text{Subst}\left(\int \frac{\cos(2c+2dx)}{x^3}\,dx,x\right.\\
&= -\frac{2a^2+b^2}{8x^4}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}-\frac{ab\sin\left(c+dx^2\right)}{2x^4}+\frac{1}{2}(abd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x^2}\,d\right.\\
&= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}-\frac{ab\sin\left(c+dx^2\right)}{2x^4}-\frac{b^2d\sin\left(2\left(\right.\right.}{4x^2}\\
&= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}-\frac{ab\sin\left(c+dx^2\right)}{2x^4}-\frac{b^2d\sin\left(2\left(\right.\right.}{4x^2}\\
&= -\frac{2a^2+b^2}{8x^4}-\frac{abd\cos\left(c+dx^2\right)}{2x^2}+\frac{b^2\cos\left(2\left(c+dx^2\right)\right)}{8x^4}+\frac{1}{2}b^2d^2\cos(2c)\text{Ci}\left(2dx^2\right)-\frac{1}{2}ab
\end{aligned}$$

Mathematica [A] time = 0.46481, size = 158, normalized size = 0.93

$$-\frac{2a^2+4abd^2x^4\sin(c)\text{CosIntegral}\left(dx^2\right)+4abd^2x^4\cos(c)\text{Si}\left(dx^2\right)+4ab\sin\left(c+dx^2\right)+4abdx^2\cos\left(c+dx^2\right)-4b^2d}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^5,x]
```

```
[Out] -(2*a^2 + b^2 + 4*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] - 4*b^2
*d^2*x^4*Cos[2*c]*CosIntegral[2*d*x^2] + 4*a*b*d^2*x^4*CosIntegral[d*x^2]*S
in[c] + 4*a*b*Sin[c + d*x^2] + 2*b^2*d*x^2*Sin[2*(c + d*x^2)] + 4*a*b*d^2*x
^4*Cos[c]*SinIntegral[d*x^2] + 4*b^2*d^2*x^4*Sin[2*c]*SinIntegral[2*d*x^2])
/(8*x^4)
```

Maple [C] time = 0.227, size = 262, normalized size = 1.6

$$-\frac{\frac{i}{4}ab\mathrm{e}^{-i\left(dx^2+c\right)}}{x^4}-\frac{bad\mathrm{e}^{-i\left(dx^2+c\right)}}{4\,x^2}+\frac{i}{4}ab\mathrm{e}^{-ic}d^2\mathrm{Ei}\left(1,idx^2\right)-\frac{a^2}{4\,x^4}-\frac{b^2}{8\,x^4}+\frac{b^2\mathrm{e}^{-2\,i\left(dx^2+c\right)}}{16\,x^4}-\frac{\frac{i}{8}b^2d\mathrm{e}^{-2\,i\left(dx^2+c\right)}}{x^2}-\frac{b^2\mathrm{e}^{-2\,ic}d^2\mathrm{Ei}\left(1,\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^5,x)
```

```
[Out] -1/4*I*a*b/x^4*exp(-I*(d*x^2+c))-1/4*a*b*d/x^2*exp(-I*(d*x^2+c))+1/4*I*a*b*
exp(-I*c)*d^2*Ei(1,I*d*x^2)-1/4/x^4*a^2-1/8*b^2/x^4+1/16*b^2/x^4*exp(-2*I*(
d*x^2+c))-1/8*I*b^2*d/x^2*exp(-2*I*(d*x^2+c))-1/4*b^2*exp(-2*I*c)*d^2*Ei(1,
2*I*d*x^2)+1/16*b^2/x^4*exp(2*I*(d*x^2+c))+1/8*I*b^2*d/x^2*exp(2*I*(d*x^2+c
))-1/4*b^2*exp(2*I*c)*d^2*Ei(1,-2*I*d*x^2)+1/4*I*a*b/x^4*exp(I*(d*x^2+c))-1
/4*a*b*d/x^2*exp(I*(d*x^2+c))-1/4*I*a*b*exp(I*c)*d^2*Ei(1,-I*d*x^2)
```

Maxima [C] time = 1.22135, size = 174, normalized size = 1.03

$$\frac{1}{2}\left(\left(i\,\Gamma\left(-2,i\,dx^2\right)-i\,\Gamma\left(-2,-i\,dx^2\right)\right)\cos\left(c\right)+\left(\Gamma\left(-2,i\,dx^2\right)+\Gamma\left(-2,-i\,dx^2\right)\right)\sin\left(c\right)\right)abd^2-\frac{\left(\left(4\left(\Gamma\left(-2,2i\,dx^2\right)+\Gamma\left(-2,-\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((I*gamma(-2, I*d*x^2) - I*gamma(-2, -I*d*x^2))*cos(c) + (gamma(-2, I*d
*x^2) + gamma(-2, -I*d*x^2))*sin(c))*a*b*d^2 - 1/8*((4*(gamma(-2, 2*I*d*x^2
) + gamma(-2, -2*I*d*x^2))*cos(2*c) - (4*I*gamma(-2, 2*I*d*x^2) - 4*I*gamma
(-2, -2*I*d*x^2))*sin(2*c))*d^2*x^4 + 1)*b^2/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 2.16567, size = 510, normalized size = 3.02

$$-\frac{2\,b^2d^2x^4\sin\left(2\,c\right)\mathrm{Si}\left(2\,dx^2\right)+2\,abd^2x^4\cos\left(c\right)\mathrm{Si}\left(dx^2\right)+2\,abdx^2\cos\left(dx^2+c\right)-b^2\cos\left(dx^2+c\right)^2+a^2+b^2-\left(b^2d^2x^4\,\mathrm{C}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*d^2*x^4*sin(2*c)*sin_integral(2*d*x^2) + 2*a*b*d^2*x^4*cos(c)*s
in_integral(d*x^2) + 2*a*b*d*x^2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c)^2 + a^
2 + b^2 - (b^2*d^2*x^4*cos_integral(2*d*x^2) + b^2*d^2*x^4*cos_integral(-2*
d*x^2))*cos(2*c) + 2*(b^2*d*x^2*cos(d*x^2 + c) + a*b)*sin(d*x^2 + c) + (a*b
*d^2*x^4*cos_integral(d*x^2) + a*b*d^2*x^4*cos_integral(-d*x^2))*sin(c))/x^
4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**5, x)
```

Giac [B] time = 1.4964, size = 605, normalized size = 3.58

$$\frac{4\left(dx^2+c\right)^2 b^2 d^3 \cos\left(2\,c\right) \operatorname{Ci}\left(2\,dx^2\right) - 8\left(dx^2+c\right) b^2 c d^3 \cos\left(2\,c\right) \operatorname{Ci}\left(2\,dx^2\right) + 4\,b^2 c^2 d^3 \cos\left(2\,c\right) \operatorname{Ci}\left(2\,dx^2\right) - 4\left(dx^2+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x^2 + c)^2*b^2*d^3*cos(2*c)*cos_integral(2*d*x^2) - 8*(d*x^2 + c)
*b^2*c*d^3*cos(2*c)*cos_integral(2*d*x^2) + 4*b^2*c^2*d^3*cos(2*c)*cos_inte
gral(2*d*x^2) - 4*(d*x^2 + c)^2*a*b*d^3*cos_integral(d*x^2)*sin(c) + 8*(d*x
^2 + c)*a*b*c*d^3*cos_integral(d*x^2)*sin(c) - 4*a*b*c^2*d^3*cos_integral(d
*x^2)*sin(c) - 4*(d*x^2 + c)^2*a*b*d^3*cos(c)*sin_integral(d*x^2) + 8*(d*x^
2 + c)*a*b*c*d^3*cos(c)*sin_integral(d*x^2) - 4*a*b*c^2*d^3*cos(c)*sin_inte
gral(d*x^2) + 4*(d*x^2 + c)^2*b^2*d^3*sin(2*c)*sin_integral(-2*d*x^2) - 8*(
d*x^2 + c)*b^2*c*d^3*sin(2*c)*sin_integral(-2*d*x^2) + 4*b^2*c^2*d^3*sin(2*
c)*sin_integral(-2*d*x^2) - 4*(d*x^2 + c)*a*b*d^3*cos(d*x^2 + c) + 4*a*b*c*
d^3*cos(d*x^2 + c) - 2*(d*x^2 + c)*b^2*d^3*sin(2*d*x^2 + 2*c) + 2*b^2*c*d^3
*sin(2*d*x^2 + 2*c) + b^2*d^3*cos(2*d*x^2 + 2*c) - 4*a*b*d^3*sin(d*x^2 + c)
 - 2*a^2*d^3 - b^2*d^3)/(((d*x^2 + c)^2 - 2*(d*x^2 + c)*c + c^2)*d)
```

3.18 $\int x^4 \left(a+b\sin\left(c+dx^2\right)\right)^2 dx$

Optimal. Leaf size=247

$$\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{3\sqrt{\frac{\pi}{2}}ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{2d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2}-\frac{abx^3\cos\left(c+\right.}{d}$$

```
[Out] ((2*a^2 + b^2)*x^5)/10 - (a*b*x^3*Cos[c + d*x^2])/d - (3*b^2*x*Cos[2*c + 2*
d*x^2])/(32*d^2) + (3*b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
])/(64*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/
(2*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/(2*d
^(5/2)) - (3*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(64*d^
(5/2)) + (3*a*b*x*Sin[c + d*x^2])/(2*d^2) - (b^2*x^3*Sin[2*c + 2*d*x^2])/(8
*d)
```

Rubi [A] time = 0.241754, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3386, 3385, 3354, 3352, 3351, 3353}

$$\frac{1}{10}x^5\left(2a^2+b^2\right)-\frac{3\sqrt{\frac{\pi}{2}}ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{2d^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{2d^{5/2}}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2}-\frac{abx^3\cos\left(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^2])^2,x]
```


```
[Out] ((2*a^2 + b^2)*x^5)/10 - (a*b*x^3*Cos[c + d*x^2])/d - (3*b^2*x*Cos[2*c + 2*
d*x^2])/(32*d^2) + (3*b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
])/(64*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/
(2*d^(5/2)) - (3*a*b*Sqrt[Pi/2]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/(2*d
^(5/2)) - (3*b^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(64*d^
(5/2)) + (3*a*b*x*Sin[c + d*x^2])/(2*d^2) - (b^2*x^3*Sin[2*c + 2*d*x^2])/(8
*d)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
```

```
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx &= \int\left(a^2x^4+\frac{b^2x^4}{2}-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+2abx^4\sin\left(c+dx^2\right)\right)dx \\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x^4-\frac{1}{2}b^2x^4\cos\left(2c+2dx^2\right)+2abx^4\sin\left(c+dx^2\right)\right)dx \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5+(2ab)\int x^4\sin\left(c+dx^2\right)dx-\frac{1}{2}b^2\int x^4\cos\left(2c+2dx^2\right)dx \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{b^2x^3\sin\left(2c+2dx^2\right)}{8d}+\frac{(3ab)\int x^2\cos\left(c+a\right.}{d} \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2} \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3abx\sin\left(c+dx^2\right)}{2d^2} \\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{abx^3\cos\left(c+dx^2\right)}{d}-\frac{3b^2x\cos\left(2c+2dx^2\right)}{32d^2}+\frac{3b^2\sqrt{\pi}\cos(2c)C\left(\right.}{64d^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.572672, size = 234, normalized size = 0.95

$$64a^2d^{5/2}x^5-320abd^{3/2}x^3\cos\left(c+dx^2\right)-240\sqrt{2\pi}ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-240\sqrt{2\pi}ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+48$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (64*a^2*d^(5/2)*x^5 + 32*b^2*d^(5/2)*x^5 - 320*a*b*d^(3/2)*x^3*Cos[c + d*x^
2] - 30*b^2*Sqrt[d]*x*Cos[2*(c + d*x^2)] + 15*b^2*Sqrt[Pi]*Cos[2*c]*Fresnel
C[(2*Sqrt[d]*x)/Sqrt[Pi]] - 240*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt
```

```
[2/Pi]*x] - 240*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] - 15*b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 480*a*b*Sqrt[d]*x*S
in[c + d*x^2] - 40*b^2*d^(3/2)*x^3*Sin[2*(c + d*x^2)])/(320*d^(5/2))
```

Maple [A] time = 0.016, size = 189, normalized size = 0.8

$$\frac{x^5a^2}{5}+\frac{x^5b^2}{10}-\frac{b^2}{2}\left(\frac{x^3\sin\left(2\,dx^2+2\,c\right)}{4\,d}-\frac{3}{4\,d}\left(-\frac{x\cos\left(2\,dx^2+2\,c\right)}{4\,d}+\frac{\sqrt{\pi}}{8}\left(\cos\left(2\,c\right)\mathrm{FresnelC}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)-\sin\left(2\,c\right)\mathrm{Fresne}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] 1/5*x^5*a^2+1/10*x^5*b^2-1/2*b^2*(1/4/d*x^3*sin(2*d*x^2+2*c)-3/4/d*(-1/4/d*
x*cos(2*d*x^2+2*c)+1/8/d^(3/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(
1/2))-sin(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))))+2*a*b*(-1/2/d*x^3*cos(d*x^
2+c)+3/2/d*(1/2/d*x*sin(d*x^2+c)-1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(c)*Fresn
elS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))
)))
```

Maxima [C] time = 1.85333, size = 807, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - 1/16*(16*d*x^3*abs(d)*cos(d*x^2 + c) - 24*x*abs(d)*sin(d*x^2
+ c) - sqrt(pi)*(((-3*I*cos(1/4*pi + 1/2*arctan2(0, d)) - 3*I*cos(-1/4*pi +
 1/2*arctan2(0, d)) - 3*sin(1/4*pi + 1/2*arctan2(0, d)) + 3*sin(-1/4*pi + 1
/2*arctan2(0, d)))*cos(c) - (3*cos(1/4*pi + 1/2*arctan2(0, d)) + 3*cos(-1/4
*pi + 1/2*arctan2(0, d)) - 3*I*sin(1/4*pi + 1/2*arctan2(0, d)) + 3*I*sin(-1
/4*pi + 1/2*arctan2(0, d)))*sin(c))*erf(sqrt(I*d)*x) + ((3*I*cos(1/4*pi + 1
/2*arctan2(0, d)) + 3*I*cos(-1/4*pi + 1/2*arctan2(0, d)) - 3*sin(1/4*pi + 1
/2*arctan2(0, d)) + 3*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) - (3*cos(1/4
*pi + 1/2*arctan2(0, d)) + 3*cos(-1/4*pi + 1/2*arctan2(0, d)) + 3*I*sin(1/4
*pi + 1/2*arctan2(0, d)) - 3*I*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(c))*er
f(sqrt(-I*d)*x))*sqrt(abs(d)))*a*b/(d^2*abs(d)) + 1/2560*(256*d^2*x^5*abs(d
) - 320*d*x^3*abs(d)*sin(2*d*x^2 + 2*c) - 240*x*abs(d)*cos(2*d*x^2 + 2*c) +
 sqrt(2)*sqrt(pi)*(((15*cos(1/4*pi + 1/2*arctan2(0, d)) + 15*cos(-1/4*pi +
1/2*arctan2(0, d)) - 15*I*sin(1/4*pi + 1/2*arctan2(0, d)) + 15*I*sin(-1/4*p
i + 1/2*arctan2(0, d)))*cos(2*c) - (15*I*cos(1/4*pi + 1/2*arctan2(0, d)) +
15*I*cos(-1/4*pi + 1/2*arctan2(0, d)) + 15*sin(1/4*pi + 1/2*arctan2(0, d))
- 15*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(2*c))*erf(sqrt(2*I*d)*x) + ((15*
cos(1/4*pi + 1/2*arctan2(0, d)) + 15*cos(-1/4*pi + 1/2*arctan2(0, d)) + 15*
I*sin(1/4*pi + 1/2*arctan2(0, d)) - 15*I*sin(-1/4*pi + 1/2*arctan2(0, d)))*
cos(2*c) - (-15*I*cos(1/4*pi + 1/2*arctan2(0, d)) - 15*I*cos(-1/4*pi + 1/2*
arctan2(0, d)) + 15*sin(1/4*pi + 1/2*arctan2(0, d)) - 15*sin(-1/4*pi + 1/2*
arctan2(0, d)))*sin(2*c))*erf(sqrt(-2*I*d)*x))*sqrt(abs(d)))*b^2/(d^2*abs(d
))
```

Fricas [A] time = 2.25266, size = 595, normalized size = 2.41

$$32\left(2a^2+b^2\right)d^3x^5-320\,abd^2x^3\cos\left(dx^2+c\right)-60\,b^2dx\cos\left(dx^2+c\right)^2-240\,\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-240\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/320*(32*(2*a^2 + b^2)*d^3*x^5 - 320*a*b*d^2*x^3*cos(d*x^2 + c) - 60*b^2*d
*x*cos(d*x^2 + c)^2 - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt
(2)*x*sqrt(d/pi)) - 240*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqr
t(d/pi))*sin(c) + 15*pi*b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi))
 - 15*pi*b^2*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 30*b^2*d*x -
 80*(b^2*d^2*x^3*cos(d*x^2 + c) - 6*a*b*d*x)*sin(d*x^2 + c))/d^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**2))**2, x)
```

Giac [C] time = 1.42665, size = 444, normalized size = 1.8

$$\frac{1}{5}\,a^2x^5+\frac{1}{10}\,b^2x^5-\frac{3i\,\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(i\,c)}}{8\,d^2\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}+\frac{3i\,\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}\right)e^{(-i\,c)}}{8\,d^2\left(\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}-\frac{3\,\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/5*a^2*x^5 + 1/10*b^2*x^5 - 3/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*
(-I*d/abs(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d^2*(-I*d/abs(d) + 1)*sqrt(abs(d))
) + 3/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs
(d)))*e^(-I*c)/(d^2*(I*d/abs(d) + 1)*sqrt(abs(d))) - 3/128*sqrt(pi)*b^2*erf
(-sqrt(d)*x*(-I*d/abs(d) + 1))*e^(2*I*c)/(d^(5/2)*(-I*d/abs(d) + 1)) - 3/12
8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(I*d/abs(d) + 1))*e^(-2*I*c)/(d^(5/2)*(I*d/ab
s(d) + 1)) - 1/64*(-4*I*b^2*d*x^3 + 3*b^2*x)*e^(2*I*d*x^2 + 2*I*c)/d^2 + 1/
4*I*(2*I*a*b*d*x^3 - 3*a*b*x)*e^(I*d*x^2 + I*c)/d^2 + 1/4*I*(2*I*a*b*d*x^3
+ 3*a*b*x)*e^(-I*d*x^2 - I*c)/d^2 - 1/64*(4*I*b^2*d*x^3 + 3*b^2*x)*e^(-2*I*
d*x^2 - 2*I*c)/d^2
```

3.19 $\int x^2 \left(a + b\sin\left(c + dx^2\right)\right)^2 dx$

Optimal. Leaf size=198

$$\frac{1}{6}x^3\left(2a^2+b^2\right)+\frac{\sqrt{\frac{\pi}{2}}ab\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}ab\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{\sqrt{\pi}b^2\sin(2c)\text{Fres}}{16d^{3/2}}$$

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (a*b*x*Cos[c + d*x^2])/d + (a*b*Sqrt[Pi/2]*Cos[c]*F
resnelC[Sqrt[d]*Sqrt[2/Pi]*x])/d^(3/2) + (b^2*Sqrt[Pi]*Cos[2*c]*FresnelS[(2
*Sqrt[d]*x)/Sqrt[Pi]])/(16*d^(3/2)) - (a*b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c])/d^(3/2) + (b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*
Sin[2*c])/(16*d^(3/2)) - (b^2*x*Sin[2*c + 2*d*x^2])/(8*d)
```

Rubi [A] time = 0.15663, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3386, 3353, 3352, 3351, 3385, 3354}

$$\frac{1}{6}x^3\left(2a^2+b^2\right)+\frac{\sqrt{\frac{\pi}{2}}ab\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{d^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}ab\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{\sqrt{\pi}b^2\sin(2c)\text{Fres}}{16d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^2])^2,x]
```


```
[Out] ((2*a^2 + b^2)*x^3)/6 - (a*b*x*Cos[c + d*x^2])/d + (a*b*Sqrt[Pi/2]*Cos[c]*F
resnelC[Sqrt[d]*Sqrt[2/Pi]*x])/d^(3/2) + (b^2*Sqrt[Pi]*Cos[2*c]*FresnelS[(2
*Sqrt[d]*x)/Sqrt[Pi]])/(16*d^(3/2)) - (a*b*Sqrt[Pi/2]*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c])/d^(3/2) + (b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*
Sin[2*c])/(16*d^(3/2)) - (b^2*x*Sin[2*c + 2*d*x^2])/(8*d)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\sin\left(c+dx^2\right)\right)^2 dx &= \int\left(a^2x^2+\frac{b^2x^2}{2}-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+2abx^2\sin\left(c+dx^2\right)\right)dx \\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x^2-\frac{1}{2}b^2x^2\cos\left(2c+2dx^2\right)+2abx^2\sin\left(c+dx^2\right)\right)dx \\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3+(2ab)\int x^2\sin\left(c+dx^2\right)dx-\frac{1}{2}b^2\int x^2\cos\left(2c+2dx^2\right)dx \\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}+\frac{(ab)\int\cos\left(c+dx^2\right)dx}{d} \\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}-\frac{b^2x\sin\left(2c+2dx^2\right)}{8d}+\frac{(ab\cos(c))\int\cos\left(dx^2\right)}{d} \\
&= \frac{1}{6}\left(2a^2+b^2\right)x^3-\frac{abx\cos\left(c+dx^2\right)}{d}+\frac{ab\sqrt{\frac{\pi}{2}}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{d^{3/2}}+\frac{b^2\sqrt{\pi}\cos(2c)S\left(\right.}{16d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.53196, size = 191, normalized size = 0.96

$$
\frac{16a^2d^{3/2}x^3+24\sqrt{2\pi}ab\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-24\sqrt{2\pi}ab\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-48ab\sqrt{d}x\cos\left(c+dx^2\right)+3\sqrt{\pi}b^2\,\mathrm{s}}{48d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (16*a^2*d^(3/2)*x^3 + 8*b^2*d^(3/2)*x^3 - 48*a*b*Sqrt[d]*x*Cos[c + d*x^2] +
 24*a*b*Sqrt[2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 3*b^2*Sqrt[Pi]*C
os[2*c]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 24*a*b*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + 3*b^2*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]]*S
in[2*c] - 6*b^2*Sqrt[d]*x*Sin[2*(c + d*x^2)])/(48*d^(3/2))
```

Maple [A] time = 0.016, size = 142, normalized size = 0.7

$$\frac{x^3a^2}{3}+\frac{x^3b^2}{6}-\frac{b^2}{2}\left(\frac{x\sin\left(2\,dx^2+2\,c\right)}{4\,d}-\frac{\sqrt{\pi}}{8}\left(\cos\left(2\,c\right)\mathrm{FresnelS}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)+\sin\left(2\,c\right)\mathrm{FresnelC}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)\right)d^{-\frac{3}{2}}\right)+2\,ab\left(-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/6*x^3*b^2-1/2*b^2*(1/4/d*x*sin(2*d*x^2+2*c)-1/8/d^(3/2)*Pi^(1
/2)*(cos(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2))+sin(2*c)*FresnelC(2*x*d^(1/2)/
Pi^(1/2))))+2*a*b*(-1/2/d*x*cos(d*x^2+c)+1/4/d^(3/2)*2^(1/2)*Pi^(1/2)*(cos(
c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/P
i^(1/2))))
```

Maxima [C] time = 1.81792, size = 743, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 - 1/8*(8*x*abs(d)*cos(d*x^2 + c) - sqrt(pi)*(((cos(1/4*pi + 1/2
*arctan2(0, d)) + cos(-1/4*pi + 1/2*arctan2(0, d)) - I*sin(1/4*pi + 1/2*arc
tan2(0, d)) + I*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) - (I*cos(1/4*pi +
1/2*arctan2(0, d)) + I*cos(-1/4*pi + 1/2*arctan2(0, d)) + sin(1/4*pi + 1/2*
arctan2(0, d)) - sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(c))*erf(sqrt(I*d)*x)
 + ((cos(1/4*pi + 1/2*arctan2(0, d)) + cos(-1/4*pi + 1/2*arctan2(0, d)) + I
*sin(1/4*pi + 1/2*arctan2(0, d)) - I*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(
c) - (-I*cos(1/4*pi + 1/2*arctan2(0, d)) - I*cos(-1/4*pi + 1/2*arctan2(0, d
)) + sin(1/4*pi + 1/2*arctan2(0, d)) - sin(-1/4*pi + 1/2*arctan2(0, d)))*si
n(c))*erf(sqrt(-I*d)*x))*sqrt(abs(d)))*a*b/(d*abs(d)) + 1/384*(64*d*x^3*abs
(d) - 48*x*abs(d)*sin(2*d*x^2 + 2*c) - sqrt(2)*sqrt(pi)*(((-3*I*cos(1/4*pi
+ 1/2*arctan2(0, d)) - 3*I*cos(-1/4*pi + 1/2*arctan2(0, d)) - 3*sin(1/4*pi
+ 1/2*arctan2(0, d)) + 3*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(2*c) - (3*co
s(1/4*pi + 1/2*arctan2(0, d)) + 3*cos(-1/4*pi + 1/2*arctan2(0, d)) - 3*I*si
n(1/4*pi + 1/2*arctan2(0, d)) + 3*I*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(2
*c))*erf(sqrt(2*I*d)*x) + ((3*I*cos(1/4*pi + 1/2*arctan2(0, d)) + 3*I*cos(-
1/4*pi + 1/2*arctan2(0, d)) - 3*sin(1/4*pi + 1/2*arctan2(0, d)) + 3*sin(-1/
4*pi + 1/2*arctan2(0, d)))*cos(2*c) - (3*cos(1/4*pi + 1/2*arctan2(0, d)) +
3*cos(-1/4*pi + 1/2*arctan2(0, d)) + 3*I*sin(1/4*pi + 1/2*arctan2(0, d)) -
3*I*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(2*c))*erf(sqrt(-2*I*d)*x))*sqrt(a
bs(d)))*b^2/(d*abs(d))
```

Fricas [A] time = 2.30346, size = 498, normalized size = 2.52

$$\frac{8\left(2\,a^2+b^2\right)d^2x^3+24\,\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-12\,b^2dx\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)-24\,\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\cdots}{48\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(8*(2*a^2 + b^2)*d^2*x^3 + 24*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel
_cos(sqrt(2)*x*sqrt(d/pi)) - 12*b^2*d*x*cos(d*x^2 + c)*sin(d*x^2 + c) - 24*
sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + 3*pi*b
^2*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + 3*pi*b^2*sqrt(d/pi)*fr
esnel_cos(2*x*sqrt(d/pi))*sin(2*c) - 48*a*b*d*x*cos(d*x^2 + c))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b\sin\left(c + dx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**2*(a + b*sin(c + d*x**2))**2, x)
```

Giac [C] time = 1.52749, size = 382, normalized size = 1.93

$$\frac{1}{3}\,a^2x^3 + \frac{1}{6}\,b^2x^3 + \frac{i\,b^2xe^{\left(2i\,dx^2+2i\,c\right)}}{16\,d} - \frac{abxe^{\left(i\,dx^2+i\,c\right)}}{2\,d} - \frac{abxe^{\left(-i\,dx^2-i\,c\right)}}{2\,d} - \frac{i\,b^2xe^{\left(-2i\,dx^2-2i\,c\right)}}{16\,d} - \frac{\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{2}x\left(-\frac{i\,d}{|d|} + \right.\right.}{4\,d\left(-\frac{i\,d}{|d|}+1\right)\sqrt{|d|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*x^3 + 1/6*b^2*x^3 + 1/16*I*b^2*x*e^(2*I*d*x^2 + 2*I*c)/d - 1/2*a*b*
x*e^(I*d*x^2 + I*c)/d - 1/2*a*b*x*e^(-I*d*x^2 - I*c)/d - 1/16*I*b^2*x*e^(-2
*I*d*x^2 - 2*I*c)/d - 1/4*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(-I*d/abs
(d) + 1)*sqrt(abs(d)))*e^(I*c)/(d*(-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/4*sqr
t(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c
)/(d*(I*d/abs(d) + 1)*sqrt(abs(d))) + 1/32*I*sqrt(pi)*b^2*erf(-sqrt(d)*x*(-
I*d/abs(d) + 1))*e^(2*I*c)/(d^(3/2)*(-I*d/abs(d) + 1)) - 1/32*I*sqrt(pi)*b^
2*erf(-sqrt(d)*x*(I*d/abs(d) + 1))*e^(-2*I*c)/(d^(3/2)*(I*d/abs(d) + 1))
```

3.20 $\int \left(a+b\sin\left(c+dx^2\right)\right)^2 dx$

Optimal. Leaf size=153

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{\sqrt{2\pi}ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{\sqrt{d}}+\frac{\sqrt{2\pi}ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}-\frac{\sqrt{\pi}b^2\cos(2c)\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{\sqrt{\pi}}{}$$

```
[Out] ((2*a^2 + b^2)*x)/2 - (b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi
]])/(4*Sqrt[d]) + (a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sq
rt[d] + (a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d] + (b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Rubi [A] time = 0.109163, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3357, 3354, 3352, 3351, 3353}

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{\sqrt{2\pi}ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)}{\sqrt{d}}+\frac{\sqrt{2\pi}ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}-\frac{\sqrt{\pi}b^2\cos(2c)\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2,x]
```


```
[Out] ((2*a^2 + b^2)*x)/2 - (b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi
]])/(4*Sqrt[d]) + (a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x])/Sq
rt[d] + (a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/Sqrt[d] + (b
^2*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```


Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin\left(c+dx^2\right)\right)^2\,dx = \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2dx^2\right)+2ab\sin\left(c+dx^2\right)\right)dx$$
$$=\frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+dx^2\right)dx-\frac{1}{2}b^2\int \cos\left(2c+2dx^2\right)dx$$
$$=\frac{1}{2}\left(2a^2+b^2\right)x+(2ab\cos(c))\int \sin\left(dx^2\right)dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \cos\left(2dx^2\right)dx+(2ab\,\text{s}$$
$$=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{b^2\sqrt{\pi}\cos(2c)C\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)}{4\sqrt{d}}+\frac{ab\sqrt{2\pi}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{d}}+\frac{ab\sqrt{2\pi}C\left(\sqrt{d}\right.}{\sqrt{d}}$$

Mathematica [A] time = 0.323231, size = 147, normalized size = 0.96

$$\frac{4a^2\sqrt{d}x+4\sqrt{2\pi}ab\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)+4\sqrt{2\pi}ab\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{\pi}b^2\cos(2c)\text{FresnelC}\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)+\sqrt{\pi}b^2}{4\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (4*a^2*Sqrt[d]*x + 2*b^2*Sqrt[d]*x - b^2*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt
[d]*x)/Sqrt[Pi]] + 4*a*b*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] +
 4*a*b*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[Pi]*Fres
nelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/(4*Sqrt[d])
```

Maple [A] time = 0.015, size = 99, normalized size = 0.7

$$a^2x+\frac{b^2x}{2}-\frac{b^2\sqrt{\pi}}{4}\left(\cos(2c)\,\text{FresnelC}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)-\sin(2c)\,\text{FresnelS}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)\right)\frac{1}{\sqrt{d}}+ab\sqrt{2}\sqrt{\pi}\left(\cos(c)\,\text{FresnelS}\left(\frac{x\sqrt{}}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2,x)
```

```
[Out] a^2*x+1/2*b^2*x-1/4*b^2*Pi^(1/2)/d^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^
(1/2))-sin(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))+a*b*2^(1/2)*Pi^(1/2)/d^(1/2
)*(cos(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^
(1/2)/Pi^(1/2)))
```

Maxima [C] time = 1.78175, size = 660, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/4*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, d)) - I*cos(-1/4*pi
+ 1/2*arctan2(0, d)) - sin(1/4*pi + 1/2*arctan2(0, d)) + sin(-1/4*pi + 1/2*
arctan2(0, d)))*cos(c) - (cos(1/4*pi + 1/2*arctan2(0, d)) + cos(-1/4*pi + 1
/2*arctan2(0, d)) - I*sin(1/4*pi + 1/2*arctan2(0, d)) + I*sin(-1/4*pi + 1/2
*arctan2(0, d)))*sin(c))*erf(sqrt(I*d)*x) + ((I*cos(1/4*pi + 1/2*arctan2(0,
 d)) + I*cos(-1/4*pi + 1/2*arctan2(0, d)) - sin(1/4*pi + 1/2*arctan2(0, d))
 + sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) - (cos(1/4*pi + 1/2*arctan2(0,
d)) + cos(-1/4*pi + 1/2*arctan2(0, d)) + I*sin(1/4*pi + 1/2*arctan2(0, d))
- I*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(c))*erf(sqrt(-I*d)*x))*a*b/sqrt(a
bs(d)) - 1/32*(sqrt(2)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, d)) + cos(-1
/4*pi + 1/2*arctan2(0, d)) - I*sin(1/4*pi + 1/2*arctan2(0, d)) + I*sin(-1/4
*pi + 1/2*arctan2(0, d)))*cos(2*c) - (I*cos(1/4*pi + 1/2*arctan2(0, d)) + I
*cos(-1/4*pi + 1/2*arctan2(0, d)) + sin(1/4*pi + 1/2*arctan2(0, d)) - sin(-
1/4*pi + 1/2*arctan2(0, d)))*sin(2*c))*erf(sqrt(2*I*d)*x) + ((cos(1/4*pi +
1/2*arctan2(0, d)) + cos(-1/4*pi + 1/2*arctan2(0, d)) + I*sin(1/4*pi + 1/2*
arctan2(0, d)) - I*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(2*c) - (-I*cos(1/4
*pi + 1/2*arctan2(0, d)) - I*cos(-1/4*pi + 1/2*arctan2(0, d)) + sin(1/4*pi
+ 1/2*arctan2(0, d)) - sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(2*c))*erf(sqrt
(-2*I*d)*x))*sqrt(abs(d)) - 16*x*abs(d))*b^2/abs(d)
```

Fricas [A] time = 2.02911, size = 385, normalized size = 2.52

$$\frac{4\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\cos(c)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\pi ab\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin(c)-\pi b^2\sqrt{\frac{d}{\pi}}\cos(2c)\,\mathrm{C}\left(2x\sqrt{\frac{d}{\pi}}\right)+\pi b^2\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(2x\sqrt{\frac{d}{\pi}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*pi*a*b*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d/pi)) +
 4*sqrt(2)*pi*a*b*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))*sin(c) - pi*
b^2*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) + pi*b^2*sqrt(d/pi)*fre
snel_sin(2*x*sqrt(d/pi))*sin(2*c) + 2*(2*a^2 + b^2)*d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(c + dx^2\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2, x)
```

Giac [C] time = 1.4476, size = 263, normalized size = 1.72

$$\frac{i\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(ic)}}{2\left(-\frac{id}{|d|}+1\right)\sqrt{|d|}}-\frac{i\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{id}{|d|}+1\right)\sqrt{|d|}\right)e^{(-ic)}}{2\left(\frac{id}{|d|}+1\right)\sqrt{|d|}}+\frac{\sqrt{\pi}b^2\,\mathrm{erf}\left(-\sqrt{d}x\left(-\frac{id}{|d|}+1\right)\right)e^{(2}}{8\sqrt{d}\left(-\frac{id}{|d|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*sqrt(2)*x*(-I*d/abs(d) + 1)*sqrt(abs(d)
))*e^(I*c)/((-I*d/abs(d) + 1)*sqrt(abs(d))) - 1/2*I*sqrt(2)*sqrt(pi)*a*b*er
f(-1/2*sqrt(2)*x*(I*d/abs(d) + 1)*sqrt(abs(d)))*e^(-I*c)/((I*d/abs(d) + 1)*
sqrt(abs(d))) + 1/8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(-I*d/abs(d) + 1))*e^(2*I*c
)/(sqrt(d)*(-I*d/abs(d) + 1)) + 1/8*sqrt(pi)*b^2*erf(-sqrt(d)*x*(I*d/abs(d)
 + 1))*e^(-2*I*c)/(sqrt(d)*(I*d/abs(d) + 1)) + 1/2*(2*a^2 + b^2)*x
```

3.21 $\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=187

$$-\frac{2a^2+b^2}{2x}+2\sqrt{2\pi}ab\sqrt{d}\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-2\sqrt{2\pi}ab\sqrt{d}\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2ab\sin\left(c+dx^2\right)}{x}+\sqrt{\pi}b^2\sqrt{d}\,\text{sin}$$

```
[Out] -(2*a^2 + b^2)/(2*x) + (b^2*Cos[2*c + 2*d*x^2])/(2*x) + 2*a*b*Sqrt[d]*Sqrt[
2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + b^2*Sqrt[d]*Sqrt[Pi]*Cos[2*c]
*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 2*a*b*Sqrt[d]*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[d]*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[
Pi]]*Sin[2*c] - (2*a*b*Sin[c + d*x^2])/x
```

Rubi [A] time = 0.16243, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3388, 3353, 3352, 3351, 3387, 3354}

$$-\frac{2a^2+b^2}{2x}+2\sqrt{2\pi}ab\sqrt{d}\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-2\sqrt{2\pi}ab\sqrt{d}\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2ab\sin\left(c+dx^2\right)}{x}+\sqrt{\pi}b^2\sqrt{d}\,\text{sin}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

```
[Out] -(2*a^2 + b^2)/(2*x) + (b^2*Cos[2*c + 2*d*x^2])/(2*x) + 2*a*b*Sqrt[d]*Sqrt[
2*Pi]*Cos[c]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + b^2*Sqrt[d]*Sqrt[Pi]*Cos[2*c]
*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]] - 2*a*b*Sqrt[d]*Sqrt[2*Pi]*FresnelS[Sqrt[
d]*Sqrt[2/Pi]*x]*Sin[c] + b^2*Sqrt[d]*Sqrt[Pi]*FresnelC[(2*Sqrt[d]*x)/Sqrt[
Pi]]*Sin[2*c] - (2*a*b*Sin[c + d*x^2])/x
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx &= \int \left(\frac{a^2}{x^2}+\frac{b^2}{2x^2}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^2}+\frac{2ab\sin\left(c+dx^2\right)}{x^2}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^2}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^2}+\frac{2ab\sin\left(c+dx^2\right)}{x^2}\right)dx\\
&= -\frac{2a^2+b^2}{2x}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^2}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^2}\,dx\\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}-\frac{2ab\sin\left(c+dx^2\right)}{x}+(4abd)\int \cos\left(c+dx^2\right)dx+\left(2l\right.\\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}-\frac{2ab\sin\left(c+dx^2\right)}{x}+(4abd\cos(c))\int \cos\left(dx^2\right)dx+\\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^2\right)}{2x}+2ab\sqrt{d}\sqrt{2\pi}\cos(c)C\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+b^2\sqrt{d}\sqrt{\pi}\cos(2c)S
\end{aligned}$$

Mathematica [A] time = 0.514054, size = 184, normalized size = 0.98

$$\frac{-2a^2+4\sqrt{2\pi}ab\sqrt{d}x\cos(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-4\sqrt{2\pi}ab\sqrt{d}x\sin(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-4ab\sin\left(c+dx^2\right)+2\sqrt{\pi}b^2\sqrt{d}x\,\text{si}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^2)] + 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*Cos[c]*
FresnelC[Sqrt[d]*Sqrt[2/Pi]*x] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*Cos[2*c]*FresnelS
[(2*Sqrt[d]*x)/Sqrt[Pi]] - 4*a*b*Sqrt[d]*Sqrt[2*Pi]*x*FresnelS[Sqrt[d]*Sqrt
[2/Pi]*x]*Sin[c] + 2*b^2*Sqrt[d]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]
]*Sin[2*c] - 4*a*b*Sin[c + d*x^2])/(2*x)
```

Maple [A] time = 0.014, size = 137, normalized size = 0.7

$$-\frac{1}{x}\left(a^2+\frac{b^2}{2}\right)-\frac{b^2}{2}\left(-\frac{\cos\left(2\,dx^2+2\,c\right)}{x}-2\,\sqrt{d}\sqrt{\pi}\left(\cos\left(2\,c\right)\mathrm{FresnelS}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)+\sin\left(2\,c\right)\mathrm{FresnelC}\left(2\,\frac{x\sqrt{d}}{\sqrt{\pi}}\right)\right)\right)+2\,ab\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^2,x)
```

```
[Out] -(a^2+1/2*b^2)/x-1/2*b^2*(-1/x*cos(2*d*x^2+2*c)-2*d^(1/2)*Pi^(1/2)*(cos(2*c
)*FresnelS(2*x*d^(1/2)/Pi^(1/2))+sin(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))))+
2*a*b*(-1/x*sin(d*x^2+c)+d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelC(x*d^(1/2
)*2^(1/2)/Pi^(1/2))-sin(c)*FresnelS(x*d^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] time = 1.25833, size = 743, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^2*abs(d))*(((I*gamma(-1/2, I*d*x^2) - I*gamma(-1/2, -I*d*x^2))*
cos(1/4*pi + 1/2*arctan2(0, d)) + (I*gamma(-1/2, I*d*x^2) - I*gamma(-1/2, -
I*d*x^2))*cos(-1/4*pi + 1/2*arctan2(0, d)) - (gamma(-1/2, I*d*x^2) + gamma(
-1/2, -I*d*x^2))*sin(1/4*pi + 1/2*arctan2(0, d)) + (gamma(-1/2, I*d*x^2) +
gamma(-1/2, -I*d*x^2))*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(c) + ((gamma(-
1/2, I*d*x^2) + gamma(-1/2, -I*d*x^2))*cos(1/4*pi + 1/2*arctan2(0, d)) + (g
amma(-1/2, I*d*x^2) + gamma(-1/2, -I*d*x^2))*cos(-1/4*pi + 1/2*arctan2(0, d
)) + (I*gamma(-1/2, I*d*x^2) - I*gamma(-1/2, -I*d*x^2))*sin(1/4*pi + 1/2*ar
ctan2(0, d)) + (-I*gamma(-1/2, I*d*x^2) + I*gamma(-1/2, -I*d*x^2))*sin(-1/4
*pi + 1/2*arctan2(0, d)))*sin(c))*a*b/x + 1/16*(sqrt(2)*sqrt(x^2*abs(d))*((
(gamma(-1/2, 2*I*d*x^2) + gamma(-1/2, -2*I*d*x^2))*cos(1/4*pi + 1/2*arctan2
(0, d)) + (gamma(-1/2, 2*I*d*x^2) + gamma(-1/2, -2*I*d*x^2))*cos(-1/4*pi +
1/2*arctan2(0, d)) + (I*gamma(-1/2, 2*I*d*x^2) - I*gamma(-1/2, -2*I*d*x^2))
*sin(1/4*pi + 1/2*arctan2(0, d)) + (-I*gamma(-1/2, 2*I*d*x^2) + I*gamma(-1/
2, -2*I*d*x^2))*sin(-1/4*pi + 1/2*arctan2(0, d)))*cos(2*c) + ((-I*gamma(-1/
2, 2*I*d*x^2) + I*gamma(-1/2, -2*I*d*x^2))*cos(1/4*pi + 1/2*arctan2(0, d))
+ (-I*gamma(-1/2, 2*I*d*x^2) + I*gamma(-1/2, -2*I*d*x^2))*cos(-1/4*pi + 1/2
*arctan2(0, d)) + (gamma(-1/2, 2*I*d*x^2) + gamma(-1/2, -2*I*d*x^2))*sin(1/
4*pi + 1/2*arctan2(0, d)) - (gamma(-1/2, 2*I*d*x^2) + gamma(-1/2, -2*I*d*x^
2))*sin(-1/4*pi + 1/2*arctan2(0, d)))*sin(2*c)) - 8)*b^2/x - a^2/x
```

Fricas [A] time = 2.16197, size = 439, normalized size = 2.35

$$\frac{2\,\sqrt{2}\pi abx\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)-2\,\sqrt{2}\pi abx\sqrt{\frac{d}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)+\pi b^2x\sqrt{\frac{d}{\pi}}\cos\left(2\,c\right)\mathrm{S}\left(2\,x\sqrt{\frac{d}{\pi}}\right)+\pi b^2x\sqrt{\frac{d}{\pi}}\,\mathrm{C}\left(2\,x\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*sqrt(2)*pi*a*b*x*sqrt(d/pi)*cos(c)*fresnel_cos(sqrt(2)*x*sqrt(d/pi)) - 2
*sqrt(2)*pi*a*b*x*sqrt(d/pi)*fresnel_sin(sqrt(2)*x*sqrt(d/pi))*sin(c) + pi*
b^2*x*sqrt(d/pi)*cos(2*c)*fresnel_sin(2*x*sqrt(d/pi)) + pi*b^2*x*sqrt(d/pi)
*fresnel_cos(2*x*sqrt(d/pi))*sin(2*c) + b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*
x^2 + c) - a^2 - b^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx^2+c\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^2/x^2, x)
```

3.22 $\int \frac{(a+b\sin(c+dx^2))^2}{x^4}\,dx$

Optimal. Leaf size=239

$$-\frac{2a^2+b^2}{6x^3}-\frac{4}{3}\sqrt{2\pi}abd^{3/2}\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\frac{4}{3}\sqrt{2\pi}abd^{3/2}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2ab\sin(c+dx^2)}{3x^3}-\frac{4abd\cos(}{3}$$

```
[Out] -(2*a^2 + b^2)/(6*x^3) - (4*a*b*d*Cos[c + d*x^2])/(3*x) + (b^2*Cos[2*c + 2*
d*x^2])/(6*x^3) + (4*b^2*d^(3/2)*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/S
qrt[Pi]])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/3
- (4*b^2*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/3 - (2
*a*b*Sin[c + d*x^2])/(3*x^3) - (2*b^2*d*Sin[2*c + 2*d*x^2])/(3*x)
```

Rubi [A] time = 0.19728, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3388, 3387, 3354, 3352, 3351, 3353}

$$-\frac{2a^2+b^2}{6x^3}-\frac{4}{3}\sqrt{2\pi}abd^{3/2}\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)-\frac{4}{3}\sqrt{2\pi}abd^{3/2}\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)-\frac{2ab\sin(c+dx^2)}{3x^3}-\frac{4abd\cos(}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^2])^2/x^4,x]
```


```
[Out] -(2*a^2 + b^2)/(6*x^3) - (4*a*b*d*Cos[c + d*x^2])/(3*x) + (b^2*Cos[2*c + 2*
d*x^2])/(6*x^3) + (4*b^2*d^(3/2)*Sqrt[Pi]*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/S
qrt[Pi]])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*
x])/3 - (4*a*b*d^(3/2)*Sqrt[2*Pi]*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c])/3
- (4*b^2*d^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c])/3 - (2
*a*b*Sin[c + d*x^2])/(3*x^3) - (2*b^2*d*Sin[2*c + 2*d*x^2])/(3*x)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
```

```
LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^2\right)\right)^2}{x^4}\,dx &= \int \left(\frac{a^2}{x^4}+\frac{b^2}{2x^4}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^4}+\frac{2ab\sin\left(c+dx^2\right)}{x^4}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^4}-\frac{b^2\cos\left(2c+2dx^2\right)}{2x^4}+\frac{2ab\sin\left(c+dx^2\right)}{x^4}\right)dx \\
&= -\frac{2a^2+b^2}{6x^3}+(2ab)\int \frac{\sin\left(c+dx^2\right)}{x^4}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^2\right)}{x^4}\,dx \\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}-\frac{2ab\sin\left(c+dx^2\right)}{3x^3}+\frac{1}{3}(4abd)\int \frac{\cos\left(c+dx^2\right)}{x^2}\,dx+\frac{1}{3} \\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos\left(c+dx^2\right)}{3x}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}-\frac{2ab\sin\left(c+dx^2\right)}{3x^3}-\frac{2b^2d\sin\left(\right.}{3} \\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos\left(c+dx^2\right)}{3x}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}-\frac{2ab\sin\left(c+dx^2\right)}{3x^3}-\frac{2b^2d\sin\left(\right.}{3} \\
&= -\frac{2a^2+b^2}{6x^3}-\frac{4abd\cos\left(c+dx^2\right)}{3x}+\frac{b^2\cos\left(2c+2dx^2\right)}{6x^3}+\frac{4}{3}b^2d^{3/2}\sqrt{\pi}\cos(2c)C\left(\frac{2\sqrt{d}x}{\sqrt{\pi}}\right)
\end{aligned}$$

Mathematica [A] time = 0.663367, size = 226, normalized size = 0.95

$$-\frac{2a^2+8\sqrt{2\pi}abd^{3/2}x^3\sin(c)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{d}x\right)+8\sqrt{2\pi}abd^{3/2}x^3\cos(c)S\left(\sqrt{d}\sqrt{\frac{2}{\pi}}x\right)+4ab\sin\left(c+dx^2\right)+8abdx^2\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^2/x^4,x]
```

```
[Out] -(2*a^2 + b^2 + 8*a*b*d*x^2*Cos[c + d*x^2] - b^2*Cos[2*(c + d*x^2)] - 8*b^2
*d^(3/2)*Sqrt[Pi]*x^3*Cos[2*c]*FresnelC[(2*Sqrt[d]*x)/Sqrt[Pi]] + 8*a*b*d^(
```

```
3/2)*Sqrt[2*Pi]*x^3*Cos[c]*FresnelS[Sqrt[d]*Sqrt[2/Pi]*x] + 8*a*b*d^(3/2)*S
qrt[2*Pi]*x^3*FresnelC[Sqrt[d]*Sqrt[2/Pi]*x]*Sin[c] + 8*b^2*d^(3/2)*Sqrt[Pi
]*x^3*FresnelS[(2*Sqrt[d]*x)/Sqrt[Pi]]*Sin[2*c] + 4*a*b*Sin[c + d*x^2] + 4*
b^2*d*x^2*Sin[2*(c + d*x^2)])/(6*x^3)
```

Maple [A] time = 0.014, size = 175, normalized size = 0.7

$$-\frac{1}{3x^3}\left(a^2+\frac{b^2}{2}\right)-\frac{b^2}{2}\left(-\frac{\cos\left(2dx^2+2c\right)}{3x^3}-\frac{4d}{3}\left(-\frac{\sin\left(2dx^2+2c\right)}{x}+2\sqrt{d}\sqrt{\pi}\left(\cos\left(2c\right)\mathrm{FresnelC}\left(2\frac{x\sqrt{d}}{\sqrt{\pi}}\right)-\sin\left(2c\right)\mathrm{F}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^2+c))^2/x^4,x)
```

```
[Out] -1/3*(a^2+1/2*b^2)/x^3-1/2*b^2*(-1/3/x^3*cos(2*d*x^2+2*c)-4/3*d*(-1/x*sin(2
*d*x^2+2*c)+2*d^(1/2)*Pi^(1/2)*(cos(2*c)*FresnelC(2*x*d^(1/2)/Pi^(1/2))-sin
(2*c)*FresnelS(2*x*d^(1/2)/Pi^(1/2)))))+2*a*b*(-1/3/x^3*sin(d*x^2+c)+2/3*d*
(-1/x*cos(d*x^2+c)-d^(1/2)*2^(1/2)*Pi^(1/2)*(cos(c)*FresnelS(x*d^(1/2)*2^(1
/2)/Pi^(1/2))+sin(c)*FresnelC(x*d^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Maxima [C] time = 1.25199, size = 756, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^2*abs(d))*(((I*gamma(-3/2, I*d*x^2) - I*gamma(-3/2, -I*d*x^2))*
cos(3/4*pi + 3/2*arctan2(0, d)) + (I*gamma(-3/2, I*d*x^2) - I*gamma(-3/2, -
I*d*x^2))*cos(-3/4*pi + 3/2*arctan2(0, d)) - (gamma(-3/2, I*d*x^2) + gamma(
-3/2, -I*d*x^2))*sin(3/4*pi + 3/2*arctan2(0, d)) + (gamma(-3/2, I*d*x^2) +
gamma(-3/2, -I*d*x^2))*sin(-3/4*pi + 3/2*arctan2(0, d)))*cos(c) + ((gamma(-
3/2, I*d*x^2) + gamma(-3/2, -I*d*x^2))*cos(3/4*pi + 3/2*arctan2(0, d)) + (g
amma(-3/2, I*d*x^2) + gamma(-3/2, -I*d*x^2))*cos(-3/4*pi + 3/2*arctan2(0, d
)) + (I*gamma(-3/2, I*d*x^2) - I*gamma(-3/2, -I*d*x^2))*sin(3/4*pi + 3/2*ar
ctan2(0, d)) + (-I*gamma(-3/2, I*d*x^2) + I*gamma(-3/2, -I*d*x^2))*sin(-3/4
*pi + 3/2*arctan2(0, d)))*sin(c))*a*b*abs(d)/x + 1/24*(sqrt(2)*sqrt(x^2*abs
(d))*((3*(gamma(-3/2, 2*I*d*x^2) + gamma(-3/2, -2*I*d*x^2))*cos(3/4*pi + 3/
2*arctan2(0, d)) + 3*(gamma(-3/2, 2*I*d*x^2) + gamma(-3/2, -2*I*d*x^2))*cos
(-3/4*pi + 3/2*arctan2(0, d)) + (3*I*gamma(-3/2, 2*I*d*x^2) - 3*I*gamma(-3/
2, -2*I*d*x^2))*sin(3/4*pi + 3/2*arctan2(0, d)) + (-3*I*gamma(-3/2, 2*I*d*x
^2) + 3*I*gamma(-3/2, -2*I*d*x^2))*sin(-3/4*pi + 3/2*arctan2(0, d)))*cos(2*
c) + ((-3*I*gamma(-3/2, 2*I*d*x^2) + 3*I*gamma(-3/2, -2*I*d*x^2))*cos(3/4*p
i + 3/2*arctan2(0, d)) + (-3*I*gamma(-3/2, 2*I*d*x^2) + 3*I*gamma(-3/2, -2*
I*d*x^2))*cos(-3/4*pi + 3/2*arctan2(0, d)) + 3*(gamma(-3/2, 2*I*d*x^2) + ga
mma(-3/2, -2*I*d*x^2))*sin(3/4*pi + 3/2*arctan2(0, d)) - 3*(gamma(-3/2, 2*I
*d*x^2) + gamma(-3/2, -2*I*d*x^2))*sin(-3/4*pi + 3/2*arctan2(0, d)))*sin(2*
c))*x^2*abs(d) - 4)*b^2/x^3 - 1/3*a^2/x^3
```

Fricas [A] time = 2.44602, size = 556, normalized size = 2.33

$$-\frac{4\sqrt{2}\pi abdx^3\sqrt{\frac{d}{\pi}}\cos\left(c\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)+4\sqrt{2}\pi abdx^3\sqrt{\frac{d}{\pi}}\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{d}{\pi}}\right)\sin\left(c\right)-4\pi b^2dx^3\sqrt{\frac{d}{\pi}}\cos\left(2c\right)\mathrm{C}\left(2x\sqrt{\frac{d}{\pi}}\right)+4\pi b^2 d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*cos(c)*fresnel_sin(sqrt(2)*x*sqrt(d
/pi)) + 4*sqrt(2)*pi*a*b*d*x^3*sqrt(d/pi)*fresnel_cos(sqrt(2)*x*sqrt(d/pi))
*sin(c) - 4*pi*b^2*d*x^3*sqrt(d/pi)*cos(2*c)*fresnel_cos(2*x*sqrt(d/pi)) +
4*pi*b^2*d*x^3*sqrt(d/pi)*fresnel_sin(2*x*sqrt(d/pi))*sin(2*c) + 4*a*b*d*x^
2*cos(d*x^2 + c) - b^2*cos(d*x^2 + c)^2 + a^2 + b^2 + 2*(2*b^2*d*x^2*cos(d*
x^2 + c) + a*b)*sin(d*x^2 + c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(c + dx^2\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**2+c))**2/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx^2 + c\right) + a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^2+c))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^2/x^4, x)
```

3.23 $\int x^5 \sin^3\left(a+bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{x^2\sin^3\left(a+bx^2\right)}{9b^2}+\frac{2x^2\sin\left(a+bx^2\right)}{3b^2}-\frac{\cos^3\left(a+bx^2\right)}{27b^3}+\frac{7\cos\left(a+bx^2\right)}{9b^3}-\frac{x^4\cos\left(a+bx^2\right)}{3b}-\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+\right.}{6b}$$

```
[Out] (7*Cos[a + b*x^2])/(9*b^3) - (x^4*Cos[a + b*x^2])/(3*b) - Cos[a + b*x^2]^3/
(27*b^3) + (2*x^2*Sin[a + b*x^2])/(3*b^2) - (x^4*Cos[a + b*x^2]*Sin[a + b*x
^2]^2)/(6*b) + (x^2*Sin[a + b*x^2]^3)/(9*b^2)
```

Rubi [A] time = 0.130316, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3379, 3311, 3296, 2638, 2633}

$$\frac{x^2\sin^3\left(a+bx^2\right)}{9b^2}+\frac{2x^2\sin\left(a+bx^2\right)}{3b^2}-\frac{\cos^3\left(a+bx^2\right)}{27b^3}+\frac{7\cos\left(a+bx^2\right)}{9b^3}-\frac{x^4\cos\left(a+bx^2\right)}{3b}-\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+\right.}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sin[a + b*x^2]^3,x]
```

```
[Out] (7*Cos[a + b*x^2])/(9*b^3) - (x^4*Cos[a + b*x^2])/(3*b) - Cos[a + b*x^2]^3/
(27*b^3) + (2*x^2*Sin[a + b*x^2])/(3*b^2) - (x^4*Cos[a + b*x^2]*Sin[a + b*x
^2]^2)/(6*b) + (x^2*Sin[a + b*x^2]^3)/(9*b^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x^5 \sin^3\left(a+bx^2\right) dx = \frac{1}{2}\text{Subst}\left(\int x^2 \sin^3(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{x^4\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{x^2\sin^3\left(a+bx^2\right)}{9b^2} + \frac{1}{3}\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{x^4\cos\left(a+bx^2\right)}{3b} - \frac{x^4\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{x^2\sin^3\left(a+bx^2\right)}{9b^2} + \frac{\text{Subst}\left(\int\left(1-x^2\right.\right.}{}$$
$$= \frac{\cos\left(a+bx^2\right)}{9b^3} - \frac{x^4\cos\left(a+bx^2\right)}{3b} - \frac{\cos^3\left(a+bx^2\right)}{27b^3} + \frac{2x^2\sin\left(a+bx^2\right)}{3b^2} - \frac{x^4\cos\left(a+bx^2\right)\text{si}}{6b}$$
$$= \frac{7\cos\left(a+bx^2\right)}{9b^3} - \frac{x^4\cos\left(a+bx^2\right)}{3b} - \frac{\cos^3\left(a+bx^2\right)}{27b^3} + \frac{2x^2\sin\left(a+bx^2\right)}{3b^2} - \frac{x^4\cos\left(a+bx^2\right)}{6b}$$

Mathematica [A] time = 0.266078, size = 75, normalized size = 0.64

$$\frac{-81\left(b^2x^4-2\right)\cos\left(a+bx^2\right)+\left(9b^2x^4-2\right)\cos\left(3\left(a+bx^2\right)\right)-6bx^2\left(\sin\left(3\left(a+bx^2\right)\right)-27\sin\left(a+bx^2\right)\right)}{216b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sin[a + b*x^2]^3,x]
```

```
[Out] (-81*(-2 + b^2*x^4)*Cos[a + b*x^2] + (-2 + 9*b^2*x^4)*Cos[3*(a + b*x^2)] -
6*b*x^2*(-27*Sin[a + b*x^2] + Sin[3*(a + b*x^2)]))/(216*b^3)
```

Maple [A] time = 0.013, size = 113, normalized size = 1.

$$-\frac{3x^4\cos\left(bx^2+a\right)}{8b} + \frac{3}{2b}\left(\frac{x^2\sin\left(bx^2+a\right)}{2b} + \frac{\cos\left(bx^2+a\right)}{2b^2}\right) + \frac{x^4\cos\left(3bx^2+3a\right)}{24b} - \frac{1}{6b}\left(\frac{x^2\sin\left(3bx^2+3a\right)}{6b} + \frac{\cos}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*sin(b*x^2+a)^3,x)
```

```
[Out] -3/8*x^4*cos(b*x^2+a)/b+3/2/b*(1/2/b*x^2*sin(b*x^2+a)+1/2/b^2*cos(b*x^2+a))
+1/24/b*x^4*cos(3*b*x^2+3*a)-1/6/b*(1/6/b*x^2*sin(3*b*x^2+3*a)+1/18/b^2*cos
(3*b*x^2+3*a))
```

Maxima [A] time = 0.990429, size = 107, normalized size = 0.91

$$-\frac{6bx^2\sin\left(3bx^2+3a\right)-162bx^2\sin\left(bx^2+a\right)-\left(9b^2x^4-2\right)\cos\left(3bx^2+3a\right)+81\left(b^2x^4-2\right)\cos\left(bx^2+a\right)}{216b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/216*(6*b*x^2*sin(3*b*x^2 + 3*a) - 162*b*x^2*sin(b*x^2 + a) - (9*b^2*x^4
- 2)*cos(3*b*x^2 + 3*a) + 81*(b^2*x^4 - 2)*cos(b*x^2 + a))/b^3
```

Fricas [A] time = 2.23135, size = 182, normalized size = 1.56

$$\frac{\left(9\,b^2x^4-2\right)\cos\left(bx^2+a\right)^3-3\left(9\,b^2x^4-14\right)\cos\left(bx^2+a\right)-6\left(bx^2\cos\left(bx^2+a\right)^2-7\,bx^2\right)\sin\left(bx^2+a\right)}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/54*((9*b^2*x^4 - 2)*cos(b*x^2 + a)^3 - 3*(9*b^2*x^4 - 14)*cos(b*x^2 + a)
- 6*(b*x^2*cos(b*x^2 + a)^2 - 7*b*x^2)*sin(b*x^2 + a))/b^3
```

Sympy [A] time = 11.958, size = 143, normalized size = 1.22

$$\begin{cases} -\frac{x^4\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{x^4\cos^3\left(a+bx^2\right)}{3b}+\frac{7x^2\sin^3\left(a+bx^2\right)}{9b^2}+\frac{2x^2\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{3b^2}+\frac{7\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{9b^3}+\frac{20\cos^3\left(a+bx^2\right)}{27b^3} \\ \frac{x^6\sin^3(a)}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**4*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**4*cos(a + b*
x**2)**3/(3*b) + 7*x**2*sin(a + b*x**2)**3/(9*b**2) + 2*x**2*sin(a + b*x**2
)*cos(a + b*x**2)**2/(3*b**2) + 7*sin(a + b*x**2)**2*cos(a + b*x**2)/(9*b**
3) + 20*cos(a + b*x**2)**3/(27*b**3), Ne(b, 0)), (x**6*sin(a)**3/6, True))
```

Giac [A] time = 1.12512, size = 165, normalized size = 1.41

$$-\frac{\frac{6\,x^2\sin\left(3\,bx^2+3\,a\right)}{b}-\frac{162\,x^2\sin\left(bx^2+a\right)}{b}-\frac{\left(9\left(bx^2+a\right)^2-18\left(bx^2+a\right)a+9\,a^2-2\right)\cos\left(3\,bx^2+3\,a\right)}{b^2}+\frac{81\left(\left(bx^2+a\right)^2-2\left(bx^2+a\right)a+a^2-2\right)\cos\left(bx^2+a\right)}{b^2}}{216\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/216*(6*x^2*sin(3*b*x^2 + 3*a)/b - 162*x^2*sin(b*x^2 + a)/b - (9*(b*x^2 +
 a)^2 - 18*(b*x^2 + a)*a + 9*a^2 - 2)*cos(3*b*x^2 + 3*a)/b^2 + 81*((b*x^2 +
 a)^2 - 2*(b*x^2 + a)*a + a^2 - 2)*cos(b*x^2 + a)/b^2)/b
```

3.24 $\int x^3 \sin^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=79

$$\frac{\sin^3\left(a+bx^2\right)}{18b^2}+\frac{\sin\left(a+bx^2\right)}{3b^2}-\frac{x^2\cos\left(a+bx^2\right)}{3b}-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{6b}$$

```
[Out] -(x^2*Cos[a + b*x^2])/(3*b) + Sin[a + b*x^2]/(3*b^2) - (x^2*Cos[a + b*x^2]*
Sin[a + b*x^2]^2)/(6*b) + Sin[a + b*x^2]^3/(18*b^2)
```

Rubi [A] time = 0.073866, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3379, 3310, 3296, 2637}

$$\frac{\sin^3\left(a+bx^2\right)}{18b^2}+\frac{\sin\left(a+bx^2\right)}{3b^2}-\frac{x^2\cos\left(a+bx^2\right)}{3b}-\frac{x^2\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[a + b*x^2]^3,x]
```

```
[Out] -(x^2*Cos[a + b*x^2])/(3*b) + Sin[a + b*x^2]/(3*b^2) - (x^2*Cos[a + b*x^2]*
Sin[a + b*x^2]^2)/(6*b) + Sin[a + b*x^2]^3/(18*b^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3 \sin^3\left(a+bx^2\right) dx = \frac{1}{2}\text{Subst}\left(\int x\sin^3(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{\sin^3\left(a+bx^2\right)}{18b^2} + \frac{1}{3}\text{Subst}\left(\int x\sin(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{x^2\cos\left(a+bx^2\right)}{3b} - \frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{\sin^3\left(a+bx^2\right)}{18b^2} + \frac{\text{Subst}\left(\int \cos(a+bx)\,d\right.}{3b}$$
$$= -\frac{x^2\cos\left(a+bx^2\right)}{3b} + \frac{\sin\left(a+bx^2\right)}{3b^2} - \frac{x^2\cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)}{6b} + \frac{\sin^3\left(a+bx^2\right)}{18b^2}$$

Mathematica [A] time = 0.153363, size = 58, normalized size = 0.73

$$-\frac{-27\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)+27bx^2\cos\left(a+bx^2\right)-3bx^2\cos\left(3\left(a+bx^2\right)\right)}{72b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[a + b*x^2]^3,x]
```

```
[Out] -(27*b*x^2*Cos[a + b*x^2] - 3*b*x^2*Cos[3*(a + b*x^2)] - 27*Sin[a + b*x^2]
+ Sin[3*(a + b*x^2)])/(72*b^2)
```

Maple [A] time = 0.011, size = 66, normalized size = 0.8

$$-\frac{3x^2\cos\left(bx^2+a\right)}{8b} + \frac{3\sin\left(bx^2+a\right)}{8b^2} + \frac{x^2\cos\left(3bx^2+3a\right)}{24b} - \frac{\sin\left(3bx^2+3a\right)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(b*x^2+a)^3,x)
```

```
[Out] -3/8*x^2*cos(b*x^2+a)/b+3/8*sin(b*x^2+a)/b^2+1/24/b*x^2*cos(3*b*x^2+3*a)-1/
72/b^2*sin(3*b*x^2+3*a)
```

Maxima [A] time = 0.984789, size = 81, normalized size = 1.03

$$\frac{3bx^2\cos\left(3bx^2+3a\right)-27bx^2\cos\left(bx^2+a\right)-\sin\left(3bx^2+3a\right)+27\sin\left(bx^2+a\right)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x^2*cos(3*b*x^2 + 3*a) - 27*b*x^2*cos(b*x^2 + a) - sin(3*b*x^2 +
3*a) + 27*sin(b*x^2 + a))/b^2
```

Fricas [A] time = 2.20764, size = 138, normalized size = 1.75

$$\frac{3bx^2\cos\left(bx^2+a\right)^3-9bx^2\cos\left(bx^2+a\right)-\left(\cos\left(bx^2+a\right)^2-7\right)\sin\left(bx^2+a\right)}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b*x^2*cos(b*x^2 + a)^3 - 9*b*x^2*cos(b*x^2 + a) - (cos(b*x^2 + a)^2
 - 7)*sin(b*x^2 + a))/b^2
```

Sympy [A] time = 4.04069, size = 92, normalized size = 1.16

$$\begin{cases} -\frac{x^2 \sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b} - \frac{x^2\cos^3\left(a+bx^2\right)}{3b} + \frac{7\sin^3\left(a+bx^2\right)}{18b^2} + \frac{\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{3b^2} & \text{for } b \neq 0 \\ \frac{x^4\sin^3(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**2*sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - x**2*cos(a + b*
x**2)**3/(3*b) + 7*sin(a + b*x**2)**3/(18*b**2) + sin(a + b*x**2)*cos(a + b
*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sin(a)**3/4, True))
```

Giac [A] time = 1.12334, size = 81, normalized size = 1.03

$$\frac{3\,bx^2\cos\left(3\,bx^2+3\,a\right)-27\,bx^2\cos\left(bx^2+a\right)-\sin\left(3\,bx^2+3\,a\right)+27\,\sin\left(bx^2+a\right)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x^2*cos(3*b*x^2 + 3*a) - 27*b*x^2*cos(b*x^2 + a) - sin(3*b*x^2 +
3*a) + 27*sin(b*x^2 + a))/b^2
```

3.25 $\int x \sin^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{\cos^3\left(a+bx^2\right)}{6b}-\frac{\cos\left(a+bx^2\right)}{2b}$$

```
[Out] -Cos[a + b*x^2]/(2*b) + Cos[a + b*x^2]^3/(6*b)
```

Rubi [A] time = 0.0307496, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3379, 2633}

$$\frac{\cos^3\left(a+bx^2\right)}{6b}-\frac{\cos\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x^2]^3,x]
```

```
[Out] -Cos[a + b*x^2]/(2*b) + Cos[a + b*x^2]^3/(6*b)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin^3\left(a+bx^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sin^3(a+bx)\,dx,x,x^2\right)\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos\left(a+bx^2\right)\right)}{2b}\\ &= -\frac{\cos\left(a+bx^2\right)}{2b}+\frac{\cos^3\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [A] time = 0.0281061, size = 33, normalized size = 1.

$$\frac{\cos\left(3\left(a+bx^2\right)\right)}{24b}-\frac{3\cos\left(a+bx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x^2]^3,x]
```

```
[Out] (-3*Cos[a + b*x^2])/(8*b) + Cos[3*(a + b*x^2)]/(24*b)
```

Maple [A] time = 0.006, size = 26, normalized size = 0.8

$$-\frac{\left(2+\left(\sin\left(bx^2+a\right)\right)^2\right)\cos\left(bx^2+a\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x^2+a)^3,x)
```

```
[Out] -1/6/b*(2+sin(b*x^2+a)^2)*cos(b*x^2+a)
```

Maxima [A] time = 0.963564, size = 36, normalized size = 1.09

$$\frac{\cos\left(3\,bx^2+3\,a\right)-9\,\cos\left(bx^2+a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(cos(3*b*x^2 + 3*a) - 9*cos(b*x^2 + a))/b
```

Fricas [A] time = 2.16631, size = 61, normalized size = 1.85

$$\frac{\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

Sympy [A] time = 1.03725, size = 46, normalized size = 1.39

$$\begin{cases} -\frac{\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{\cos^3\left(a+bx^2\right)}{3b} & \text{for } b\neq 0 \\ \frac{x^2\sin^3\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x**2)**2*cos(a + b*x**2)/(2*b) - cos(a + b*x**2)**3/(
3*b), Ne(b, 0)), (x**2*sin(a)**3/2, True))
```

Giac [A] time = 1.10891, size = 35, normalized size = 1.06

$$\frac{\cos\left(bx^2+a\right)^3-3\,\cos\left(bx^2+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/6*(cos(b*x^2 + a)^3 - 3*cos(b*x^2 + a))/b
```

3.26 $\int \frac{\sin^3(a+bx^2)}{x} \, dx$

Optimal. Leaf size=55

$$\frac{3}{8}\sin(a)\text{CosIntegral}\left(bx^2\right)-\frac{1}{8}\sin(3a)\text{CosIntegral}\left(3bx^2\right)+\frac{3}{8}\cos(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\cos(3a)\text{Si}\left(3bx^2\right)$$

```
[Out] (3*CosIntegral[b*x^2]*Sin[a])/8 - (CosIntegral[3*b*x^2]*Sin[3*a])/8 + (3*Co
s[a]*SinIntegral[b*x^2])/8 - (Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rubi [A] time = 0.0953006, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3403, 3377, 3376, 3375}

$$\frac{3}{8}\sin(a)\text{CosIntegral}\left(bx^2\right)-\frac{1}{8}\sin(3a)\text{CosIntegral}\left(3bx^2\right)+\frac{3}{8}\cos(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\cos(3a)\text{Si}\left(3bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^2]*Sin[a])/8 - (CosIntegral[3*b*x^2]*Sin[3*a])/8 + (3*Co
s[a]*SinIntegral[b*x^2])/8 - (Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sin^3\left(a+bx^2\right)}{x}\,dx = \int \left(\frac{3\sin\left(a+bx^2\right)}{4x} - \frac{\sin\left(3a+3bx^2\right)}{4x}\right)dx$$
$$= -\left(\frac{1}{4}\int \frac{\sin\left(3a+3bx^2\right)}{x}\,dx\right) + \frac{3}{4}\int \frac{\sin\left(a+bx^2\right)}{x}\,dx$$
$$= \frac{1}{4}(3\cos(a))\int \frac{\sin\left(bx^2\right)}{x}\,dx - \frac{1}{4}\cos(3a)\int \frac{\sin\left(3bx^2\right)}{x}\,dx + \frac{1}{4}(3\sin(a))\int \frac{\cos\left(bx^2\right)}{x}\,dx - \frac{1}{4}\sin(3a$$
$$= \frac{3}{8}\text{Ci}\left(bx^2\right)\sin(a) - \frac{1}{8}\text{Ci}\left(3bx^2\right)\sin(3a) + \frac{3}{8}\cos(a)\text{Si}\left(bx^2\right) - \frac{1}{8}\cos(3a)\text{Si}\left(3bx^2\right)$$

Mathematica [A] time = 0.0738422, size = 51, normalized size = 0.93

$$\frac{1}{8}\left(3\sin(a)\text{CosIntegral}\left(bx^2\right) - \sin(3a)\text{CosIntegral}\left(3bx^2\right) + 3\cos(a)\text{Si}\left(bx^2\right) - \cos(3a)\text{Si}\left(3bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^2]*Sin[a] - CosIntegral[3*b*x^2]*Sin[3*a] + 3*Cos[a]*Sin
Integral[b*x^2] - Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Maple [A] time = 0.027, size = 48, normalized size = 0.9

$$\frac{3\cos(a)\,\text{Si}\left(bx^2\right)}{8} - \frac{\cos(3a)\,\text{Si}\left(3bx^2\right)}{8} + \frac{3\,\text{Ci}\left(bx^2\right)\sin(a)}{8} - \frac{\text{Ci}\left(3bx^2\right)\sin(3a)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3/x,x)
```

```
[Out] 3/8*cos(a)*Si(b*x^2)-1/8*cos(3*a)*Si(3*b*x^2)+3/8*Ci(b*x^2)*sin(a)-1/8*Ci(3
*b*x^2)*sin(3*a)
```

Maxima [C] time = 1.22061, size = 120, normalized size = 2.18

$$\frac{1}{16}\left(i\,\text{Ei}\left(3i\,bx^2\right) - i\,\text{Ei}\left(-3i\,bx^2\right)\right)\cos(3a) + \frac{1}{16}\left(-3i\,\text{Ei}\left(i\,bx^2\right) + 3i\,\text{Ei}\left(-i\,bx^2\right)\right)\cos(a) - \frac{1}{16}\left(\text{Ei}\left(3i\,bx^2\right) + \text{Ei}\left(-3i\,bx^2\right)\right)s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*(I*Ei(3*I*b*x^2) - I*Ei(-3*I*b*x^2))*cos(3*a) + 1/16*(-3*I*Ei(I*b*x^2)
 + 3*I*Ei(-I*b*x^2))*cos(a) - 1/16*(Ei(3*I*b*x^2) + Ei(-3*I*b*x^2))*sin(3*a
) + 3/16*(Ei(I*b*x^2) + Ei(-I*b*x^2))*sin(a)
```

Fricas [A] time = 2.30419, size = 262, normalized size = 4.76

$$-\frac{1}{16}\left(\text{Ci}\left(3bx^2\right) + \text{Ci}\left(-3bx^2\right)\right)\sin(3a) + \frac{3}{16}\left(\text{Ci}\left(bx^2\right) + \text{Ci}\left(-bx^2\right)\right)\sin(a) - \frac{1}{8}\cos(3a)\,\text{Si}\left(3bx^2\right) + \frac{3}{8}\cos(a)\,\text{Si}\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] -1/16*(cos_integral(3*b*x^2) + cos_integral(-3*b*x^2))*sin(3*a) + 3/16*(cos
_integral(b*x^2) + cos_integral(-b*x^2))*sin(a) - 1/8*cos(3*a)*sin_integral
(3*b*x^2) + 3/8*cos(a)*sin_integral(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3/x,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x, x)
```

Giac [A] time = 1.12242, size = 63, normalized size = 1.15

$$-\frac{1}{8}\,\mathrm{Ci}\left(3\,bx^2\right)\sin\left(3\,a\right)+\frac{3}{8}\,\mathrm{Ci}\left(bx^2\right)\sin\left(a\right)+\frac{3}{8}\,\cos\left(a\right)\mathrm{Si}\left(bx^2\right)+\frac{1}{8}\,\cos\left(3\,a\right)\mathrm{Si}\left(-3\,bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] -1/8*cos_integral(3*b*x^2)*sin(3*a) + 3/8*cos_integral(b*x^2)*sin(a) + 3/8*
cos(a)*sin_integral(b*x^2) + 1/8*cos(3*a)*sin_integral(-3*b*x^2)
```

3.27 $\int \frac{\sin^3(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=91

$$\frac{3}{8}b\cos(a)\text{CosIntegral}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\text{CosIntegral}\left(3bx^2\right)-\frac{3}{8}b\sin(a)\text{Si}\left(bx^2\right)+\frac{3}{8}b\sin(3a)\text{Si}\left(3bx^2\right)-\frac{3\sin\left(a+bx\right.}{8x^2}$$

```
[Out] (3*b*Cos[a]*CosIntegral[b*x^2])/8 - (3*b*Cos[3*a]*CosIntegral[3*b*x^2])/8 -
 (3*Sin[a + b*x^2])/(8*x^2) + Sin[3*(a + b*x^2)]/(8*x^2) - (3*b*Sin[a]*SinI
ntegral[b*x^2])/8 + (3*b*Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Rubi [A] time = 0.22005, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3403, 3379, 3297, 3303, 3299, 3302}

$$\frac{3}{8}b\cos(a)\text{CosIntegral}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\text{CosIntegral}\left(3bx^2\right)-\frac{3}{8}b\sin(a)\text{Si}\left(bx^2\right)+\frac{3}{8}b\sin(3a)\text{Si}\left(3bx^2\right)-\frac{3\sin\left(a+bx\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3/x^3,x]
```


```
[Out] (3*b*Cos[a]*CosIntegral[b*x^2])/8 - (3*b*Cos[3*a]*CosIntegral[3*b*x^2])/8 -
 (3*Sin[a + b*x^2])/(8*x^2) + Sin[3*(a + b*x^2)]/(8*x^2) - (3*b*Sin[a]*SinI
ntegral[b*x^2])/8 + (3*b*Sin[3*a]*SinIntegral[3*b*x^2])/8
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx &= \int \left(\frac{3\sin\left(a+bx^2\right)}{4x^3}-\frac{\sin\left(3a+3bx^2\right)}{4x^3}\right)dx\\ &= -\left(\frac{1}{4}\int \frac{\sin\left(3a+3bx^2\right)}{x^3}\,dx\right)+\frac{3}{4}\int \frac{\sin\left(a+bx^2\right)}{x^3}\,dx\\ &= -\left(\frac{1}{8}\operatorname{Subst}\left(\int \frac{\sin(3a+3bx)}{x^2}\,dx,x,x^2\right)\right)+\frac{3}{8}\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,x^2\right)-\frac{1}{8}(3b)\operatorname{Subst}\left(\right.\\ &= -\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{1}{8}(3b\cos(a))\operatorname{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,x^2\right)-\frac{1}{8}(3b\cos(3\\ &= \frac{3}{8}b\cos(a)\operatorname{Ci}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\operatorname{Ci}\left(3bx^2\right)-\frac{3\sin\left(a+bx^2\right)}{8x^2}+\frac{\sin\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{3}{8}b\sin(a)\operatorname{Si}\left(\right.\end{aligned}$$

Mathematica [A] time = 0.1296, size = 90, normalized size = 0.99

$$\frac{3bx^2\cos(a)\operatorname{CosIntegral}\left(bx^2\right)-3bx^2\cos(3a)\operatorname{CosIntegral}\left(3bx^2\right)-3bx^2\sin(a)\operatorname{Si}\left(bx^2\right)+3bx^2\sin(3a)\operatorname{Si}\left(3bx^2\right)-3\,\mathrm{s}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3/x^3,x]
```

```
[Out] (3*b*x^2*Cos[a]*CosIntegral[b*x^2] - 3*b*x^2*Cos[3*a]*CosIntegral[3*b*x^2]
- 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)] - 3*b*x^2*Sin[a]*SinIntegral[b*x^2]
 + 3*b*x^2*Sin[3*a]*SinIntegral[3*b*x^2])/(8*x^2)
```

Maple [C] time = 0.194, size = 162, normalized size = 1.8

$$-\frac{3i}{16}e^{-3ia}\operatorname{csgn}\left(bx^2\right)\pi b+\frac{3i}{8}e^{-3ia}\operatorname{Si}\left(3bx^2\right)b+\frac{3e^{-3ia}\operatorname{Ei}\left(1,-3ibx^2\right)b}{16}+\frac{3e^{3ia}b\operatorname{Ei}\left(1,-3ibx^2\right)}{16}-\frac{3e^{ia}b\operatorname{Ei}\left(1,-ibx^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3/x^3,x)
```

```
[Out] -3/16*I*exp(-3*I*a)*csgn(b*x^2)*Pi*b+3/8*I*exp(-3*I*a)*Si(3*b*x^2)*b+3/16*e
xp(-3*I*a)*Ei(1,-3*I*b*x^2)*b+3/16*exp(3*I*a)*b*Ei(1,-3*I*b*x^2)-3/16*exp(I
*a)*b*Ei(1,-I*b*x^2)+3/16*I*csgn(b*x^2)*exp(-I*a)*Pi*b-3/8*I*exp(-I*a)*Si(b
*x^2)*b-3/16*Ei(1,-I*b*x^2)*exp(-I*a)*b-3/8*sin(b*x^2+a)/x^2+1/8*sin(3*b*x^
```

2+3*a)/x^2

Maxima [C] time = 1.2149, size = 135, normalized size = 1.48

$$-\frac{1}{16}\left(3\left(\Gamma\left(-1,3i\,bx^2\right)+\Gamma\left(-1,-3i\,bx^2\right)\right)\cos\left(3\,a\right)-3\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos\left(a\right)-\left(3i\,\Gamma\left(-1,3i\,bx^2\right)-3i\,\Gamma\left(-1,\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(gamma(-1, 3*I*b*x^2) + gamma(-1, -3*I*b*x^2))*cos(3*a) - 3*(gamma
(-1, I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) - (3*I*gamma(-1, 3*I*b*x^2) - 3
*I*gamma(-1, -3*I*b*x^2))*sin(3*a) - (-3*I*gamma(-1, I*b*x^2) + 3*I*gamma(-
1, -I*b*x^2))*sin(a))*b
```

Fricas [A] time = 2.29476, size = 367, normalized size = 4.03

$$\frac{6\,bx^2\sin\left(3\,a\right)\mathrm{Si}\left(3\,bx^2\right)-6\,bx^2\sin\left(a\right)\mathrm{Si}\left(bx^2\right)-3\left(bx^2\,\mathrm{Ci}\left(3\,bx^2\right)+bx^2\,\mathrm{Ci}\left(-3\,bx^2\right)\right)\cos\left(3\,a\right)+3\left(bx^2\,\mathrm{Ci}\left(bx^2\right)+bx^2\,\mathrm{Ci}\right.}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*b*x^2*sin(3*a)*sin_integral(3*b*x^2) - 6*b*x^2*sin(a)*sin_integral(
b*x^2) - 3*(b*x^2*cos_integral(3*b*x^2) + b*x^2*cos_integral(-3*b*x^2))*cos
(3*a) + 3*(b*x^2*cos_integral(b*x^2) + b*x^2*cos_integral(-b*x^2))*cos(a) +
 8*(cos(b*x^2 + a)^2 - 1)*sin(b*x^2 + a))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x**3, x)
```

Giac [B] time = 1.10732, size = 251, normalized size = 2.76

$$-\frac{3\left(bx^2+a\right)b^2\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^2\right)-3\,ab^2\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^2\right)-3\left(bx^2+a\right)b^2\cos\left(a\right)\mathrm{Ci}\left(bx^2\right)+3\,ab^2\cos\left(a\right)\mathrm{Ci}\left(bx^2\right)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(b*x^2 + a)*b^2*cos(3*a)*cos_integral(3*b*x^2) - 3*a*b^2*cos(3*a)*c
os_integral(3*b*x^2) - 3*(b*x^2 + a)*b^2*cos(a)*cos_integral(b*x^2) + 3*a*b
^2*cos(a)*cos_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(a)*sin_integral(b*x^2
) - 3*a*b^2*sin(a)*sin_integral(b*x^2) + 3*(b*x^2 + a)*b^2*sin(3*a)*sin_int
egral(-3*b*x^2) - 3*a*b^2*sin(3*a)*sin_integral(-3*b*x^2) - b^2*sin(3*b*x^2
 + 3*a) + 3*b^2*sin(b*x^2 + a))/(b^2*x^2)
```

3.28 $\int x^2 \sin^3\left(a+bx^2\right) dx$

Optimal. Leaf size=188

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sin(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right.}{24b^{3/2}}$$

```
[Out] (-3*x*Cos[a + b*x^2])/(8*b) + (x*Cos[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi/2]
*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[Pi/6]*Cos[3*a]*
FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) + (Sqrt[Pi/6]*FresnelS[Sqrt[b]*Sqrt[6/
Pi]*x]*Sin[3*a])/(24*b^(3/2))
```

Rubi [A] time = 0.224809, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3403, 3385, 3354, 3352, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sin(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right.}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x^2]^3,x]
```



```
[Out] (-3*x*Cos[a + b*x^2])/(8*b) + (x*Cos[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi/2]
*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[Pi/6]*Cos[3*a]*
FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) + (Sqrt[Pi/6]*FresnelS[Sqrt[b]*Sqrt[6/
Pi]*x]*Sin[3*a])/(24*b^(3/2))
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^2 \sin^3\left(a+bx^2\right) dx = \int \left(\frac{3}{4}x^2 \sin\left(a+bx^2\right) - \frac{1}{4}x^2 \sin\left(3a+3bx^2\right)\right) dx$$
$$= -\left(\frac{1}{4}\int x^2 \sin\left(3a+3bx^2\right) dx\right) + \frac{3}{4}\int x^2 \sin\left(a+bx^2\right) dx$$
$$= -\frac{3x\cos\left(a+bx^2\right)}{8b} + \frac{x\cos\left(3a+3bx^2\right)}{24b} - \frac{\int \cos\left(3a+3bx^2\right) dx}{24b} + \frac{3\int \cos\left(a+bx^2\right) dx}{8b}$$
$$= -\frac{3x\cos\left(a+bx^2\right)}{8b} + \frac{x\cos\left(3a+3bx^2\right)}{24b} + \frac{(3\cos(a))\int \cos\left(bx^2\right) dx}{8b} - \frac{\cos(3a)\int \cos\left(3bx^2\right)}{24b}$$
$$= -\frac{3x\cos\left(a+bx^2\right)}{8b} + \frac{x\cos\left(3a+3bx^2\right)}{24b} + \frac{3\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}\right.}{24b^{3/2}}$$

Mathematica [A] time = 0.433139, size = 159, normalized size = 0.85

$$\frac{27\sqrt{2\pi}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right) - \sqrt{6\pi}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right) - 27\sqrt{2\pi}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) + \sqrt{6\pi}\sin(3a)S\left(\right.}{144b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x^2]^3,x]
```

```
[Out] (-54*Sqrt[b]*x*Cos[a + b*x^2] + 6*Sqrt[b]*x*Cos[3*(a + b*x^2)] + 27*Sqrt[2*
Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]*FresnelC[Sq
rt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] +
 Sqrt[6*Pi]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(144*b^(3/2))
```

Maple [A] time = 0.013, size = 132, normalized size = 0.7

$$-\frac{3\,x\cos\left(bx^2+a\right)}{8\,b} + \frac{3\sqrt{2}\sqrt{\pi}}{16}\left(\cos\left(a\right)\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{b}\right) - \sin\left(a\right)\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{b}\right)\right)b^{-\frac{3}{2}} + \frac{x\cos\left(3\,bx^2+3\,a\right)}{24\,b} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(b*x^2+a)^3,x)
```

```
[Out] -3/8*x*cos(b*x^2+a)/b+3/16/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x*b^(1
/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*x*c
os(3*b*x^2+3*a)/b-1/144/b^(3/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(
1/2)*b^(1/2)*x))
```

Maxima [C] time = 2.24511, size = 695, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(24*x*abs(b)*cos(3*b*x^2 + 3*a) - 216*x*abs(b)*cos(b*x^2 + a) - sqrt(
3)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(
0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan2(0,
 b)))*cos(3*a) - (I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*a
rctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2
(0, b)))*sin(3*a))*erf(sqrt(3*I*b)*x) + ((cos(1/4*pi + 1/2*arctan2(0, b)) +
 cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arctan2(0, b)) - I*s
in(-1/4*pi + 1/2*arctan2(0, b)))*cos(3*a) - (-I*cos(1/4*pi + 1/2*arctan2(0,
 b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b))
 - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(3*a))*erf(sqrt(-3*I*b)*x))*sqrt(ab
s(b)) + sqrt(pi)*(((27*cos(1/4*pi + 1/2*arctan2(0, b)) + 27*cos(-1/4*pi + 1
/2*arctan2(0, b)) - 27*I*sin(1/4*pi + 1/2*arctan2(0, b)) + 27*I*sin(-1/4*pi
 + 1/2*arctan2(0, b)))*cos(a) + (-27*I*cos(1/4*pi + 1/2*arctan2(0, b)) - 27
*I*cos(-1/4*pi + 1/2*arctan2(0, b)) - 27*sin(1/4*pi + 1/2*arctan2(0, b)) +
27*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf(sqrt(I*b)*x) + ((27*cos(1/
4*pi + 1/2*arctan2(0, b)) + 27*cos(-1/4*pi + 1/2*arctan2(0, b)) + 27*I*sin(
1/4*pi + 1/2*arctan2(0, b)) - 27*I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a)
 + (27*I*cos(1/4*pi + 1/2*arctan2(0, b)) + 27*I*cos(-1/4*pi + 1/2*arctan2(0
, b)) - 27*sin(1/4*pi + 1/2*arctan2(0, b)) + 27*sin(-1/4*pi + 1/2*arctan2(0
, b)))*sin(a))*erf(sqrt(-I*b)*x))*sqrt(abs(b)))/(b*abs(b))
```

Fricas [A] time = 2.46357, size = 447, normalized size = 2.38

$$\frac{24\,bx\cos\left(bx^2+a\right)^3-\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)+27\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/144*(24*b*x*cos(b*x^2 + a)^3 - sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_cos
(sqrt(6)*x*sqrt(b/pi)) + 27*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2
)*x*sqrt(b/pi)) + sqrt(6)*pi*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*s
in(3*a) - 27*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a)
 - 72*b*x*cos(b*x^2 + a))/b^2
```

Sympy [B] time = 6.3889, size = 439, normalized size = 2.34

$$-\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}+\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\cos(3a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{3\sqrt{b}x^3\sqrt{\frac{1}{b}}\sin(a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(b*x**2+a)**3,x)
```

```
[Out] -3*b**(3/2)*x**5*sqrt(1/b)*cos(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (
3/2, 7/4, 9/4), -b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) + 3*b**(3/2)*x**5*
```

```
sqrt(1/b)*cos(3*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4),
 -9*b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) - 3*sqrt(b)*x**3*sqrt(1/b)*sin(
a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4/4)/(
32*gamma(5/4)*gamma(7/4)) + sqrt(b)*x**3*sqrt(1/b)*sin(3*a)*gamma(1/4)*gamm
a(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*gamma(5/4)*ga
mma(7/4)) + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fresnelc(sqrt(2)*sqrt(
b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*sin(3*a)*fresnelc(sqrt(6
)*sqrt(b)*x/sqrt(pi))/24 + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(a)*fresnel
s(sqrt(2)*sqrt(b)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*cos(3*a)*
fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi))/24
```

Giac [C] time = 1.14382, size = 350, normalized size = 1.86

$$\frac{xe^{\left(3i\,bx^2+3i\,a\right)}}{48\,b} - \frac{3\,xe^{\left(i\,bx^2+i\,a\right)}}{16\,b} - \frac{3\,xe^{\left(-i\,bx^2-i\,a\right)}}{16\,b} + \frac{xe^{\left(-3i\,bx^2-3i\,a\right)}}{48\,b} + \frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{6}\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(3i\,a)}}{288\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)} - \frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\cdots\right.}{32\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/48*x*e^(3*I*b*x^2 + 3*I*a)/b - 3/16*x*e^(I*b*x^2 + I*a)/b - 3/16*x*e^(-I*
b*x^2 - I*a)/b + 1/48*x*e^(-3*I*b*x^2 - 3*I*a)/b + 1/288*sqrt(6)*sqrt(pi)*e
rf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3*I*a)/(b^(3/2)*(-I*b/abs(b
) + 1)) - 3/32*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(a
bs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) - 3/32*sqrt(2)*sqrt(pi)*
erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) +
 1)*sqrt(abs(b))) + 1/288*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(I*b/
abs(b) + 1))*e^(-3*I*a)/(b^(3/2)*(I*b/abs(b) + 1))
```

3.29 $\int \sin^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=153

$$\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}$$

```
[Out] (3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) - (Sqrt[Pi
/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) + (3*Sqrt[Pi/2]*Fr
esnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelC[Sqr
t[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```

Rubi [A] time = 0.0818357, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3357, 3353, 3352, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3,x]
```


```
[Out] (3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) - (Sqrt[Pi
/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) + (3*Sqrt[Pi/2]*Fr
esnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelC[Sqr
t[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```


Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin^3\left(a+bx^2\right)dx = \int \left(\frac{3}{4}\sin\left(a+bx^2\right)-\frac{1}{4}\sin\left(3a+3bx^2\right)\right)dx$$
$$= -\left(\frac{1}{4}\int \sin\left(3a+3bx^2\right)dx\right)+\frac{3}{4}\int \sin\left(a+bx^2\right)dx$$
$$= \frac{1}{4}(3\cos(a))\int \sin\left(bx^2\right)dx-\frac{1}{4}\cos(3a)\int \sin\left(3bx^2\right)dx+\frac{1}{4}(3\sin(a))\int \cos\left(bx^2\right)dx-\frac{1}{4}\sin$$
$$= \frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}+\frac{3\sqrt{\frac{\pi}{2}}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}C\left(\sqrt{b}\sqrt{\right.}\right.}{4\sqrt{}}$$

Mathematica [A] time = 0.229715, size = 117, normalized size = 0.76

$$\frac{\sqrt{\frac{\pi}{6}}\left(3\sqrt{3}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)-\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)+3\sqrt{3}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/6]*(3*Sqrt[3]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Cos[3*a]*Fre
snelS[Sqrt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[3]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[
a] - FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a]))/(4*Sqrt[b])
```

Maple [A] time = 0.01, size = 99, normalized size = 0.7

$$\frac{3\sqrt{2}\sqrt{\pi}}{8}\left(\cos(a)\,\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{b}\right)+\sin(a)\,\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{b}\right)\right)\frac{1}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{\pi}\sqrt{3}}{24}\left(\cos(3a)\,\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{3}x}{\sqrt{\pi}}\sqrt{\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3,x)
```

```
[Out] 3/8*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))+s
in(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))-1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/b
^(1/2)*(cos(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x))
```

Maxima [C] time = 1.79261, size = 651, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(sqrt(3)*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*cos(-1/4*p
i + 1/2*arctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + sin(-1/4*pi + 1/
2*arctan2(0, b)))*cos(3*a) - (cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi
 + 1/2*arctan2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi +
 1/2*arctan2(0, b)))*sin(3*a))*erf(sqrt(3*I*b)*x) + ((I*cos(1/4*pi + 1/2*ar
```

```
ctan2(0, b)) + I*cos(-1/4*pi + 1/2*arctan2(0, b)) - sin(1/4*pi + 1/2*arctan
2(0, b)) + sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(3*a) - (cos(1/4*pi + 1/2*a
rctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arcta
n2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(3*a))*erf(sqrt(-3*I*b)*
x))*sqrt(abs(b)) + sqrt(pi)*(((9*I*cos(1/4*pi + 1/2*arctan2(0, b)) + 9*I*co
s(-1/4*pi + 1/2*arctan2(0, b)) + 9*sin(1/4*pi + 1/2*arctan2(0, b)) - 9*sin(
-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + (9*cos(1/4*pi + 1/2*arctan2(0, b)) +
 9*cos(-1/4*pi + 1/2*arctan2(0, b)) - 9*I*sin(1/4*pi + 1/2*arctan2(0, b)) +
 9*I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf(sqrt(I*b)*x) + ((-9*I*co
s(1/4*pi + 1/2*arctan2(0, b)) - 9*I*cos(-1/4*pi + 1/2*arctan2(0, b)) + 9*si
n(1/4*pi + 1/2*arctan2(0, b)) - 9*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a)
+ (9*cos(1/4*pi + 1/2*arctan2(0, b)) + 9*cos(-1/4*pi + 1/2*arctan2(0, b)) +
 9*I*sin(1/4*pi + 1/2*arctan2(0, b)) - 9*I*sin(-1/4*pi + 1/2*arctan2(0, b))
)*sin(a))*erf(sqrt(-I*b)*x))*sqrt(abs(b)))/abs(b)
```

Fricas [A] time = 2.2009, size = 374, normalized size = 2.44

$$-\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos(3a)\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right) - 9\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right) + \sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin(3a) - 9\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\cdots}\right.}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) - 9
*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)*p
i*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) - 9*sqrt(2)*pi*sqrt
(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

Sympy [A] time = 2.45712, size = 129, normalized size = 0.84

$$\frac{3\sqrt{2}\sqrt{\pi}\left(\sin(a)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right) + \cos(a)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8} - \frac{\sqrt{6}\sqrt{\pi}\left(\sin(3a)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right) + \cos(3a)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*(sin(a)*fresnelc(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*fr
esnels(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/8 - sqrt(6)*sqrt(pi)*(sin(3*a
)*fresnelc(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnels(sqrt(6)*sqrt(b)*
x/sqrt(pi)))*sqrt(1/b)/24
```

Giac [C] time = 1.12275, size = 250, normalized size = 1.63

$$-\frac{i\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b}x\left(-\frac{ib}{|b|}+1\right)\right)e^{(3ia)}}{48\sqrt{b}\left(-\frac{ib}{|b|}+1\right)} + \frac{3i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{16\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{3i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}x\left(\frac{ib}{|b|}+1\right)\sqrt{\cdots}\right.}{16\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*I*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3
*I*a)/(sqrt(b)*(-I*b/abs(b) + 1)) + 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2
)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))
) - 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)
))*e^(-I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) + 1/48*I*sqrt(6)*sqrt(pi)*erf(-
1/2*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(I*b/abs(b) + 1
))
```

3.30 $\int \frac{\sin^3(a+bx^2)}{x^2}\,dx$

Optimal. Leaf size=168

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\frac{1}{2}\sqrt{\frac{3\pi}{2}}$$

```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/2 - (Sqrt[b]*S
qrt[(3*Pi)/2]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/2 - (3*Sqrt[b]*Sqrt[
Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Fr
esnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2 - Sin[a + b*x^2]^3/x
```

Rubi [A] time = 0.145827, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3393, 4574, 3354, 3352, 3351}

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\frac{1}{2}\sqrt{\frac{3\pi}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^2]^3/x^2,x]
```



```
[Out] (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/2 - (Sqrt[b]*S
qrt[(3*Pi)/2]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/2 - (3*Sqrt[b]*Sqrt[
Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Fr
esnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2 - Sin[a + b*x^2]^3/x
```


Rule 3393

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[(x^(m +
 1)*Sin[a + b*x^n]^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^3\left(a+bx^2\right)}{x^2}\,dx = -\frac{\sin^3\left(a+bx^2\right)}{x} + (6b)\int \cos\left(a+bx^2\right)\sin^2\left(a+bx^2\right)\,dx$$
$$= -\frac{\sin^3\left(a+bx^2\right)}{x} + (6b)\int \left(\frac{1}{4}\cos\left(a+bx^2\right) - \frac{1}{4}\cos\left(3a+3bx^2\right)\right)dx$$
$$= -\frac{\sin^3\left(a+bx^2\right)}{x} + \frac{1}{2}(3b)\int \cos\left(a+bx^2\right)dx - \frac{1}{2}(3b)\int \cos\left(3a+3bx^2\right)dx$$
$$= -\frac{\sin^3\left(a+bx^2\right)}{x} + \frac{1}{2}(3b\cos(a))\int \cos\left(bx^2\right)dx - \frac{1}{2}(3b\cos(3a))\int \cos\left(3bx^2\right)dx - \frac{1}{2}(3b\sin(a$$
$$= \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a$$

Mathematica [A] time = 0.433062, size = 167, normalized size = 0.99

$$\frac{3\sqrt{2\pi}\sqrt{b}x\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right) - \sqrt{6\pi}\sqrt{b}x\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right) - 3\sqrt{2\pi}\sqrt{b}x\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) + \sqrt{6\pi}}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^2]^3/x^2,x]
```

```
[Out] (3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[b]*Sqr
t[6*Pi]*x*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x] - 3*Sqrt[b]*Sqrt[2*Pi]*x*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelS[Sqrt[
b]*Sqrt[6/Pi]*x]*Sin[3*a] - 3*Sin[a + b*x^2] + Sin[3*(a + b*x^2)])/(4*x)
```

Maple [A] time = 0.01, size = 130, normalized size = 0.8

$$-\frac{3\sin\left(bx^2+a\right)}{4x} + \frac{3\sqrt{2}\sqrt{\pi}}{4}\sqrt{b}\left(\cos(a)\,\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{b}\right) - \sin(a)\,\text{FresnelS}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\sqrt{b}\right)\right) + \frac{\sin\left(3bx^2+3a\right)}{4x} - \frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x^2+a)^3/x^2,x)
```

```
[Out] -3/4/x*sin(b*x^2+a)+3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x*b^(1/2)
*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/4*sin(3*b
*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelC(2^(1/2)
/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(
1/2)*x))
```

Maxima [C] time = 1.2606, size = 725, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(3)*sqrt(x^2*abs(b))*(((I*gamma(-1/2, 3*I*b*x^2) - I*gamma(-1/2,
-3*I*b*x^2))*cos(1/4*pi + 1/2*arctan2(0, b)) + (I*gamma(-1/2, 3*I*b*x^2) -
I*gamma(-1/2, -3*I*b*x^2))*cos(-1/4*pi + 1/2*arctan2(0, b)) - (gamma(-1/2,
3*I*b*x^2) + gamma(-1/2, -3*I*b*x^2))*sin(1/4*pi + 1/2*arctan2(0, b)) + (ga
mma(-1/2, 3*I*b*x^2) + gamma(-1/2, -3*I*b*x^2))*sin(-1/4*pi + 1/2*arctan2(0
, b)))*cos(3*a) + ((gamma(-1/2, 3*I*b*x^2) + gamma(-1/2, -3*I*b*x^2))*cos(1
/4*pi + 1/2*arctan2(0, b)) + (gamma(-1/2, 3*I*b*x^2) + gamma(-1/2, -3*I*b*x
^2))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (I*gamma(-1/2, 3*I*b*x^2) - I*gamma
(-1/2, -3*I*b*x^2))*sin(1/4*pi + 1/2*arctan2(0, b)) + (-I*gamma(-1/2, 3*I*b
*x^2) + I*gamma(-1/2, -3*I*b*x^2))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(3*
a)) + sqrt(x^2*abs(b))*(((-3*I*gamma(-1/2, I*b*x^2) + 3*I*gamma(-1/2, -I*b*
x^2))*cos(1/4*pi + 1/2*arctan2(0, b)) + (-3*I*gamma(-1/2, I*b*x^2) + 3*I*ga
mma(-1/2, -I*b*x^2))*cos(-1/4*pi + 1/2*arctan2(0, b)) + 3*(gamma(-1/2, I*b*
x^2) + gamma(-1/2, -I*b*x^2))*sin(1/4*pi + 1/2*arctan2(0, b)) - 3*(gamma(-1
/2, I*b*x^2) + gamma(-1/2, -I*b*x^2))*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos
(a) - (3*(gamma(-1/2, I*b*x^2) + gamma(-1/2, -I*b*x^2))*cos(1/4*pi + 1/2*ar
ctan2(0, b)) + 3*(gamma(-1/2, I*b*x^2) + gamma(-1/2, -I*b*x^2))*cos(-1/4*pi
 + 1/2*arctan2(0, b)) - (-3*I*gamma(-1/2, I*b*x^2) + 3*I*gamma(-1/2, -I*b*x
^2))*sin(1/4*pi + 1/2*arctan2(0, b)) - (3*I*gamma(-1/2, I*b*x^2) - 3*I*gamm
a(-1/2, -I*b*x^2))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a)))/x
```

Fricas [A] time = 2.48165, size = 440, normalized size = 2.62

$$-\frac{\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right) - 3\,\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right) - \sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right) + 3\,\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\cdots\right.}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_cos(sqrt(6)*x*sqrt(b/pi)) -
3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(6
)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)*pi
*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a) - 4*(cos(b*x^2 + a)^
2 - 1)*sin(b*x^2 + a))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{3}\left(a + bx^{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x**2+a)**3/x**2,x)
```

```
[Out] Integral(sin(a + b*x**2)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bx^2+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^2 + a)^3/x^2, x)
```

3.31 $\int x^2 \sin^3\left(x^2\right) dx$

Optimal. Leaf size=71

$$\frac{3}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{24}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)+\frac{1}{6}x\cos^3\left(x^2\right)-\frac{1}{2}x\cos\left(x^2\right)$$

```
[Out] -(x*Cos[x^2])/2 + (x*Cos[x^2]^3)/6 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x])/
8 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/24
```

Rubi [A] time = 0.0532944, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3403, 3385, 3352}

$$\frac{3}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{24}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)-\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{24}x\cos\left(3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x^2]^3,x]
```

```
[Out] (-3*x*Cos[x^2])/8 + (x*Cos[3*x^2])/24 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x
])/8 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/24
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sin^3\left(x^2\right)dx &= \int\left(\frac{3}{4}x^2\sin\left(x^2\right)-\frac{1}{4}x^2\sin\left(3x^2\right)\right)dx\\
&= -\left(\frac{1}{4}\int x^2\sin\left(3x^2\right)dx\right)+\frac{3}{4}\int x^2\sin\left(x^2\right)dx\\
&= -\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{24}x\cos\left(3x^2\right)-\frac{1}{24}\int\cos\left(3x^2\right)dx+\frac{3}{8}\int\cos\left(x^2\right)dx\\
&= -\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{24}x\cos\left(3x^2\right)+\frac{3}{8}\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{24}\sqrt{\frac{\pi}{6}}C\left(\sqrt{\frac{6}{\pi}}x\right)
\end{aligned}$$

Mathematica [A] time = 0.0672248, size = 63, normalized size = 0.89

$$\frac{1}{144}\left(27\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)+6x\left(\cos\left(3x^2\right)-9\cos\left(x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x^2]^3,x]
```

```
[Out] (6*x*(-9*Cos[x^2] + Cos[3*x^2]) + 27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] - Sq
rt[6*Pi]*FresnelC[Sqrt[6/Pi]*x])/144
```

Maple [A] time = 0.011, size = 58, normalized size = 0.8

$$-\frac{3x\cos\left(x^2\right)}{8}+\frac{3\sqrt{2}\sqrt{\pi}}{16}\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\right)+\frac{x\cos\left(3x^2\right)}{24}-\frac{\sqrt{2}\sqrt{\pi}\sqrt{3}}{144}\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}x}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^2)^3,x)
```

```
[Out] -3/8*x*cos(x^2)+3/16*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)+1/24*x*c
os(3*x^2)-1/144*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*
x)
```

Maxima [C] time = 1.52743, size = 131, normalized size = 1.85

$$\frac{1}{24}x\cos\left(3x^2\right)-\frac{3}{8}x\cos\left(x^2\right)+\frac{1}{1152}\sqrt{\pi}\left((2i-2)\sqrt{3}\sqrt{2}\,\text{erf}\left(\sqrt{3i}x\right)-(2i+2)\sqrt{3}\sqrt{2}\,\text{erf}\left(\sqrt{-3i}x\right)-(27i-27)\sqrt{2}\,\text{erf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*x*cos(3*x^2) - 3/8*x*cos(x^2) + 1/1152*sqrt(pi)*((2*I - 2)*sqrt(3)*sqr
t(2)*erf(sqrt(3*I)*x) - (2*I + 2)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I
 - 27)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*
I - 1/2)*sqrt(2)*x) + (27*I + 27)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqr
t(2)*erf((-1)^(1/4)*x))
```

Fricas [A] time = 2.18939, size = 200, normalized size = 2.82

$$\frac{1}{6}x\cos\left(x^2\right)^3-\frac{1}{2}x\cos\left(x^2\right)-\frac{1}{144}\sqrt{6}\sqrt{\pi}\,C\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)+\frac{3}{16}\sqrt{2}\sqrt{\pi}\,C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x*cos(x^2)^3 - 1/2*x*cos(x^2) - 1/144*sqrt(6)*sqrt(pi)*fresnel_cos(sqrt
(6)*x/sqrt(pi)) + 3/16*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi))
```

Sympy [A] time = 4.98922, size = 116, normalized size = 1.63

$$-\frac{15x\cos\left(x^2\right)\Gamma\left(\frac{5}{4}\right)}{32\Gamma\left(\frac{9}{4}\right)}+\frac{5x\cos\left(3x^2\right)\Gamma\left(\frac{5}{4}\right)}{96\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt{6}\sqrt{\pi}C\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{576\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x**2)**3,x)
```

```
[Out] -15*x*cos(x**2)*gamma(5/4)/(32*gamma(9/4)) + 5*x*cos(3*x**2)*gamma(5/4)/(96
*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*gamma(5/4)/
(64*gamma(9/4)) - 5*sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt(pi))*gamma(5/4
)/(576*gamma(9/4))
```

Giac [C] time = 1.10356, size = 131, normalized size = 1.85

$$\left(\frac{1}{576}i+\frac{1}{576}\right)\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}x\right)-\left(\frac{1}{576}i-\frac{1}{576}\right)\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}x\right)-\left(\frac{3}{64}i+\frac{3}{64}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] (1/576*I + 1/576)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) - (1/576*I
- 1/576)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) - (3/64*I + 3/64)*s
qrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) + (3/64*I - 3/64)*sqrt(2)*sqrt
(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) + 1/48*x*e^(3*I*x^2) - 3/16*x*e^(I*x^2)
- 3/16*x*e^(-I*x^2) + 1/48*x*e^(-3*I*x^2)
```

3.32 $\int x^4 \cos\left(x^2\right) \sin^2\left(x^2\right) dx$

Optimal. Leaf size=84

$$-\frac{3}{16}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)+\frac{1}{48}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{12}x\cos^3\left(x^2\right)+\frac{1}{4}x\cos\left(x^2\right)$$

```
[Out] (x*Cos[x^2])/4 - (x*Cos[x^2]^3)/12 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x])/
16 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/48 + (x^3*Sin[x^2]^3)/6
```

Rubi [A] time = 0.0776766, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3443, 3403, 3385, 3352}

$$-\frac{3}{16}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)+\frac{1}{48}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

```
[Out] (3*x*Cos[x^2])/16 - (x*Cos[3*x^2])/48 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*x
])/16 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*x])/48 + (x^3*Sin[x^2]^3)/6
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int x^4\cos\left(x^2\right)\sin^2\left(x^2\right)\,dx &= \frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{2}\int x^2\sin^3\left(x^2\right)\,dx\\ &= \frac{1}{6}x^3\sin^3\left(x^2\right)-\frac{1}{2}\int\left(\frac{3}{4}x^2\sin\left(x^2\right)-\frac{1}{4}x^2\sin\left(3x^2\right)\right)dx\\ &= \frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{1}{8}\int x^2\sin\left(3x^2\right)\,dx-\frac{3}{8}\int x^2\sin\left(x^2\right)\,dx\\ &= \frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)+\frac{1}{48}\int\cos\left(3x^2\right)\,dx-\frac{3}{16}\int\cos\left(x^2\right)\,dx\\ &= \frac{3}{16}x\cos\left(x^2\right)-\frac{1}{48}x\cos\left(3x^2\right)-\frac{3}{16}\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}x\right)+\frac{1}{48}\sqrt{\frac{\pi}{6}}C\left(\sqrt{\frac{6}{\pi}}x\right)+\frac{1}{6}x^3\sin^3\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.148881, size = 75, normalized size = 0.89

$$\frac{1}{288}\left(-27\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}x\right)+\sqrt{6\pi}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}x\right)+6x\left(8x^2\sin^3\left(x^2\right)+9\cos\left(x^2\right)-\cos\left(3x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Cos[x^2]*Sin[x^2]^2,x]
```

```
[Out] (-27*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*x] + Sqrt[6*Pi]*FresnelC[Sqrt[6/Pi]*x]
+ 6*x*(9*Cos[x^2] - Cos[3*x^2] + 8*x^2*Sin[x^2]^3))/288
```

Maple [A] time = 0.019, size = 78, normalized size = 0.9

$$\frac{x^3\sin\left(x^2\right)}{8}+\frac{3\,x\cos\left(x^2\right)}{16}-\frac{3\,\sqrt{2}\sqrt{\pi}}{32}\text{FresnelC}\left(\frac{x\sqrt{2}}{\sqrt{\pi}}\right)-\frac{x^3\sin\left(3\,x^2\right)}{24}-\frac{x\cos\left(3\,x^2\right)}{48}+\frac{\sqrt{2}\sqrt{\pi}\sqrt{3}}{288}\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}x}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(x^2)*sin(x^2)^2,x)
```

```
[Out] 1/8*x^3*sin(x^2)+3/16*x*cos(x^2)-3/32*FresnelC(x*2^(1/2)/Pi^(1/2))*2^(1/2)*
Pi^(1/2)-1/24*x^3*sin(3*x^2)-1/48*x*cos(3*x^2)+1/288*2^(1/2)*Pi^(1/2)*3^(1/
2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*x)
```

Maxima [C] time = 1.53842, size = 158, normalized size = 1.88

$$-\frac{1}{24}\,x^3\sin\left(3\,x^2\right)+\frac{1}{8}\,x^3\sin\left(x^2\right)-\frac{1}{48}\,x\cos\left(3\,x^2\right)+\frac{3}{16}\,x\cos\left(x^2\right)-\frac{1}{2304}\,\sqrt{\pi}\left((2i-2)\,\sqrt{3}\sqrt{2}\,\text{erf}\left(\sqrt{3i}x\right)-(2i+2)\,\sqrt{3}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*x^3*sin(3*x^2) + 1/8*x^3*sin(x^2) - 1/48*x*cos(3*x^2) + 3/16*x*cos(x^
2) - 1/2304*sqrt(pi)*((2*I - 2)*sqrt(3)*sqrt(2)*erf(sqrt(3*I)*x) - (2*I + 2
)*sqrt(3)*sqrt(2)*erf(sqrt(-3*I)*x) - (27*I - 27)*sqrt(2)*erf((1/2*I + 1/2)
*sqrt(2)*x) - (27*I + 27)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*x) + (27*I + 27
```

```
)*sqrt(2)*erf(sqrt(-I)*x) - (27*I - 27)*sqrt(2)*erf((-1)^(1/4)*x))
```

Fricas [A] time = 2.19345, size = 254, normalized size = 3.02

$$-\frac{1}{12}x\cos\left(x^2\right)^3+\frac{1}{4}x\cos\left(x^2\right)+\frac{1}{288}\sqrt{6}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{6}x}{\sqrt{\pi}}\right)-\frac{3}{32}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}x}{\sqrt{\pi}}\right)-\frac{1}{6}\left(x^3\cos\left(x^2\right)^2-x^3\right)\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*x*cos(x^2)^3 + 1/4*x*cos(x^2) + 1/288*sqrt(6)*sqrt(pi)*fresnel_cos(sq
rt(6)*x/sqrt(pi)) - 3/32*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*x/sqrt(pi)) -
 1/6*(x^3*cos(x^2)^2 - x^3)*sin(x^2)
```

Sympy [B] time = 6.09948, size = 291, normalized size = 3.46

$$-\frac{9x^5\Gamma\left(-\frac{9}{4}\right)}{40\Gamma\left(-\frac{5}{4}\right)}+\frac{9x^3\sin\left(x^2\right)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)}-\frac{5x^3\sin\left(x^2\right)\Gamma\left(-\frac{5}{4}\right)}{16\Gamma\left(-\frac{1}{4}\right)}+\frac{3x^3\sin\left(3x^2\right)\Gamma\left(-\frac{9}{4}\right)}{32\Gamma\left(-\frac{5}{4}\right)}+\frac{27x\cos\left(x^2\right)\Gamma\left(-\frac{9}{4}\right)}{64\Gamma\left(-\frac{5}{4}\right)}-\frac{15x\cos\left(x^2\right)\Gamma}{32\Gamma\left(-\frac{1}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(x**2)*sin(x**2)**2,x)
```

```
[Out] -9*x**5*gamma(-9/4)/(40*gamma(-5/4)) + 9*x**3*sin(x**2)*gamma(-9/4)/(32*gam
ma(-5/4)) - 5*x**3*sin(x**2)*gamma(-5/4)/(16*gamma(-1/4)) + 3*x**3*sin(3*x*
*2)*gamma(-9/4)/(32*gamma(-5/4)) + 27*x*cos(x**2)*gamma(-9/4)/(64*gamma(-5/
4)) - 15*x*cos(x**2)*gamma(-5/4)/(32*gamma(-1/4)) + 3*x*cos(3*x**2)*gamma(-
9/4)/(64*gamma(-5/4)) + 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi))*ga
mma(-5/4)/(64*gamma(-1/4)) - 27*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*x/sqrt(pi
))*gamma(-9/4)/(128*gamma(-5/4)) - sqrt(6)*sqrt(pi)*fresnelc(sqrt(6)*x/sqrt
(pi))*gamma(-9/4)/(128*gamma(-5/4))
```

Giac [C] time = 1.13131, size = 169, normalized size = 2.01

$$-\left(\frac{1}{1152}i+\frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}x\right)+\left(\frac{1}{1152}i-\frac{1}{1152}\right)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}x\right)+\left(\frac{3}{128}i+\frac{3}{128}\right)\sqrt{2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(x^2)*sin(x^2)^2,x, algorithm="giac")
```

```
[Out] -(1/1152*I + 1/1152)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*x) + (1/115
2*I - 1/1152)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*x) + (3/128*I + 3
/128)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*x) - (3/128*I - 3/128)*sqr
t(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*x) - 1/96*(-2*I*x^3 + x)*e^(3*I*x^
2) - 1/32*(2*I*x^3 - 3*x)*e^(I*x^2) - 1/32*(-2*I*x^3 - 3*x)*e^(-I*x^2) - 1/
96*(2*I*x^3 + x)*e^(-3*I*x^2)
```

3.33 $\int x \sin^7\left(a+bx^2\right) dx$

Optimal. Leaf size=67

$$\frac{\cos^7\left(a+bx^2\right)}{14b}-\frac{3\cos^5\left(a+bx^2\right)}{10b}+\frac{\cos^3\left(a+bx^2\right)}{2b}-\frac{\cos\left(a+bx^2\right)}{2b}$$

```
[Out] -Cos[a + b*x^2]/(2*b) + Cos[a + b*x^2]^3/(2*b) - (3*Cos[a + b*x^2]^5)/(10*b
) + Cos[a + b*x^2]^7/(14*b)
```

Rubi [A] time = 0.0446329, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3379, 2633}

$$\frac{\cos^7\left(a+bx^2\right)}{14b}-\frac{3\cos^5\left(a+bx^2\right)}{10b}+\frac{\cos^3\left(a+bx^2\right)}{2b}-\frac{\cos\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x^2]^7,x]
```

```
[Out] -Cos[a + b*x^2]/(2*b) + Cos[a + b*x^2]^3/(2*b) - (3*Cos[a + b*x^2]^5)/(10*b
) + Cos[a + b*x^2]^7/(14*b)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin^7\left(a+bx^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int \sin^7(a+bx)\,dx,x,x^2\right)\\ &= -\frac{\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,\cos\left(a+bx^2\right)\right)}{2b}\\ &= -\frac{\cos\left(a+bx^2\right)}{2b}+\frac{\cos^3\left(a+bx^2\right)}{2b}-\frac{3\cos^5\left(a+bx^2\right)}{10b}+\frac{\cos^7\left(a+bx^2\right)}{14b}\end{aligned}$$

Mathematica [A] time = 0.0426408, size = 67, normalized size = 1.

$$-\frac{35\cos\left(a+bx^2\right)}{128b}+\frac{7\cos\left(3\left(a+bx^2\right)\right)}{128b}-\frac{7\cos\left(5\left(a+bx^2\right)\right)}{640b}+\frac{\cos\left(7\left(a+bx^2\right)\right)}{896b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x^2]^7,x]
```

```
[Out] (-35*Cos[a + b*x^2])/(128*b) + (7*Cos[3*(a + b*x^2)])/(128*b) - (7*Cos[5*(a
 + b*x^2)])/(640*b) + Cos[7*(a + b*x^2)]/(896*b)
```

Maple [A] time = 0.005, size = 50, normalized size = 0.8

$$-\frac{\cos\left(bx^2+a\right)}{14\,b}\left(\frac{16}{5}+\left(\sin\left(bx^2+a\right)\right)^6+\frac{6\left(\sin\left(bx^2+a\right)\right)^4}{5}+\frac{8\left(\sin\left(bx^2+a\right)\right)^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x^2+a)^7,x)
```

```
[Out] -1/14/b*(16/5+sin(b*x^2+a)^6+6/5*sin(b*x^2+a)^4+8/5*sin(b*x^2+a)^2)*cos(b*x
^2+a)
```

Maxima [A] time = 0.983536, size = 74, normalized size = 1.1

$$\frac{5\cos\left(7\,bx^2+7\,a\right)-49\cos\left(5\,bx^2+5\,a\right)+245\cos\left(3\,bx^2+3\,a\right)-1225\cos\left(bx^2+a\right)}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/4480*(5*cos(7*b*x^2 + 7*a) - 49*cos(5*b*x^2 + 5*a) + 245*cos(3*b*x^2 + 3*
a) - 1225*cos(b*x^2 + a))/b
```

Fricas [A] time = 2.30148, size = 126, normalized size = 1.88

$$\frac{5\cos\left(bx^2+a\right)^7-21\cos\left(bx^2+a\right)^5+35\cos\left(bx^2+a\right)^3-35\cos\left(bx^2+a\right)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*c
os(b*x^2 + a))/b
```

Sympy [A] time = 12.6649, size = 95, normalized size = 1.42

$$\begin{cases} -\frac{\sin^6\left(a+bx^2\right)\cos\left(a+bx^2\right)}{2b}-\frac{\sin^4\left(a+bx^2\right)\cos^3\left(a+bx^2\right)}{b}-\frac{4\sin^2\left(a+bx^2\right)\cos^5\left(a+bx^2\right)}{5b}-\frac{8\cos^7\left(a+bx^2\right)}{35b} & \text{for } b\neq 0 \\ \frac{x^2\sin^7(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x**2+a)**7,x)
```

```
[Out] Piecewise((-sin(a + b*x**2)**6*cos(a + b*x**2)/(2*b) - sin(a + b*x**2)**4*c
os(a + b*x**2)**3/b - 4*sin(a + b*x**2)**2*cos(a + b*x**2)**5/(5*b) - 8*cos
(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sin(a)**7/2, True))
```

Giac [A] time = 1.10784, size = 70, normalized size = 1.04

$$\frac{5\cos\left(bx^2+a\right)^7-21\cos\left(bx^2+a\right)^5+35\cos\left(bx^2+a\right)^3-35\cos\left(bx^2+a\right)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^7 - 21*cos(b*x^2 + a)^5 + 35*cos(b*x^2 + a)^3 - 35*c
os(b*x^2 + a))/b
```

3.34 $\int \frac{(1+\sin(x^2))^2}{x^3} \, dx$

Optimal. Leaf size=44

$$\text{CosIntegral}\left(x^2\right)+\frac{\text{Si}\left(2x^2\right)}{2}-\frac{3}{4x^2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{\cos\left(2x^2\right)}{4x^2}$$

```
[Out] -3/(4*x^2) + Cos[2*x^2]/(4*x^2) + CosIntegral[x^2] - Sin[x^2]/x^2 + SinInte
gral[2*x^2]/2
```

Rubi [A] time = 0.100006, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3403, 3380, 3297, 3299, 3379, 3302}

$$\text{CosIntegral}\left(x^2\right)+\frac{\text{Si}\left(2x^2\right)}{2}-\frac{3}{4x^2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{\cos\left(2x^2\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x^2])^2/x^3,x]
```


```
[Out] -3/(4*x^2) + Cos[2*x^2]/(4*x^2) + CosIntegral[x^2] - Sin[x^2]/x^2 + SinInte
gral[2*x^2]/2
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+\sin\left(x^2\right)\right)^2}{x^3}\,dx &= \int \left(\frac{3}{2x^3}-\frac{\cos\left(2x^2\right)}{2x^3}+\frac{2\sin\left(x^2\right)}{x^3}\right)dx \\
&= -\frac{3}{4x^2}-\frac{1}{2}\int \frac{\cos\left(2x^2\right)}{x^3}\,dx+2\int \frac{\sin\left(x^2\right)}{x^3}\,dx \\
&= -\frac{3}{4x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{\cos(2x)}{x^2}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{\sin(x)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{3}{4x^2}+\frac{\cos\left(2x^2\right)}{4x^2}-\frac{\sin\left(x^2\right)}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx,x,x^2\right) \\
&= -\frac{3}{4x^2}+\frac{\cos\left(2x^2\right)}{4x^2}+\text{Ci}\left(x^2\right)-\frac{\sin\left(x^2\right)}{x^2}+\frac{\text{Si}\left(2x^2\right)}{2}
\end{aligned}$$

Mathematica [A] time = 0.101377, size = 41, normalized size = 0.93

$$\frac{4x^2\text{CosIntegral}\left(x^2\right)+2x^2\text{Si}\left(2x^2\right)-4\sin\left(x^2\right)+\cos\left(2x^2\right)-3}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x^2])^2/x^3,x]
```

```
[Out] (-3 + Cos[2*x^2] + 4*x^2*CosIntegral[x^2] - 4*Sin[x^2] + 2*x^2*SinIntegral[
2*x^2])/(4*x^2)
```

Maple [A] time = 0.019, size = 39, normalized size = 0.9

$$-\frac{3}{4\,x^2}+\text{Ci}\left(x^2\right)+\frac{\cos\left(2\,x^2\right)}{4\,x^2}+\frac{\text{Si}\left(2\,x^2\right)}{2}-\frac{\sin\left(x^2\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x^2))^2/x^3,x)
```

```
[Out] -3/4/x^2+Ci(x^2)+1/4*cos(2*x^2)/x^2+1/2*Si(2*x^2)-sin(x^2)/x^2
```

Maxima [C] time = 1.11866, size = 73, normalized size = 1.66

$$\frac{x^2\left(i\,\Gamma\left(-1,2i\,x^2\right)-i\,\Gamma\left(-1,-2i\,x^2\right)\right)-1}{4\,x^2}-\frac{1}{2\,x^2}+\frac{1}{2}\Gamma\left(-1,i\,x^2\right)+\frac{1}{2}\Gamma\left(-1,-i\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(x^2*(I*gamma(-1, 2*I*x^2) - I*gamma(-1, -2*I*x^2)) - 1)/x^2 - 1/2/x^2
+ 1/2*gamma(-1, I*x^2) + 1/2*gamma(-1, -I*x^2)
```

Fricas [A] time = 2.3233, size = 154, normalized size = 3.5

$$\frac{x^2 \operatorname{Ci}\left(-x^2\right) + x^2 \operatorname{Ci}\left(x^2\right) + x^2 \operatorname{Si}\left(2\,x^2\right) + \cos\left(x^2\right)^2 - 2\,\sin\left(x^2\right) - 2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*cos_integral(-x^2) + x^2*cos_integral(x^2) + x^2*sin_integral(2*x^
2) + cos(x^2)^2 - 2*sin(x^2) - 2)/x^2
```

Sympy [A] time = 5.82142, size = 51, normalized size = 1.16

$$-\log\left(x^2\right) + \frac{\log\left(x^4\right)}{2} + \operatorname{Ci}\left(x^2\right) + \frac{\operatorname{Si}\left(2x^2\right)}{2} - \frac{\sin\left(x^2\right)}{x^2} + \frac{\cos\left(2x^2\right)}{4x^2} - \frac{3}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x**2))**2/x**3,x)
```

```
[Out] -log(x**2) + log(x**4)/2 + Ci(x**2) + Si(2*x**2)/2 - sin(x**2)/x**2 + cos(2
*x**2)/(4*x**2) - 3/(4*x**2)
```

Giac [A] time = 1.09745, size = 53, normalized size = 1.2

$$\frac{4\,x^2 \operatorname{Ci}\left(x^2\right) + 2\,x^2 \operatorname{Si}\left(2\,x^2\right) + \cos\left(2\,x^2\right) - 4\,\sin\left(x^2\right) - 3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*x^2*cos_integral(x^2) + 2*x^2*sin_integral(2*x^2) + cos(2*x^2) - 4*s
in(x^2) - 3)/x^2
```

3.35 $\int \frac{x^5}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=362

$$-\frac{x^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{x^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\sqrt{a^2-b^2}}-\frac{i\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{i\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\sqrt{a^2-b^2}}-\frac{ix^4\log\left(1\right.}{2d\sqrt{}}$$

```
[Out] ((-I/2)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) + ((I/2)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b
^2])])/(Sqrt[a^2 - b^2]*d) - (x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - S
qrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) + (x^2*PolyLog[2, (I*b*E^(I*(c + d*
x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) - (I*PolyLog[3, (I*b*E
^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3) + (I*PolyLo
g[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3)
```

Rubi [A] time = 0.878822, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3379, 3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\sqrt{a^2-b^2}}+\frac{x^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\sqrt{a^2-b^2}}-\frac{i\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\sqrt{a^2-b^2}}+\frac{i\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\sqrt{a^2-b^2}}-\frac{ix^4\log\left(1\right.}{2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-I/2)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) + ((I/2)*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b
^2])])/(Sqrt[a^2 - b^2]*d) - (x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - S
qrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) + (x^2*PolyLog[2, (I*b*E^(I*(c + d*
x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^2) - (I*PolyLog[3, (I*b*E
^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3) + (I*PolyLo
g[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]*d^3)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\sin(c+dx)}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)$$
$$= -\frac{(ib)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}} + \frac{(ib)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{a^2-b^2}}$$
$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{i\,\text{Subst}\left(\int x\log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx, x, x^2\right.}{\sqrt{a^2-b^2}d}$$
$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{\text{S}}{}$$
$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{i\,\text{S}}{}$$
$$= -\frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} + \frac{ix^4\log\left(1-\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d} - \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} + \frac{x^2\text{Li}_2\left(\frac{ibe^{i(c+dx^2)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{i\text{L}}{}$$

Mathematica [A] time = 0.209812, size = 289, normalized size = 0.8

$$\frac{-i\left(2idx^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)+2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)-2\text{PolyLog}\left(3,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)+d^2x^4\log\left(1+\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}-a}\right)-d^2x^4\text{lo}\right.}{2d^3\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (-2*d*x^2*PolyLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - I
*(d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - d^2*x^4
*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*x^2*PolyL
og[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, (I*b*E^
(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])] - 2*PolyLog[3, (I*b*E^(I*(c + d*x^2
)))/(a + Sqrt[a^2 - b^2])]))/(2*Sqrt[a^2 - b^2]*d^3)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^5}{a+b\sin\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*sin(d*x^2+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

Fricas [C] time = 3.32103, size = 3376, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(-4*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x^2 + c)
+ 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2
 - b^2)/b^2) + 2*b)/b + 1) + 4*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*
(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) - I*b*sin(
d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 4*I*b*d*x^2*sqrt(-(a^2 -
 b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*co
s(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 4
*I*b*d*x^2*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*s
in(d*x^2 + c) - 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)
/b^2) + 2*b)/b + 1) - 2*b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c)
 + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*b*c^2*sqr
t(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqr
t(-(a^2 - b^2)/b^2) - 2*I*a) + 2*b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(
d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2
*b*c^2*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c
) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - 2*(b*d^2*x^4 - b*c^2)*sqrt(-(a^2
- b^2)/b^2)*log(1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d
*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^2
*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x^2 + c) + 2*a*si
n(d*x^2 + c) - 2*(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/
b^2) + 2*b)/b) - 2*(b*d^2*x^4 - b*c^2)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I
*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^
2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d^2*x^4 - b*c^2)*sqrt(-(a^2
 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos
(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 4*b*sq
rt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x^2 + c) - 2*a*sin(d*x^2 +
 c) + 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b)
- 4*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x^2 + c) - 2*a*sin
(d*x^2 + c) - 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b
^2))/b) - 4*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x^2 + c) + a*si
n(d*x^2 + c) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^
2))/b) + 4*b*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x^2 + c) + a*sin
(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2
))/b))/((a^2 - b^2)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(x**5/(a + b*sin(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a), x)
```

3.36 $\int \frac{x^3}{a+b\sin\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=245

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\sqrt{a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d^2\sqrt{a^2-b^2}}-\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d\sqrt{a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d\sqrt{a^2-b^2}}$$

```
[Out] ((-I/2)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) + ((I/2)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b
^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a
^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a
 + Sqrt[a^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] time = 0.514179, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 3323, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\sqrt{a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d^2\sqrt{a^2-b^2}}-\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d\sqrt{a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-I/2)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) + ((I/2)*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b
^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a
^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a
 + Sqrt[a^2 - b^2])]/(2*Sqrt[a^2 - b^2]*d^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3}{a+b\sin\left(c+dx^2\right)}\,dx=\frac{1}{2}\text{Subst}\left(\int\frac{x}{a+b\sin(c+dx)}\,dx,x,x^2\right)$$
$$=\text{Subst}\left(\int\frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx,x,x^2\right)$$
$$=-\frac{(ib)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx,x,x^2\right)}{\sqrt{a^2-b^2}}+\frac{(ib)\,\text{Subst}\left(\int\frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx,x,x^2\right)}{\sqrt{a^2-b^2}}$$
$$=-\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{i\,\text{Subst}\left(\int\log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx,x,x^2\right)}{2\sqrt{a^2-b^2}d}$$
$$=-\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{2\sqrt{a^2-b^2}d^2}$$
$$=-\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d}-\frac{\text{Li}_2\left(\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2}+\frac{\text{Li}_2\left(\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}d^2}$$

Mathematica [A] time = 0.0645226, size = 188, normalized size = 0.77

$$\frac{-\text{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}-a}\right)+\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)-idx^2\left(\log\left(1+\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}-a}\right)-\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)\right)}{2d^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ((-I)*d*x^2*(Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] - Log[
1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^
(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x^2)
))/(a + Sqrt[a^2 - b^2])])/(2*Sqrt[a^2 - b^2]*d^2)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\sin\left(dx^2 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^3/(a+b*sin(d*x^2+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\sin\left(dx^2 + c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

Fricas [B] time = 3.44699, size = 2481, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2
 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*b*c*sqrt(-(a^2 - b^2)/b^2)*
log(2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
- 2*I*a) - 2*b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin
(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*b*c*sqrt(-(a^2 - b^2)
/b^2)*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) - 2*I*b*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x^2
+ c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(
2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) - I*b*sin(d
*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*sqrt(-(a^2 - b^2)/b
^2)*dilog(-1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2
 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b*sq
rt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c)
 - 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/
b + 1) - 2*(b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x^2
+ c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(
-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(
1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) - I*b*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b*d*x^2 + b*c)*sqrt(-
(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b
*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*
(b*d*x^2 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x^2 + c) + 2*a
```

```
*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^
2)/b^2) + 2*b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b\sin\left(c + dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(x**3/(a + b*sin(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b\sin\left(dx^2 + c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a), x)
```

3.37 $\int \frac{x}{a+b\sin\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

```
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Rubi [A] time = 0.0690306, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 2660, 618, 204}

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{a+b\sin\left(c+dx^2\right)}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^2\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{d}$$
$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{d}$$
$$= \frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

Mathematica [A] time = 0.0813741, size = 48, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]]/(Sqrt[a^2 - b^2]*d)
```

Maple [A] time = 0.017, size = 48, normalized size = 1.

$$\frac{1}{d}\arctan\left(\frac{1}{2}\left(2\,a\tan\left(1/2\,dx^2+c/2\right)+2\,b\right)\frac{1}{\sqrt{a^2-b^2}}\right)\frac{1}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^2+c)),x)
```

```
[Out] 1/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^2+1/2*c)+2*b)/(a^2-b^2)^(1/
2))
```

Maxima [B] time = 155.146, size = 10905, normalized size = 227.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^4*cos(c)*sin(c) - 4*(a^2
*b^4 - b^6)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c)^
3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 4*(3*(a^3*b^3
 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^6)*c
os(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^3 +
```

```
4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 +
 b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6
)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 + 3*((a^3
*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c
))*sin(d*x^2 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5 + 2*(2*a^
5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)
^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3 + (2*a^5*b -
3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^2*b^4 - b^6)*s
in(c)^2)*cos(d*x^2 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*sin(c) - (a^3*b
^3 - a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + ((4*a^4*b^2 - 5*a^2*b^4 + b^6)*c
os(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x
^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^5*cos(c) - 4*a*b^4*cos(d*x^2 + 2*c)^4*cos
(c)*sin(c) + b^5*sin(d*x^2 + 2*c)^5*sin(c) + (b^5*cos(d*x^2 + 2*c)*cos(c) +
 4*a*b^4*cos(c)*sin(c))*sin(d*x^2 + 2*c)^4 + 2*((2*a^2*b^3 - b^5)*cos(c)^3
+ 3*(2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^3 + 2*(b^5*cos(d*x^
2 + 2*c)^2*sin(c) + 3*(2*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (2*a^2*b^3 - b^5)
*sin(c)^3 + 2*(a*b^4*cos(c)^2 - a*b^4*sin(c)^2)*cos(d*x^2 + 2*c))*sin(d*x^2
 + 2*c)^3 - 4*((4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + (4*a^3*b^2 - 3*a*b^4
)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 2*(b^5*cos(d*x^2 + 2*c)^3*cos(c) +
2*(4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + 2*(4*a^3*b^2 - 3*a*b^4)*cos(c)*si
n(c)^3 + 3*((2*a^2*b^3 - b^5)*cos(c)^3 - (2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)
*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 + ((8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)
^5 + 2*(8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (8*a^4*b - 8*a^2*b^3
 + b^5)*cos(c)*sin(c)^4)*cos(d*x^2 + 2*c) + (b^5*cos(d*x^2 + 2*c)^4*sin(c)
+ (8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(8*a^4*b - 8*a^2*b^3 + b^
5)*cos(c)^2*sin(c)^3 + (8*a^4*b - 8*a^2*b^3 + b^5)*sin(c)^5 + 4*(a*b^4*cos(
c)^2 - a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 6*((2*a^2*b^3 - b^5)*cos(c)^2*s
in(c) - (2*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^2 + 4*((4*a^3*b^2 - 3*
a*b^4)*cos(c)^4 - (4*a^3*b^2 - 3*a*b^4)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x
^2 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*
x^2 + 2*c)^5 + b^6*sin(d*x^2 + 2*c)^6 - 6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) +
 (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2
 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^
4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c
)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x
^2 + 2*c)^4 + 3*(b^6*cos(d*x^2 + 2*c)^2 - 2*a*b^5*cos(d*x^2 + 2*c)*sin(c) +
 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c
)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*
sin(c)^3)*cos(d*x^2 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(4*
a^3*b^3 - 3*a*b^5)*cos(c)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*s
in(c) + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 3*((8
*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(
c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^
2 + 3*(b^6*cos(d*x^2 + 2*c)^4 - 4*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^
4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^
2*sin(c)^2 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*
cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*(3*(4*a^3*b^3
 - 3*a*b^5)*cos(c)^2*sin(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2
*c))*sin(d*x^2 + 2*c)^2 - 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin
(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 2
0*a^3*b^3 + 5*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 6*(a*b^5*cos(d*x^2 + 2*c)
^4*cos(c) + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6
)*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*co
s(c)^3*sin(c)^2 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4
*a^3*b^3 - 3*a*b^5)*cos(c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos
(d*x^2 + 2*c)^2 - 4*((8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4
*b^2 - 8*a^2*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)
 - 2*(3*b^5*cos(c)*sin(d*x^2 + 2*c)^5 - 3*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (
```

16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)
*cos(c)^4*sin(c)^2 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 +
(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(
c)^2)*cos(d*x^2 + 2*c)^4 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^2 + 2*c)*sin(c
) + a*b^4*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*si
n(c) + 5*(4*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c)^3 + 2*(3*b^5*cos(d*x^
2 + 2*c)^2*cos(c) - 12*a*b^4*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3
- b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 +
6*((2*a^3*b^2 - a*b^4)*cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2
+ 5*(2*a^3*b^2 - a*b^4)*sin(c)^4)*cos(d*x^2 + 2*c)^2 - 6*(b^5*cos(d*x^2 + 2
*c)^3*sin(c) - 5*(2*a^3*b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c
)^2*sin(c)^2 - (2*a^3*b^2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin
(c)^2)*cos(d*x^2 + 2*c)^2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b
^3 - b^5)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 3*((16*a^4*b - 1
2*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2
*sin(c)^3 + (16*a^4*b - 12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 3*(b
^5*cos(d*x^2 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (
16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos
(c)^3*sin(c)^2 + (16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*
b^3 - b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)
^2 - 16*((2*a^3*b^2 - a*b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*s
in(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(a^2 - b^2)), (b^6*cos(d*x
^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^2 + 2*c)^5 + b^6*sin(d*x^2 + 2*c)^6 -
6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^6
+ 3*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4*sin(c)^2 + 3*(8*a^4*b^2 - 8*a^2*
b^4 + b^6)*cos(c)^2*sin(c)^4 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^6 + ((4
*a^2*b^4 - b^6)*cos(c)^2 + 5*(4*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^4
+ (3*b^6*cos(d*x^2 + 2*c)^2 - 6*a*b^5*cos(d*x^2 + 2*c)*sin(c) + 5*(4*a^2*b
^4 - b^6)*cos(c)^2 + (4*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 4*(3*
(2*a^3*b^3 - a*b^5)*cos(c)^2*sin(c) + 5*(2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d
*x^2 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(2*a^3*b^3 - a*b^5
)*cos(c)^3 - 2*(4*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*(2*a^3*
b^3 - a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + ((8*a^4*b^2 - 4*a^2*b^4
- b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + 5*(8*
a^4*b^2 - 4*a^2*b^4 - b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + (3*b^6*cos(d*x^2
+ 2*c)^4 - 12*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 4*a^2*b^4 -
b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + (8*a^4*
b^2 - 4*a^2*b^4 - b^6)*sin(c)^4 + 6*((4*a^2*b^4 - b^6)*cos(c)^2 + (4*a^2*b^
4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 12*(3*(2*a^3*b^3 - a*b^5)*cos(c)^2*
sin(c) + (2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2
- 2*((8*a^5*b - 5*a*b^5)*cos(c)^4*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^2*
sin(c)^3 + (8*a^5*b - 5*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 2*(3*a*b^5*cos(
d*x^2 + 2*c)^4*cos(c) + (8*a^5*b - 5*a*b^5)*cos(c)^5 - 4*(4*a^2*b^4 - b^6)*
cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^3*sin(c)^2
+ (8*a^5*b - 5*a*b^5)*cos(c)*sin(c)^4 + 6*((2*a^3*b^3 - a*b^5)*cos(c)^3 + 3
*(2*a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*((8*a^4*b^2 -
4*a^2*b^4 - b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)*sin
(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) - 4*(b^5*cos(c)*sin(d*x^2 + 2*c)^
5 - b^5*cos(d*x^2 + 2*c)^5*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)^6 + 3*(2*a^3
*b^2 - a*b^4)*cos(c)^4*sin(c)^2 + 3*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^4 +
(2*a^3*b^2 - a*b^4)*sin(c)^6 + (a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x
^2 + 2*c)^4 + (5*a*b^4*cos(c)^2 - b^5*cos(d*x^2 + 2*c)*sin(c) + a*b^4*sin(c
)^2)*sin(d*x^2 + 2*c)^4 - 2*(3*a^2*b^3*cos(c)^2*sin(c) + 5*a^2*b^3*sin(c)^3
)*cos(d*x^2 + 2*c)^3 + 2*(b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*a^2*b^3*cos(c)^
3 - 4*a*b^4*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*a^2*b^3*cos(c)*sin(c)^2)*sin
(d*x^2 + 2*c)^3 + 2*(a^3*b^2*cos(c)^4 + 6*a^3*b^2*cos(c)^2*sin(c)^2 + 5*a^3
*b^2*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 2*(5*a^3*b^2*cos(c)^4 - b^5*cos(d*x^2 +
2*c)^3*sin(c) + 6*a^3*b^2*cos(c)^2*sin(c)^2 + a^3*b^2*sin(c)^4 + 3*(a*b^4*
cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 3*(3*a^2*b^3*cos(c)^2*sin(c

```
) + a^2*b^3*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - ((4*a^4*b + 2*
a^2*b^3 - b^5)*cos(c)^4*sin(c) + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^2*sin
(c)^3 + (4*a^4*b + 2*a^2*b^3 - b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + (b^5*cos(d
*x^2 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (4*a^4*b
+ 2*a^2*b^3 - b^5)*cos(c)^5 + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^3*sin(c)
^2 + (4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)*sin(c)^4 + 6*(a^2*b^3*cos(c)^3 + 3*
a^2*b^3*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 16*(a^3*b^2*cos(c)^3*sin(c) +
 a^3*b^2*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c))*sqrt(a^2 - b^
2))/(b^6*cos(d*x^2 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^2 + 2*c)^5 + b^6*sin(d
*x^2 + 2*c)^6 - 6*a*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (32*a^6 - 48*a^4*b^2 +
18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos
(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^2*sin(c)
^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c)^6 + 3*((2*a^2*b^4 - b^
6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^4 + 3*(b^6*cos
(d*x^2 + 2*c)^2 - 2*a*b^5*cos(d*x^2 + 2*c)*sin(c) + 5*(2*a^2*b^4 - b^6)*cos
(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^2 + 2*c)^4 - 4*(3*(4*a^3*b^3 -
3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2*
c)^3 + 4*(3*a*b^5*cos(d*x^2 + 2*c)^2*cos(c) + 5*(4*a^3*b^3 - 3*a*b^5)*cos(c
)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 3*(4*a^3*b^3 - 3
*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 3*((8*a^4*b^2 - 8*a^2*b^4 + b
^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + 5*(8*a^4
*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^2 + 2*c)^2 + 3*(b^6*cos(d*x^2 + 2
*c)^4 - 4*a*b^5*cos(d*x^2 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)
*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + (8*a^4*b^2
- 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 -
b^6)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin
(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2
- 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin(c) + 2*(16*a^5*b - 20*a
^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*sin
(c)^5)*cos(d*x^2 + 2*c) + 6*(a*b^5*cos(d*x^2 + 2*c)^4*cos(c) + (16*a^5*b -
20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6)*cos(d*x^2 + 2*c)^3*cos
(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^3*sin(c)^2 + (16*a^
5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4*a^3*b^3 - 3*a*b^5)*cos(
c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 4*((8*
a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*
cos(c)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) - 2*(3*b^5*cos(c)*sin(d
*x^2 + 2*c)^5 - 3*b^5*cos(d*x^2 + 2*c)^5*sin(c) + (16*a^5 - 16*a^3*b^2 + 3*
a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^4*sin(c)^2 + 3*(
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 + (16*a^5 - 16*a^3*b^2 + 3
*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^4
 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^2 + 2*c)*sin(c) + a*b^4*sin(c)^2)*sin(
d*x^2 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + 5*(4*a^2*b^3 - b^
5)*sin(c)^3)*cos(d*x^2 + 2*c)^3 + 2*(3*b^5*cos(d*x^2 + 2*c)^2*cos(c) - 12*a
*b^4*cos(d*x^2 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3 - b^5)*cos(c)^3 + 3*(4*a
^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^2 + 2*c)^3 + 6*((2*a^3*b^2 - a*b^4)*
cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 + 5*(2*a^3*b^2 - a*b^4)*
sin(c)^4)*cos(d*x^2 + 2*c)^2 - 6*(b^5*cos(d*x^2 + 2*c)^3*sin(c) - 5*(2*a^3*
b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 - (2*a^3*b^
2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^2 + 2*c)^
2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b^3 - b^5)*sin(c)^3)*cos(
d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 3*((16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^
4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2*sin(c)^3 + (16*a^4*b -
12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^2 + 2*c) + 3*(b^5*cos(d*x^2 + 2*c)^4*co
s(c) - 8*a*b^4*cos(d*x^2 + 2*c)^3*cos(c)*sin(c) + (16*a^4*b - 12*a^2*b^3 +
b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (16*a^4
*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*b^3 - b^5)*cos(c)^3 + 3*
(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^2 + 2*c)^2 - 16*((2*a^3*b^2 - a*
b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c
))*sin(d*x^2 + 2*c))*sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Fricas [A] time = 2.10674, size = 458, normalized size = 9.54

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(dx^2+c\right)^2-2ab\sin\left(dx^2+c\right)-a^2-b^2+2\left(a\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)+b\cos\left(dx^2+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx^2+c\right)^2-2ab\sin\left(dx^2+c\right)-a^2-b^2}\right)}{4\left(a^2-b^2\right)d}, -\frac{\arctan\left(-\frac{a\sin\left(dx^2+c\right)}{\sqrt{a^2-b^2}\cos\left(dx\right.}\right.}{2\sqrt{a^2-b^2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^
2 + c) - a^2 - b^2 + 2*(a*cos(d*x^2 + c)*sin(d*x^2 + c) + b*cos(d*x^2 + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2
))/((a^2 - b^2)*d), -1/2*arctan(-(a*sin(d*x^2 + c) + b)/(sqrt(a^2 - b^2)*co
s(d*x^2 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [A] time = 16.0437, size = 192, normalized size = 4.

$$\begin{cases} \frac{x^2}{2(a+b\sin(c))} & \text{for } d=0 \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)\right)}{2bd} & \text{for } a=0 \\ \frac{\sqrt{b^2}}{b^2 d\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)-bd\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{b^2 d\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)+bd\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^2}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{2d\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Piecewise((x**2/(2*(a + b*sin(c))), Eq(d, 0)), (log(tan(c/2 + d*x**2/2))/(2
*b*d), Eq(a, 0)), (sqrt(b**2)/(b**2*d*tan(c/2 + d*x**2/2) - b*d*sqrt(b**2))
, Eq(a, -sqrt(b**2))), (-sqrt(b**2)/(b**2*d*tan(c/2 + d*x**2/2) + b*d*sqrt(
b**2)), Eq(a, sqrt(b**2))), (log(tan(c/2 + d*x**2/2) + b/a - sqrt(-a**2 + b
**2)/a)/(2*d*sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**2/2) + b/a + sqrt(-a*
*2 + b**2)/a)/(2*d*sqrt(-a**2 + b**2)), True))
```

Giac [A] time = 1.11353, size = 85, normalized size = 1.77

$$\frac{\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2*
c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

3.38 $\int \frac{1}{x(a+b\sin(c+dx^2))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Sin[c + d*x^2])), x]
```

Rubi [A] time = 0.0275942, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^2])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [A] time = 0.394565, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^2])),x]
```

```
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^2])), x]
```

Maple [A] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(dx^2+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(1/x/(a+b*sin(d*x^2+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx\sin\left(dx^2+c\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*sin(d*x^2 + c) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**2))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x), x)
```

3.39 $\int \frac{1}{x^3(a+b\sin(c+dx^2))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

[Out] Unintegrable[1/(x^3*(a + b*Sin[c + d*x^2])), x]

Rubi [A] time = 0.0274094, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^3*(a + b*Sin[c + d*x^2])),x]

[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^2])), x]

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [A] time = 0.350616, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])),x]

[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])), x]

Maple [A] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(dx^2+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b*sin(d*x^2+c)),x)

[Out] int(1/x^3/(a+b*sin(d*x^2+c)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^3\sin\left(dx^2+c\right)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^3*sin(d*x^2 + c) + a*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**2))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^3), x)
```

3.40 $\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^2}{a+b\sin\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable[x^2/(a + b*Sin[c + d*x^2]), x]
```

Rubi [A] time = 0.0288569, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^2/(a + b*Sin[c + d*x^2]), x]
```

Rubi steps

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Mathematica [A] time = 0.364938, size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^2]),x]
```

```
[Out] Integrate[x^2/(a + b*Sin[c + d*x^2]), x]
```

Maple [A] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\sin\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(x^2/(a+b*sin(d*x^2+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{b\sin\left(dx^2+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*sin(d*x^2 + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(x**2/(a + b*sin(c + d*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a), x)
```

3.41 $\int \frac{1}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{a+b\sin\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x^2])^(-1), x]
```

Rubi [A] time = 0.0052529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^2])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^2])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Mathematica [A] time = 0.0234969, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^(-1),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x^2])^(-1), x]
```

Maple [A] time = 0.032, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(dx^2+c\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(1/(a+b*sin(d*x^2+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x^2 + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\sin\left(dx^2+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*sin(d*x^2 + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x^2 + c) + a), x)
```

3.42 $\int \frac{1}{x^2(a+b\sin(c+dx^2))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Rubi [A] time = 0.0270504, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^2])),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Mathematica [A] time = 0.267629, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])),x]
```

```
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])), x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(dx^2+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

```
[Out] int(1/x^2/(a+b*sin(d*x^2+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^2\sin\left(dx^2+c\right)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*sin(d*x^2 + c) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**2))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)*x^2), x)
```

3.43 $\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=663

$$-\frac{ax^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{ax^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{i\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\left(a^2-b^2\right)}+\frac{i\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\left(a^2-b^2\right)}-\frac{ia\text{Poly}}{d^3}$$

```
[Out] ((I/2)*x^4)/((a^2 - b^2)*d) - (x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqr
t[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*a*x^4*Log[1 - (I*b*E^(I*(c + d*x
^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (x^2*Log[1 - (I*b*E^(
I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + ((I/2)*a*x^4*Lo
g[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d)
 + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d^3) - (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])
/((a^2 - b^2)^(3/2)*d^2) + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
a^2 - b^2])])/((a^2 - b^2)*d^3) + (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (I*a*PolyLog[3, (I*b*E^(
I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) + (I*a*Poly
Log[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d
^3) + (b*x^4*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rubi [A] time = 1.30189, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3379, 3324, 3323, 2264, 2190, 2531, 2282, 6589, 4519, 2279, 2391}

$$-\frac{ax^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{ax^2\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^2\left(a^2-b^2\right)^{3/2}}+\frac{i\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{a-\sqrt{a^2-b^2}}\right)}{d^3\left(a^2-b^2\right)}+\frac{i\text{PolyLog}\left(2,\frac{ibe^{i(c+dx^2)}}{\sqrt{a^2-b^2}+a}\right)}{d^3\left(a^2-b^2\right)}-\frac{ia\text{Poly}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^2])^2,x]
```


```
[Out] ((I/2)*x^4)/((a^2 - b^2)*d) - (x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqr
t[a^2 - b^2])])/((a^2 - b^2)*d^2) - ((I/2)*a*x^4*Log[1 - (I*b*E^(I*(c + d*x
^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (x^2*Log[1 - (I*b*E^(
I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*d^2) + ((I/2)*a*x^4*Lo
g[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d)
 + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^
2)*d^3) - (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])
/((a^2 - b^2)^(3/2)*d^2) + (I*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[
a^2 - b^2])])/((a^2 - b^2)*d^3) + (a*x^2*PolyLog[2, (I*b*E^(I*(c + d*x^2)))
/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^2) - (I*a*PolyLog[3, (I*b*E^(
I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d^3) + (I*a*Poly
Log[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d
^3) + (b*x^4*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
```

```
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{bx^4\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2\left(a^2-b^2\right)} - \frac{b\,\mathrm{Subst}\left(\int \frac{x\cos(\cdots}{a+b\sin\cdots}\right.}{\left(a^2-b\cdots\right.}$$

$$= \frac{ix^4}{2\left(a^2-b^2\right)d} + \frac{bx^4\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{a\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x\cdots\right.}{a^2-b^2}$$

$$= \frac{ix^4}{2\left(a^2-b^2\right)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} + \frac{bx^4\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+\cdots\right.\right.}$$

$$= \frac{ix^4}{2\left(a^2-b^2\right)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} + \cdots$$

$$= \frac{ix^4}{2\left(a^2-b^2\right)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} + \cdots$$

$$= \frac{ix^4}{2\left(a^2-b^2\right)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} + \cdots$$

$$= \frac{ix^4}{2\left(a^2-b^2\right)d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} - \frac{iax^4\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{x^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)d^2} + \cdots$$

Mathematica [A] time = 2.3089, size = 513, normalized size = 0.77

$$\frac{\left(-\frac{2adx^2}{\sqrt{a^2-b^2}}+2i\right)\mathrm{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}-a}\right)+\left(\frac{2adx^2}{\sqrt{a^2-b^2}}+2i\right)\mathrm{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)-\frac{2ia\mathrm{PolyLog}\left(3,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{2ia\mathrm{PolyLog}\left(3,\frac{ibe^{i\left(c+\cdots\right)}}{\sqrt{a^2-b\cdots}}\right)}{\sqrt{a^2-b^2}}\cdots}{2d^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (I*d^2*x^4 - 2*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])
] - (I*a*d^2*x^4*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/S
qrt[a^2 - b^2] - 2*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^
2])] + (I*a*d^2*x^4*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])
/Sqrt[a^2 - b^2] + (2*I - (2*a*d*x^2)/Sqrt[a^2 - b^2])*PolyLog[2, ((-I)*b*E
^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])] + (2*I + (2*a*d*x^2)/Sqrt[a^2 - b
^2])*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])] - ((2*I)*a*P
olyLog[3, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2] +
 ((2*I)*a*PolyLog[3, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/Sqrt[a
^2 - b^2] + (b*d^2*x^4*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a^2 - b^
2)*d^3)
```

Maple [F] time = 0.669, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a + b\sin\left(dx^2 + c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^5/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.06118, size = 5577, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a^2*b - b^3)*d^2*x^4*cos(d*x^2 + c) - 4*(a*b^2*sin(d*x^2 + c) + a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, 1/2*(2*I*a*cos(d*x^2 + c) - 2*a*sin(d
*x^2 + c) + 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2
))/b) + 4*(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3,
1/2*(2*I*a*cos(d*x^2 + c) - 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) + I*b*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) + 4*(a*b^2*sin(d*x^2 + c) + a^2*
```

```
b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(d*x^2 + c) + a*sin(d*x^2 + c
) + (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))/b) - 4*
(a*b^2*sin(d*x^2 + c) + a^2*b)*sqrt(-(a^2 - b^2)/b^2)*polylog(3, -(I*a*cos(
d*x^2 + c) + a*sin(d*x^2 + c) - (b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2))/b) + (-4*I*a^3 + 4*I*a*b^2 + (-4*I*a^2*b + 4*I*b^3)*si
n(d*x^2 + c) + 2*(2*I*a*b^2*d*x^2*sin(d*x^2 + c) + 2*I*a^2*b*d*x^2)*sqrt(-(
a^2 - b^2)/b^2))*dilog(-1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*
(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1
) + (-4*I*a^3 + 4*I*a*b^2 + (-4*I*a^2*b + 4*I*b^3)*sin(d*x^2 + c) + 2*(-2*I
*a*b^2*d*x^2*sin(d*x^2 + c) - 2*I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilo
g(-1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) - I
*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (4*I*a^3 - 4*I*a*
b^2 + (4*I*a^2*b - 4*I*b^3)*sin(d*x^2 + c) + 2*(-2*I*a*b^2*d*x^2*sin(d*x^2
 + c) - 2*I*a^2*b*d*x^2)*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(-2*I*a*cos(d*x^
2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (4*I*a^3 - 4*I*a*b^2 + (4*I*a^2*b - 4*I
*b^3)*sin(d*x^2 + c) + 2*(2*I*a*b^2*d*x^2*sin(d*x^2 + c) + 2*I*a^2*b*d*x^2)
*sqrt(-(a^2 - b^2)/b^2))*dilog(-1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2
 + c) - 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b + 1) + 2*(2*(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b^2)*c + (a*b
^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^
2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(2*
(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*(a^3 - a*b^2)*c + (a*b^2*c^2*sin(d*x^2 +
 c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^2 + c) - 2*I*b*sin
(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) + 2*(2*(a^2*b - b^3)*c*si
n(d*x^2 + c) + 2*(a^3 - a*b^2)*c - (a*b^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*s
qrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b
*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*(2*(a^2*b - b^3)*c*sin(d*x^2 + c) + 2*
(a^3 - a*b^2)*c - (a*b^2*c^2*sin(d*x^2 + c) + a^2*b*c^2)*sqrt(-(a^2 - b^2)/
b^2))*log(-2*b*cos(d*x^2 + c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) - 2*I*a) - 2*(2*(a^3 - a*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c + 2*((a^2*b
 - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) - (a^2*b*d^2*x^4 - a^2*b*c^2
 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))*log(
1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) - I*b*
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(2*(a^3 - a*b^2)*d*x^2
 + 2*(a^3 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2
 + c) + (a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 +
 c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2
 + c) - 2*(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2
*b)/b) - 2*(2*(a^3 - a*b^2)*d*x^2 + 2*(a^3 - a*b^2)*c + 2*((a^2*b - b^3)*d*
x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) - (a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2
*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2))*log(1/2*(-2*I
*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^
2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(2*(a^3 - a*b^2)*d*x^2 + 2*(a^
3 - a*b^2)*c + 2*((a^2*b - b^3)*d*x^2 + (a^2*b - b^3)*c)*sin(d*x^2 + c) + (
a^2*b*d^2*x^4 - a^2*b*c^2 + (a*b^2*d^2*x^4 - a*b^2*c^2)*sin(d*x^2 + c))*sqr
t(-(a^2 - b^2)/b^2))*log(1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) -
2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b))
/((a^4*b - 2*a^2*b^3 + b^5)*d^3*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*
d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sin(d*x^2 + c) + a)^2, x)
```

3.44 $\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=324

$$-\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\left(a^2-b^2\right)^{3/2}}+\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d^2\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2d^2\left(a^2-b^2\right)}-\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d\left(a^2-b^2\right)^{3/2}}+\frac{iax^2\log\left(1-\right.}{2d\left(a^2-\right.}$$

```
[Out] ((-I/2)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + ((I/2)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a
^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^2]]/(2*(a^2 - b^
2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(2*
(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^
2 - b^2])])/(2*(a^2 - b^2)^(3/2)*d^2) + (b*x^2*Cos[c + d*x^2])/(2*(a^2 - b^
2)*d*(a + b*Sin[c + d*x^2]))
```

Rubi [A] time = 0.596076, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3379, 3324, 3323, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d^2\left(a^2-b^2\right)^{3/2}}+\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{2d^2\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2d^2\left(a^2-b^2\right)}-\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2d\left(a^2-b^2\right)^{3/2}}+\frac{iax^2\log\left(1-\right.}{2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((-I/2)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + ((I/2)*a*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a
^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^2]]/(2*(a^2 - b^
2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a - Sqrt[a^2 - b^2])])/(2*
(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^
2 - b^2])])/(2*(a^2 - b^2)^(3/2)*d^2) + (b*x^2*Cos[c + d*x^2])/(2*(a^2 - b^
2)*d*(a + b*Sin[c + d*x^2]))
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^3}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{bx^2\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2\left(a^2-b^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{\cos(c\cdot}{a+b\sin}\right.}{2\left(a^2-\right.}$$

$$= \frac{bx^2\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2-b^2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2\left(a^2-b^2\right)d^2} + \frac{bx^2\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)} - \frac{(iab)\,\text{Subst}\left(\int \frac{e^{i(c+}}{2a-2\sqrt{a^2-b}}\right.}{\left(a^2-b^2\right.}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2\left(a^2-b^2\right)d^2} + \frac{b}{2\left(a^2-b\right.}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2\left(a^2-b^2\right)d^2} + \frac{b}{2\left(a^2-b\right.}$$

$$= -\frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a-\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} + \frac{iax^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{a+\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}d} - \frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{2\left(a^2-b^2\right)d^2} - \frac{a\text{Li}_2\left(\frac{ibe^{i}}{a-}\right.}{2\left(a^2-b\right.}$$

Mathematica [A] time = 0.949466, size = 302, normalized size = 0.93

$$\frac{-\frac{a\text{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}-a}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{iadx^2\log\left(1+\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}-a}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{iadx^2\log\left(1-\frac{ibe^{i\left(c+dx^2\right)}}{\sqrt{a^2-b^2}+a}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{\log\left(a+b\sin\left(c+dx^2\right)\right)}{a^2-b^2} + \frac{bdx^2\cos(c+}{\left(a^2-b^2\right)\left(a+b\sin\right.}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (((-I)*a*d*x^2*Log[1 + (I*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (I*a*d*x^2*Log[1 - (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 -
 b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^2]]/(a^2 - b^2) - (a*Pol
yLog[2, ((-I)*b*E^(I*(c + d*x^2)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)^(3/
2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^2)))/(a + Sqrt[a^2 - b^2])])/(a^2 - b
^2)^(3/2) + (b*d*x^2*Cos[c + d*x^2])/((a^2 - b^2)*(a + b*Sin[c + d*x^2])))/
(2*d^2)
```

Maple [F] time = 0.586, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+b\sin\left(dx^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^3/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.97458, size = 3438, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*b - b^3)*d*x^2*cos(d*x^2 + c) + (I*a*b^2*sin(d*x^2 + c) + I*a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2
 + c) + 2*(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + (-I*a*b^2*sin(d*x^2 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*di
log(-1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) -
 I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-I*a*b^2*sin(d
*x^2 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^2 +
c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(
a^2 - b^2)/b^2) + 2*b)/b + 1) + (I*a*b^2*sin(d*x^2 + c) + I*a^2*b)*sqrt(-(a
^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(
b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
 + (a^2*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(
a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*c
os(d*x^2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^2
*b*d*x^2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b
^2)/b^2)*log(1/2*(2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^
2 + c) - I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (a^2*b*d*x^
2 + a^2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2
)*log(1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) + 2*(b*cos(d*x^2 + c)
 + I*b*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^2*b*d*x^2 + a^
2*b*c + (a*b^2*d*x^2 + a*b^2*c)*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2)*log(
1/2*(-2*I*a*cos(d*x^2 + c) + 2*a*sin(d*x^2 + c) - 2*(b*cos(d*x^2 + c) + I*b
*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^3 - a*b^2 + (a^2*b -
 b^3)*sin(d*x^2 + c) + (a*b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)
/b^2))*log(2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c) + (a*b^2*c*si
n(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^2 + c) - 2*
I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^3 - a*b^2 + (
a^2*b - b^3)*sin(d*x^2 + c) - (a*b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2
 - b^2)/b^2))*log(-2*b*cos(d*x^2 + c) + 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a
^2 - b^2)/b^2) + 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^2 + c) - (a*
b^2*c*sin(d*x^2 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^2
+ c) - 2*I*b*sin(d*x^2 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b
- 2*a^2*b^3 + b^5)*d^2*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sin(d*x^2 + c) + a)^2, x)
```

3.45 $\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^2\right)}{2d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}$$

```
[Out] (a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*d
) + (b*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```

Rubi [A] time = 0.102029, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 2664, 12, 2660, 618, 204}

$$\frac{a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^2\right)}{2d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sin[c + d*x^2])^2,x]
```


```
[Out] (a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*d
) + (b*Cos[c + d*x^2])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x^2]))
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+b\sin(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{\text{Subst}\left(\int \frac{a}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2\left(a^2-b^2\right)}\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{a\,\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx, x, x^2\right)}{2\left(a^2-b^2\right)}\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}+\frac{a\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{\left(a^2-b^2\right)d}\\
&= \frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}\left(c+d\right.\right.\right.}{\left(a^2-b^2\right)d}\\
&= \frac{a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}+\frac{b\cos\left(c+dx^2\right)}{2\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^2\right)\right)}
\end{aligned}$$

Mathematica [A] time = 0.205112, size = 91, normalized size = 1.

$$\frac{\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b\cos\left(c+dx^2\right)}{a+b\sin\left(c+dx^2\right)}}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (b*Cos[c + d*x^2])/(a + b*Sin[c + d*x^2]))/(2*(a - b)*(a + b)*d)
```

Maple [A] time = 0.041, size = 164, normalized size = 1.8

$$\frac{b^2}{da\left(a^2-b^2\right)}\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\left(\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)^2a+2\tan\left(1/2\,dx^2+c/2\right)b+a\right)^{-1}+\frac{b}{d\left(a^2-b^2\right)}\left(\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)^2a+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] 1/d/(tan(1/2*d*x^2+1/2*c)^2*a+2*tan(1/2*d*x^2+1/2*c)*b+a)*b^2/a/(a^2-b^2)*t
an(1/2*d*x^2+1/2*c)+1/d/(tan(1/2*d*x^2+1/2*c)^2*a+2*tan(1/2*d*x^2+1/2*c)*b+
a)/(a^2-b^2)*b+1/d*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x^2+1/2*c)+2
*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.10745, size = 798, normalized size = 8.77

$$\left[\frac{\left(ab\sin\left(dx^2+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(dx^2+c\right)^2-2ab\sin\left(dx^2+c\right)-a^2-b^2-2\left(a\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)+b\cos\left(dx^2+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx^2+c\right)^2-2ab\sin\left(dx^2+c\right)-a^2-b^2}\right.}{4\left(\left(a^4b-2a^2b^3+b^5\right)d\sin\left(dx^2+c\right)+\left(a^5-2a^3b^2+ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*sin(d*x^2 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d
*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2 - 2*(a*cos(d*x^2 + c)*sin(d*
x^2 + c) + b*cos(d*x^2 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^2 + c)^2 - 2*a*
b*sin(d*x^2 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^2 + c))/((a^4*b -
2*a^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/2*((a*
b*sin(d*x^2 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^2 + c) + b)/(sqr
t(a^2 - b^2)*cos(d*x^2 + c))) - (a^2*b - b^3)*cos(d*x^2 + c))/((a^4*b - 2*a
^2*b^3 + b^5)*d*sin(d*x^2 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11736, size = 194, normalized size = 2.13

$$\frac{\left(\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2d-b^2d\right)\sqrt{a^2-b^2}}+\frac{b^2\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+ab}{\left(a^3d-ab^2d\right)\left(a\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^2 + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + (b^2*tan(1/
2*d*x^2 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^2 + 1/2*c)^2 + 2*
b*tan(1/2*d*x^2 + 1/2*c) + a))
```

3.46 $\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi [A] time = 0.0463497, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 5.92258, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [A] time = 0.598, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(dx^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(1/x/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x\cos\left(dx^2+c\right)^2-2\,abx\sin\left(dx^2+c\right)-\left(a^2+b^2\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x*cos(d*x^2 + c)^2 - 2*a*b*x*sin(d*x^2 + c) - (a^2 + b^2)*
x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**2))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x), x)
```

3.47 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi [A] time = 0.0275243, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 8.16858, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [A] time = 0.921, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(dx^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^2+c))^2,x)
```

[Out] int(1/x^3/(a+b*sin(d*x^2+c))^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x^3\cos\left(dx^2+c\right)^2-2\,abx^3\sin\left(dx^2+c\right)-\left(a^2+b^2\right)x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")

[Out] integral(-1/(b^2*x^3*cos(d*x^2 + c)^2 - 2*a*b*x^3*sin(d*x^2 + c) - (a^2 + b^2)*x^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b*sin(d*x**2+c))**2,x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")

[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x^3), x)

3.48 $\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[x^2/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi [A] time = 0.027027, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^2/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 4.19227, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] Integrate[x^2/(a + b*Sin[c + d*x^2])^2, x]
```

Maple [A] time = 0.371, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+b\sin\left(dx^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(x^2/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2}{b^2\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-x^2/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral(x**2/(a + b*sin(c + d*x**2))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin(d*x^2 + c) + a)^2, x)
```

3.49 $\int \frac{1}{(a+b\sin(c+dx^2))^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x^2])^(-2), x]
```

Rubi [A] time = 0.0065561, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^2])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^2])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 4.80739, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^2])^(-2),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x^2])^(-2), x]
```

Maple [A] time = 0.509, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(dx^2+c\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(1/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{b^2\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**2))**(-2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^(-2), x)
```

3.50 $\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi [A] time = 0.0254166, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 7.19191, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2),x]
```

```
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^2])^2), x]
```

Maple [A] time = 0.696, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(dx^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int(1/x^2/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x^2\cos\left(dx^2+c\right)^2-2\,abx^2\sin\left(dx^2+c\right)-\left(a^2+b^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos(d*x^2 + c)^2 - 2*a*b*x^2*sin(d*x^2 + c) - (a^2 + b
^2)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(dx^2+c\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^2 + c) + a)^2*x^2), x)
```

3.51 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Rubi [A] time = 0.0267236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]

[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx=\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx$$

Mathematica [A] time = 0.846408, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p,x]

[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^p, x]
```

Maple [A] time = 0.723, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^2+c\right)\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)

[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**p,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(b\sin\left(dx^2+c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^2 + c) + a)^p, x)
```

3.52 $\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\,dx$

Optimal. Leaf size=444

$$\frac{3ibe^{ic}\left(4a^2+b^2\right)\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{16e}-\frac{3ibe^{-ic}\left(4a^2+b^2\right)\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+}{2}\right.}{16e}$$

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + (((3*I)/16)*b*(4*a^2 + b^
2)*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*
x^2])/e - (((3*I)/16)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c)) + (3*2^(-7/2 - m/2)*a*b^2*E^((2*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2])/e
 + (3*2^(-7/2 - m/2)*a*b^2*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 +
m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c)) - ((I/16)*3^(-1/2 - m/2)*b^3*E^((3*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2])/e
 + ((I/16)*3^(-1/2 - m/2)*b^3*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1
 + m)/2, (3*I)*d*x^2])/(e*E^((3*I)*c))
```

Rubi [A] time = 0.479603, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3403, 6, 3390, 2218, 3389}

$$\frac{3ibe^{ic}\left(4a^2+b^2\right)\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{16e}-\frac{3ibe^{-ic}\left(4a^2+b^2\right)\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+}{2}\right.}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + (((3*I)/16)*b*(4*a^2 + b^
2)*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*
x^2])/e - (((3*I)/16)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c)) + (3*2^(-7/2 - m/2)*a*b^2*E^((2*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2])/e
 + (3*2^(-7/2 - m/2)*a*b^2*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 +
m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c)) - ((I/16)*3^(-1/2 - m/2)*b^3*E^((3*I)*c
)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2])/e
 + ((I/16)*3^(-1/2 - m/2)*b^3*(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1
 + m)/2, (3*I)*d*x^2])/(e*E^((3*I)*c))
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3 dx = \int \left(a^3(ex)^m + \frac{3}{2}ab^2(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right) + 3a^2b(ex)^m\sin\left(c+dx^2\right) + \frac{3}{4}\right.$$

$$= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right) + 3a^2b(ex)^m\sin\left(c+dx^2\right) + \frac{3}{4}b^3(\right.$$

$$= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^2\right) + \left(3a^2b+\frac{3b^3}{4}\right)(ex)^m\sin\left(c+dx\right.\right.$$

$$= \frac{a\left(2a^2+3b^2\right)(ex)^{1+m}}{2e(1+m)} - \frac{1}{2}\left(3ab^2\right)\int (ex)^m\cos\left(2c+2dx^2\right)dx - \frac{1}{4}b^3\int (ex)^m\sin\left(3c+\right.$$

$$= \frac{a\left(2a^2+3b^2\right)(ex)^{1+m}}{2e(1+m)} - \frac{1}{4}\left(3ab^2\right)\int e^{-2ic-2idx^2}(ex)^m\,dx - \frac{1}{4}\left(3ab^2\right)\int e^{2ic+2idx^2}(ex)^m\,d$$

$$= \frac{a\left(2a^2+3b^2\right)(ex)^{1+m}}{2e(1+m)} + \frac{3ib\left(4a^2+b^2\right)e^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{16e} - \frac{3i}{}$$

Mathematica [A] time = 8.56236, size = 373, normalized size = 0.84

$$\frac{1}{16}ix(ex)^m\left(3be^{ic}\left(4a^2+b^2\right)\left(-idx^2\right)^{-\frac{m}{2}-\frac{1}{2}}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right) - 3be^{-ic}\left(4a^2+b^2\right)\left(idx^2\right)^{-\frac{m}{2}-\frac{1}{2}}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^3,x]
```

```
[Out] (I/16)*x*(e*x)^m*(((-8*I)*a*(2*a^2 + 3*b^2))/(1 + m) + 3*b*(4*a^2 + b^2)*E^
(I*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-I)*d*x^2] - (3*b*(4*a^2
+ b^2)*(I*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, I*d*x^2])/E^(I*c) - (3*I)*2^
(1/2 - m/2)*a*b^2*E^((2*I)*c)*((-I)*d*x^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-
2*I)*d*x^2] - ((3*I)*2^(1/2 - m/2)*a*b^2*(I*d*x^2)^(-1/2 - m/2)*Gamma[(1 +
m)/2, (2*I)*d*x^2])/E^((2*I)*c) - 3^(-1/2 - m/2)*b^3*E^((3*I)*c)*((-I)*d*x^
2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (-3*I)*d*x^2] + (3^(-1/2 - m/2)*b^3*(I*d*x
^2)^(-1/2 - m/2)*Gamma[(1 + m)/2, (3*I)*d*x^2])/E^((3*I)*c))
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^2+c\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35579, size = 894, normalized size = 2.01

$$\frac{24\left(2a^3+3ab^2\right)(ex)^m dx+\left(b^3em+b^3e\right)e^{\left(-\frac{1}{2}(m-1)\log\left(\frac{3id}{e^2}\right)-3ic\right)}\Gamma\left(\frac{1}{2}m+\frac{1}{2},3idx^2\right)+\left(-9iab^2em-9iab^2e\right)e^{\left(-\frac{1}{2}(m-1)\log\left(\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(24*(2*a^3 + 3*a*b^2)*(e*x)^m*d*x + (b^3*e*m + b^3*e)*e^(-1/2*(m - 1)*
log(3*I*d/e^2) - 3*I*c)*gamma(1/2*m + 1/2, 3*I*d*x^2) + (-9*I*a*b^2*e*m - 9
*I*a*b^2*e)*e^(-1/2*(m - 1)*log(2*I*d/e^2) - 2*I*c)*gamma(1/2*m + 1/2, 2*I*
d*x^2) - 9*((4*a^2*b + b^3)*e*m + (4*a^2*b + b^3)*e)*e^(-1/2*(m - 1)*log(I*
d/e^2) - I*c)*gamma(1/2*m + 1/2, I*d*x^2) - 9*((4*a^2*b + b^3)*e*m + (4*a^2
*b + b^3)*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I*c)*gamma(1/2*m + 1/2, -I*d*x
^2) + (9*I*a*b^2*e*m + 9*I*a*b^2*e)*e^(-1/2*(m - 1)*log(-2*I*d/e^2) + 2*I*c
)*gamma(1/2*m + 1/2, -2*I*d*x^2) + (b^3*e*m + b^3*e)*e^(-1/2*(m - 1)*log(-3
*I*d/e^2) + 3*I*c)*gamma(1/2*m + 1/2, -3*I*d*x^2))/(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**3,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^2+c\right)+a\right)^3 (ex)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^3*(e*x)^m, x)
```

3.53 $\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right)^2 \, dx$

Optimal. Leaf size=279

$$\frac{iabe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{2e}-\frac{iabe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)}{2e}+\frac{b^2e^{2ic}2^{-\frac{m}{2}-}}{}$$

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/2)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/2)*a*b*
(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
 + (2^(-7/2 - m/2)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, (-2*I)*d*x^2])/e + (2^(-7/2 - m/2)*b^2*(e*x)^(1 + m)*(I*d*
x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c))
```

Rubi [A] time = 0.2631, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3403, 6, 3390, 2218, 3389}

$$\frac{iabe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-idx^2\right)}{2e}-\frac{iabe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)}{2e}+\frac{b^2e^{2ic}2^{-\frac{m}{2}-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/2)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/2)*a*b*
(e*x)^(1 + m)*(I*d*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
 + (2^(-7/2 - m/2)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^2)^((-1 - m)/2)*
Gamma[(1 + m)/2, (-2*I)*d*x^2])/e + (2^(-7/2 - m/2)*b^2*(e*x)^(1 + m)*(I*d*
x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2])/(e*E^((2*I)*c))
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)^2 dx = \int \left(a^2(ex)^m + \frac{1}{2}b^2(ex)^m - \frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^2\right) + 2ab(ex)^m\sin\left(c+dx^2\right)\right) dx$$

$$= \int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m - \frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^2\right) + 2ab(ex)^m\sin\left(c+dx^2\right)\right) dx$$

$$= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + (2ab)\int (ex)^m\sin\left(c+dx^2\right) dx - \frac{1}{2}b^2\int (ex)^m\cos\left(2c+2dx^2\right) d$$

$$= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + (iab)\int e^{-ic-idx^2}(ex)^m dx - (iab)\int e^{ic+idx^2}(ex)^m dx - \frac{1}{4}b^2\int e^{-2i}$$

$$= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{2e} - \frac{iabe^{-ic}(ex)^{1+m}\left(i\right.}{}$$

Mathematica [A] time = 6.51348, size = 551, normalized size = 1.97

$$\frac{2^{\frac{1}{2}(-m-7)}x\left(d^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left(-iab2^{\frac{m+5}{2}}(m+1)(\cos(c)-i\sin(c))\left(-idx^2\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)+iab2^{\frac{m+5}{2}}(m+1)(\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] (2^((-7 - m)/2)*x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(2^((7 + m)/2)*a^2*(d^2*x^
4)^((1 + m)/2) + 2^((5 + m)/2)*b^2*(d^2*x^4)^((1 + m)/2) + b^2*(I*d*x^2)^((
1 + m)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*m*(I*d*x^2)^((1 + m
)/2)*Cos[2*c]*Gamma[(1 + m)/2, (-2*I)*d*x^2] + b^2*((-I)*d*x^2)^((1 + m)/2)
*Cos[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] + b^2*m*((-I)*d*x^2)^((1 + m)/2)*Co
s[2*c]*Gamma[(1 + m)/2, (2*I)*d*x^2] - I*2^((5 + m)/2)*a*b*(1 + m)*((-I)*d*
x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*2^((5 +
m)/2)*a*b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c
] + I*Sin[c]) + I*b^2*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2]*
Sin[2*c] + I*b^2*m*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-2*I)*d*x^2]*Sin
[2*c] - I*b^2*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2]*Sin[2*
c] - I*b^2*m*((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (2*I)*d*x^2]*Sin[2*c
]))/(1 + m)
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^2+c\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03255, size = 555, normalized size = 1.99

$$8\left(2a^2+b^2\right)(ex)^m\,dx+\left(-ib^2em-ib^2e\right)e^{\left(-\frac{1}{2}(m-1)\log\left(\frac{2id}{e^2}\right)-2ic\right)}\Gamma\left(\frac{1}{2}m+\frac{1}{2},2idx^2\right)-8\left(abem+abe\right)e^{\left(-\frac{1}{2}(m-1)\log\left(\frac{id}{e^2}\right)-ic\right)}\Gamma$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*(2*a^2 + b^2)*(e*x)^m*d*x + (-I*b^2*e*m - I*b^2*e)*e^(-1/2*(m - 1)*
log(2*I*d/e^2) - 2*I*c)*gamma(1/2*m + 1/2, 2*I*d*x^2) - 8*(a*b*e*m + a*b*e)
*e^(-1/2*(m - 1)*log(I*d/e^2) - I*c)*gamma(1/2*m + 1/2, I*d*x^2) - 8*(a*b*e
*m + a*b*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I*c)*gamma(1/2*m + 1/2, -I*d*x^
2) + (I*b^2*e*m + I*b^2*e)*e^(-1/2*(m - 1)*log(-2*I*d/e^2) + 2*I*c)*gamma(1
/2*m + 1/2, -2*I*d*x^2))/(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m\left(a+b\sin\left(c+dx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^2+c\right)+a\right)^2(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)^2*(e*x)^m, x)
```

3.54 $\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right) dx$

Optimal. Leaf size=134

$$\frac{ibe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2}, -idx^2\right)}{4e} - \frac{ibe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2}, idx^2\right)}{4e} + \frac{a(ex)^{m+1}}{e(m+1)}$$

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/4)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)
^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/4)*b*(e*x)^(1 + m)*(I*d
*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
```

Rubi [A] time = 0.118672, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {14, 3389, 2218}

$$\frac{ibe^{ic}\left(-idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2}, -idx^2\right)}{4e} - \frac{ibe^{-ic}\left(idx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2}, idx^2\right)}{4e} + \frac{a(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/4)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^2)
^((-1 - m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2])/e - ((I/4)*b*(e*x)^(1 + m)*(I*d
*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, I*d*x^2])/(e*E^(I*c))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^2\right)\right)dx = \int \left(a(ex)^m + b(ex)^m \sin\left(c+dx^2\right)\right)dx$$
$$= \frac{a(ex)^{1+m}}{e(1+m)} + b\int (ex)^m \sin\left(c+dx^2\right)dx$$
$$= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{1}{2}(ib)\int e^{-ic-idx^2}(ex)^m\,dx - \frac{1}{2}(ib)\int e^{ic+idx^2}(ex)^m\,dx$$
$$= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{ibe^{ic}(ex)^{1+m}\left(-idx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-idx^2\right)}{4e} - \frac{ibe^{-ic}(ex)^{1+m}\left(idx^2\right)^{\frac{1}{2}(-1-m)}}{4e}$$

Mathematica [A] time = 1.57058, size = 149, normalized size = 1.11

$$\frac{x\left(d^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left(-ib(m+1)(\cos(c)-i\sin(c))\left(-idx^2\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2},idx^2\right)+ib(m+1)(\cos(c)+i\sin(c))\left(id\right.\right.}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^2]),x]
```

```
[Out] (x*(e*x)^m*(d^2*x^4)^((-1 - m)/2)*(4*a*(d^2*x^4)^((1 + m)/2) - I*b*(1 + m)*
((-I)*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, I*d*x^2]*(Cos[c] - I*Sin[c]) + I*
b*(1 + m)*(I*d*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, (-I)*d*x^2]*(Cos[c] + I*Si
n[c])))/(4*(1 + m))
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^2+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^2+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69436, size = 263, normalized size = 1.96

$$\frac{4\,(ex)^m\,adx-(bem+be)e^{\left(-\frac{1}{2}\,(m-1)\log\left(\frac{i\,d}{e^2}\right)-i\,c\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},i\,dx^2\right)-(bem+be)e^{\left(-\frac{1}{2}\,(m-1)\log\left(-\frac{i\,d}{e^2}\right)+i\,c\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-i\,dx^2\right)}{4\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(e*x)^m*a*d*x - (b*e*m + b*e)*e^(-1/2*(m - 1)*log(I*d/e^2) - I*c)*ga
mma(1/2*m + 1/2, I*d*x^2) - (b*e*m + b*e)*e^(-1/2*(m - 1)*log(-I*d/e^2) + I
*c)*gamma(1/2*m + 1/2, -I*d*x^2))/(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a + b\sin\left(c + dx^2\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^2 + c\right) + a\right)(ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^2 + c) + a)*(e*x)^m, x)
```

3.55 $\int \frac{(ex)^m}{a+b\sin(c+dx^2)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(ex)^m}{a+b\sin\left(c+dx^2\right)},x\right)$$

[Out] Unintegrable[(e*x)^m/(a + b*Sin[c + d*x^2]), x]

Rubi [A] time = 0.0277162, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(e*x)^m/(a + b*Sin[c + d*x^2]),x]

[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^2]), x]

Rubi steps

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx=\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Mathematica [A] time = 0.4123, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]),x]

[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2]), x]

Maple [A] time = 0.08, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{a+b\sin\left(dx^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m/(a+b*sin(d*x^2+c)),x)

[Out] int((e*x)^m/(a+b*sin(d*x^2+c)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{b\sin\left(dx^2+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**2+c)),x)
```

```
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{b\sin\left(dx^2+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^2 + c) + a), x)
```

3.56 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi [A] time = 0.02626, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [A] time = 0.658531, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2,x]
```

```
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^2])^2, x]
```

Maple [A] time = 0.224, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(a+b\sin\left(dx^2+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

```
[Out] int((e*x)^m/(a+b*sin(d*x^2+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ex)^m}{b^2\cos\left(dx^2+c\right)^2-2\,ab\sin\left(dx^2+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e*x)^m/(b^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2)
, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex)^m}{\left(a+b\sin\left(c+dx^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**2+c))**2,x)
```

```
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**2))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex)^m}{\left(b\sin\left(dx^2+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^2 + c) + a)^2, x)
```

3.57 $\int x^5 \left(a + b \sin \left(c + dx^3\right)\right) dx$

Optimal. Leaf size=44

$$\frac{ax^6}{6} + \frac{b \sin \left(c + dx^3\right)}{3d^2} - \frac{bx^3 \cos \left(c + dx^3\right)}{3d}$$

```
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Rubi [A] time = 0.0519132, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3379, 3296, 2637}

$$\frac{ax^6}{6} + \frac{b \sin \left(c + dx^3\right)}{3d^2} - \frac{bx^3 \cos \left(c + dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\sin\left(c+dx^3\right)\right)dx &= \int\left(ax^5+bx^5\sin\left(c+dx^3\right)\right)dx\\ &= \frac{ax^6}{6}+b\int x^5\sin\left(c+dx^3\right)dx\\ &= \frac{ax^6}{6}+\frac{1}{3}b\,\text{Subst}\left(\int x\sin(c+dx)\,dx,x,x^3\right)\\ &= \frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\,\text{Subst}\left(\int\cos(c+dx)\,dx,x,x^3\right)}{3d}\\ &= \frac{ax^6}{6}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}+\frac{b\sin\left(c+dx^3\right)}{3d^2}\end{aligned}$$

Mathematica [A] time = 0.0084399, size = 44, normalized size = 1.

$$\frac{ax^6}{6}+\frac{b\sin\left(c+dx^3\right)}{3d^2}-\frac{bx^3\cos\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^6)/6 - (b*x^3*Cos[c + d*x^3])/(3*d) + (b*Sin[c + d*x^3])/(3*d^2)
```

Maple [A] time = 0.019, size = 73, normalized size = 1.7

$$\frac{ax^6}{6}+b\left(-\frac{x^3}{3\,d}+\frac{2}{3\,d^2}\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)+\frac{x^3}{3\,d}\left(\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)^2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^3+c)),x)
```

```
[Out] 1/6*a*x^6+b*(-1/3*x^3/d+2/3/d^2*tan(1/2*d*x^3+1/2*c)+1/3*x^3/d*tan(1/2*d*x^
3+1/2*c)^2)/(1+tan(1/2*d*x^3+1/2*c)^2)
```

Maxima [A] time = 0.97892, size = 50, normalized size = 1.14

$$\frac{1}{6}\,ax^6-\frac{\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)b}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 - 1/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*b/d^2
```

Fricas [A] time = 1.75342, size = 93, normalized size = 2.11

$$\frac{ad^2x^6-2\,bdx^3\cos\left(dx^3+c\right)+2\,b\sin\left(dx^3+c\right)}{6\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^2*x^6 - 2*b*d*x^3*cos(d*x^3 + c) + 2*b*sin(d*x^3 + c))/d^2
```

Sympy [A] time = 4.5573, size = 49, normalized size = 1.11

$$\begin{cases} \frac{ax^6}{6} - \frac{bx^3\cos\left(c+dx^3\right)}{3d} + \frac{b\sin\left(c+dx^3\right)}{3d^2} & \text{for } d \neq 0 \\ \frac{x^6(a+b\sin(c))}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((a*x**6/6 - b*x**3*cos(c + d*x**3)/(3*d) + b*sin(c + d*x**3)/(3*d
**2), Ne(d, 0)), (x**6*(a + b*sin(c))/6, True))
```

Giac [A] time = 1.09509, size = 82, normalized size = 1.86

$$\frac{\frac{\left(\left(dx^3+c\right)^2-2\left(dx^3+c\right)c\right)a}{d} - \frac{2\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)b}{d}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 1/6*(((d*x^3 + c)^2 - 2*(d*x^3 + c)*c)*a/d - 2*(d*x^3*cos(d*x^3 + c) - sin(
d*x^3 + c))*b/d)/d
```

3.58 $\int x^2 \left(a+b \sin \left(c+d x^3\right)\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d}$$

```
[Out] (a*x^3)/3 - (b*Cos[c + d*x^3])/(3*d)
```

Rubi [A] time = 0.0269652, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {14, 3379, 2638}

$$\frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 - (b*Cos[c + d*x^3])/(3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a+b \sin \left(c+d x^3\right)\right) \, dx &= \int \left(ax^2+bx^2 \sin \left(c+dx^3\right)\right) \, dx\\
&= \frac{ax^3}{3}+b \int x^2 \sin \left(c+dx^3\right) \, dx\\
&= \frac{ax^3}{3}+\frac{1}{3} b \operatorname{Subst}\left(\int \sin (c+dx) \, dx,x,x^3\right)\\
&= \frac{ax^3}{3}-\frac{b \cos \left(c+dx^3\right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0215153, size = 41, normalized size = 1.64

$$\frac{ax^3}{3}+\frac{b\sin(c)\sin\left(dx^3\right)}{3d}-\frac{b\cos(c)\cos\left(dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 - (b*Cos[c]*Cos[d*x^3])/(3*d) + (b*Sin[c]*Sin[d*x^3])/(3*d)
```

Maple [A] time = 0.001, size = 27, normalized size = 1.1

$$\frac{a\left(dx^3+c\right)-b\cos\left(dx^3+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^3+c)),x)
```

```
[Out] 1/3/d*(a*(d*x^3+c)-b*cos(d*x^3+c))
```

Maxima [A] time = 0.965724, size = 28, normalized size = 1.12

$$\frac{1}{3}ax^3-\frac{b\cos\left(dx^3+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 - 1/3*b*cos(d*x^3 + c)/d
```

Fricas [A] time = 1.66614, size = 49, normalized size = 1.96

$$\frac{adx^3-b\cos\left(dx^3+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d*x^3 - b*cos(d*x^3 + c))/d
```

Sympy [A] time = 0.56678, size = 31, normalized size = 1.24

$$\begin{cases}\frac{ax^3}{3}-\frac{b\cos\left(c+dx^3\right)}{3d} & \text{for } d\neq 0\\ \frac{x^3(a+b\sin(c))}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((a*x**3/3 - b*cos(c + d*x**3)/(3*d), Ne(d, 0)), (x**3*(a + b*sin(
c))/3, True))
```

Giac [A] time = 1.09267, size = 35, normalized size = 1.4

$$\frac{\left(dx^3+c\right)a-b\cos\left(dx^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 1/3*((d*x^3 + c)*a - b*cos(d*x^3 + c))/d
```

3.59 $\int \frac{a+b\sin(c+dx^3)}{x}\,dx$

Optimal. Leaf size=31

$$a\log(x)+\frac{1}{3}b\sin(c)\text{CosIntegral}\left(dx^3\right)+\frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

```
[Out] a*Log[x] + (b*CosIntegral[d*x^3]*Sin[c])/3 + (b*Cos[c]*SinIntegral[d*x^3])/
3
```

Rubi [A] time = 0.0387281, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3377, 3376, 3375}

$$a\log(x)+\frac{1}{3}b\sin(c)\text{CosIntegral}\left(dx^3\right)+\frac{1}{3}b\cos(c)\text{Si}\left(dx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x,x]
```

```
[Out] a*Log[x] + (b*CosIntegral[d*x^3]*Sin[c])/3 + (b*Cos[c]*SinIntegral[d*x^3])/
3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx = \int \left(\frac{a}{x}+\frac{b\sin\left(c+dx^3\right)}{x}\right)dx$$
$$= a\log(x)+b\int \frac{\sin\left(c+dx^3\right)}{x}\,dx$$
$$= a\log(x)+(b\cos(c))\int \frac{\sin\left(dx^3\right)}{x}\,dx+(b\sin(c))\int \frac{\cos\left(dx^3\right)}{x}\,dx$$
$$= a\log(x)+\frac{1}{3}b\mathbf{Ci}\left(dx^3\right)\sin(c)+\frac{1}{3}b\cos(c)\mathbf{Si}\left(dx^3\right)$$

Mathematica [A] time = 0.0511973, size = 29, normalized size = 0.94

$$a\log(x)+\frac{1}{3}b\left(\sin(c)\mathbf{CosIntegral}\left(dx^3\right)+\cos(c)\mathbf{Si}\left(dx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x,x]
```

```
[Out] a*Log[x] + (b*(CosIntegral[d*x^3]*Sin[c] + Cos[c]*SinIntegral[d*x^3]))/3
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(dx^3+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x,x)
```

Maxima [C] time = 1.51283, size = 68, normalized size = 2.19

$$-\frac{1}{6}\left(\left(i\,\mathrm{Ei}\left(i\,dx^3\right)-i\,\mathrm{Ei}\left(-i\,dx^3\right)\right)\cos\left(c\right)-\left(\mathrm{Ei}\left(i\,dx^3\right)+\mathrm{Ei}\left(-i\,dx^3\right)\right)\sin\left(c\right)\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="maxima")
```

```
[Out] -1/6*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3)
)*sin(c))*b + a*log(x)
```

Fricas [A] time = 1.66128, size = 144, normalized size = 4.65

$$\frac{1}{3}\,b\cos\left(c\right)\mathrm{Si}\left(dx^3\right)+a\log\left(x\right)+\frac{1}{6}\left(b\,\mathrm{Ci}\left(dx^3\right)+b\,\mathrm{Ci}\left(-dx^3\right)\right)\sin\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*cos(c)*sin_integral(d*x^3) + a*log(x) + 1/6*(b*cos_integral(d*x^3) +
b*cos_integral(-d*x^3))*sin(c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x, x)
```

Giac [A] time = 1.14297, size = 43, normalized size = 1.39

$$\frac{1}{3}\,b\,\mathrm{Ci}\left(dx^3\right)\sin\left(c\right)+\frac{1}{3}\,b\cos\left(c\right)\mathrm{Si}\left(dx^3\right)+\frac{1}{3}\,a\log\left(dx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x,x, algorithm="giac")
```

```
[Out] 1/3*b*cos_integral(d*x^3)*sin(c) + 1/3*b*cos(c)*sin_integral(d*x^3) + 1/3*a
*log(d*x^3)
```

3.60 $\int \frac{a+b\sin(c+dx^3)}{x^4}\,dx$

Optimal. Leaf size=53

$$-\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{CosIntegral}\left(dx^3\right)-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)-\frac{b\sin\left(c+dx^3\right)}{3x^3}$$

```
[Out] -a/(3*x^3) + (b*d*Cos[c]*CosIntegral[d*x^3])/3 - (b*Sin[c + d*x^3])/(3*x^3)
 - (b*d*Sin[c]*SinIntegral[d*x^3])/3
```

Rubi [A] time = 0.101523, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 3379, 3297, 3303, 3299, 3302}

$$-\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{CosIntegral}\left(dx^3\right)-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)-\frac{b\sin\left(c+dx^3\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^4,x]
```

```
[Out] -a/(3*x^3) + (b*d*Cos[c]*CosIntegral[d*x^3])/3 - (b*Sin[c + d*x^3])/(3*x^3)
 - (b*d*Sin[c]*SinIntegral[d*x^3])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b\sin\left(c+dx^3\right)}{x^4}\right)dx \\
&= -\frac{a}{3x^3}+b\int \frac{\sin\left(c+dx^3\right)}{x^4}\,dx \\
&= -\frac{a}{3x^3}+\frac{1}{3}b\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx,x,x^3\right) \\
&= -\frac{a}{3x^3}-\frac{b\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}(bd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx,x,x^3\right) \\
&= -\frac{a}{3x^3}-\frac{b\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}(bd\cos(c))\,\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx,x,x^3\right)-\frac{1}{3}(bd\sin(c))\,\text{Subst} \\
&= -\frac{a}{3x^3}+\frac{1}{3}bd\cos(c)\text{Ci}\left(dx^3\right)-\frac{b\sin\left(c+dx^3\right)}{3x^3}-\frac{1}{3}bd\sin(c)\text{Si}\left(dx^3\right)
\end{aligned}$$

Mathematica [A] time = 0.0866847, size = 48, normalized size = 0.91

$$-\frac{a-bdx^3\cos(c)\text{CosIntegral}\left(dx^3\right)+bdx^3\sin(c)\text{Si}\left(dx^3\right)+b\sin\left(c+dx^3\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^4,x]
```

```
[Out] -(a - b*d*x^3*Cos[c]*CosIntegral[d*x^3] + b*Sin[c + d*x^3] + b*d*x^3*Sin[c]
*SinIntegral[d*x^3])/(3*x^3)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(dx^3+c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^4,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^4,x)
```

Maxima [C] time = 1.13674, size = 77, normalized size = 1.45

$$\frac{1}{6}\left(\left(\Gamma\left(-1,i\,dx^3\right)+\Gamma\left(-1,-i\,dx^3\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1,i\,dx^3\right)-i\,\Gamma\left(-1,-i\,dx^3\right)\right)\sin\left(c\right)\right)bd-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*x
^3) - I*gamma(-1, -I*d*x^3))*sin(c))*b*d - 1/3*a/x^3
```

Fricas [A] time = 1.70861, size = 197, normalized size = 3.72

$$-\frac{2\,bdx^3\sin\left(c\right)\operatorname{Si}\left(dx^3\right)-\left(bdx^3\operatorname{Ci}\left(dx^3\right)+bdx^3\operatorname{Ci}\left(-dx^3\right)\right)\cos\left(c\right)+2\,b\sin\left(dx^3+c\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*d*x^3*sin(c)*sin_integral(d*x^3) - (b*d*x^3*cos_integral(d*x^3) +
 b*d*x^3*cos_integral(-d*x^3))*cos(c) + 2*b*sin(d*x^3 + c) + 2*a)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**4, x)
```

Giac [B] time = 1.11348, size = 134, normalized size = 2.53

$$\frac{\left(dx^3+c\right)bd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-bcd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-\left(dx^3+c\right)bd^2\sin\left(c\right)\operatorname{Si}\left(dx^3\right)+bcd^2\sin\left(c\right)\operatorname{Si}\left(dx^3\right)-bd^2\sin\left(dx^3+\right.}{3\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^4,x, algorithm="giac")
```

```
[Out] 1/3*((d*x^3 + c)*b*d^2*cos(c)*cos_integral(d*x^3) - b*c*d^2*cos(c)*cos_inte
gral(d*x^3) - (d*x^3 + c)*b*d^2*sin(c)*sin_integral(d*x^3) + b*c*d^2*sin(c)
*sin_integral(d*x^3) - b*d^2*sin(d*x^3 + c) - a*d^2)/(d^2*x^3)
```

3.61 $\int x^4 \left(a+b\sin\left(c+dx^3\right)\right) dx$

Optimal. Leaf size=112

$$-\frac{be^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}+\frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}$$

```
[Out] (a*x^5)/5 - (b*x^2*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x^2*Gamma[2/3, (-I)*d
*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (b*x^2*Gamma[2/3, I*d*x^3])/(9*d*E^(I*c)*
(I*d*x^3)^(2/3))
```

Rubi [A] time = 0.085472, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3385, 3390, 2218}

$$-\frac{be^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}+\frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^5)/5 - (b*x^2*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x^2*Gamma[2/3, (-I)*d
*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (b*x^2*Gamma[2/3, I*d*x^3])/(9*d*E^(I*c)*
(I*d*x^3)^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)\,dx &= \int \left(ax^4+bx^4\sin\left(c+dx^3\right)\right)\,dx \\
&= \frac{ax^5}{5}+b\int x^4\sin\left(c+dx^3\right)\,dx \\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}+\frac{(2b)\int x\cos\left(c+dx^3\right)\,dx}{3d} \\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}+\frac{b\int e^{-ic-idx^3}x\,dx}{3d}+\frac{b\int e^{ic+idx^3}x\,dx}{3d} \\
&= \frac{ax^5}{5}-\frac{bx^2\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.227819, size = 124, normalized size = 1.11

$$
\frac{dx^8\left(-5b\left(-idx^3\right)^{2/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)-5b\left(idx^3\right)^{2/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+3\left(d^2x^6\right)^{2/3}\right.}{45\left(d^2x^6\right)^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (d*x^8*(3*(d^2*x^6)^(2/3)*(3*a*d*x^3 - 5*b*Cos[c + d*x^3]) - 5*b*((-I)*d*x^
3)^(2/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 5*b*(I*d*x^3)^(2/3)*Gamm
a[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(45*(d^2*x^6)^(5/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$
\int x^4\left(a+b\sin\left(dx^3+c\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^4*(a+b*sin(d*x^3+c)),x)
```

Maxima [B] time = 1.17716, size = 402, normalized size = 3.59

$$
\frac{1}{5}\,ax^5-\frac{\left(6\,x^3|d|\cos\left(dx^3+c\right)+\left(x^3|d|\right)^{\frac{1}{3}}\left(\left(\left(\Gamma\left(\frac{2}{3},i\,dx^3\right)+\Gamma\left(\frac{2}{3},-i\,dx^3\right)\right)\cos\left(\frac{1}{3}\,\pi+\frac{2}{3}\,\arctan\left(0,d\right)\right)+\left(\Gamma\left(\frac{2}{3},i\,dx^3\right)+\Gamma\left(\frac{2}{3}\right.\right.\right.\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 - 1/18*(6*x^3*abs(d)*cos(d*x^3 + c) + (x^3*abs(d))^(1/3)*(((gamma
(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (g
amma(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d))
 - (I*gamma(2/3, I*d*x^3) - I*gamma(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan
```

```
2(0, d)) - (-I*gamma(2/3, I*d*x^3) + I*gamma(2/3, -I*d*x^3))*sin(-1/3*pi +
2/3*arctan2(0, d)))*cos(c) - ((I*gamma(2/3, I*d*x^3) - I*gamma(2/3, -I*d*x^
3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (I*gamma(2/3, I*d*x^3) - I*gamma(2/3,
 -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) + (gamma(2/3, I*d*x^3) + gamma
(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) - (gamma(2/3, I*d*x^3) + g
amma(2/3, -I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*sin(c)))*b/(d*x*abs(
d))
```

Fricas [A] time = 1.6596, size = 204, normalized size = 1.82

$$\frac{9\,ad^{2}x^{5}-15\,bdx^{2}\cos\left(dx^{3}+c\right)+5i\,b\,(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^{3}\right)-5i\,b\,(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)}{45\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/45*(9*a*d^2*x^5 - 15*b*d*x^2*cos(d*x^3 + c) + 5*I*b*(I*d)^(1/3)*e^(-I*c)*
gamma(2/3, I*d*x^3) - 5*I*b*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -I*d*x^3))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{4}\left(a+b\sin\left(c+dx^{3}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(dx^{3}+c\right)+a\right)x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*x^4, x)
```

3.62 $\int x\left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal. Leaf size=91

$$\frac{ibe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}+\frac{ax^2}{2}$$

```
[Out] (a*x^2)/2 + ((I/6)*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3)
 - ((I/6)*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3))
```

Rubi [A] time = 0.0642006, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 3389, 2218}

$$\frac{ibe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}+\frac{ax^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^2)/2 + ((I/6)*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3)
 - ((I/6)*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\sin\left(c+dx^3\right)\right)dx &= \int\left(ax+bx\sin\left(c+dx^3\right)\right)dx\\ &= \frac{ax^2}{2}+b\int x\sin\left(c+dx^3\right)dx\\ &= \frac{ax^2}{2}+\frac{1}{2}(ib)\int e^{-ic-idx^3}x\,dx-\frac{1}{2}(ib)\int e^{ic+idx^3}x\,dx\\ &= \frac{ax^2}{2}+\frac{ibe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{6\left(-idx^3\right)^{2/3}}-\frac{ibe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{6\left(idx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.12647, size = 108, normalized size = 1.19

$$\frac{x^2\left(b\left(-idx^3\right)^{2/3}(-\sin(c)-i\cos(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)+ib\left(idx^3\right)^{2/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+3a\left(d^2x^6\right)^{2/3}\right)}{6\left(d^2x^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (x^2*(3*a*(d^2*x^6)^(2/3) + b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*((-I)*
Cos[c] - Sin[c]) + I*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*S
in[c])))/(6*(d^2*x^6)^(2/3))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int x\left(a+b\sin\left(dx^3+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x*(a+b*sin(d*x^3+c)),x)
```

Maxima [B] time = 1.15415, size = 375, normalized size = 4.12

$$\frac{1}{2}ax^2+\frac{\left(x^3|d|\right)^{\frac{1}{3}}\left(\left(\left(-i\Gamma\left(\frac{2}{3},idx^3\right)+i\Gamma\left(\frac{2}{3},-idx^3\right)\right)\cos\left(\frac{1}{3}\pi+\frac{2}{3}\arctan\left(0,d\right)\right)+\left(-i\Gamma\left(\frac{2}{3},idx^3\right)+i\Gamma\left(\frac{2}{3},-idx^3\right)\right)\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/12*(x^3*abs(d))^(1/3)*(((-I*gamma(2/3, I*d*x^3) + I*gamma(2/3
, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (-I*gamma(2/3, I*d*x^3) + I*
gamma(2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) - (gamma(2/3, I*d*x^
3) + gamma(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) + (gamma(2/3, I*
d*x^3) + gamma(2/3, -I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*cos(c) - (
(gamma(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)
```

```
) + (gamma(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(
0, d)) - (I*gamma(2/3, I*d*x^3) - I*gamma(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*
arctan2(0, d)) - (-I*gamma(2/3, I*d*x^3) + I*gamma(2/3, -I*d*x^3))*sin(-1/3
*pi + 2/3*arctan2(0, d)))*sin(c))*b/(x*abs(d))
```

Fricas [A] time = 1.73832, size = 149, normalized size = 1.64

$$\frac{3\,adx^2 - b\,(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3}, i\,dx^3\right) - b\,(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3}, -i\,dx^3\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x^2 - b*(I*d)^(1/3)*e^(-I*c)*gamma(2/3, I*d*x^3) - b*(-I*d)^(1/3
)*e^(I*c)*gamma(2/3, -I*d*x^3))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x*(a + b*sin(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3 + c\right) + a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*x, x)
```

3.63 $\int \frac{a+b\sin(c+dx^3)}{x^2}\,dx$

Optimal. Leaf size=101

$$-\frac{be^{ic}dx^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}dx^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}$$

```
[Out] -(a/x) - (b*d*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(2/3)) -
(b*d*x^2*Gamma[2/3, I*d*x^3])/(2*E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^
3])/x
```

Rubi [A] time = 0.0780452, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3387, 3390, 2218}

$$-\frac{be^{ic}dx^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{be^{-ic}dx^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^2,x]
```

```
[Out] -(a/x) - (b*d*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(2/3)) -
(b*d*x^2*Gamma[2/3, I*d*x^3])/(2*E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^
3])/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^2}\,dx = \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+dx^3\right)}{x^2}\right)dx$$
$$= -\frac{a}{x}+b\int \frac{\sin\left(c+dx^3\right)}{x^2}\,dx$$
$$= -\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}+(3bd)\int x\cos\left(c+dx^3\right)dx$$
$$= -\frac{a}{x}-\frac{b\sin\left(c+dx^3\right)}{x}+\frac{1}{2}(3bd)\int e^{-ic-idx^3}x\,dx+\frac{1}{2}(3bd)\int e^{ic+idx^3}x\,dx$$
$$= -\frac{a}{x}-\frac{bde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{2\left(-idx^3\right)^{2/3}}-\frac{bde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{2\left(idx^3\right)^{2/3}}-\frac{b\sin\left(c+dx^3\right)}{x}$$

Mathematica [A] time = 0.210745, size = 120, normalized size = 1.19

$$\frac{-ib\left(-idx^3\right)^{5/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)+ib\left(idx^3\right)^{5/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)-2\left(d^2x^6\right)^{2/3}\left(a+b\right.}{2x\left(d^2x^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^2,x]
```

```
[Out] ((-I)*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(I
*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(2/3
)*(a + b*Sin[c + d*x^3]))/(2*x*(d^2*x^6)^(2/3))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(dx^3+c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^2,x)
```

Maxima [B] time = 1.15554, size = 366, normalized size = 3.62

$$-\frac{\left(x^3|d|\right)^{\frac{1}{3}}\left(\left(\left(i\Gamma\left(-\frac{1}{3},i\,dx^3\right)-i\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\cos\left(\frac{1}{6}\pi+\frac{1}{3}\arctan\left(0,d\right)\right)+\left(i\Gamma\left(-\frac{1}{3},i\,dx^3\right)-i\Gamma\left(-\frac{1}{3},-i\,dx^3\right)\right)\cos\left(-\frac{1}{6}\pi\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="maxima")
```

```
[Out] -1/12*(x^3*abs(d))^(1/3)*(((I*gamma(-1/3, I*d*x^3) - I*gamma(-1/3, -I*d*x^3
))*cos(1/6*pi + 1/3*arctan2(0, d)) + (I*gamma(-1/3, I*d*x^3) - I*gamma(-1/3
```

```
, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (gamma(-1/3, I*d*x^3) + gam
ma(-1/3, -I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (gamma(-1/3, I*d*x^3)
 + gamma(-1/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*cos(c) + ((gamm
a(-1/3, I*d*x^3) + gamma(-1/3, -I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) +
 (gamma(-1/3, I*d*x^3) + gamma(-1/3, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0
, d)) + (I*gamma(-1/3, I*d*x^3) - I*gamma(-1/3, -I*d*x^3))*sin(1/6*pi + 1/3
*arctan2(0, d)) + (-I*gamma(-1/3, I*d*x^3) + I*gamma(-1/3, -I*d*x^3))*sin(-
1/6*pi + 1/3*arctan2(0, d)))*sin(c))*b/x - a/x
```

Fricas [A] time = 1.7596, size = 180, normalized size = 1.78

$$\frac{i\,b\,(i\,d)^{\frac{1}{3}}\,xe^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)-i\,b\,(-i\,d)^{\frac{1}{3}}\,xe^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)-2\,b\sin\left(dx^3+c\right)-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*b*(I*d)^(1/3)*x*e^(-I*c)*gamma(2/3, I*d*x^3) - I*b*(-I*d)^(1/3)*x*e^
(I*c)*gamma(2/3, -I*d*x^3) - 2*b*sin(d*x^3 + c) - 2*a)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sin\left(c+dx^3\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\sin\left(dx^3+c\right)+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^2, x)
```

3.64 $\int \frac{a+b\sin(c+dx^3)}{x^5}\,dx$

Optimal. Leaf size=130

$$-\frac{3ibe^{ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibe^{-ic}d^2x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{a}{4x^4}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}$$

```
[Out] -a/(4*x^4) - (3*b*d*Cos[c + d*x^3])/(4*x) - (((3*I)/8)*b*d^2*E^(I*c)*x^2*Ga
mma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) + (((3*I)/8)*b*d^2*x^2*Gamma[2/3,
I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^3])/(4*x^4)
```

Rubi [A] time = 0.101653, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3387, 3388, 3389, 2218}

$$-\frac{3ibe^{ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibe^{-ic}d^2x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{a}{4x^4}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^5,x]
```

```
[Out] -a/(4*x^4) - (3*b*d*Cos[c + d*x^3])/(4*x) - (((3*I)/8)*b*d^2*E^(I*c)*x^2*Ga
mma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) + (((3*I)/8)*b*d^2*x^2*Gamma[2/3,
I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/3)) - (b*Sin[c + d*x^3])/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+dx^3\right)}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b\sin\left(c+dx^3\right)}{x^5}\right)dx \\
&= -\frac{a}{4x^4}+b\int \frac{\sin\left(c+dx^3\right)}{x^5}\,dx \\
&= -\frac{a}{4x^4}-\frac{b\sin\left(c+dx^3\right)}{4x^4}+\frac{1}{4}(3bd)\int \frac{\cos\left(c+dx^3\right)}{x^2}\,dx \\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{1}{4}\left(9bd^2\right)\int x\sin\left(c+dx^3\right)\,dx \\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{b\sin\left(c+dx^3\right)}{4x^4}-\frac{1}{8}\left(9ibd^2\right)\int e^{-ic-idx^3}x\,dx+\frac{1}{8}\left(9ibd^2\right)\int e^{ic+ \\
&= -\frac{a}{4x^4}-\frac{3bd\cos\left(c+dx^3\right)}{4x}-\frac{3ibd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{8\left(-idx^3\right)^{2/3}}+\frac{3ibd^2e^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{8\left(idx^3\right)^{2/3}}-\frac{b\sin\left(c+a\right.}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.372953, size = 143, normalized size = 1.1

$$\frac{3bd^2x^6\left(idx^3\right)^{2/3}(\sin(c)-i\cos(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+3bd^2x^6\left(-idx^3\right)^{2/3}(\sin(c)+i\cos(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)-2\left(d^2\right.}{8x^4\left(d^2x^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^5,x]
```

```
[Out] (3*b*d^2*x^6*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
+ 3*b*d^2*x^6*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(I*Cos[c] + Sin[c]) -
2*(d^2*x^6)^(2/3)*(a + 3*b*d*x^3*Cos[c + d*x^3] + b*Sin[c + d*x^3]))/(8*x^4
*(d^2*x^6)^(2/3))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(dx^3+c\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^5,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^5,x)
```

Maxima [B] time = 1.1634, size = 369, normalized size = 2.84

$$-\frac{\left(x^3|d|\right)^{\frac{1}{3}}\left(\left(\left(i\Gamma\left(-\frac{4}{3},idx^3\right)-i\Gamma\left(-\frac{4}{3},-idx^3\right)\right)\cos\left(\frac{2}{3}\pi+\frac{4}{3}\arctan(0,d)\right)+\left(i\Gamma\left(-\frac{4}{3},idx^3\right)-i\Gamma\left(-\frac{4}{3},-idx^3\right)\right)\cos\left(-\frac{2}{3}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(x^3*abs(d))^(1/3)*(((I*gamma(-4/3, I*d*x^3) - I*gamma(-4/3, -I*d*x^3
))*cos(2/3*pi + 4/3*arctan2(0, d)) + (I*gamma(-4/3, I*d*x^3) - I*gamma(-4/3
, -I*d*x^3))*cos(-2/3*pi + 4/3*arctan2(0, d)) - (gamma(-4/3, I*d*x^3) + gam
ma(-4/3, -I*d*x^3))*sin(2/3*pi + 4/3*arctan2(0, d)) + (gamma(-4/3, I*d*x^3)
 + gamma(-4/3, -I*d*x^3))*sin(-2/3*pi + 4/3*arctan2(0, d)))*cos(c) + ((gamm
a(-4/3, I*d*x^3) + gamma(-4/3, -I*d*x^3))*cos(2/3*pi + 4/3*arctan2(0, d)) +
 (gamma(-4/3, I*d*x^3) + gamma(-4/3, -I*d*x^3))*cos(-2/3*pi + 4/3*arctan2(0
, d)) + (I*gamma(-4/3, I*d*x^3) - I*gamma(-4/3, -I*d*x^3))*sin(2/3*pi + 4/3
*arctan2(0, d)) + (-I*gamma(-4/3, I*d*x^3) + I*gamma(-4/3, -I*d*x^3))*sin(-
2/3*pi + 4/3*arctan2(0, d)))*sin(c))*b*abs(d)/x - 1/4*a/x^4
```

Fricas [A] time = 1.77573, size = 230, normalized size = 1.77

$$\frac{3\,b\,(i\,d)^{\frac{1}{3}}\,dx^4e^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right)+3\,b\,(-i\,d)^{\frac{1}{3}}\,dx^4e^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right)-6\,bdx^3\cos\left(dx^3+c\right)-2\,b\sin\left(dx^3+c\right)-2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*(I*d)^(1/3)*d*x^4*e^(-I*c)*gamma(2/3, I*d*x^3) + 3*b*(-I*d)^(1/3)*
d*x^4*e^(I*c)*gamma(2/3, -I*d*x^3) - 6*b*d*x^3*cos(d*x^3 + c) - 2*b*sin(d*x
^3 + c) - 2*a)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(dx^3+c\right)+a}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^5, x)
```

3.65 $\int x^3 \left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal. Leaf size=106

$$-\frac{be^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}+\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}$$

```
[Out] (a*x^4)/4 - (b*x*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3
])/(18*d*((-I)*d*x^3)^(1/3)) - (b*x*Gamma[1/3, I*d*x^3])/(18*d*E^(I*c)*(I*d
*x^3)^(1/3))
```

Rubi [A] time = 0.0723359, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3385, 3356, 2208}

$$-\frac{be^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}+\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*x^4)/4 - (b*x*Cos[c + d*x^3])/(3*d) - (b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3
])/(18*d*((-I)*d*x^3)^(1/3)) - (b*x*Gamma[1/3, I*d*x^3])/(18*d*E^(I*c)*(I*d
*x^3)^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\int x^3\left(a+b\sin\left(c+dx^3\right)\right)dx=\int\left(ax^3+bx^3\sin\left(c+dx^3\right)\right)dx$$
$$=\frac{ax^4}{4}+b\int x^3\sin\left(c+dx^3\right)dx$$
$$=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}+\frac{b\int\cos\left(c+dx^3\right)dx}{3d}$$
$$=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}+\frac{b\int e^{-ic-idx^3}dx}{6d}+\frac{b\int e^{ic+idx^3}dx}{6d}$$
$$=\frac{ax^4}{4}-\frac{bx\cos\left(c+dx^3\right)}{3d}-\frac{be^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{18d\sqrt[3]{-idx^3}}-\frac{be^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{18d\sqrt[3]{idx^3}}$$

Mathematica [A] time = 0.191393, size = 124, normalized size = 1.17

$$\frac{dx^7\left(-2b\sqrt[3]{-idx^3}(\cos(c)-i\sin(c))\mathrm{Gamma}\left(\frac{1}{3},idx^3\right)-2b\sqrt[3]{idx^3}(\cos(c)+i\sin(c))\mathrm{Gamma}\left(\frac{1}{3},-idx^3\right)+3\sqrt[3]{d^2x^6}\left(3adx^3-\right.\right.}{36\left(d^2x^6\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (d*x^7*(3*(d^2*x^6)^(1/3)*(3*a*d*x^3 - 4*b*Cos[c + d*x^3]) - 2*b*((-I)*d*x^
3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) - 2*b*(I*d*x^3)^(1/3)*Gamm
a[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c])))/(36*(d^2*x^6)^(4/3))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int x^3\left(a+b\sin\left(dx^3+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^3*(a+b*sin(d*x^3+c)),x)
```

Maxima [B] time = 1.13706, size = 396, normalized size = 3.74

$$\frac{1}{4}ax^4-\frac{\left(12\left(x^3|d|\right)^{\frac{1}{3}}x\cos\left(dx^3+c\right)+\left(\left(\left(\Gamma\left(\frac{1}{3},i\,dx^3\right)+\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(\frac{1}{6}\pi+\frac{1}{3}\arctan\left(0,d\right)\right)+\left(\Gamma\left(\frac{1}{3},i\,dx^3\right)+\Gamma\left(\frac{1}{3},-\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 - 1/36*(12*(x^3*abs(d))^(1/3)*x*cos(d*x^3 + c) + (((gamma(1/3, I*
d*x^3) + gamma(1/3, -I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3
, I*d*x^3) + gamma(1/3, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) + (-I*g
```

```
amma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)
) + (I*gamma(1/3, I*d*x^3) - I*gamma(1/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arct
an2(0, d)))*cos(c) + ((-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*cos
(1/6*pi + 1/3*arctan2(0, d)) + (-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*
x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (gamma(1/3, I*d*x^3) + gamma(1/3,
-I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I*d*x^3) + gamma(1
/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(c))*x)*b/((x^3*abs(d))
^(1/3)*d)
```

Fricas [A] time = 1.8875, size = 201, normalized size = 1.9

$$\frac{9\,ad^2x^4 - 12\,bdx\cos\left(dx^3 + c\right) + 2i\,b\,(i\,d)^{\frac{2}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{1}{3}, i\,dx^3\right) - 2i\,b\,(-i\,d)^{\frac{2}{3}}\,e^{(i\,c)}\Gamma\left(\frac{1}{3}, -i\,dx^3\right)}{36\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/36*(9*a*d^2*x^4 - 12*b*d*x*cos(d*x^3 + c) + 2*I*b*(I*d)^(2/3)*e^(-I*c)*ga
mma(1/3, I*d*x^3) - 2*I*b*(-I*d)^(2/3)*e^(I*c)*gamma(1/3, -I*d*x^3))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x**3*(a + b*sin(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3 + c\right) + a\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*x^3, x)
```

3.66 $\int \left(a+b \sin \left(c+d x^3\right)\right) \, dx$

Optimal. Leaf size=82

$$\frac{ibe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}+ax$$

```
[Out] a*x + ((I/6)*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - ((I/6
)*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3))
```

Rubi [A] time = 0.0294216, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3355, 2208}

$$\frac{ibe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}+ax$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x^3],x]
```

```
[Out] a*x + ((I/6)*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) - ((I/6
)*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3))
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b \sin \left(c+d x^3\right)\right) \, dx &= ax+b\int \sin \left(c+dx^3\right) \, dx\\ &= ax+\frac{1}{2}(ib)\int e^{-ic-idx^3} \, dx-\frac{1}{2}(ib)\int e^{ic+idx^3} \, dx\\ &= ax+\frac{ibe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{6\sqrt[3]{-idx^3}}-\frac{ibe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{6\sqrt[3]{idx^3}}\end{aligned}$$

Mathematica [A] time = 0.100292, size = 138, normalized size = 1.68

$$-\frac{1}{2}ib\cos(c)\left(\frac{x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}-\frac{x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}\right)+\frac{1}{2}b\sin(c)\left(-\frac{x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x^3],x]
```

```
[Out] a*x - (I/2)*b*Cos[c]*(-(x*Gamma[1/3, (-I)*d*x^3])/(3*((-I)*d*x^3)^(1/3)) +
(x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3))) + (b*(-(x*Gamma[1/3, (-I)*d*x^
3])/(3*((-I)*d*x^3)^(1/3)) - (x*Gamma[1/3, I*d*x^3])/(3*(I*d*x^3)^(1/3)))*S
in[c])/2
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int a+b\sin\left(dx^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(d*x^3+c),x)
```

```
[Out] int(a+b*sin(d*x^3+c),x)
```

Maxima [B] time = 1.12915, size = 363, normalized size = 4.43

$$\frac{\left(\left(\left(-i\Gamma\left(\frac{1}{3},i\,dx^3\right)+i\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(\frac{1}{6}\,\pi+\frac{1}{3}\,\arctan\left(0,d\right)\right)+\left(-i\Gamma\left(\frac{1}{3},i\,dx^3\right)+i\Gamma\left(\frac{1}{3},-i\,dx^3\right)\right)\cos\left(-\frac{1}{6}\,\pi+\frac{1}{3}\,\text{arct}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/12*(((-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*cos(1/6*pi + 1/3*a
rctan2(0, d)) + (-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*cos(-1/6*
pi + 1/3*arctan2(0, d)) - (gamma(1/3, I*d*x^3) + gamma(1/3, -I*d*x^3))*sin(
1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I*d*x^3) + gamma(1/3, -I*d*x^3))*
sin(-1/6*pi + 1/3*arctan2(0, d)))*cos(c) - ((gamma(1/3, I*d*x^3) + gamma(1/
3, -I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I*d*x^3) + gamm
a(1/3, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (I*gamma(1/3, I*d*x^3)
 - I*gamma(1/3, -I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) - (-I*gamma(1/3,
 I*d*x^3) + I*gamma(1/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(c
))*b*x/(x^3*abs(d))^(1/3) + a*x
```

Fricas [A] time = 1.62296, size = 147, normalized size = 1.79

$$-\frac{b\left(i\,d\right)^{\frac{2}{3}}e^{\left(-i\,c\right)}\Gamma\left(\frac{1}{3},i\,dx^3\right)+b\left(-i\,d\right)^{\frac{2}{3}}e^{\left(i\,c\right)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-6\,adx}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(b*(I*d)^(2/3)*e^(-I*c)*gamma(1/3, I*d*x^3) + b*(-I*d)^(2/3)*e^(I*c)*g
amma(1/3, -I*d*x^3) - 6*a*d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(c + dx^3\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x**3+c),x)
```

```
[Out] Integral(a + b*sin(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b\sin\left(dx^3 + c\right) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(b*sin(d*x^3 + c) + a, x)
```

3.67 $\int \frac{a+b\sin(c+dx^3)}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{be^{ic}dx\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{be^{-ic}dx\text{Gamma}\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}$$

```
[Out] -a/(2*x^2) - (b*d*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/(4*((-I)*d*x^3)^(1/3))
- (b*d*x*Gamma[1/3, I*d*x^3])/(4*E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^
3])/(2*x^2)
```

Rubi [A] time = 0.0549399, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {14, 3387, 3356, 2208}

$$-\frac{be^{ic}dx\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{be^{-ic}dx\text{Gamma}\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*d*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/(4*((-I)*d*x^3)^(1/3))
- (b*d*x*Gamma[1/3, I*d*x^3])/(4*E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^
3])/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b\sin\left(c+dx^3\right)}{x^3}\right)dx\\ &= -\frac{a}{2x^2}+b\int \frac{\sin\left(c+dx^3\right)}{x^3}\,dx\\ &= -\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}+\frac{1}{2}(3bd)\int \cos\left(c+dx^3\right)\,dx\\ &= -\frac{a}{2x^2}-\frac{b\sin\left(c+dx^3\right)}{2x^2}+\frac{1}{4}(3bd)\int e^{-ic-idx^3}\,dx+\frac{1}{4}(3bd)\int e^{ic+idx^3}\,dx\\ &= -\frac{a}{2x^2}-\frac{bde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{4\sqrt[3]{-idx^3}}-\frac{bde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{4\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.192598, size = 120, normalized size = 1.19

$$\frac{-ib\left(-idx^3\right)^{4/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{1}{3},idx^3\right)+ib\left(idx^3\right)^{4/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{1}{3},-idx^3\right)-2\sqrt[3]{d^2x^6}\left(a+b\sin\right.}{4x^2\sqrt[3]{d^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^3,x]
```

```
[Out] ((-I)*b*((-I)*d*x^3)^(4/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*b*(I
*d*x^3)^(4/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 2*(d^2*x^6)^(1/3
)*(a + b*Sin[c + d*x^3]))/(4*x^2*(d^2*x^6)^(1/3))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(dx^3+c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^3,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^3,x)
```

Maxima [B] time = 1.17775, size = 366, normalized size = 3.62

$$-\frac{\left(x^3|d|\right)^{\frac{2}{3}}\left(\left(\left(i\Gamma\left(-\frac{2}{3},i\,dx^3\right)-i\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos\left(\frac{1}{3}\pi+\frac{2}{3}\arctan\left(0,d\right)\right)+\left(i\Gamma\left(-\frac{2}{3},i\,dx^3\right)-i\Gamma\left(-\frac{2}{3},-i\,dx^3\right)\right)\cos\left(-\frac{1}{3}\pi\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(x^3*abs(d))^(2/3)*(((I*gamma(-2/3, I*d*x^3) - I*gamma(-2/3, -I*d*x^3
))*cos(1/3*pi + 2/3*arctan2(0, d)) + (I*gamma(-2/3, I*d*x^3) - I*gamma(-2/3
, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) - (gamma(-2/3, I*d*x^3) + gam
```

```
ma(-2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) + (gamma(-2/3, I*d*x^3)
 + gamma(-2/3, -I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*cos(c) + ((gamm
a(-2/3, I*d*x^3) + gamma(-2/3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) +
 (gamma(-2/3, I*d*x^3) + gamma(-2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0
, d)) + (I*gamma(-2/3, I*d*x^3) - I*gamma(-2/3, -I*d*x^3))*sin(1/3*pi + 2/3
*arctan2(0, d)) + (-I*gamma(-2/3, I*d*x^3) + I*gamma(-2/3, -I*d*x^3))*sin(-
1/3*pi + 2/3*arctan2(0, d)))*sin(c))*b/x^2 - 1/2*a/x^2
```

Fricas [A] time = 1.73536, size = 188, normalized size = 1.86

$$\frac{i\,b\,(i\,d)^{\frac{2}{3}}\,x^2e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)-i\,b\,(-i\,d)^{\frac{2}{3}}\,x^2e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-2\,b\sin\left(dx^3+c\right)-2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(I*b*(I*d)^(2/3)*x^2*e^(-I*c)*gamma(1/3, I*d*x^3) - I*b*(-I*d)^(2/3)*x^
2*e^(I*c)*gamma(1/3, -I*d*x^3) - 2*b*sin(d*x^3 + c) - 2*a)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(dx^3+c\right)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^3, x)
```

3.68 $\int \frac{a+b\sin(c+dx^3)}{x^6}\,dx$

Optimal. Leaf size=126

$$-\frac{3ibe^{ic}d^2x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibe^{-ic}d^2x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{a}{5x^5}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}$$

```
[Out] -a/(5*x^5) - (3*b*d*Cos[c + d*x^3])/(10*x^2) - (((3*I)/20)*b*d^2*E^(I*c)*x*
Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (((3*I)/20)*b*d^2*x*Gamma[1/3,
 I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^3])/(5*x^5)
```

Rubi [A] time = 0.0711301, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 3387, 3388, 3355, 2208}

$$-\frac{3ibe^{ic}d^2x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibe^{-ic}d^2x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{a}{5x^5}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])/x^6,x]
```

```
[Out] -a/(5*x^5) - (3*b*d*Cos[c + d*x^3])/(10*x^2) - (((3*I)/20)*b*d^2*E^(I*c)*x*
Gamma[1/3, (-I)*d*x^3])/((-I)*d*x^3)^(1/3) + (((3*I)/20)*b*d^2*x*Gamma[1/3,
 I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) - (b*Sin[c + d*x^3])/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\int \frac{a+b\sin\left(c+dx^3\right)}{x^6}\,dx = \int \left(\frac{a}{x^6}+\frac{b\sin\left(c+dx^3\right)}{x^6}\right)dx$$
$$= -\frac{a}{5x^5}+b\int \frac{\sin\left(c+dx^3\right)}{x^6}\,dx$$
$$= -\frac{a}{5x^5}-\frac{b\sin\left(c+dx^3\right)}{5x^5}+\frac{1}{5}(3bd)\int \frac{\cos\left(c+dx^3\right)}{x^3}\,dx$$
$$= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{1}{10}\left(9bd^2\right)\int \sin\left(c+dx^3\right)\,dx$$
$$= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{b\sin\left(c+dx^3\right)}{5x^5}-\frac{1}{20}\left(9ibd^2\right)\int e^{-ic-idx^3}\,dx+\frac{1}{20}\left(9ibd^2\right)\int e^{ic}$$
$$= -\frac{a}{5x^5}-\frac{3bd\cos\left(c+dx^3\right)}{10x^2}-\frac{3ibd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{20\sqrt[3]{-idx^3}}+\frac{3ibd^2e^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{20\sqrt[3]{idx^3}}-\frac{b\sin\left(c+dx^3\right)}{5x^5}$$

Mathematica [A] time = 0.44704, size = 146, normalized size = 1.16

$$\frac{3bd^2x^6\sqrt[3]{idx^3}(\sin(c)-i\cos(c))\text{Gamma}\left(\frac{1}{3},-idx^3\right)+3bd^2x^6\sqrt[3]{-idx^3}(\sin(c)+i\cos(c))\text{Gamma}\left(\frac{1}{3},idx^3\right)-2\sqrt[3]{d^2x^6}\left(2a\right.}{20x^5\sqrt[3]{d^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])/x^6,x]
```

```
[Out] (3*b*d^2*x^6*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*((-I)*Cos[c] + Sin[c])
+ 3*b*d^2*x^6*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(I*Cos[c] + Sin[c]) -
2*(d^2*x^6)^(1/3)*(2*a + 3*b*d*x^3*Cos[c + d*x^3] + 2*b*Sin[c + d*x^3]))/(2
0*x^5*(d^2*x^6)^(1/3))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(dx^3+c\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))/x^6,x)
```

```
[Out] int((a+b*sin(d*x^3+c))/x^6,x)
```

Maxima [B] time = 1.16373, size = 369, normalized size = 2.93

$$-\frac{\left(x^3|d|\right)^{\frac{2}{3}}\left(\left(\left(i\Gamma\left(-\frac{5}{3},idx^3\right)-i\Gamma\left(-\frac{5}{3},-idx^3\right)\right)\cos\left(\frac{5}{6}\pi+\frac{5}{3}\arctan(0,d)\right)+\left(i\Gamma\left(-\frac{5}{3},idx^3\right)-i\Gamma\left(-\frac{5}{3},-idx^3\right)\right)\cos\left(-\frac{5}{6}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="maxima")
```

```
[Out] -1/12*(x^3*abs(d))^(2/3)*(((I*gamma(-5/3, I*d*x^3) - I*gamma(-5/3, -I*d*x^3
))*cos(5/6*pi + 5/3*arctan2(0, d)) + (I*gamma(-5/3, I*d*x^3) - I*gamma(-5/3
, -I*d*x^3))*cos(-5/6*pi + 5/3*arctan2(0, d)) - (gamma(-5/3, I*d*x^3) + gam
ma(-5/3, -I*d*x^3))*sin(5/6*pi + 5/3*arctan2(0, d)) + (gamma(-5/3, I*d*x^3)
 + gamma(-5/3, -I*d*x^3))*sin(-5/6*pi + 5/3*arctan2(0, d)))*cos(c) + ((gamm
a(-5/3, I*d*x^3) + gamma(-5/3, -I*d*x^3))*cos(5/6*pi + 5/3*arctan2(0, d)) +
 (gamma(-5/3, I*d*x^3) + gamma(-5/3, -I*d*x^3))*cos(-5/6*pi + 5/3*arctan2(0
, d)) + (I*gamma(-5/3, I*d*x^3) - I*gamma(-5/3, -I*d*x^3))*sin(5/6*pi + 5/3
*arctan2(0, d)) + (-I*gamma(-5/3, I*d*x^3) + I*gamma(-5/3, -I*d*x^3))*sin(-
5/6*pi + 5/3*arctan2(0, d)))*sin(c))*b*abs(d)/x^2 - 1/5*a/x^5
```

Fricas [A] time = 1.71829, size = 231, normalized size = 1.83

$$\frac{3\,b\,(i\,d)^{\frac{2}{3}}\,dx^5e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)+3\,b\,(-i\,d)^{\frac{2}{3}}\,dx^5e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)-6\,bdx^3\cos\left(dx^3+c\right)-4\,b\sin\left(dx^3+c\right)-4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(3*b*(I*d)^(2/3)*d*x^5*e^(-I*c)*gamma(1/3, I*d*x^3) + 3*b*(-I*d)^(2/3)
*d*x^5*e^(I*c)*gamma(1/3, -I*d*x^3) - 6*b*d*x^3*cos(d*x^3 + c) - 4*b*sin(d*
x^3 + c) - 4*a)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\sin\left(c+dx^3\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))/x**6,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\sin\left(dx^3+c\right)+a}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)/x^6, x)
```

3.69 $\int x^5 \left(a+b\sin\left(c+dx^3\right)\right)^2 dx$

Optimal. Leaf size=107

$$\frac{a^2x^6}{6}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}-\frac{b^2x^3\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d}+\frac{b^2x^6}{12}$$

```
[Out] (a^2*x^6)/6 + (b^2*x^6)/12 - (2*a*b*x^3*Cos[c + d*x^3])/(3*d) + (2*a*b*Sin[
c + d*x^3])/(3*d^2) - (b^2*x^3*Cos[c + d*x^3]*Sin[c + d*x^3])/(6*d) + (b^2*
Sin[c + d*x^3]^2)/(12*d^2)
```

Rubi [A] time = 0.132765, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 3317, 3296, 2637, 3310, 30}

$$\frac{a^2x^6}{6}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}-\frac{b^2x^3\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d}+\frac{b^2x^6}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (a^2*x^6)/6 + (b^2*x^6)/12 - (2*a*b*x^3*Cos[c + d*x^3])/(3*d) + (2*a*b*Sin[
c + d*x^3])/(3*d^2) - (b^2*x^3*Cos[c + d*x^3]*Sin[c + d*x^3])/(6*d) + (b^2*
Sin[c + d*x^3]^2)/(12*d^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx&=\frac{1}{3}\operatorname{Subst}\left(\int x(a+b\sin(c+dx))^2\,dx,x,x^3\right)\\&=\frac{1}{3}\operatorname{Subst}\left(\int\left(a^2x+2abx\sin(c+dx)+b^2x\sin^2(c+dx)\right)dx,x,x^3\right)\\&=\frac{a^2x^6}{6}+\frac{1}{3}(2ab)\operatorname{Subst}\left(\int x\sin(c+dx)\,dx,x,x^3\right)+\frac{1}{3}b^2\operatorname{Subst}\left(\int x\sin^2(c+dx)\,dx,x,x\right.\\&=\frac{a^2x^6}{6}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^3\cos\left(c+dx^3\right)\sin\left(c+dx^3\right)}{6d}+\frac{b^2\sin^2\left(c+dx^3\right)}{12d^2}+\frac{1}{6}b\\&=\frac{a^2x^6}{6}+\frac{b^2x^6}{12}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}-\frac{b^2x^3\cos\left(c+dx^3\right)\sin\left(c+dx\right.}{6d}\end{aligned}$$

Mathematica [A] time = 0.281155, size = 92, normalized size = 0.86

$$\frac{4a^2d^2x^6+16ab\sin\left(c+dx^3\right)-16abdx^3\cos\left(c+dx^3\right)-2b^2dx^3\sin\left(2\left(c+dx^3\right)\right)-b^2\cos\left(2\left(c+dx^3\right)\right)+2b^2d^2x^6}{24d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (4*a^2*d^2*x^6 + 2*b^2*d^2*x^6 - 16*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c
 + d*x^3)] + 16*a*b*Sin[c + d*x^3] - 2*b^2*d*x^3*Sin[2*(c + d*x^3)])/(24*d^
2)
```

Maple [A] time = 0.095, size = 137, normalized size = 1.3

$$\frac{a^2x^6}{6}+\frac{b^2x^6}{6}-\frac{b^2}{2}\left(\frac{x^6}{6}+\frac{1}{1+\left(\tan\left(dx^3+c\right)\right)^2}\left(\frac{1}{6d^2}+\frac{x^3\tan\left(dx^3+c\right)}{3d}\right)\right)+\frac{1}{2}\left(\frac{8ab}{3d^2}\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)-\frac{4abx^3}{3d}+\frac{4abx^3}{3d}\left(\mathrm{t}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] 1/6*a^2*x^6+1/6*b^2*x^6-1/2*b^2*(1/6*x^6+(1/6/d^2+1/3*x^3/d*tan(d*x^3+c))/(
1+tan(d*x^3+c)^2))+1/2*(8/3*a*b/d^2*tan(1/2*d*x^3+1/2*c)-4/3/d*a*b*x^3+4/3/
d*a*b*x^3*tan(1/2*d*x^3+1/2*c)^2)/(1+tan(1/2*d*x^3+1/2*c)^2)
```

Maxima [A] time = 1.01646, size = 117, normalized size = 1.09

$$\frac{1}{6}a^2x^6-\frac{2\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)ab}{3d^2}+\frac{\left(2d^2x^6-2dx^3\sin\left(2dx^3+2c\right)-\cos\left(2dx^3+2c\right)\right)b^2}{24d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 - 2/3*(d*x^3*cos(d*x^3 + c) - sin(d*x^3 + c))*a*b/d^2 + 1/24*(2
*d^2*x^6 - 2*d*x^3*sin(2*d*x^3 + 2*c) - cos(2*d*x^3 + 2*c))*b^2/d^2
```

Fricas [A] time = 1.76182, size = 189, normalized size = 1.77

$$\frac{\left(2\,a^2+b^2\right)d^2x^6-8\,abdx^3\cos\left(dx^3+c\right)-b^2\cos\left(dx^3+c\right)^2-2\left(b^2dx^3\cos\left(dx^3+c\right)-4\,ab\right)\sin\left(dx^3+c\right)}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*((2*a^2 + b^2)*d^2*x^6 - 8*a*b*d*x^3*cos(d*x^3 + c) - b^2*cos(d*x^3 +
c)^2 - 2*(b^2*d*x^3*cos(d*x^3 + c) - 4*a*b)*sin(d*x^3 + c))/d^2
```

Sympy [A] time = 8.30871, size = 143, normalized size = 1.34

$$\begin{cases} \frac{a^2x^6}{6}-\frac{2abx^3\cos\left(c+dx^3\right)}{3d}+\frac{2ab\sin\left(c+dx^3\right)}{3d^2}+\frac{b^2x^6\sin^2\left(c+dx^3\right)}{12}+\frac{b^2x^6\cos^2\left(c+dx^3\right)}{12}-\frac{b^2x^3\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d}-\frac{b^2\cos^2\left(c+dx^3\right)}{12d^2} & \text{for } \\ \frac{x^6(a+b\sin(c))^2}{6} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 - 2*a*b*x**3*cos(c + d*x**3)/(3*d) + 2*a*b*sin(c + d
*x**3)/(3*d**2) + b**2*x**6*sin(c + d*x**3)**2/12 + b**2*x**6*cos(c + d*x**
3)**2/12 - b**2*x**3*sin(c + d*x**3)*cos(c + d*x**3)/(6*d) - b**2*cos(c + d
*x**3)**2/(12*d**2), Ne(d, 0)), (x**6*(a + b*sin(c))**2/6, True))
```

Giac [A] time = 1.10524, size = 166, normalized size = 1.55

$$\frac{\frac{4\left(\left(dx^3+c\right)^2-2\left(dx^3+c\right)c\right)a^2}{d}-\frac{16\left(dx^3\cos\left(dx^3+c\right)-\sin\left(dx^3+c\right)\right)ab}{d}-\frac{\left(2\,dx^3\sin\left(2\,dx^3+2\,c\right)-2\left(dx^3+c\right)^2+4\left(dx^3+c\right)c+\cos\left(2\,dx^3+2\,c\right)\right)b^2}{d}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(4*((d*x^3 + c)^2 - 2*(d*x^3 + c)*c)*a^2/d - 16*(d*x^3*cos(d*x^3 + c)
- sin(d*x^3 + c))*a*b/d - (2*d*x^3*sin(2*d*x^3 + 2*c) - 2*(d*x^3 + c)^2 + 4
*(d*x^3 + c)*c + cos(2*d*x^3 + 2*c))*b^2/d)/d
```

3.70 $\int x^2 \left(a+b \sin \left(c+d x^3\right)\right)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{6} x^3 \left(2 a^2+b^2\right)-\frac{2 a b \cos \left(c+d x^3\right)}{3 d}-\frac{b^2 \sin \left(c+d x^3\right) \cos \left(c+d x^3\right)}{6 d}$$

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (2*a*b*Cos[c + d*x^3])/(3*d) - (b^2*Cos[c + d*x^3]*
Sin[c + d*x^3])/(6*d)
```

Rubi [A] time = 0.0569236, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3379, 2644}

$$\frac{1}{6} x^3 \left(2 a^2+b^2\right)-\frac{2 a b \cos \left(c+d x^3\right)}{3 d}-\frac{b^2 \sin \left(c+d x^3\right) \cos \left(c+d x^3\right)}{6 d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^3)/6 - (2*a*b*Cos[c + d*x^3])/(3*d) - (b^2*Cos[c + d*x^3]*
Sin[c + d*x^3])/(6*d)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(a+b \sin \left(c+d x^3\right)\right)^2 \, dx &= \frac{1}{3} \text{Subst}\left(\int (a+b \sin (c+d x))^2 \, dx,x,x^3\right)\\ &= \frac{1}{6} \left(2 a^2+b^2\right) x^3-\frac{2 a b \cos \left(c+d x^3\right)}{3 d}-\frac{b^2 \cos \left(c+d x^3\right) \sin \left(c+d x^3\right)}{6 d} \end{aligned}$$

Mathematica [A] time = 0.146364, size = 52, normalized size = 0.87

$$-\frac{-2 \left(2 a^2+b^2\right) \left(c+d x^3\right)+8 a b \cos \left(c+d x^3\right)+b^2 \sin \left(2 \left(c+d x^3\right)\right)}{12 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] -(-2*(2*a^2 + b^2)*(c + d*x^3) + 8*a*b*Cos[c + d*x^3] + b^2*Sin[2*(c + d*x^
3)])/(12*d)
```

Maple [A] time = 0.014, size = 62, normalized size = 1.

$$\frac{1}{3d}\left(b^2\left(-\frac{\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)}{2}+\frac{dx^3}{2}+\frac{c}{2}\right)-2\,ab\cos\left(dx^3+c\right)+a^2\left(dx^3+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] 1/3/d*(b^2*(-1/2*cos(d*x^3+c)*sin(d*x^3+c)+1/2*d*x^3+1/2*c)-2*a*b*cos(d*x^3
+c)+a^2*(d*x^3+c))
```

Maxima [A] time = 0.98571, size = 70, normalized size = 1.17

$$\frac{1}{3}\,a^2x^3+\frac{\left(2\,dx^3-\sin\left(2\,dx^3+2\,c\right)\right)b^2}{12\,d}-\frac{2\,ab\cos\left(dx^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/12*(2*d*x^3 - sin(2*d*x^3 + 2*c))*b^2/d - 2/3*a*b*cos(d*x^3
 + c)/d
```

Fricas [A] time = 1.66223, size = 119, normalized size = 1.98

$$\frac{\left(2\,a^2+b^2\right)dx^3-b^2\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)-4\,ab\cos\left(dx^3+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2 + b^2)*d*x^3 - b^2*cos(d*x^3 + c)*sin(d*x^3 + c) - 4*a*b*cos(d*
x^3 + c))/d
```

Sympy [A] time = 1.24779, size = 99, normalized size = 1.65

$$\begin{cases}\frac{a^2x^3}{3}-\frac{2ab\cos\left(c+dx^3\right)}{3d}+\frac{b^2x^3\sin^2\left(c+dx^3\right)}{6}+\frac{b^2x^3\cos^2\left(c+dx^3\right)}{6}-\frac{b^2\sin\left(c+dx^3\right)\cos\left(c+dx^3\right)}{6d} & \text{for } d\neq 0\\ \frac{x^3(a+b\sin(c))^2}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Piecewise((a**2*x**3/3 - 2*a*b*cos(c + d*x**3)/(3*d) + b**2*x**3*sin(c + d*
x**3)**2/6 + b**2*x**3*cos(c + d*x**3)**2/6 - b**2*sin(c + d*x**3)*cos(c +
d*x**3)/(6*d), Ne(d, 0)), (x**3*(a + b*sin(c))**2/3, True))
```

Giac [A] time = 1.0961, size = 77, normalized size = 1.28

$$\frac{4\left(dx^3+c\right)a^2+\left(2\,dx^3+2\,c-\sin\left(2\,dx^3+2\,c\right)\right)b^2-8\,ab\cos\left(dx^3+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(4*(d*x^3 + c)*a^2 + (2*d*x^3 + 2*c - sin(2*d*x^3 + 2*c))*b^2 - 8*a*b*
cos(d*x^3 + c))/d
```

3.71 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+\frac{2}{3}ab\sin(c)\text{CosIntegral}\left(dx^3\right)+\frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right)-\frac{1}{6}b^2\cos(2c)\text{CosIntegral}\left(2dx^3\right)+\frac{1}{6}b^2\,\mathrm{si}$$

```
[Out] -(b^2*Cos[2*c]*CosIntegral[2*d*x^3])/6 + ((2*a^2 + b^2)*Log[x])/2 + (2*a*b*
CosIntegral[d*x^3]*Sin[c])/3 + (2*a*b*Cos[c]*SinIntegral[d*x^3])/3 + (b^2*S
in[2*c]*SinIntegral[2*d*x^3])/6
```

Rubi [A] time = 0.0927945, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3403, 6, 3378, 3376, 3375, 3377}

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)+\frac{2}{3}ab\sin(c)\text{CosIntegral}\left(dx^3\right)+\frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right)-\frac{1}{6}b^2\cos(2c)\text{CosIntegral}\left(2dx^3\right)+\frac{1}{6}b^2\,\mathrm{si}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x,x]
```

```
[Out] -(b^2*Cos[2*c]*CosIntegral[2*d*x^3])/6 + ((2*a^2 + b^2)*Log[x])/2 + (2*a*b*
CosIntegral[d*x^3]*Sin[c])/3 + (2*a*b*Cos[c]*SinIntegral[d*x^3])/3 + (b^2*S
in[2*c]*SinIntegral[2*d*x^3])/6
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
```

]

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx = \int \left(\frac{a^2}{x}+\frac{b^2}{2x}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x}+\frac{2ab\sin\left(c+dx^3\right)}{x}\right)dx$$
$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x}+\frac{2ab\sin\left(c+dx^3\right)}{x}\right)dx$$
$$= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x}\,dx$$
$$= \frac{1}{2}\left(2a^2+b^2\right)\log(x)+(2ab\cos(c))\int \frac{\sin\left(dx^3\right)}{x}\,dx-\frac{1}{2}\left(b^2\cos(2c)\right)\int \frac{\cos\left(2dx^3\right)}{x}\,dx+(2a$$
$$= -\frac{1}{6}b^2\cos(2c)\text{Ci}\left(2dx^3\right)+\frac{1}{2}\left(2a^2+b^2\right)\log(x)+\frac{2}{3}ab\text{Ci}\left(dx^3\right)\sin(c)+\frac{2}{3}ab\cos(c)\text{Si}\left(dx^3\right)+$$

Mathematica [A] time = 0.180826, size = 71, normalized size = 0.89

$$\frac{1}{2}\left(2a^2+b^2\right)\log(x)-\frac{1}{6}b\left(-4a\sin(c)\text{CosIntegral}\left(dx^3\right)-4a\cos(c)\text{Si}\left(dx^3\right)+b\cos(2c)\text{CosIntegral}\left(2dx^3\right)-b\sin(2c)\text{Si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x,x]
```

```
[Out] ((2*a^2 + b^2)*Log[x])/2 - (b*(b*Cos[2*c]*CosIntegral[2*d*x^3] - 4*a*CosInt
egral[d*x^3]*Sin[c] - 4*a*Cos[c]*SinIntegral[d*x^3] - b*Sin[2*c]*SinIntegra
l[2*d*x^3]))/6
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(dx^3+c\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x,x)
```

Maxima [C] time = 1.22112, size = 146, normalized size = 1.82

$$-\frac{1}{3}\left(\left(i\,\text{Ei}\left(i\,dx^3\right)-i\,\text{Ei}\left(-i\,dx^3\right)\right)\cos(c)-\left(\text{Ei}\left(i\,dx^3\right)+\text{Ei}\left(-i\,dx^3\right)\right)\sin(c)\right)ab-\frac{1}{12}\left(\left(\text{Ei}\left(2i\,dx^3\right)+\text{Ei}\left(-2i\,dx^3\right)\right)\cos(2c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/3*((I*Ei(I*d*x^3) - I*Ei(-I*d*x^3))*cos(c) - (Ei(I*d*x^3) + Ei(-I*d*x^3)
)*sin(c))*a*b - 1/12*((Ei(2*I*d*x^3) + Ei(-2*I*d*x^3))*cos(2*c) - (-I*Ei(2*
```

```
I*d*x^3) + I*Ei(-2*I*d*x^3))*sin(2*c) - 6*log(x))*b^2 + a^2*log(x)
```

Fricas [A] time = 1.70371, size = 328, normalized size = 4.1

$$\frac{1}{6}b^2\sin(2c)\,\mathrm{Si}\left(2dx^3\right)+\frac{2}{3}ab\cos(c)\,\mathrm{Si}\left(dx^3\right)-\frac{1}{12}\left(b^2\,\mathrm{Ci}\left(2dx^3\right)+b^2\,\mathrm{Ci}\left(-2dx^3\right)\right)\cos(2c)+\frac{1}{2}\left(2a^2+b^2\right)\log(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*sin(2*c)*sin_integral(2*d*x^3) + 2/3*a*b*cos(c)*sin_integral(d*x^3)
 - 1/12*(b^2*cos_integral(2*d*x^3) + b^2*cos_integral(-2*d*x^3))*cos(2*c) +
 1/2*(2*a^2 + b^2)*log(x) + 1/3*(a*b*cos_integral(d*x^3) + a*b*cos_integral
(-d*x^3))*sin(c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x, x)
```

Giac [A] time = 1.10639, size = 107, normalized size = 1.34

$$-\frac{1}{6}b^2\cos(2c)\,\mathrm{Ci}\left(2dx^3\right)+\frac{2}{3}ab\,\mathrm{Ci}\left(dx^3\right)\sin(c)+\frac{2}{3}ab\cos(c)\,\mathrm{Si}\left(dx^3\right)-\frac{1}{6}b^2\sin(2c)\,\mathrm{Si}\left(-2dx^3\right)+\frac{1}{3}a^2\log\left(dx^3\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x,x, algorithm="giac")
```

```
[Out] -1/6*b^2*cos(2*c)*cos_integral(2*d*x^3) + 2/3*a*b*cos_integral(d*x^3)*sin(c
) + 2/3*a*b*cos(c)*sin_integral(d*x^3) - 1/6*b^2*sin(2*c)*sin_integral(-2*d
*x^3) + 1/3*a^2*log(d*x^3) + 1/6*b^2*log(d*x^3)
```

3.72 $\int \frac{(a+b\sin(c+dx^3))^2}{x^4}\,dx$

Optimal. Leaf size=122

$$-\frac{2a^2+b^2}{6x^3}+\frac{2}{3}abd\cos(c)\text{CosIntegral}\left(dx^3\right)-\frac{2}{3}abd\sin(c)\text{Si}\left(dx^3\right)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}b^2d\sin(2c)\text{CosIntegral}\left(2dx\right.$$

```
[Out] -(2*a^2 + b^2)/(6*x^3) + (b^2*Cos[2*(c + d*x^3)])/(6*x^3) + (2*a*b*d*Cos[c]
*CosIntegral[d*x^3])/3 + (b^2*d*CosIntegral[2*d*x^3]*Sin[2*c])/3 - (2*a*b*S
in[c + d*x^3])/(3*x^3) - (2*a*b*d*Sin[c]*SinIntegral[d*x^3])/3 + (b^2*d*Cos
[2*c]*SinIntegral[2*d*x^3])/3
```

Rubi [A] time = 0.218841, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3403, 6, 3380, 3297, 3303, 3299, 3302, 3379}

$$-\frac{2a^2+b^2}{6x^3}+\frac{2}{3}abd\cos(c)\text{CosIntegral}\left(dx^3\right)-\frac{2}{3}abd\sin(c)\text{Si}\left(dx^3\right)-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}b^2d\sin(2c)\text{CosIntegral}\left(2dx\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^4,x]
```


```
[Out] -(2*a^2 + b^2)/(6*x^3) + (b^2*Cos[2*(c + d*x^3)])/(6*x^3) + (2*a*b*d*Cos[c]
*CosIntegral[d*x^3])/3 + (b^2*d*CosIntegral[2*d*x^3]*Sin[2*c])/3 - (2*a*b*S
in[c + d*x^3])/(3*x^3) - (2*a*b*d*Sin[c]*SinIntegral[d*x^3])/3 + (b^2*d*Cos
[2*c]*SinIntegral[2*d*x^3])/3
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx &= \int\left(\frac{a^2}{x^4}+\frac{b^2}{2x^4}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^4}+\frac{2ab\sin\left(c+dx^3\right)}{x^4}\right)dx\\
&= \int\left(\frac{a^2+\frac{b^2}{2}}{x^4}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^4}+\frac{2ab\sin\left(c+dx^3\right)}{x^4}\right)dx\\
&= -\frac{2a^2+b^2}{6x^3}+(2ab)\int\frac{\sin\left(c+dx^3\right)}{x^4}\,dx-\frac{1}{2}b^2\int\frac{\cos\left(2c+2dx^3\right)}{x^4}\,dx\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{1}{3}(2ab)\,\text{Subst}\left(\int\frac{\sin(c+dx)}{x^2}\,dx,x,x^3\right)-\frac{1}{6}b^2\,\text{Subst}\left(\int\frac{\cos(2c+2dx)}{x^2}\,d\right.\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}(2abd)\,\text{Subst}\left(\int\frac{\cos(c+dx}{x}\right.\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}-\frac{2ab\sin\left(c+dx^3\right)}{3x^3}+\frac{1}{3}(2abd\cos(c))\,\text{Subst}\left(\int\frac{\cos(}{x}\right.\\
&= -\frac{2a^2+b^2}{6x^3}+\frac{b^2\cos\left(2\left(c+dx^3\right)\right)}{6x^3}+\frac{2}{3}abd\cos(c)\text{Ci}\left(dx^3\right)+\frac{1}{3}b^2d\text{Ci}\left(2dx^3\right)\sin(2c)-\frac{2a}{}
\end{aligned}$$

Mathematica [A] time = 0.26715, size = 116, normalized size = 0.95

$$\frac{-2a^2+4abdx^3\cos(c)\text{CosIntegral}\left(dx^3\right)-4abdx^3\sin(c)\text{Si}\left(dx^3\right)-4ab\sin\left(c+dx^3\right)+2b^2dx^3\sin(2c)\text{CosIntegral}\left(2\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^4,x]
```

```
[Out] (-2*a^2 - b^2 + b^2*Cos[2*(c + d*x^3)] + 4*a*b*d*x^3*Cos[c]*CosIntegral[d*x
^3] + 2*b^2*d*x^3*CosIntegral[2*d*x^3]*Sin[2*c] - 4*a*b*Sin[c + d*x^3] - 4*
```

```
a*b*d*x^3*Sin[c]*SinIntegral[d*x^3] + 2*b^2*d*x^3*Cos[2*c]*SinIntegral[2*d*
x^3])/(6*x^3)
```

Maple [F] time = 0.225, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(dx^3+c\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^4,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^4,x)
```

Maxima [C] time = 1.24392, size = 167, normalized size = 1.37

$$\frac{1}{3}\left(\left(\Gamma\left(-1,i\,dx^3\right)+\Gamma\left(-1,-i\,dx^3\right)\right)\cos\left(c\right)-\left(i\,\Gamma\left(-1,i\,dx^3\right)-i\,\Gamma\left(-1,-i\,dx^3\right)\right)\sin\left(c\right)\right)abd+\frac{\left(\left(\left(i\,\Gamma\left(-1,2i\,dx^3\right)-i\,\Gamma\left(-1,-2i\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((gamma(-1, I*d*x^3) + gamma(-1, -I*d*x^3))*cos(c) - (I*gamma(-1, I*d*x
^3) - I*gamma(-1, -I*d*x^3))*sin(c))*a*b*d + 1/6*(((I*gamma(-1, 2*I*d*x^3)
- I*gamma(-1, -2*I*d*x^3))*cos(2*c) + (gamma(-1, 2*I*d*x^3) + gamma(-1, -2*
I*d*x^3))*sin(2*c))*d*x^3 - 1)*b^2/x^3 - 1/3*a^2/x^3
```

Fricas [A] time = 1.7884, size = 425, normalized size = 3.48

$$\frac{2\,b^2dx^3\cos\left(2\,c\right)\mathrm{Si}\left(2\,dx^3\right)-4\,abdx^3\sin\left(c\right)\mathrm{Si}\left(dx^3\right)+2\,b^2\cos\left(dx^3+c\right)^2-4\,ab\sin\left(dx^3+c\right)-2\,a^2-2\,b^2+2\left(abdx^3\,\mathrm{Ci}\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d*x^3*cos(2*c)*sin_integral(2*d*x^3) - 4*a*b*d*x^3*sin(c)*sin_in
tegral(d*x^3) + 2*b^2*cos(d*x^3 + c)^2 - 4*a*b*sin(d*x^3 + c) - 2*a^2 - 2*b
^2 + 2*(a*b*d*x^3*cos_integral(d*x^3) + a*b*d*x^3*cos_integral(-d*x^3))*cos
(c) + (b^2*d*x^3*cos_integral(2*d*x^3) + b^2*d*x^3*cos_integral(-2*d*x^3))*
sin(2*c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**4, x)
```

Giac [B] time = 1.12372, size = 305, normalized size = 2.5

$$\frac{4\left(dx^3+c\right)abd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)-4\,abcd^2\cos\left(c\right)\operatorname{Ci}\left(dx^3\right)+2\left(dx^3+c\right)b^2d^2\operatorname{Ci}\left(2\,dx^3\right)\sin\left(2\,c\right)-2\,b^2cd^2\operatorname{Ci}\left(2\,dx^3\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(4*(d*x^3 + c)*a*b*d^2*cos(c)*cos_integral(d*x^3) - 4*a*b*c*d^2*cos(c)*
cos_integral(d*x^3) + 2*(d*x^3 + c)*b^2*d^2*cos_integral(2*d*x^3)*sin(2*c)
- 2*b^2*c*d^2*cos_integral(2*d*x^3)*sin(2*c) - 4*(d*x^3 + c)*a*b*d^2*sin(c)
*sin_integral(d*x^3) + 4*a*b*c*d^2*sin(c)*sin_integral(d*x^3) - 2*(d*x^3 +
c)*b^2*d^2*cos(2*c)*sin_integral(-2*d*x^3) + 2*b^2*c*d^2*cos(2*c)*sin_integ
ral(-2*d*x^3) + b^2*d^2*cos(2*d*x^3 + 2*c) - 4*a*b*d^2*sin(d*x^3 + c) - 2*a
^2*d^2 - b^2*d^2)/(d^2*x^3)
```

3.73 $\int x^4 \left(a+b\sin\left(c+dx^3\right)\right)^2 dx$

Optimal. Leaf size=249

$$-\frac{2abe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}+\frac{ib^2e^{2ic}x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}x^2\text{Gamma}\left(\frac{2}{3},2i\right.}{36\ 2^{2/3}d\left(idx^3\right)^{2/3}}$$

```
[Out] ((2*a^2 + b^2)*x^5)/10 - (2*a*b*x^2*Cos[c + d*x^3])/(3*d) - (2*a*b*E^(I*c)*
x^2*Gamma[2/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (2*a*b*x^2*Gamma[2/3
, I*d*x^3])/(9*d*E^(I*c)*(I*d*x^3)^(2/3)) + ((I/36)*b^2*E^((2*I)*c)*x^2*Gam
ma[2/3, (-2*I)*d*x^3])/(2^(2/3)*d*((-I)*d*x^3)^(2/3)) - ((I/36)*b^2*x^2*Gam
ma[2/3, (2*I)*d*x^3])/(2^(2/3)*d*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (b^2*x^2*Si
n[2*c + 2*d*x^3])/(12*d)
```

Rubi [A] time = 0.206465, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3403, 6, 3386, 3389, 2218, 3385, 3390}

$$-\frac{2abe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{9d\left(idx^3\right)^{2/3}}+\frac{ib^2e^{2ic}x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{36\ 2^{2/3}d\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}x^2\text{Gamma}\left(\frac{2}{3},2i\right.}{36\ 2^{2/3}d\left(idx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*Sin[c + d*x^3])^2,x]
```



```
[Out] ((2*a^2 + b^2)*x^5)/10 - (2*a*b*x^2*Cos[c + d*x^3])/(3*d) - (2*a*b*E^(I*c)*
x^2*Gamma[2/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(2/3)) - (2*a*b*x^2*Gamma[2/3
, I*d*x^3])/(9*d*E^(I*c)*(I*d*x^3)^(2/3)) + ((I/36)*b^2*E^((2*I)*c)*x^2*Gam
ma[2/3, (-2*I)*d*x^3])/(2^(2/3)*d*((-I)*d*x^3)^(2/3)) - ((I/36)*b^2*x^2*Gam
ma[2/3, (2*I)*d*x^3])/(2^(2/3)*d*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (b^2*x^2*Si
n[2*c + 2*d*x^3])/(12*d)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int\left(a^2x^4+\frac{b^2x^4}{2}-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+2abx^4\sin\left(c+dx^3\right)\right)dx\\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x^4-\frac{1}{2}b^2x^4\cos\left(2c+2dx^3\right)+2abx^4\sin\left(c+dx^3\right)\right)dx\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5+(2ab)\int x^4\sin\left(c+dx^3\right)dx-\frac{1}{2}b^2\int x^4\cos\left(2c+2dx^3\right)dx\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}+\frac{(4ab)\int x\cos\left(c+d\right.}{3d}\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{b^2x^2\sin\left(2c+2dx^3\right)}{12d}+\frac{(2ab)\int e^{-ic-idx^3}x\,d}{3d}\\
&= \frac{1}{10}\left(2a^2+b^2\right)x^5-\frac{2abx^2\cos\left(c+dx^3\right)}{3d}-\frac{2abe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{9d\left(-idx^3\right)^{2/3}}-\frac{2abe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right.}{9d\left(idx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.630199, size = 339, normalized size = 1.36

$$\frac{dx^8\left(-80ab\left(-idx^3\right)^{2/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)-80ab\left(idx^3\right)^{2/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+5i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (d*x^8*(72*a^2*d*x^3*(d^2*x^6)^(2/3) + 36*b^2*d*x^3*(d^2*x^6)^(2/3) - 240*a
*b*(d^2*x^6)^(2/3)*Cos[c + d*x^3] + (5*I)*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Cos[2
*c]*Gamma[2/3, (-2*I)*d*x^3] - (5*I)*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Cos[2*c
]*Gamma[2/3, (2*I)*d*x^3] - 80*a*b*((-I)*d*x^3)^(2/3)*Gamma[2/3, I*d*x^3]*(
Cos[c] - I*Sin[c]) - 80*a*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c]
+ I*Sin[c]) - 5*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*
c] - 5*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 30
```

```
*b^2*(d^2*x^6)^(2/3)*Sin[2*(c + d*x^3)]))/(360*(d^2*x^6)^(5/3))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int x^4 \left(a + b\sin\left(dx^3 + c\right)\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^4*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [B] time = 1.28706, size = 830, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - 1/9*(6*x^3*abs(d)*cos(d*x^3 + c) + (x^3*abs(d))^(1/3)*(((gamm
a(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (
gamma(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)
) - (I*gamma(2/3, I*d*x^3) - I*gamma(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arcta
n2(0, d)) - (-I*gamma(2/3, I*d*x^3) + I*gamma(2/3, -I*d*x^3))*sin(-1/3*pi +
 2/3*arctan2(0, d)))*cos(c) - ((I*gamma(2/3, I*d*x^3) - I*gamma(2/3, -I*d*x
^3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (I*gamma(2/3, I*d*x^3) - I*gamma(2/3
, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) + (gamma(2/3, I*d*x^3) + gamm
a(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) - (gamma(2/3, I*d*x^3) +
gamma(2/3, -I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*sin(c))*a*b/(d*x*a
bs(d)) + 1/720*(72*d*x^6*abs(d) - 60*x^3*abs(d)*sin(2*d*x^3 + 2*c) + 2^(1/3
)*(x^3*abs(d))^(1/3)*(((-5*I*gamma(2/3, 2*I*d*x^3) + 5*I*gamma(2/3, -2*I*d*
x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (-5*I*gamma(2/3, 2*I*d*x^3) + 5*I*g
amma(2/3, -2*I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) - 5*(gamma(2/3, 2*I
*d*x^3) + gamma(2/3, -2*I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) + 5*(gamm
a(2/3, 2*I*d*x^3) + gamma(2/3, -2*I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)
))*cos(2*c) - (5*(gamma(2/3, 2*I*d*x^3) + gamma(2/3, -2*I*d*x^3))*cos(1/3*p
i + 2/3*arctan2(0, d)) + 5*(gamma(2/3, 2*I*d*x^3) + gamma(2/3, -2*I*d*x^3))
*cos(-1/3*pi + 2/3*arctan2(0, d)) - (5*I*gamma(2/3, 2*I*d*x^3) - 5*I*gamma(
2/3, -2*I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) - (-5*I*gamma(2/3, 2*I*d*
x^3) + 5*I*gamma(2/3, -2*I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*sin(2*
c)))*b^2/(d*x*abs(d))
```

Fricas [A] time = 1.81543, size = 446, normalized size = 1.79

$$\frac{36\left(2\,a^2 + b^2\right)d^2x^5 - 60\,b^2dx^2\cos\left(dx^3 + c\right)\sin\left(dx^3 + c\right) - 240\,abdx^2\cos\left(dx^3 + c\right) - 5\,b^2\left(2i\,d\right)^{\frac{1}{3}}e^{(-2i\,c)}\Gamma\left(\frac{2}{3}, 2i\,dx^3\right) + 80i}{360\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/360*(36*(2*a^2 + b^2)*d^2*x^5 - 60*b^2*d*x^2*cos(d*x^3 + c)*sin(d*x^3 + c
) - 240*a*b*d*x^2*cos(d*x^3 + c) - 5*b^2*(2*I*d)^(1/3)*e^(-2*I*c)*gamma(2/3
, 2*I*d*x^3) + 80*I*a*b*(I*d)^(1/3)*e^(-I*c)*gamma(2/3, I*d*x^3) - 80*I*a*b
*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -I*d*x^3) - 5*b^2*(-2*I*d)^(1/3)*e^(2*I*c)
*gamma(2/3, -2*I*d*x^3))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(a + b\sin\left(c + dx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x**4*(a + b*sin(c + d*x**3))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3 + c\right) + a\right)^2 x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x^4, x)
```

3.74 $\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$

Optimal. Leaf size=193

$$\frac{iabe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}}+\frac{b^2e^{-2ic}x^2\text{Gamma}\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}$$

```
[Out] ((2*a^2 + b^2)*x^2)/4 + ((I/3)*a*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I
)*d*x^3)^(2/3) - ((I/3)*a*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/
3)) + (b^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(12*2^(2/3)*((-I)*d*x^
3)^(2/3)) + (b^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(12*2^(2/3)*E^((2*I)*c)*(I*d*
x^3)^(2/3))
```

Rubi [A] time = 0.135917, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3403, 6, 3390, 2218, 3389}

$$\frac{iabe^{ic}x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{12\ 2^{2/3}\left(-idx^3\right)^{2/3}}+\frac{b^2e^{-2ic}x^2\text{Gamma}\left(\frac{2}{3},2idx^3\right)}{12\ 2^{2/3}\left(idx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x^2)/4 + ((I/3)*a*b*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((-I
)*d*x^3)^(2/3) - ((I/3)*a*b*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(2/
3)) + (b^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(12*2^(2/3)*((-I)*d*x^
3)^(2/3)) + (b^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(12*2^(2/3)*E^((2*I)*c)*(I*d*
x^3)^(2/3))
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\sin\left(c+dx^3\right)\right)^2\,dx &= \int\left(a^2x+\frac{b^2x}{2}-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+2abx\sin\left(c+dx^3\right)\right)dx\\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x-\frac{1}{2}b^2x\cos\left(2c+2dx^3\right)+2abx\sin\left(c+dx^3\right)\right)dx\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+(2ab)\int x\sin\left(c+dx^3\right)dx-\frac{1}{2}b^2\int x\cos\left(2c+2dx^3\right)dx\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+(iab)\int e^{-ic-idx^3}x\,dx-(iab)\int e^{ic+idx^3}x\,dx-\frac{1}{4}b^2\int e^{-2ic-2idx^3}x\,dx-\\
&= \frac{1}{4}\left(2a^2+b^2\right)x^2+\frac{iabe^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{3\left(-idx^3\right)^{2/3}}-\frac{iabe^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{3\left(idx^3\right)^{2/3}}+\frac{b^2e^{2ic}x^2\Gamma\left(\frac{2}{3},-2idx^3\right)}{12\,2^{2/3}\left(-idx^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.366051, size = 283, normalized size = 1.47

$$
\frac{x^2\left(-8iab\left(-idx^3\right)^{2/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)+8iab\left(idx^3\right)^{2/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+\sqrt[3]{2}b\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (x^2*(12*a^2*(d^2*x^6)^(2/3) + 6*b^2*(d^2*x^6)^(2/3) + 2^(1/3)*b^2*(I*d*x^3
)^(2/3)*Cos[2*c]*Gamma[2/3, (-2*I)*d*x^3] + 2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*
Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] - (8*I)*a*b*((-I)*d*x^3)^(2/3)*Gamma[2/3,
I*d*x^3]*(Cos[c] - I*Sin[c]) + (8*I)*a*b*(I*d*x^3)^(2/3)*Gamma[2/3, (-I)*d*
x^3]*(Cos[c] + I*Sin[c]) + I*2^(1/3)*b^2*(I*d*x^3)^(2/3)*Gamma[2/3, (-2*I)*
d*x^3]*Sin[2*c] - I*2^(1/3)*b^2*((-I)*d*x^3)^(2/3)*Gamma[2/3, (2*I)*d*x^3]*
Sin[2*c]))/(24*(d^2*x^6)^(2/3))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$
\int x\left(a+b\sin\left(dx^3+c\right)\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [B] time = 1.25813, size = 770, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/6*(x^3*abs(d))^(1/3)*(((-I*gamma(2/3, I*d*x^3) + I*gamma(2/
3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) + (-I*gamma(2/3, I*d*x^3) + I
*gamma(2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) - (gamma(2/3, I*d*x
^3) + gamma(2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) + (gamma(2/3, I
*d*x^3) + gamma(2/3, -I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*cos(c) -
((gamma(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d
)) + (gamma(2/3, I*d*x^3) + gamma(2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2
(0, d)) - (I*gamma(2/3, I*d*x^3) - I*gamma(2/3, -I*d*x^3))*sin(1/3*pi + 2/3
*arctan2(0, d)) - (-I*gamma(2/3, I*d*x^3) + I*gamma(2/3, -I*d*x^3))*sin(-1/
3*pi + 2/3*arctan2(0, d)))*sin(c))*a*b/(x*abs(d)) + 1/48*(12*x^3*abs(d) + 2
^(1/3)*(x^3*abs(d))^(1/3)*(((gamma(2/3, 2*I*d*x^3) + gamma(2/3, -2*I*d*x^3)
)*cos(1/3*pi + 2/3*arctan2(0, d)) + (gamma(2/3, 2*I*d*x^3) + gamma(2/3, -2*
I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) - (I*gamma(2/3, 2*I*d*x^3) - I*g
amma(2/3, -2*I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) - (-I*gamma(2/3, 2*I
*d*x^3) + I*gamma(2/3, -2*I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*cos(2
*c) - ((I*gamma(2/3, 2*I*d*x^3) - I*gamma(2/3, -2*I*d*x^3))*cos(1/3*pi + 2/
3*arctan2(0, d)) + (I*gamma(2/3, 2*I*d*x^3) - I*gamma(2/3, -2*I*d*x^3))*cos
(-1/3*pi + 2/3*arctan2(0, d)) + (gamma(2/3, 2*I*d*x^3) + gamma(2/3, -2*I*d*
x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) - (gamma(2/3, 2*I*d*x^3) + gamma(2/3,
 -2*I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*sin(2*c)))*b^2/(x*abs(d))
```

Fricas [A] time = 1.69592, size = 328, normalized size = 1.7

$$\frac{-i\,b^2\,(2i\,d)^{\frac{1}{3}}\,e^{(-2i\,c)}\Gamma\left(\frac{2}{3},2i\,dx^3\right) - 8\,ab\,(i\,d)^{\frac{1}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{2}{3},i\,dx^3\right) - 8\,ab\,(-i\,d)^{\frac{1}{3}}\,e^{(i\,c)}\Gamma\left(\frac{2}{3},-i\,dx^3\right) + i\,b^2\,(-2i\,d)^{\frac{1}{3}}\,e^{(2i\,c)}\Gamma\left(\frac{2}{3},-2i\,d\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*b^2*(2*I*d)^(1/3)*e^(-2*I*c)*gamma(2/3, 2*I*d*x^3) - 8*a*b*(I*d)^(
1/3)*e^(-I*c)*gamma(2/3, I*d*x^3) - 8*a*b*(-I*d)^(1/3)*e^(I*c)*gamma(2/3, -
I*d*x^3) + I*b^2*(-2*I*d)^(1/3)*e^(2*I*c)*gamma(2/3, -2*I*d*x^3) + 6*(2*a^2
 + b^2)*d*x^2)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\sin\left(c + dx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x*(a + b*sin(c + d*x**3))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(dx^3 + c\right) + a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x, x)
```

3.75 $\int \frac{(a+b\sin(c+dx^3))^2}{x^2}\,dx$

Optimal. Leaf size=229

$$-\frac{abe^{ic}dx^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{\left(-idx^3\right)^{2/3}}-\frac{abe^{-ic}dx^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{\left(idx^3\right)^{2/3}}+\frac{ib^2e^{2ic}dx^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{2\,2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}dx^2\text{Gamma}\left(\frac{2}{3},\right.}{2\,2^{2/3}\left(idx^3\right)^{2/3}}$$

```
[Out] -(2*a^2 + b^2)/(2*x) + (b^2*Cos[2*c + 2*d*x^3])/(2*x) - (a*b*d*E^(I*c)*x^2*
Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) - (a*b*d*x^2*Gamma[2/3, I*d*x^3]
)/(E^(I*c)*(I*d*x^3)^(2/3)) + ((I/2)*b^2*d*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I
)*d*x^3])/(2^(2/3)*((-I)*d*x^3)^(2/3)) - ((I/2)*b^2*d*x^2*Gamma[2/3, (2*I)*
d*x^3])/(2^(2/3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (2*a*b*Sin[c + d*x^3])/x
```

Rubi [A] time = 0.189251, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3403, 6, 3388, 3389, 2218, 3387, 3390}

$$-\frac{abe^{ic}dx^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{\left(-idx^3\right)^{2/3}}-\frac{abe^{-ic}dx^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{\left(idx^3\right)^{2/3}}+\frac{ib^2e^{2ic}dx^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{2\,2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{ib^2e^{-2ic}dx^2\text{Gamma}\left(\frac{2}{3},\right.}{2\,2^{2/3}\left(idx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^2,x]
```


```
[Out] -(2*a^2 + b^2)/(2*x) + (b^2*Cos[2*c + 2*d*x^3])/(2*x) - (a*b*d*E^(I*c)*x^2*
Gamma[2/3, (-I)*d*x^3])/((-I)*d*x^3)^(2/3) - (a*b*d*x^2*Gamma[2/3, I*d*x^3]
)/(E^(I*c)*(I*d*x^3)^(2/3)) + ((I/2)*b^2*d*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I
)*d*x^3])/(2^(2/3)*((-I)*d*x^3)^(2/3)) - ((I/2)*b^2*d*x^2*Gamma[2/3, (2*I)*
d*x^3])/(2^(2/3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (2*a*b*Sin[c + d*x^3])/x
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^2}\,dx &= \int \left(\frac{a^2}{x^2}+\frac{b^2}{2x^2}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^2}+\frac{2ab\sin\left(c+dx^3\right)}{x^2}\right)dx\\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^2}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^2}+\frac{2ab\sin\left(c+dx^3\right)}{x^2}\right)dx\\
&= -\frac{2a^2+b^2}{2x}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^2}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^2}\,dx\\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{2x}-\frac{2ab\sin\left(c+dx^3\right)}{x}+(6abd)\int x\cos\left(c+dx^3\right)\,dx+\Big(\\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{2x}-\frac{2ab\sin\left(c+dx^3\right)}{x}+(3abd)\int e^{-ic-idx^3}x\,dx+(3abd)\\
&= -\frac{2a^2+b^2}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{2x}-\frac{abde^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{\left(-idx^3\right)^{2/3}}-\frac{abde^{-ic}x^2\Gamma\left(\frac{2}{3},idx^3\right)}{\left(idx^3\right)^{2/3}}+\frac{ib^2de}{2}
\end{aligned}$$

Mathematica [A] time = 0.584463, size = 332, normalized size = 1.45

$$-4iab\left(-idx^3\right)^{5/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)+4iab\left(idx^3\right)^{5/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+\sqrt[3]{2}b^2\text{co}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^2,x]
```

```
[Out] (-4*a^2*(d^2*x^6)^(2/3) - 2*b^2*(d^2*x^6)^(2/3) + 2*b^2*(d^2*x^6)^(2/3)*Cos
[2*(c + d*x^3)] + 2^(1/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (-2*I)*d*
x^3] + 2^(1/3)*b^2*((-I)*d*x^3)^(5/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] - (4
*I)*a*b*((-I)*d*x^3)^(5/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (4*I)*
a*b*(I*d*x^3)^(5/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*2^(1/3)*
b^2*(I*d*x^3)^(5/3)*Gamma[2/3, (-2*I)*d*x^3]*Sin[2*c] - I*2^(1/3)*b^2*((-I)
*d*x^3)^(5/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] - 8*a*b*(d^2*x^6)^(2/3)*Sin[
```

```
c + d*x^3])/(4*x*(d^2*x^6)^(2/3))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(dx^{3}+c\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^2,x)
```

Maxima [B] time = 1.27987, size = 743, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(x^3*abs(d))^(1/3)*(((I*gamma(-1/3, I*d*x^3) - I*gamma(-1/3, -I*d*x^3)
)*cos(1/6*pi + 1/3*arctan2(0, d)) + (I*gamma(-1/3, I*d*x^3) - I*gamma(-1/3,
 -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (gamma(-1/3, I*d*x^3) + gamm
a(-1/3, -I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (gamma(-1/3, I*d*x^3)
+ gamma(-1/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*cos(c) + ((gamma
(-1/3, I*d*x^3) + gamma(-1/3, -I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) +
(gamma(-1/3, I*d*x^3) + gamma(-1/3, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0,
 d)) + (I*gamma(-1/3, I*d*x^3) - I*gamma(-1/3, -I*d*x^3))*sin(1/6*pi + 1/3*
arctan2(0, d)) + (-I*gamma(-1/3, I*d*x^3) + I*gamma(-1/3, -I*d*x^3))*sin(-1
/6*pi + 1/3*arctan2(0, d)))*sin(c))*a*b/x + 1/24*(2^(1/3)*(x^3*abs(d))^(1/3
)*(((gamma(-1/3, 2*I*d*x^3) + gamma(-1/3, -2*I*d*x^3))*cos(1/6*pi + 1/3*arc
tan2(0, d)) + (gamma(-1/3, 2*I*d*x^3) + gamma(-1/3, -2*I*d*x^3))*cos(-1/6*p
i + 1/3*arctan2(0, d)) + (I*gamma(-1/3, 2*I*d*x^3) - I*gamma(-1/3, -2*I*d*x
^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (-I*gamma(-1/3, 2*I*d*x^3) + I*gamma
(-1/3, -2*I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*cos(2*c) + ((-I*gamma
(-1/3, 2*I*d*x^3) + I*gamma(-1/3, -2*I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0,
d)) + (-I*gamma(-1/3, 2*I*d*x^3) + I*gamma(-1/3, -2*I*d*x^3))*cos(-1/6*pi +
 1/3*arctan2(0, d)) + (gamma(-1/3, 2*I*d*x^3) + gamma(-1/3, -2*I*d*x^3))*si
n(1/6*pi + 1/3*arctan2(0, d)) - (gamma(-1/3, 2*I*d*x^3) + gamma(-1/3, -2*I*
d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(2*c)) - 12)*b^2/x - a^2/x
```

Fricas [A] time = 1.8324, size = 392, normalized size = 1.71

$$-\frac{b^{2}\left(2i\,d\right)^{\frac{1}{3}}xe^{\left(-2i\,c\right)}\Gamma\left(\frac{2}{3},2i\,dx^{3}\right)-4i\,ab\left(i\,d\right)^{\frac{1}{3}}xe^{\left(-i\,c\right)}\Gamma\left(\frac{2}{3},i\,dx^{3}\right)+4i\,ab\left(-i\,d\right)^{\frac{1}{3}}xe^{\left(i\,c\right)}\Gamma\left(\frac{2}{3},-i\,dx^{3}\right)+b^{2}\left(-2i\,d\right)^{\frac{1}{3}}xe^{\left(2i\,c\right)}\Gamma\left(\frac{2}{3},-\right.}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*(2*I*d)^(1/3)*x*e^(-2*I*c)*gamma(2/3, 2*I*d*x^3) - 4*I*a*b*(I*d)^
(1/3)*x*e^(-I*c)*gamma(2/3, I*d*x^3) + 4*I*a*b*(-I*d)^(1/3)*x*e^(I*c)*gamma
(2/3, -I*d*x^3) + b^2*(-2*I*d)^(1/3)*x*e^(2*I*c)*gamma(2/3, -2*I*d*x^3) - 4
*b^2*cos(d*x^3 + c)^2 + 8*a*b*sin(d*x^3 + c) + 4*a^2 + 4*b^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(c + dx^3\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx^3 + c\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^2, x)
```

3.76 $\int \frac{(a+b\sin(c+dx^3))^2}{x^5}\,dx$

Optimal. Leaf size=283

$$-\frac{3iabe^{ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iabe^{-ic}d^2x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}}-\frac{3b^2e^{2ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{4\,2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{3b^2e^{-2ic}d^2x^2\text{Ga}}{4\,2^{2/3}(}$$

```
[Out] -(2*a^2 + b^2)/(8*x^4) - (3*a*b*d*Cos[c + d*x^3])/(2*x) + (b^2*Cos[2*c + 2*
d*x^3])/(8*x^4) - (((3*I)/4)*a*b*d^2*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((
-I)*d*x^3)^(2/3) + (((3*I)/4)*a*b*d^2*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*
d*x^3)^(2/3)) - (3*b^2*d^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(4*2^(
2/3)*((-I)*d*x^3)^(2/3)) - (3*b^2*d^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(4*2^(2/
3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (a*b*Sin[c + d*x^3])/(2*x^4) - (3*b^2*d*S
in[2*c + 2*d*x^3])/(4*x)
```

Rubi [A] time = 0.235946, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3403, 6, 3388, 3387, 3390, 2218, 3389}

$$-\frac{3iabe^{ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iabe^{-ic}d^2x^2\text{Gamma}\left(\frac{2}{3},idx^3\right)}{4\left(idx^3\right)^{2/3}}-\frac{3b^2e^{2ic}d^2x^2\text{Gamma}\left(\frac{2}{3},-2idx^3\right)}{4\,2^{2/3}\left(-idx^3\right)^{2/3}}-\frac{3b^2e^{-2ic}d^2x^2\text{Ga}}{4\,2^{2/3}(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^5,x]
```


```
[Out] -(2*a^2 + b^2)/(8*x^4) - (3*a*b*d*Cos[c + d*x^3])/(2*x) + (b^2*Cos[2*c + 2*
d*x^3])/(8*x^4) - (((3*I)/4)*a*b*d^2*E^(I*c)*x^2*Gamma[2/3, (-I)*d*x^3])/((
-I)*d*x^3)^(2/3) + (((3*I)/4)*a*b*d^2*x^2*Gamma[2/3, I*d*x^3])/(E^(I*c)*(I*
d*x^3)^(2/3)) - (3*b^2*d^2*E^((2*I)*c)*x^2*Gamma[2/3, (-2*I)*d*x^3])/(4*2^(
2/3)*((-I)*d*x^3)^(2/3)) - (3*b^2*d^2*x^2*Gamma[2/3, (2*I)*d*x^3])/(4*2^(2/
3)*E^((2*I)*c)*(I*d*x^3)^(2/3)) - (a*b*Sin[c + d*x^3])/(2*x^4) - (3*b^2*d*S
in[2*c + 2*d*x^3])/(4*x)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^5}\,dx = \int \left(\frac{a^2}{x^5}+\frac{b^2}{2x^5}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^5}+\frac{2ab\sin\left(c+dx^3\right)}{x^5}\right)dx$$

$$= \int \left(\frac{a^2+\frac{b^2}{2}}{x^5}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^5}+\frac{2ab\sin\left(c+dx^3\right)}{x^5}\right)dx$$

$$= -\frac{2a^2+b^2}{8x^4}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^5}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^5}\,dx$$

$$= -\frac{2a^2+b^2}{8x^4}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}-\frac{ab\sin\left(c+dx^3\right)}{2x^4}+\frac{1}{2}(3abd)\int \frac{\cos\left(c+dx^3\right)}{x^2}\,dx+\frac{1}{4}$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos\left(c+dx^3\right)}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}-\frac{ab\sin\left(c+dx^3\right)}{2x^4}-\frac{3b^2d\sin\left(2c\right.}{4x}$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos\left(c+dx^3\right)}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}-\frac{ab\sin\left(c+dx^3\right)}{2x^4}-\frac{3b^2d\sin\left(2c\right.}{4x}$$

$$= -\frac{2a^2+b^2}{8x^4}-\frac{3abd\cos\left(c+dx^3\right)}{2x}+\frac{b^2\cos\left(2c+2dx^3\right)}{8x^4}-\frac{3iabd^2e^{ic}x^2\Gamma\left(\frac{2}{3},-idx^3\right)}{4\left(-idx^3\right)^{2/3}}+\frac{3iab}{}$$

Mathematica [A] time = 2.5053, size = 292, normalized size = 1.03

$$-\frac{6iab\left(idx^3\right)^{2/3}\sqrt[3]{d^2x^6}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{2}{3},-idx^3\right)+6iab\left(idx^3\right)^{4/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{2}{3},idx^3\right)-3\sqrt[3]{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^5,x]
```

```
[Out] -(2*a^2 + b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - b^2*Cos[2*(c + d*x^3)] - 3*2^
(1/3)*b^2*(I*d*x^3)^(4/3)*Cos[2*c]*Gamma[2/3, (2*I)*d*x^3] + (6*I)*a*b*(I*d
```

```
*x^3)^(4/3)*Gamma[2/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I*d*x^3)^(
2/3)*(d^2*x^6)^(1/3)*Gamma[2/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 3*2^(1/3)
*b^2*((-I)*d*x^3)^(4/3)*Gamma[2/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]) +
(3*I)*2^(1/3)*b^2*(I*d*x^3)^(4/3)*Gamma[2/3, (2*I)*d*x^3]*Sin[2*c] + 4*a*b*
Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/(8*x^4)
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(d x^3 + c\right)\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^5,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^5,x)
```

Maxima [B] time = 1.26353, size = 756, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*(x^3*abs(d))^(1/3)*(((I*gamma(-4/3, I*d*x^3) - I*gamma(-4/3, -I*d*x^3)
)*cos(2/3*pi + 4/3*arctan2(0, d)) + (I*gamma(-4/3, I*d*x^3) - I*gamma(-4/3,
 -I*d*x^3))*cos(-2/3*pi + 4/3*arctan2(0, d)) - (gamma(-4/3, I*d*x^3) + gamm
a(-4/3, -I*d*x^3))*sin(2/3*pi + 4/3*arctan2(0, d)) + (gamma(-4/3, I*d*x^3)
+ gamma(-4/3, -I*d*x^3))*sin(-2/3*pi + 4/3*arctan2(0, d)))*cos(c) + ((gamma
(-4/3, I*d*x^3) + gamma(-4/3, -I*d*x^3))*cos(2/3*pi + 4/3*arctan2(0, d)) +
(gamma(-4/3, I*d*x^3) + gamma(-4/3, -I*d*x^3))*cos(-2/3*pi + 4/3*arctan2(0,
 d)) + (I*gamma(-4/3, I*d*x^3) - I*gamma(-4/3, -I*d*x^3))*sin(2/3*pi + 4/3*
arctan2(0, d)) + (-I*gamma(-4/3, I*d*x^3) + I*gamma(-4/3, -I*d*x^3))*sin(-2
/3*pi + 4/3*arctan2(0, d)))*sin(c))*a*b*abs(d)/x + 1/24*(2^(1/3)*(x^3*abs(d
))^(1/3)*((2*(gamma(-4/3, 2*I*d*x^3) + gamma(-4/3, -2*I*d*x^3))*cos(2/3*pi
+ 4/3*arctan2(0, d)) + 2*(gamma(-4/3, 2*I*d*x^3) + gamma(-4/3, -2*I*d*x^3))
*cos(-2/3*pi + 4/3*arctan2(0, d)) + (2*I*gamma(-4/3, 2*I*d*x^3) - 2*I*gamma
(-4/3, -2*I*d*x^3))*sin(2/3*pi + 4/3*arctan2(0, d)) + (-2*I*gamma(-4/3, 2*I
*d*x^3) + 2*I*gamma(-4/3, -2*I*d*x^3))*sin(-2/3*pi + 4/3*arctan2(0, d)))*co
s(2*c) + ((-2*I*gamma(-4/3, 2*I*d*x^3) + 2*I*gamma(-4/3, -2*I*d*x^3))*cos(2
/3*pi + 4/3*arctan2(0, d)) + (-2*I*gamma(-4/3, 2*I*d*x^3) + 2*I*gamma(-4/3,
 -2*I*d*x^3))*cos(-2/3*pi + 4/3*arctan2(0, d)) + 2*(gamma(-4/3, 2*I*d*x^3)
+ gamma(-4/3, -2*I*d*x^3))*sin(2/3*pi + 4/3*arctan2(0, d)) - 2*(gamma(-4/3,
 2*I*d*x^3) + gamma(-4/3, -2*I*d*x^3))*sin(-2/3*pi + 4/3*arctan2(0, d)))*si
n(2*c))*x^3*abs(d) - 3)*b^2/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 1.85463, size = 502, normalized size = 1.77

$$\frac{3 i b^2 (2 i d)^{\frac{1}{3}} d x^4 e^{(-2 i c)} \Gamma\left(\frac{2}{3}, 2 i d x^3\right) + 6 a b (i d)^{\frac{1}{3}} d x^4 e^{(-i c)} \Gamma\left(\frac{2}{3}, i d x^3\right) + 6 a b (-i d)^{\frac{1}{3}} d x^4 e^{(i c)} \Gamma\left(\frac{2}{3}, -i d x^3\right) - 3 i b^2 (-2 i d)^{\frac{1}{3}} d x^4 e}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*b^2*(2*I*d)^(1/3)*d*x^4*e^(-2*I*c)*gamma(2/3, 2*I*d*x^3) + 6*a*b*(
I*d)^(1/3)*d*x^4*e^(-I*c)*gamma(2/3, I*d*x^3) + 6*a*b*(-I*d)^(1/3)*d*x^4*e^
(I*c)*gamma(2/3, -I*d*x^3) - 3*I*b^2*(-2*I*d)^(1/3)*d*x^4*e^(2*I*c)*gamma(2
/3, -2*I*d*x^3) - 12*a*b*d*x^3*cos(d*x^3 + c) + 2*b^2*cos(d*x^3 + c)^2 - 2*
a^2 - 2*b^2 - 4*(3*b^2*d*x^3*cos(d*x^3 + c) + a*b)*sin(d*x^3 + c))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \sin\left(c + dx^3\right)\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**5,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(dx^3 + c\right) + a\right)^2}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^5, x)
```

3.77 $\int x^3 \left(a+b\sin\left(c+dx^3\right)\right)^2 dx$

Optimal. Leaf size=237

$$-\frac{abe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}+$$

```
[Out] ((2*a^2 + b^2)*x^4)/8 - (2*a*b*x*Cos[c + d*x^3])/(3*d) - (a*b*E^(I*c)*x*Gam
ma[1/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(1/3)) - (a*b*x*Gamma[1/3, I*d*x^3])
/(9*d*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/72)*b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I
)*d*x^3])/(2^(1/3)*d*((-I)*d*x^3)^(1/3)) - ((I/72)*b^2*x*Gamma[1/3, (2*I)*d
*x^3])/(2^(1/3)*d*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (b^2*x*Sin[2*c + 2*d*x^3])
/(12*d)
```

Rubi [A] time = 0.150236, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3403, 6, 3386, 3355, 2208, 3385, 3356}

$$-\frac{abe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{72\sqrt[3]{2}d\sqrt[3]{idx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*x^3])^2,x]
```


```
[Out] ((2*a^2 + b^2)*x^4)/8 - (2*a*b*x*Cos[c + d*x^3])/(3*d) - (a*b*E^(I*c)*x*Gam
ma[1/3, (-I)*d*x^3])/(9*d*((-I)*d*x^3)^(1/3)) - (a*b*x*Gamma[1/3, I*d*x^3])
/(9*d*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/72)*b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I
)*d*x^3])/(2^(1/3)*d*((-I)*d*x^3)^(1/3)) - ((I/72)*b^2*x*Gamma[1/3, (2*I)*d
*x^3])/(2^(1/3)*d*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (b^2*x*Sin[2*c + 2*d*x^3])
/(12*d)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
```

```
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\sin\left(c+dx^3\right)\right)^2 dx &= \int\left(a^2x^3+\frac{b^2x^3}{2}-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+2abx^3\sin\left(c+dx^3\right)\right)dx \\
&= \int\left(\left(a^2+\frac{b^2}{2}\right)x^3-\frac{1}{2}b^2x^3\cos\left(2c+2dx^3\right)+2abx^3\sin\left(c+dx^3\right)\right)dx \\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4+(2ab)\int x^3\sin\left(c+dx^3\right)dx-\frac{1}{2}b^2\int x^3\cos\left(2c+2dx^3\right)dx \\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}+\frac{(2ab)\int\cos\left(c+dx^3\right)}{3d} \\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{b^2x\sin\left(2c+2dx^3\right)}{12d}+\frac{(ab)\int e^{-ic-idx^3}dx}{3d}+ \\
&= \frac{1}{8}\left(2a^2+b^2\right)x^4-\frac{2abx\cos\left(c+dx^3\right)}{3d}-\frac{abe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{9d\sqrt[3]{-idx^3}}-\frac{abe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{9d\sqrt[3]{idx^3}}+
\end{aligned}
$$

Mathematica [A] time = 0.578417, size = 339, normalized size = 1.43

$$
\frac{dx^7\left(-16ab\sqrt[3]{-idx^3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{1}{3},idx^3\right)-16ab\sqrt[3]{idx^3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{1}{3},-idx^3\right)+i2^{2/3}b^2\text{co}\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (d*x^7*(36*a^2*d*x^3*(d^2*x^6)^(1/3) + 18*b^2*d*x^3*(d^2*x^6)^(1/3) - 96*a*
b*(d^2*x^6)^(1/3)*Cos[c + d*x^3] + I*2^(2/3)*b^2*(I*d*x^3)^(1/3)*Cos[2*c]*G
amma[1/3, (-2*I)*d*x^3] - I*2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Cos[2*c]*Gamma[1
/3, (2*I)*d*x^3] - 16*a*b*((-I)*d*x^3)^(1/3)*Gamma[1/3, I*d*x^3]*(Cos[c] -
I*Sin[c]) - 16*a*b*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c
]) - 2^(2/3)*b^2*(I*d*x^3)^(1/3)*Gamma[1/3, (-2*I)*d*x^3]*Sin[2*c] - 2^(2/3
)*b^2*((-I)*d*x^3)^(1/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] - 12*b^2*(d^2*x^6
```

```
)^(1/3)*Sin[2*(c + d*x^3)]))/(144*(d^2*x^6)^(4/3))
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int x^3 \left(a + b \sin \left(dx^3 + c\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^3*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [B] time = 1.24388, size = 818, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 - 1/288*2^(2/3)*((((I*gamma(1/3, 2*I*d*x^3) - I*gamma(1/3, -2*I
*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) + (I*gamma(1/3, 2*I*d*x^3) - I*gam
ma(1/3, -2*I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, 2*I*d*x
^3) + gamma(1/3, -2*I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) - (gamma(1/3,
 2*I*d*x^3) + gamma(1/3, -2*I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*cos
(2*c) + ((gamma(1/3, 2*I*d*x^3) + gamma(1/3, -2*I*d*x^3))*cos(1/6*pi + 1/3*
arctan2(0, d)) + (gamma(1/3, 2*I*d*x^3) + gamma(1/3, -2*I*d*x^3))*cos(-1/6*
pi + 1/3*arctan2(0, d)) + (-I*gamma(1/3, 2*I*d*x^3) + I*gamma(1/3, -2*I*d*x
^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (I*gamma(1/3, 2*I*d*x^3) - I*gamma(1
/3, -2*I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(2*c))*x - 6*2^(1/3)*
(3*d*x^4 - 2*x*sin(2*d*x^3 + 2*c))*(x^3*abs(d))^(1/3))*b^2/((x^3*abs(d))^(1
/3)*d) - 1/18*(12*(x^3*abs(d))^(1/3)*x*cos(d*x^3 + c) + (((gamma(1/3, I*d*x
^3) + gamma(1/3, -I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I
*d*x^3) + gamma(1/3, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) + (-I*gamm
a(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) +
 (I*gamma(1/3, I*d*x^3) - I*gamma(1/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2
(0, d)))*cos(c) + ((-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*cos(1/
6*pi + 1/3*arctan2(0, d)) + (-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3
))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (gamma(1/3, I*d*x^3) + gamma(1/3, -I*
d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I*d*x^3) + gamma(1/3,
 -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(c))*x)*a*b/((x^3*abs(d))^
(1/3)*d)
```

Fricas [A] time = 1.86661, size = 433, normalized size = 1.83

$$\frac{18 \left(2 a^2 + b^2\right) d^2 x^4 - 24 b^2 dx \cos\left(dx^3 + c\right) \sin\left(dx^3 + c\right) - 96 abdx \cos\left(dx^3 + c\right) - b^2 \left(2i d\right)^{\frac{2}{3}} e^{(-2ic)} \Gamma\left(\frac{1}{3}, 2i dx^3\right) + 16i ab (i}{144 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(18*(2*a^2 + b^2)*d^2*x^4 - 24*b^2*d*x*cos(d*x^3 + c)*sin(d*x^3 + c)
- 96*a*b*d*x*cos(d*x^3 + c) - b^2*(2*I*d)^(2/3)*e^(-2*I*c)*gamma(1/3, 2*I*d
*x^3) + 16*I*a*b*(I*d)^(2/3)*e^(-I*c)*gamma(1/3, I*d*x^3) - 16*I*a*b*(-I*d)
^(2/3)*e^(I*c)*gamma(1/3, -I*d*x^3) - b^2*(-2*I*d)^(2/3)*e^(2*I*c)*gamma(1/
3, -2*I*d*x^3))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(a + b\sin\left(c + dx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x**3*(a + b*sin(c + d*x**3))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3 + c\right) + a\right)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*x^3, x)
```

3.78 $\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$

Optimal. Leaf size=183

$$\frac{iabe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}+$$

```
[Out] ((2*a^2 + b^2)*x)/2 + ((I/3)*a*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*
x^3)^(1/3) - ((I/3)*a*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) +
(b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(12*2^(1/3)*((-I)*d*x^3)^(1/3)
) + (b^2*x*Gamma[1/3, (2*I)*d*x^3])/(12*2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)
)
```

Rubi [A] time = 0.0746712, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3357, 3356, 2208, 3355}

$$\frac{iabe^{ic}x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e^{-2ic}x\text{Gamma}\left(\frac{1}{3},2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{idx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + ((I/3)*a*b*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])/((-I)*d*
x^3)^(1/3) - ((I/3)*a*b*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(I*d*x^3)^(1/3)) +
(b^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(12*2^(1/3)*((-I)*d*x^3)^(1/3)
) + (b^2*x*Gamma[1/3, (2*I)*d*x^3])/(12*2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)
)
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2 dx = \int \left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2dx^3\right)+2ab\sin\left(c+dx^3\right)\right) dx$$
$$= \frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int \sin\left(c+dx^3\right) dx-\frac{1}{2}b^2\int \cos\left(2c+2dx^3\right) dx$$
$$= \frac{1}{2}\left(2a^2+b^2\right)x+(iab)\int e^{-ic-idx^3} dx-(iab)\int e^{ic+idx^3} dx-\frac{1}{4}b^2\int e^{-2ic-2idx^3} dx-\frac{1}{4}b^2\int$$
$$= \frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{3\sqrt[3]{-idx^3}}-\frac{iabe^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{3\sqrt[3]{idx^3}}+\frac{b^2e^{2ic}x\Gamma\left(\frac{1}{3},-2idx^3\right)}{12\sqrt[3]{2}\sqrt[3]{-idx^3}}+\frac{b^2e}{}$$

Mathematica [A] time = 0.274625, size = 281, normalized size = 1.54

$$\frac{x\left(-8iab\sqrt[3]{-idx^3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{1}{3},idx^3\right)+8iab\sqrt[3]{idx^3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{1}{3},-idx^3\right)+2^{2/3}b^2\cos(2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (x*(24*a^2*(d^2*x^6)^(1/3) + 12*b^2*(d^2*x^6)^(1/3) + 2^(2/3)*b^2*(I*d*x^3)
^(1/3)*Cos[2*c]*Gamma[1/3, (-2*I)*d*x^3] + 2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*C
os[2*c]*Gamma[1/3, (2*I)*d*x^3] - (8*I)*a*b*((-I)*d*x^3)^(1/3)*Gamma[1/3, I
*d*x^3]*(Cos[c] - I*Sin[c]) + (8*I)*a*b*(I*d*x^3)^(1/3)*Gamma[1/3, (-I)*d*x
^3]*(Cos[c] + I*Sin[c]) + I*2^(2/3)*b^2*(I*d*x^3)^(1/3)*Gamma[1/3, (-2*I)*d
*x^3]*Sin[2*c] - I*2^(2/3)*b^2*((-I)*d*x^3)^(1/3)*Gamma[1/3, (2*I)*d*x^3]*S
in[2*c]))/(24*(d^2*x^6)^(1/3))
```

Maple [F] time = 0.13, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(dx^3+c\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2,x)
```

Maxima [B] time = 1.22682, size = 755, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(((-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*cos(1/6*pi + 1/3*ar
ctan2(0, d)) + (-I*gamma(1/3, I*d*x^3) + I*gamma(1/3, -I*d*x^3))*cos(-1/6*p
i + 1/3*arctan2(0, d)) - (gamma(1/3, I*d*x^3) + gamma(1/3, -I*d*x^3))*sin(1
```

```
/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I*d*x^3) + gamma(1/3, -I*d*x^3))*s
in(-1/6*pi + 1/3*arctan2(0, d)))*cos(c) - ((gamma(1/3, I*d*x^3) + gamma(1/3
, -I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, I*d*x^3) + gamma
(1/3, -I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (I*gamma(1/3, I*d*x^3)
- I*gamma(1/3, -I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) - (-I*gamma(1/3,
I*d*x^3) + I*gamma(1/3, -I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(c)
)*a*b*x/(x^3*abs(d))^(1/3) + 1/48*2^(2/3)*((((gamma(1/3, 2*I*d*x^3) + gamma
(1/3, -2*I*d*x^3))*cos(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, 2*I*d*x^3)
 + gamma(1/3, -2*I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) + (-I*gamma(1/3
, 2*I*d*x^3) + I*gamma(1/3, -2*I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) +
(I*gamma(1/3, 2*I*d*x^3) - I*gamma(1/3, -2*I*d*x^3))*sin(-1/6*pi + 1/3*arct
an2(0, d)))*cos(2*c) + ((-I*gamma(1/3, 2*I*d*x^3) + I*gamma(1/3, -2*I*d*x^3
))*cos(1/6*pi + 1/3*arctan2(0, d)) + (-I*gamma(1/3, 2*I*d*x^3) + I*gamma(1/
3, -2*I*d*x^3))*cos(-1/6*pi + 1/3*arctan2(0, d)) - (gamma(1/3, 2*I*d*x^3) +
 gamma(1/3, -2*I*d*x^3))*sin(1/6*pi + 1/3*arctan2(0, d)) + (gamma(1/3, 2*I*
d*x^3) + gamma(1/3, -2*I*d*x^3))*sin(-1/6*pi + 1/3*arctan2(0, d)))*sin(2*c)
)*x + 12*2^(1/3)*(x^3*abs(d))^(1/3)*x)*b^2/(x^3*abs(d))^(1/3) + a^2*x
```

Fricas [A] time = 1.83365, size = 327, normalized size = 1.79

$$\frac{-i\,b^2\,(2i\,d)^{\frac{2}{3}}\,e^{(-2i\,c)}\Gamma\left(\frac{1}{3},2i\,dx^3\right)-8\,ab\,(i\,d)^{\frac{2}{3}}\,e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)-8\,ab\,(-i\,d)^{\frac{2}{3}}\,e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)+i\,b^2\,(-2i\,d)^{\frac{2}{3}}\,e^{(2i\,c)}\Gamma\left(\frac{1}{3},-2i\,d\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*b^2*(2*I*d)^(2/3)*e^(-2*I*c)*gamma(1/3, 2*I*d*x^3) - 8*a*b*(I*d)^(
2/3)*e^(-I*c)*gamma(1/3, I*d*x^3) - 8*a*b*(-I*d)^(2/3)*e^(I*c)*gamma(1/3, -
I*d*x^3) + I*b^2*(-2*I*d)^(2/3)*e^(2*I*c)*gamma(1/3, -2*I*d*x^3) + 12*(2*a^
2 + b^2)*d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2, x)
```

3.79 $\int \frac{(a+b\sin(c+dx^3))^2}{x^3}\,dx$

Optimal. Leaf size=225

$$-\frac{abe^{ic}dx\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abe^{-ic}dx\text{Gamma}\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}dx\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}dx\text{Gamma}\left(\frac{1}{3},2\right.}{4\sqrt[3]{2}\sqrt[3]{idx^3}}$$

```
[Out] -(2*a^2 + b^2)/(4*x^2) + (b^2*Cos[2*c + 2*d*x^3])/(4*x^2) - (a*b*d*E^(I*c)*
x*Gamma[1/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(1/3)) - (a*b*d*x*Gamma[1/3, I*d*
x^3])/(2*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/4)*b^2*d*E^((2*I)*c)*x*Gamma[1/3, (
-2*I)*d*x^3])/(2^(1/3)*((-I)*d*x^3)^(1/3)) - ((I/4)*b^2*d*x*Gamma[1/3, (2*I
)*d*x^3])/(2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (a*b*Sin[c + d*x^3])/x^2
```

Rubi [A] time = 0.133969, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3403, 6, 3388, 3355, 2208, 3387, 3356}

$$-\frac{abe^{ic}dx\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abe^{-ic}dx\text{Gamma}\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}+\frac{ib^2e^{2ic}dx\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{4\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{ib^2e^{-2ic}dx\text{Gamma}\left(\frac{1}{3},2\right.}{4\sqrt[3]{2}\sqrt[3]{idx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

```
[Out] -(2*a^2 + b^2)/(4*x^2) + (b^2*Cos[2*c + 2*d*x^3])/(4*x^2) - (a*b*d*E^(I*c)*
x*Gamma[1/3, (-I)*d*x^3])/(2*((-I)*d*x^3)^(1/3)) - (a*b*d*x*Gamma[1/3, I*d*
x^3])/(2*E^(I*c)*(I*d*x^3)^(1/3)) + ((I/4)*b^2*d*E^((2*I)*c)*x*Gamma[1/3, (
-2*I)*d*x^3])/(2^(1/3)*((-I)*d*x^3)^(1/3)) - ((I/4)*b^2*d*x*Gamma[1/3, (2*I
)*d*x^3])/(2^(1/3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (a*b*Sin[c + d*x^3])/x^2
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{b^2}{2x^3}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^3}+\frac{2ab\sin\left(c+dx^3\right)}{x^3}\right)dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^3}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^3}+\frac{2ab\sin\left(c+dx^3\right)}{x^3}\right)dx \\
&= -\frac{2a^2+b^2}{4x^2}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^3}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^3}\,dx \\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{ab\sin\left(c+dx^3\right)}{x^2}+(3abd)\int \cos\left(c+dx^3\right)dx+\frac{1}{2}\left(3b^2 \right. \\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{ab\sin\left(c+dx^3\right)}{x^2}+\frac{1}{2}(3abd)\int e^{-ic-idx^3}\,dx+\frac{1}{2}(3abd)\int \\
&= -\frac{2a^2+b^2}{4x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{4x^2}-\frac{abde^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{2\sqrt[3]{-idx^3}}-\frac{abde^{-ic}x\Gamma\left(\frac{1}{3},idx^3\right)}{2\sqrt[3]{idx^3}}+\frac{ib^2de^{2ic}x\Gamma}{4\sqrt[3]{2}\sqrt{}}
\end{aligned}
$$

Mathematica [A] time = 0.562294, size = 332, normalized size = 1.48

$$
\frac{-4iab\left(-idx^3\right)^{4/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{1}{3},idx^3\right)+4iab\left(idx^3\right)^{4/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{1}{3},-idx^3\right)+2^{2/3}b^2\cos(2}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^3,x]
```

```
[Out] (-4*a^2*(d^2*x^6)^(1/3) - 2*b^2*(d^2*x^6)^(1/3) + 2*b^2*(d^2*x^6)^(1/3)*Cos
[2*(c + d*x^3)] + 2^(2/3)*b^2*(I*d*x^3)^(4/3)*Cos[2*c]*Gamma[1/3, (-2*I)*d*
x^3] + 2^(2/3)*b^2*((-I)*d*x^3)^(4/3)*Cos[2*c]*Gamma[1/3, (2*I)*d*x^3] - (4
*I)*a*b*((-I)*d*x^3)^(4/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (4*I)*
a*b*(I*d*x^3)^(4/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) + I*2^(2/3)*
b^2*(I*d*x^3)^(4/3)*Gamma[1/3, (-2*I)*d*x^3]*Sin[2*c] - I*2^(2/3)*b^2*((-I)
*d*x^3)^(4/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] - 8*a*b*(d^2*x^6)^(1/3)*Sin[
c + d*x^3])/(8*x^2*(d^2*x^6)^(1/3))
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(dx^3+c\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^3,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^3,x)
```

Maxima [B] time = 1.27861, size = 743, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(x^3*abs(d))^(2/3)*(((I*gamma(-2/3, I*d*x^3) - I*gamma(-2/3, -I*d*x^3)
)*cos(1/3*pi + 2/3*arctan2(0, d)) + (I*gamma(-2/3, I*d*x^3) - I*gamma(-2/3,
 -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0, d)) - (gamma(-2/3, I*d*x^3) + gamm
a(-2/3, -I*d*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) + (gamma(-2/3, I*d*x^3)
+ gamma(-2/3, -I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*cos(c) + ((gamma
(-2/3, I*d*x^3) + gamma(-2/3, -I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0, d)) +
(gamma(-2/3, I*d*x^3) + gamma(-2/3, -I*d*x^3))*cos(-1/3*pi + 2/3*arctan2(0,
 d)) + (I*gamma(-2/3, I*d*x^3) - I*gamma(-2/3, -I*d*x^3))*sin(1/3*pi + 2/3*
arctan2(0, d)) + (-I*gamma(-2/3, I*d*x^3) + I*gamma(-2/3, -I*d*x^3))*sin(-1
/3*pi + 2/3*arctan2(0, d)))*sin(c))*a*b/x^2 + 1/24*(2^(2/3)*(x^3*abs(d))^(2
/3)*(((gamma(-2/3, 2*I*d*x^3) + gamma(-2/3, -2*I*d*x^3))*cos(1/3*pi + 2/3*a
rctan2(0, d)) + (gamma(-2/3, 2*I*d*x^3) + gamma(-2/3, -2*I*d*x^3))*cos(-1/3
*pi + 2/3*arctan2(0, d)) + (I*gamma(-2/3, 2*I*d*x^3) - I*gamma(-2/3, -2*I*d
*x^3))*sin(1/3*pi + 2/3*arctan2(0, d)) + (-I*gamma(-2/3, 2*I*d*x^3) + I*gam
ma(-2/3, -2*I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*cos(2*c) + ((-I*gam
ma(-2/3, 2*I*d*x^3) + I*gamma(-2/3, -2*I*d*x^3))*cos(1/3*pi + 2/3*arctan2(0
, d)) + (-I*gamma(-2/3, 2*I*d*x^3) + I*gamma(-2/3, -2*I*d*x^3))*cos(-1/3*pi
 + 2/3*arctan2(0, d)) + (gamma(-2/3, 2*I*d*x^3) + gamma(-2/3, -2*I*d*x^3))*
sin(1/3*pi + 2/3*arctan2(0, d)) - (gamma(-2/3, 2*I*d*x^3) + gamma(-2/3, -2*
I*d*x^3))*sin(-1/3*pi + 2/3*arctan2(0, d)))*sin(2*c)) - 6)*b^2/x^2 - 1/2*a^
2/x^2
```

Fricas [A] time = 1.92269, size = 405, normalized size = 1.8

$$-\frac{b^2\,(2i\,d)^{\frac{2}{3}}\,x^2e^{(-2i\,c)}\Gamma\left(\frac{1}{3},2i\,dx^3\right)-4i\,ab\,(i\,d)^{\frac{2}{3}}\,x^2e^{(-i\,c)}\Gamma\left(\frac{1}{3},i\,dx^3\right)+4i\,ab\,(-i\,d)^{\frac{2}{3}}\,x^2e^{(i\,c)}\Gamma\left(\frac{1}{3},-i\,dx^3\right)+b^2\,(-2i\,d)^{\frac{2}{3}}\,x^2e^{(2i\,c}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*(2*I*d)^(2/3)*x^2*e^(-2*I*c)*gamma(1/3, 2*I*d*x^3) - 4*I*a*b*(I*d
)^(2/3)*x^2*e^(-I*c)*gamma(1/3, I*d*x^3) + 4*I*a*b*(-I*d)^(2/3)*x^2*e^(I*c)
```

```
*gamma(1/3, -I*d*x^3) + b^2*(-2*I*d)^(2/3)*x^2*e^(2*I*c)*gamma(1/3, -2*I*d*
x^3) - 4*b^2*cos(d*x^3 + c)^2 + 8*a*b*sin(d*x^3 + c) + 4*a^2 + 4*b^2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(c + dx^3\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx^3 + c\right) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^3, x)
```

3.80 $\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=275

$$-\frac{3iabe^{ic}d^2x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabe^{-ic}d^2x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}}-\frac{3b^2e^{2ic}d^2x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{3b^2e^{-2ic}d^2x\text{Gan}}{10\sqrt[3]{2}}$$

```
[Out] -(2*a^2 + b^2)/(10*x^5) - (3*a*b*d*Cos[c + d*x^3])/(5*x^2) + (b^2*Cos[2*c +
 2*d*x^3])/(10*x^5) - (((3*I)/10)*a*b*d^2*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])
/((-I)*d*x^3)^(1/3) + (((3*I)/10)*a*b*d^2*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(
I*d*x^3)^(1/3)) - (3*b^2*d^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(10*2^
(1/3)*((-I)*d*x^3)^(1/3)) - (3*b^2*d^2*x*Gamma[1/3, (2*I)*d*x^3])/(10*2^(1/
3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (2*a*b*Sin[c + d*x^3])/(5*x^5) - (3*b^2*d
*Sin[2*c + 2*d*x^3])/(10*x^2)
```

Rubi [A] time = 0.182991, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3403, 6, 3388, 3387, 3356, 2208, 3355}

$$-\frac{3iabe^{ic}d^2x\text{Gamma}\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabe^{-ic}d^2x\text{Gamma}\left(\frac{1}{3},idx^3\right)}{10\sqrt[3]{idx^3}}-\frac{3b^2e^{2ic}d^2x\text{Gamma}\left(\frac{1}{3},-2idx^3\right)}{10\sqrt[3]{2}\sqrt[3]{-idx^3}}-\frac{3b^2e^{-2ic}d^2x\text{Gan}}{10\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x^3])^2/x^6,x]
```


```
[Out] -(2*a^2 + b^2)/(10*x^5) - (3*a*b*d*Cos[c + d*x^3])/(5*x^2) + (b^2*Cos[2*c +
 2*d*x^3])/(10*x^5) - (((3*I)/10)*a*b*d^2*E^(I*c)*x*Gamma[1/3, (-I)*d*x^3])
/((-I)*d*x^3)^(1/3) + (((3*I)/10)*a*b*d^2*x*Gamma[1/3, I*d*x^3])/(E^(I*c)*(
I*d*x^3)^(1/3)) - (3*b^2*d^2*E^((2*I)*c)*x*Gamma[1/3, (-2*I)*d*x^3])/(10*2^
(1/3)*((-I)*d*x^3)^(1/3)) - (3*b^2*d^2*x*Gamma[1/3, (2*I)*d*x^3])/(10*2^(1/
3)*E^((2*I)*c)*(I*d*x^3)^(1/3)) - (2*a*b*Sin[c + d*x^3])/(5*x^5) - (3*b^2*d
*Sin[2*c + 2*d*x^3])/(10*x^2)
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sin\left(c+dx^3\right)\right)^2}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{b^2}{2x^6}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^6}+\frac{2ab\sin\left(c+dx^3\right)}{x^6}\right) dx \\
&= \int \left(\frac{a^2+\frac{b^2}{2}}{x^6}-\frac{b^2\cos\left(2c+2dx^3\right)}{2x^6}+\frac{2ab\sin\left(c+dx^3\right)}{x^6}\right) dx \\
&= -\frac{2a^2+b^2}{10x^5}+(2ab)\int \frac{\sin\left(c+dx^3\right)}{x^6}\,dx-\frac{1}{2}b^2\int \frac{\cos\left(2c+2dx^3\right)}{x^6}\,dx \\
&= -\frac{2a^2+b^2}{10x^5}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}+\frac{1}{5}(6abd)\int \frac{\cos\left(c+dx^3\right)}{x^3}\,dx+\frac{1}{5}\,(3 \\
&= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}-\frac{3b^2d\sin\left(2c+\right.}{10x^2} \\
&= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{2ab\sin\left(c+dx^3\right)}{5x^5}-\frac{3b^2d\sin\left(2c+\right.}{10x^2} \\
&= -\frac{2a^2+b^2}{10x^5}-\frac{3abd\cos\left(c+dx^3\right)}{5x^2}+\frac{b^2\cos\left(2c+2dx^3\right)}{10x^5}-\frac{3iabd^2e^{ic}x\Gamma\left(\frac{1}{3},-idx^3\right)}{10\sqrt[3]{-idx^3}}+\frac{3iabd^2e^{-}}{1}
\end{aligned}$$

Mathematica [A] time = 2.5157, size = 294, normalized size = 1.07

$$-\frac{6iab\sqrt[3]{idx^3}\left(d^2x^6\right)^{2/3}(\cos(c)+i\sin(c))\text{Gamma}\left(\frac{1}{3},-idx^3\right)+6iab\left(idx^3\right)^{5/3}(\cos(c)-i\sin(c))\text{Gamma}\left(\frac{1}{3},idx^3\right)-3\,2^{2/3}b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^2/x^6,x]
```

```
[Out] -(4*a^2 + 2*b^2 + 12*a*b*d*x^3*Cos[c + d*x^3] - 2*b^2*Cos[2*(c + d*x^3)] -
3*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Cos[2*c]*Gamma[1/3, (2*I)*d*x^3] + (6*I)*a*b*
(I*d*x^3)^(5/3)*Gamma[1/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + (6*I)*a*b*(I*d*x^
```

```
3)^(1/3)*(d^2*x^6)^(2/3)*Gamma[1/3, (-I)*d*x^3]*(Cos[c] + I*Sin[c]) - 3*2^(
2/3)*b^2*((-I)*d*x^3)^(5/3)*Gamma[1/3, (-2*I)*d*x^3]*(Cos[2*c] + I*Sin[2*c]
) + (3*I)*2^(2/3)*b^2*(I*d*x^3)^(5/3)*Gamma[1/3, (2*I)*d*x^3]*Sin[2*c] + 8*
a*b*Sin[c + d*x^3] + 6*b^2*d*x^3*Sin[2*(c + d*x^3)])/(20*x^5)
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(dx^3+c\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x^3+c))^2/x^6,x)
```

```
[Out] int((a+b*sin(d*x^3+c))^2/x^6,x)
```

Maxima [B] time = 1.27423, size = 756, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*(x^3*abs(d))^(2/3)*(((I*gamma(-5/3, I*d*x^3) - I*gamma(-5/3, -I*d*x^3)
)*cos(5/6*pi + 5/3*arctan2(0, d)) + (I*gamma(-5/3, I*d*x^3) - I*gamma(-5/3,
 -I*d*x^3))*cos(-5/6*pi + 5/3*arctan2(0, d)) - (gamma(-5/3, I*d*x^3) + gamm
a(-5/3, -I*d*x^3))*sin(5/6*pi + 5/3*arctan2(0, d)) + (gamma(-5/3, I*d*x^3)
+ gamma(-5/3, -I*d*x^3))*sin(-5/6*pi + 5/3*arctan2(0, d)))*cos(c) + ((gamma
(-5/3, I*d*x^3) + gamma(-5/3, -I*d*x^3))*cos(5/6*pi + 5/3*arctan2(0, d)) +
(gamma(-5/3, I*d*x^3) + gamma(-5/3, -I*d*x^3))*cos(-5/6*pi + 5/3*arctan2(0,
 d)) + (I*gamma(-5/3, I*d*x^3) - I*gamma(-5/3, -I*d*x^3))*sin(5/6*pi + 5/3*
arctan2(0, d)) + (-I*gamma(-5/3, I*d*x^3) + I*gamma(-5/3, -I*d*x^3))*sin(-5
/6*pi + 5/3*arctan2(0, d)))*sin(c))*a*b*abs(d)/x^2 + 1/60*(2^(2/3)*(x^3*abs
(d))^(2/3)*((5*(gamma(-5/3, 2*I*d*x^3) + gamma(-5/3, -2*I*d*x^3))*cos(5/6*p
i + 5/3*arctan2(0, d)) + 5*(gamma(-5/3, 2*I*d*x^3) + gamma(-5/3, -2*I*d*x^3
))*cos(-5/6*pi + 5/3*arctan2(0, d)) + (5*I*gamma(-5/3, 2*I*d*x^3) - 5*I*gam
ma(-5/3, -2*I*d*x^3))*sin(5/6*pi + 5/3*arctan2(0, d)) + (-5*I*gamma(-5/3, 2
*I*d*x^3) + 5*I*gamma(-5/3, -2*I*d*x^3))*sin(-5/6*pi + 5/3*arctan2(0, d)))*
cos(2*c) + ((-5*I*gamma(-5/3, 2*I*d*x^3) + 5*I*gamma(-5/3, -2*I*d*x^3))*cos
(5/6*pi + 5/3*arctan2(0, d)) + (-5*I*gamma(-5/3, 2*I*d*x^3) + 5*I*gamma(-5/
3, -2*I*d*x^3))*cos(-5/6*pi + 5/3*arctan2(0, d)) + 5*(gamma(-5/3, 2*I*d*x^3
) + gamma(-5/3, -2*I*d*x^3))*sin(5/6*pi + 5/3*arctan2(0, d)) - 5*(gamma(-5/
3, 2*I*d*x^3) + gamma(-5/3, -2*I*d*x^3))*sin(-5/6*pi + 5/3*arctan2(0, d)))*
sin(2*c))*x^3*abs(d) - 6)*b^2/x^5 - 1/5*a^2/x^5
```

Fricas [A] time = 1.91319, size = 506, normalized size = 1.84

$$3i\,b^2\,(2i\,d)^{\frac{2}{3}}\,dx^5e^{(-2i\,c)}\Gamma\left(\frac{1}{3}, 2i\,dx^3\right) + 6\,ab\,(i\,d)^{\frac{2}{3}}\,dx^5e^{(-i\,c)}\Gamma\left(\frac{1}{3}, i\,dx^3\right) + 6\,ab\,(-i\,d)^{\frac{2}{3}}\,dx^5e^{(i\,c)}\Gamma\left(\frac{1}{3}, -i\,dx^3\right) - 3i\,b^2\,(-2i\,d)^{\frac{2}{3}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(3*I*b^2*(2*I*d)^(2/3)*d*x^5*e^(-2*I*c)*gamma(1/3, 2*I*d*x^3) + 6*a*b*
(I*d)^(2/3)*d*x^5*e^(-I*c)*gamma(1/3, I*d*x^3) + 6*a*b*(-I*d)^(2/3)*d*x^5*e
^(I*c)*gamma(1/3, -I*d*x^3) - 3*I*b^2*(-2*I*d)^(2/3)*d*x^5*e^(2*I*c)*gamma(
1/3, -2*I*d*x^3) - 12*a*b*d*x^3*cos(d*x^3 + c) + 4*b^2*cos(d*x^3 + c)^2 - 4
*a^2 - 4*b^2 - 4*(3*b^2*d*x^3*cos(d*x^3 + c) + 2*a*b)*sin(d*x^3 + c))/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(c + dx^3\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x**3+c))**2/x**6,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**2/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx^3 + c\right) + a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x^3+c))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2/x^6, x)
```

3.81 $\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=245

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\sqrt{a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d^2\sqrt{a^2-b^2}}-\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d\sqrt{a^2-b^2}}$$

```
[Out] ((-I/3)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) + ((I/3)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b
^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a
^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a
 + Sqrt[a^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2)
```

Rubi [A] time = 0.502793, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 3323, 2264, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\sqrt{a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d^2\sqrt{a^2-b^2}}-\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^3]),x]
```


```
[Out] ((-I/3)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a
^2 - b^2]*d) + ((I/3)*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b
^2])])/(Sqrt[a^2 - b^2]*d) - PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a
^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2) + PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a
 + Sqrt[a^2 - b^2])]/(3*Sqrt[a^2 - b^2]*d^2)
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+b\sin\left(c+dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^3\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^3\right)\\
&= -\frac{(2ib)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a-2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}} + \frac{(2ib)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x}{2a+2\sqrt{a^2-b^2}-2ibe^{i(c+dx)}}\,dx, x, x^3\right)}{3\sqrt{a^2-b^2}}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} + \frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} + \frac{i\operatorname{Subst}\left(\int \log\left(1-\frac{2ibe^{i(c+dx)}}{2a-2\sqrt{a^2-b^2}}\right)dx, x, x^3\right)}{3\sqrt{a^2-b^2}d}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} + \frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} + \frac{\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{2ibx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^3\right)}\right)}{3\sqrt{a^2-b^2}d^2} \cdot\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} + \frac{ix^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d} - \frac{\operatorname{Li}_2\left(\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2} + \frac{\operatorname{Li}_2\left(\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d^2}
\end{aligned}$$

Mathematica [A] time = 0.164522, size = 188, normalized size = 0.77

$$\frac{-\operatorname{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}-a}\right)+\operatorname{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)-idx^3\left(\log\left(1+\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}-a}\right)-\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)\right)}{3d^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] ((-I)*d*x^3*(Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] - Log[
1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])]) - PolyLog[2, ((-I)*b*E^
(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])] + PolyLog[2, (I*b*E^(I*(c + d*x^3)
))/(a + Sqrt[a^2 - b^2])])/(3*Sqrt[a^2 - b^2]*d^2)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^5}{a+b\sin\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x^5/(a+b*sin(d*x^3+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

Fricas [B] time = 2.95407, size = 2483, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b*c*sqrt(-(a^2 - b^2)/b^2)*log(2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^
3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) + 2*b*c*sqrt(-(a^2 - b^2)/b^2)
*log(2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2)
 - 2*I*a) - 2*b*c*sqrt(-(a^2 - b^2)/b^2)*log(-2*b*cos(d*x^3 + c) + 2*I*b*si
n(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) + 2*I*a) - 2*b*c*sqrt(-(a^2 - b^2
)/b^2)*log(-2*b*cos(d*x^3 + c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^
2)/b^2) - 2*I*a) - 2*I*b*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x^3
 + c) + 2*a*sin(d*x^3 + c) + 2*(b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*
(2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) - 2*(b*cos(d*x^3 + c) - I*b*sin(
d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + 2*I*b*sqrt(-(a^2 - b^2)/
b^2)*dilog(-1/2*(-2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) + 2*(b*cos(d*x^
3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) - 2*I*b*s
qrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c
) - 2*(b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)
/b + 1) - 2*(b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x^3
 + c) + 2*a*sin(d*x^3 + c) + 2*(b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt
(-(a^2 - b^2)/b^2) + 2*b)/b) + 2*(b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log
(1/2*(2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) - 2*(b*cos(d*x^3 + c) - I*b
*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - 2*(b*d*x^3 + b*c)*sqrt(
-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) + 2*(
b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + 2
*(b*d*x^3 + b*c)*sqrt(-(a^2 - b^2)/b^2)*log(1/2*(-2*I*a*cos(d*x^3 + c) + 2*
```

```
a*sin(d*x^3 + c) - 2*(b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b
^2)/b^2) + 2*b)/b))/((a^2 - b^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{a + b\sin\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x**5/(a + b*sin(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{b\sin\left(dx^3 + c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a), x)
```

3.82 $\int \frac{x^2}{a+b\sin\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=51

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```

Rubi [A] time = 0.0778821, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3379, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sin[c + d*x^3]),x]
```


```
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{a+b\sin\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx,x,x^3\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)\right)}{3d}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)\right)}{3d}$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{3\sqrt{a^2-b^2}d}$$

Mathematica [A] time = 0.0946136, size = 51, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*Sqrt[a^2 - b^2]*d
)
```

Maple [A] time = 0.016, size = 49, normalized size = 1.

$$\frac{2}{3\,d}\arctan\left(\frac{1}{2}\left(2\,a\tan\left(1/2\,dx^3+c/2\right)+2\,b\right)\frac{1}{\sqrt{a^2-b^2}}\right)\frac{1}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^3+c)),x)
```

```
[Out] 2/3/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x^3+1/2*c)+2*b)/(a^2-b^2)^(
1/2))
```

Maxima [B] time = 154.506, size = 10905, normalized size = 213.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*arctan2(-2*(4*(a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^4*cos(c)*sin(c) - 4*(a^2
*b^4 - b^6)*cos(c)*sin(d*x^3 + 2*c)^4*sin(c) - 4*((a^3*b^3 - a*b^5)*cos(c)^
3 + 3*(a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 4*(3*(a^3*b^3
 - a*b^5)*cos(c)^2*sin(c) + (a^3*b^3 - a*b^5)*sin(c)^3 + ((a^2*b^4 - b^6)*c
```

```
os(c)^2 - (a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^3 +
4*((4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 +
 b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 - 4*((4*a^4*b^2 - 5*a^2*b^4 + b^6
)*cos(c)^3*sin(c) + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(c)*sin(c)^3 + 3*((a^3
*b^3 - a*b^5)*cos(c)^3 - (a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c
))*sin(d*x^3 + 2*c)^2 - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^5 + 2*(2*a^
5*b - 3*a^3*b^3 + a*b^5)*cos(c)^3*sin(c)^2 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*
cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) - 4*((2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)
^4*sin(c) + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(c)^2*sin(c)^3 + (2*a^5*b -
3*a^3*b^3 + a*b^5)*sin(c)^5 + ((a^2*b^4 - b^6)*cos(c)^2 - (a^2*b^4 - b^6)*s
in(c)^2)*cos(d*x^3 + 2*c)^3 - 3*((a^3*b^3 - a*b^5)*cos(c)^2*sin(c) - (a^3*b
^3 - a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + ((4*a^4*b^2 - 5*a^2*b^4 + b^6)*c
os(c)^4 - (4*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c))*sin(d*x
^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^5*cos(c) - 4*a*b^4*cos(d*x^3 + 2*c)^4*cos
(c)*sin(c) + b^5*sin(d*x^3 + 2*c)^5*sin(c) + (b^5*cos(d*x^3 + 2*c)*cos(c) +
 4*a*b^4*cos(c)*sin(c))*sin(d*x^3 + 2*c)^4 + 2*((2*a^2*b^3 - b^5)*cos(c)^3
+ 3*(2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^3 + 2*(b^5*cos(d*x^
3 + 2*c)^2*sin(c) + 3*(2*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (2*a^2*b^3 - b^5)
*sin(c)^3 + 2*(a*b^4*cos(c)^2 - a*b^4*sin(c)^2)*cos(d*x^3 + 2*c))*sin(d*x^3
 + 2*c)^3 - 4*((4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + (4*a^3*b^2 - 3*a*b^4
)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + 2*(b^5*cos(d*x^3 + 2*c)^3*cos(c) +
2*(4*a^3*b^2 - 3*a*b^4)*cos(c)^3*sin(c) + 2*(4*a^3*b^2 - 3*a*b^4)*cos(c)*si
n(c)^3 + 3*((2*a^2*b^3 - b^5)*cos(c)^3 - (2*a^2*b^3 - b^5)*cos(c)*sin(c)^2)
*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 + ((8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)
^5 + 2*(8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (8*a^4*b - 8*a^2*b^3
 + b^5)*cos(c)*sin(c)^4)*cos(d*x^3 + 2*c) + (b^5*cos(d*x^3 + 2*c)^4*sin(c)
+ (8*a^4*b - 8*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(8*a^4*b - 8*a^2*b^3 + b^
5)*cos(c)^2*sin(c)^3 + (8*a^4*b - 8*a^2*b^3 + b^5)*sin(c)^5 + 4*(a*b^4*cos(
c)^2 - a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^3 - 6*((2*a^2*b^3 - b^5)*cos(c)^2*s
in(c) - (2*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^2 + 4*((4*a^3*b^2 - 3*
a*b^4)*cos(c)^4 - (4*a^3*b^2 - 3*a*b^4)*sin(c)^4)*cos(d*x^3 + 2*c))*sin(d*x
^3 + 2*c))*sqrt(a^2 - b^2))/(b^6*cos(d*x^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*
x^3 + 2*c)^5 + b^6*sin(d*x^3 + 2*c)^6 - 6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) +
 (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2
 + 18*a^2*b^4 - b^6)*cos(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^
4 - b^6)*cos(c)^2*sin(c)^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c
)^6 + 3*((2*a^2*b^4 - b^6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x
^3 + 2*c)^4 + 3*(b^6*cos(d*x^3 + 2*c)^2 - 2*a*b^5*cos(d*x^3 + 2*c)*sin(c) +
 5*(2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c
)^4 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*
sin(c)^3)*cos(d*x^3 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(4*
a^3*b^3 - 3*a*b^5)*cos(c)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*s
in(c) + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 3*((8
*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(
c)^2*sin(c)^2 + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^
2 + 3*(b^6*cos(d*x^3 + 2*c)^4 - 4*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^
4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^
2*sin(c)^2 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*
cos(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*(3*(4*a^3*b^3
 - 3*a*b^5)*cos(c)^2*sin(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2
*c))*sin(d*x^3 + 2*c)^2 - 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin
(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 2
0*a^3*b^3 + 5*a*b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 6*(a*b^5*cos(d*x^3 + 2*c)
^4*cos(c) + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6
)*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*co
s(c)^3*sin(c)^2 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4
*a^3*b^3 - 3*a*b^5)*cos(c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos
(d*x^3 + 2*c)^2 - 4*((8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4
*b^2 - 8*a^2*b^4 + b^6)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)
```

```
 - 2*(3*b^5*cos(c)*sin(d*x^3 + 2*c)^5 - 3*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)
*cos(c)^4*sin(c)^2 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 +
(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(
c)^2)*cos(d*x^3 + 2*c)^4 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^3 + 2*c)*sin(c
) + a*b^4*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*si
n(c) + 5*(4*a^2*b^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c)^3 + 2*(3*b^5*cos(d*x^
3 + 2*c)^2*cos(c) - 12*a*b^4*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3
- b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 +
 6*((2*a^3*b^2 - a*b^4)*cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2
+ 5*(2*a^3*b^2 - a*b^4)*sin(c)^4)*cos(d*x^3 + 2*c)^2 - 6*(b^5*cos(d*x^3 + 2
*c)^3*sin(c) - 5*(2*a^3*b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c
)^2*sin(c)^2 - (2*a^3*b^2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin
(c)^2)*cos(d*x^3 + 2*c)^2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b
^3 - b^5)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - 3*((16*a^4*b - 1
2*a^2*b^3 + b^5)*cos(c)^4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2
*sin(c)^3 + (16*a^4*b - 12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 3*(b
^5*cos(d*x^3 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + (
16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos
(c)^3*sin(c)^2 + (16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*
b^3 - b^5)*cos(c)^3 + 3*(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)
^2 - 16*((2*a^3*b^2 - a*b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*s
in(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c))*sqrt(a^2 - b^2)), (b^6*cos(d*x
^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^3 + 2*c)^5 + b^6*sin(d*x^3 + 2*c)^6 -
6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^6
+ 3*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^4*sin(c)^2 + 3*(8*a^4*b^2 - 8*a^2*
b^4 + b^6)*cos(c)^2*sin(c)^4 + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*sin(c)^6 + ((4
*a^2*b^4 - b^6)*cos(c)^2 + 5*(4*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^4
 + (3*b^6*cos(d*x^3 + 2*c)^2 - 6*a*b^5*cos(d*x^3 + 2*c)*sin(c) + 5*(4*a^2*b
^4 - b^6)*cos(c)^2 + (4*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 4*(3*
(2*a^3*b^3 - a*b^5)*cos(c)^2*sin(c) + 5*(2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d
*x^3 + 2*c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(2*a^3*b^3 - a*b^5
)*cos(c)^3 - 2*(4*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 3*(2*a^3*
b^3 - a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + ((8*a^4*b^2 - 4*a^2*b^4
- b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + 5*(8*
a^4*b^2 - 4*a^2*b^4 - b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^2 + (3*b^6*cos(d*x^3
+ 2*c)^4 - 12*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 4*a^2*b^4 -
b^6)*cos(c)^4 + 6*(8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)^2*sin(c)^2 + (8*a^4*
b^2 - 4*a^2*b^4 - b^6)*sin(c)^4 + 6*((4*a^2*b^4 - b^6)*cos(c)^2 + (4*a^2*b^
4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 12*(3*(2*a^3*b^3 - a*b^5)*cos(c)^2*
sin(c) + (2*a^3*b^3 - a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2
 - 2*((8*a^5*b - 5*a*b^5)*cos(c)^4*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^2*
sin(c)^3 + (8*a^5*b - 5*a*b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 2*(3*a*b^5*cos(
d*x^3 + 2*c)^4*cos(c) + (8*a^5*b - 5*a*b^5)*cos(c)^5 - 4*(4*a^2*b^4 - b^6)*
cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + 2*(8*a^5*b - 5*a*b^5)*cos(c)^3*sin(c)^2
+ (8*a^5*b - 5*a*b^5)*cos(c)*sin(c)^4 + 6*((2*a^3*b^3 - a*b^5)*cos(c)^3 + 3
*(2*a^3*b^3 - a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*((8*a^4*b^2 -
4*a^2*b^4 - b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 4*a^2*b^4 - b^6)*cos(c)*sin
(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c) - 4*(b^5*cos(c)*sin(d*x^3 + 2*c)^
5 - b^5*cos(d*x^3 + 2*c)^5*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)^6 + 3*(2*a^3
*b^2 - a*b^4)*cos(c)^4*sin(c)^2 + 3*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^4 +
 (2*a^3*b^2 - a*b^4)*sin(c)^6 + (a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x
^3 + 2*c)^4 + (5*a*b^4*cos(c)^2 - b^5*cos(d*x^3 + 2*c)*sin(c) + a*b^4*sin(c
)^2)*sin(d*x^3 + 2*c)^4 - 2*(3*a^2*b^3*cos(c)^2*sin(c) + 5*a^2*b^3*sin(c)^3
)*cos(d*x^3 + 2*c)^3 + 2*(b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*a^2*b^3*cos(c)^
3 - 4*a*b^4*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 3*a^2*b^3*cos(c)*sin(c)^2)*sin
(d*x^3 + 2*c)^3 + 2*(a^3*b^2*cos(c)^4 + 6*a^3*b^2*cos(c)^2*sin(c)^2 + 5*a^3
*b^2*sin(c)^4)*cos(d*x^3 + 2*c)^2 + 2*(5*a^3*b^2*cos(c)^4 - b^5*cos(d*x^3 +
 2*c)^3*sin(c) + 6*a^3*b^2*cos(c)^2*sin(c)^2 + a^3*b^2*sin(c)^4 + 3*(a*b^4*
```

```
cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 3*(3*a^2*b^3*cos(c)^2*sin(c
) + a^2*b^3*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - ((4*a^4*b + 2*
a^2*b^3 - b^5)*cos(c)^4*sin(c) + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^2*sin
(c)^3 + (4*a^4*b + 2*a^2*b^3 - b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + (b^5*cos(d
*x^3 + 2*c)^4*cos(c) - 8*a*b^4*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + (4*a^4*b
+ 2*a^2*b^3 - b^5)*cos(c)^5 + 2*(4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)^3*sin(c)
^2 + (4*a^4*b + 2*a^2*b^3 - b^5)*cos(c)*sin(c)^4 + 6*(a^2*b^3*cos(c)^3 + 3*
a^2*b^3*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 16*(a^3*b^2*cos(c)^3*sin(c) +
 a^3*b^2*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c))*sqrt(a^2 - b^
2))/(b^6*cos(d*x^3 + 2*c)^6 + 6*a*b^5*cos(c)*sin(d*x^3 + 2*c)^5 + b^6*sin(d
*x^3 + 2*c)^6 - 6*a*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (32*a^6 - 48*a^4*b^2 +
18*a^2*b^4 - b^6)*cos(c)^6 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos
(c)^4*sin(c)^2 + 3*(32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*cos(c)^2*sin(c)
^4 + (32*a^6 - 48*a^4*b^2 + 18*a^2*b^4 - b^6)*sin(c)^6 + 3*((2*a^2*b^4 - b^
6)*cos(c)^2 + 5*(2*a^2*b^4 - b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^4 + 3*(b^6*cos
(d*x^3 + 2*c)^2 - 2*a*b^5*cos(d*x^3 + 2*c)*sin(c) + 5*(2*a^2*b^4 - b^6)*cos
(c)^2 + (2*a^2*b^4 - b^6)*sin(c)^2)*sin(d*x^3 + 2*c)^4 - 4*(3*(4*a^3*b^3 -
3*a*b^5)*cos(c)^2*sin(c) + 5*(4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2*
c)^3 + 4*(3*a*b^5*cos(d*x^3 + 2*c)^2*cos(c) + 5*(4*a^3*b^3 - 3*a*b^5)*cos(c
)^3 - 6*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 3*(4*a^3*b^3 - 3
*a*b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 3*((8*a^4*b^2 - 8*a^2*b^4 + b
^6)*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + 5*(8*a^4
*b^2 - 8*a^2*b^4 + b^6)*sin(c)^4)*cos(d*x^3 + 2*c)^2 + 3*(b^6*cos(d*x^3 + 2
*c)^4 - 4*a*b^5*cos(d*x^3 + 2*c)^3*sin(c) + 5*(8*a^4*b^2 - 8*a^2*b^4 + b^6)
*cos(c)^4 + 6*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^2*sin(c)^2 + (8*a^4*b^2
- 8*a^2*b^4 + b^6)*sin(c)^4 + 6*((2*a^2*b^4 - b^6)*cos(c)^2 + (2*a^2*b^4 -
b^6)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*(3*(4*a^3*b^3 - 3*a*b^5)*cos(c)^2*sin
(c) + (4*a^3*b^3 - 3*a*b^5)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2
- 6*((16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^4*sin(c) + 2*(16*a^5*b - 20*a
^3*b^3 + 5*a*b^5)*cos(c)^2*sin(c)^3 + (16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*sin
(c)^5)*cos(d*x^3 + 2*c) + 6*(a*b^5*cos(d*x^3 + 2*c)^4*cos(c) + (16*a^5*b -
20*a^3*b^3 + 5*a*b^5)*cos(c)^5 - 4*(2*a^2*b^4 - b^6)*cos(d*x^3 + 2*c)^3*cos
(c)*sin(c) + 2*(16*a^5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)^3*sin(c)^2 + (16*a^
5*b - 20*a^3*b^3 + 5*a*b^5)*cos(c)*sin(c)^4 + 2*((4*a^3*b^3 - 3*a*b^5)*cos(
c)^3 + 3*(4*a^3*b^3 - 3*a*b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 4*((8*
a^4*b^2 - 8*a^2*b^4 + b^6)*cos(c)^3*sin(c) + (8*a^4*b^2 - 8*a^2*b^4 + b^6)*
cos(c)*sin(c)^3)*cos(d*x^3 + 2*c))*sin(d*x^3 + 2*c) - 2*(3*b^5*cos(c)*sin(d
*x^3 + 2*c)^5 - 3*b^5*cos(d*x^3 + 2*c)^5*sin(c) + (16*a^5 - 16*a^3*b^2 + 3*
a*b^4)*cos(c)^6 + 3*(16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^4*sin(c)^2 + 3*(
16*a^5 - 16*a^3*b^2 + 3*a*b^4)*cos(c)^2*sin(c)^4 + (16*a^5 - 16*a^3*b^2 + 3
*a*b^4)*sin(c)^6 + 3*(a*b^4*cos(c)^2 + 5*a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^4
 + 3*(5*a*b^4*cos(c)^2 - b^5*cos(d*x^3 + 2*c)*sin(c) + a*b^4*sin(c)^2)*sin(
d*x^3 + 2*c)^4 - 2*(3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + 5*(4*a^2*b^3 - b^
5)*sin(c)^3)*cos(d*x^3 + 2*c)^3 + 2*(3*b^5*cos(d*x^3 + 2*c)^2*cos(c) - 12*a
*b^4*cos(d*x^3 + 2*c)*cos(c)*sin(c) + 5*(4*a^2*b^3 - b^5)*cos(c)^3 + 3*(4*a
^2*b^3 - b^5)*cos(c)*sin(c)^2)*sin(d*x^3 + 2*c)^3 + 6*((2*a^3*b^2 - a*b^4)*
cos(c)^4 + 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 + 5*(2*a^3*b^2 - a*b^4)*
sin(c)^4)*cos(d*x^3 + 2*c)^2 - 6*(b^5*cos(d*x^3 + 2*c)^3*sin(c) - 5*(2*a^3*
b^2 - a*b^4)*cos(c)^4 - 6*(2*a^3*b^2 - a*b^4)*cos(c)^2*sin(c)^2 - (2*a^3*b^
2 - a*b^4)*sin(c)^4 - 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*cos(d*x^3 + 2*c)^
2 + (3*(4*a^2*b^3 - b^5)*cos(c)^2*sin(c) + (4*a^2*b^3 - b^5)*sin(c)^3)*cos(
d*x^3 + 2*c))*sin(d*x^3 + 2*c)^2 - 3*((16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^
4*sin(c) + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^2*sin(c)^3 + (16*a^4*b -
12*a^2*b^3 + b^5)*sin(c)^5)*cos(d*x^3 + 2*c) + 3*(b^5*cos(d*x^3 + 2*c)^4*co
s(c) - 8*a*b^4*cos(d*x^3 + 2*c)^3*cos(c)*sin(c) + (16*a^4*b - 12*a^2*b^3 +
b^5)*cos(c)^5 + 2*(16*a^4*b - 12*a^2*b^3 + b^5)*cos(c)^3*sin(c)^2 + (16*a^4
*b - 12*a^2*b^3 + b^5)*cos(c)*sin(c)^4 + 2*((4*a^2*b^3 - b^5)*cos(c)^3 + 3*
(4*a^2*b^3 - b^5)*cos(c)*sin(c)^2)*cos(d*x^3 + 2*c)^2 - 16*((2*a^3*b^2 - a*
b^4)*cos(c)^3*sin(c) + (2*a^3*b^2 - a*b^4)*cos(c)*sin(c)^3)*cos(d*x^3 + 2*c
```

```
))*sin(d*x^3 + 2*c))*sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Fricas [A] time = 1.77605, size = 458, normalized size = 8.98

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(dx^3+c\right)^2-2ab\sin\left(dx^3+c\right)-a^2-b^2+2\left(a\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)+b\cos\left(dx^3+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx^3+c\right)^2-2ab\sin\left(dx^3+c\right)-a^2-b^2}\right)}{6\left(a^2-b^2\right)d}, -\frac{\arctan\left(-\frac{a\sin\left(dx^3+c\right)}{\sqrt{a^2-b^2}\cos\left(d\right.}\right.}{3\sqrt{a^2-b^2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^
3 + c) - a^2 - b^2 + 2*(a*cos(d*x^3 + c)*sin(d*x^3 + c) + b*cos(d*x^3 + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2
))/((a^2 - b^2)*d), -1/3*arctan(-(a*sin(d*x^3 + c) + b)/(sqrt(a^2 - b^2)*co
s(d*x^3 + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [A] time = 24.6765, size = 202, normalized size = 3.96

$$\begin{cases} \frac{x^3}{3(a+b\sin(c))} & \text{for } d=0 \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)\right)}{3bd} & \text{for } a=0 \\ \frac{2\sqrt{b^2}}{3b^2d\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)-3bd\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{3b^2d\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)+3bd\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{3d\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx^3}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{3d\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Piecewise((x**3/(3*(a + b*sin(c))), Eq(d, 0)), (log(tan(c/2 + d*x**3/2))/(3
*b*d), Eq(a, 0)), (2*sqrt(b**2)/(3*b**2*d*tan(c/2 + d*x**3/2) - 3*b*d*sqrt(
b**2)), Eq(a, -sqrt(b**2))), (-2*sqrt(b**2)/(3*b**2*d*tan(c/2 + d*x**3/2) +
 3*b*d*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(c/2 + d*x**3/2) + b/a - sq
rt(-a**2 + b**2)/a)/(3*d*sqrt(-a**2 + b**2)) - log(tan(c/2 + d*x**3/2) + b/
a + sqrt(-a**2 + b**2)/a)/(3*d*sqrt(-a**2 + b**2)), True))
```

Giac [A] time = 1.10127, size = 86, normalized size = 1.69

$$\frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{3\sqrt{a^2-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

3.83 $\int \frac{1}{x(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Sin[c + d*x^3])), x]
```

Rubi [A] time = 0.0260812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A] time = 0.444037, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Integrate[1/(x*(a + b*Sin[c + d*x^3])), x]
```

Maple [A] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(dx^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(1/x/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx\sin\left(dx^3+c\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*sin(d*x^3 + c) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**3))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x), x)
```

3.84 $\int \frac{1}{x^4(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Rubi [A] time = 0.0261377, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Defer[Int][1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A] time = 0.459977, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])), x]
```

Maple [A] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+b\sin\left(dx^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(1/x^4/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^4\sin\left(dx^3+c\right)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*sin(d*x^3 + c) + a*x^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(1/(x**4*(a + b*sin(c + d*x**3))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^4), x)
```

3.85 $\int \frac{x}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{x}{a+b\sin\left(c+dx^3\right)},x\right)$$

```
[Out] Unintegrable[x/(a + b*Sin[c + d*x^3]), x]
```

Rubi [A] time = 0.0146964, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] Defer[Int][x/(a + b*Sin[c + d*x^3]), x]
```

Rubi steps

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Mathematica [A] time = 0.606372, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] Integrate[x/(a + b*Sin[c + d*x^3]), x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\sin\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(x/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*sin(d*x^3 + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{b\sin\left(dx^3+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(x/(b*sin(d*x^3 + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(x/(a + b*sin(c + d*x**3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*sin(d*x^3 + c) + a), x)
```

3.86 $\int \frac{1}{x^2(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Rubi [A] time = 0.0257709, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A] time = 0.317911, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])), x]
```

Maple [A] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(dx^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(1/x^2/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin \left(dx^3 + c\right) + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^2 \sin \left(dx^3 + c\right) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*sin(d*x^3 + c) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + b \sin \left(c + dx^3\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**3))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin \left(dx^3 + c\right) + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^2), x)
```

3.87 $\int \frac{1}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{a+b\sin\left(c+dx^3\right)},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x^3])^(-1), x]
```

Rubi [A] time = 0.0052006, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^3])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^3])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Mathematica [A] time = 0.0267252, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^(-1),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x^3])^(-1), x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(dx^3+c\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(1/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x^3 + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\sin\left(dx^3+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*sin(d*x^3 + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x**3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x^3 + c) + a), x)
```

3.88 $\int \frac{1}{x^3(a+b\sin(c+dx^3))}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Rubi [A] time = 0.0256459, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Mathematica [A] time = 0.364034, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])),x]
```

```
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])), x]
```

Maple [A] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(dx^3+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int(1/x^3/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^3\sin\left(dx^3+c\right)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^3*sin(d*x^3 + c) + a*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral(1/(x**3*(a + b*sin(c + d*x**3))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)*x^3), x)
```

3.89 $\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=324

$$-\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\left(a^2-b^2\right)^{3/2}}+\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d^2\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3d^2\left(a^2-b^2\right)}-\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{iax^3\log\left(1-\right.}{3d\left(a^2-\right.}$$

```
[Out] ((-I/3)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + ((I/3)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a
^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^3]]/(3*(a^2 - b^
2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(3*
(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^
2 - b^2])])/(3*(a^2 - b^2)^(3/2)*d^2) + (b*x^3*Cos[c + d*x^3])/(3*(a^2 - b^
2)*d*(a + b*Sin[c + d*x^3]))
```

Rubi [A] time = 0.59335, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3379, 3324, 3323, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d^2\left(a^2-b^2\right)^{3/2}}+\frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{3d^2\left(a^2-b^2\right)^{3/2}}-\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3d^2\left(a^2-b^2\right)}-\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{iax^3\log\left(1-\right.}{3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sin[c + d*x^3])^2,x]
```


```
[Out] ((-I/3)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/((a^2
 - b^2)^(3/2)*d) + ((I/3)*a*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a
^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - Log[a + b*Sin[c + d*x^3]]/(3*(a^2 - b^
2)*d^2) - (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a - Sqrt[a^2 - b^2])])/(3*
(a^2 - b^2)^(3/2)*d^2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^
2 - b^2])])/(3*(a^2 - b^2)^(3/2)*d^2) + (b*x^3*Cos[c + d*x^3])/(3*(a^2 - b^
2)*d*(a + b*Sin[c + d*x^3]))
```


Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+b\sin(c+dx))^2}\,dx, x, x^3\right)$$

$$= \frac{bx^3\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} + \frac{a\,\text{Subst}\left(\int \frac{x}{a+b\sin(c+dx)}\,dx, x, x^3\right)}{3\left(a^2-b^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{\cos(c\cdot}{a+b\sin}\right.}{3\left(a^2-\right.}$$

$$= \frac{bx^3\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} + \frac{(2a)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+2ae^{i(c+dx)}-ibe^{2i(c+dx)}}\,dx, x, x^3\right)}{3\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3\left(a^2-b^2\right)d^2} + \frac{bx^3\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)} - \frac{(2iab)\,\text{Subst}\left(\int \frac{e^{i(}}{2a-2\sqrt{a^2-}}\right.}{3\left(a^2-b\right.}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} - \frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3\left(a^2-b^2\right)d^2} + \frac{b}{3\left(a^2-b^2\right.}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} - \frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3\left(a^2-b^2\right)d^2} + \frac{b}{3\left(a^2-b^2\right.}$$

$$= -\frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a-\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} + \frac{iax^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{a+\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d} - \frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{3\left(a^2-b^2\right)d^2} - \frac{a\text{Li}_2\left(\frac{ibe^{i}}{a-\sqrt{}}\right.}{3\left(a^2-b^2\right.}$$

Mathematica [A] time = 0.993256, size = 302, normalized size = 0.93

$$\frac{-\frac{a\text{PolyLog}\left(2,-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}-a}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\text{PolyLog}\left(2,\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{iadx^3\log\left(1+\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}-a}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{iadx^3\log\left(1-\frac{ibe^{i\left(c+dx^3\right)}}{\sqrt{a^2-b^2}+a}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{\log\left(a+b\sin\left(c+dx^3\right)\right)}{a^2-b^2} + \frac{bdx^3\cos\left(c+\right.}{\left(a^2-b^2\right)\left(a+b\sin\right.}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (((-I)*a*d*x^3*Log[1 + (I*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^
2 - b^2)^(3/2) + (I*a*d*x^3*Log[1 - (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 -
 b^2])])/(a^2 - b^2)^(3/2) - Log[a + b*Sin[c + d*x^3]]/(a^2 - b^2) - (a*Pol
yLog[2, ((-I)*b*E^(I*(c + d*x^3)))/(-a + Sqrt[a^2 - b^2])])/(a^2 - b^2)^(3/
2) + (a*PolyLog[2, (I*b*E^(I*(c + d*x^3)))/(a + Sqrt[a^2 - b^2])])/(a^2 - b
^2)^(3/2) + (b*d*x^3*Cos[c + d*x^3])/((a^2 - b^2)*(a + b*Sin[c + d*x^3])))/
(3*d^2)
```

Maple [F] time = 0.773, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+b\sin\left(dx^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x^5/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.28416, size = 3438, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2*b - b^3)*d*x^3*cos(d*x^3 + c) + (I*a*b^2*sin(d*x^3 + c) + I*a^2
*b)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3
 + c) + 2*(b*cos(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) +
2*b)/b + 1) + (-I*a*b^2*sin(d*x^3 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*di
log(-1/2*(2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) - 2*(b*cos(d*x^3 + c) -
 I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1) + (-I*a*b^2*sin(d
*x^3 + c) - I*a^2*b)*sqrt(-(a^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^3 +
c) + 2*a*sin(d*x^3 + c) + 2*(b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(
a^2 - b^2)/b^2) + 2*b)/b + 1) + (I*a*b^2*sin(d*x^3 + c) + I*a^2*b)*sqrt(-(a
^2 - b^2)/b^2)*dilog(-1/2*(-2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) - 2*(
b*cos(d*x^3 + c) + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b + 1)
 + (a^2*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(
a^2 - b^2)/b^2)*log(1/2*(2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) + 2*(b*c
os(d*x^3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^2
*b*d*x^3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b
^2)/b^2)*log(1/2*(2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) - 2*(b*cos(d*x^
3 + c) - I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) + (a^2*b*d*x^
3 + a^2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2
)*log(1/2*(-2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) + 2*(b*cos(d*x^3 + c)
 + I*b*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^2*b*d*x^3 + a^
2*b*c + (a*b^2*d*x^3 + a*b^2*c)*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2)*log(
1/2*(-2*I*a*cos(d*x^3 + c) + 2*a*sin(d*x^3 + c) - 2*(b*cos(d*x^3 + c) + I*b
*sin(d*x^3 + c))*sqrt(-(a^2 - b^2)/b^2) + 2*b)/b) - (a^3 - a*b^2 + (a^2*b -
 b^3)*sin(d*x^3 + c) + (a*b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)
/b^2))*log(2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2
)/b^2) + 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c) + (a*b^2*c*si
n(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(2*b*cos(d*x^3 + c) - 2*
I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a) - (a^3 - a*b^2 + (
a^2*b - b^3)*sin(d*x^3 + c) - (a*b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2
 - b^2)/b^2))*log(-2*b*cos(d*x^3 + c) + 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a
^2 - b^2)/b^2) + 2*I*a) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x^3 + c) - (a*
b^2*c*sin(d*x^3 + c) + a^2*b*c)*sqrt(-(a^2 - b^2)/b^2))*log(-2*b*cos(d*x^3
+ c) - 2*I*b*sin(d*x^3 + c) + 2*b*sqrt(-(a^2 - b^2)/b^2) - 2*I*a))/((a^4*b
- 2*a^2*b^3 + b^5)*d^2*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sin(d*x^3 + c) + a)^2, x)
```

3.90 $\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=94

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^3\right)}{3d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}$$

```
[Out] (2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*(a^2 - b^2)^(3/
2)*d) + (b*Cos[c + d*x^3])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x^3]))
```

Rubi [A] time = 0.108524, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3379, 2664, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{3d\left(a^2-b^2\right)^{3/2}}+\frac{b\cos\left(c+dx^3\right)}{3d\left(a^2-b^2\right)\left(a+b\sin\left(c+dx^3\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/(3*(a^2 - b^2)^(3/
2)*d) + (b*Cos[c + d*x^3])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x^3]))
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+b\sin(c+dx))^2}\,dx,x,x^3\right)$$
$$= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}+\frac{\text{Subst}\left(\int \frac{a}{a+b\sin(c+dx)}\,dx,x,x^3\right)}{3\left(a^2-b^2\right)}$$
$$= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}+\frac{a\,\text{Subst}\left(\int \frac{1}{a+b\sin(c+dx)}\,dx,x,x^3\right)}{3\left(a^2-b^2\right)}$$
$$= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)\right)}{3\left(a^2-b^2\right)d}$$
$$= \frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}-\frac{(4a)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}\left(c+d\right.\right.\right.}{3\left(a^2-b^2\right)d}$$
$$= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)}{\sqrt{a^2-b^2}}\right)}{3\left(a^2-b^2\right)^{3/2}d}+\frac{b\cos\left(c+dx^3\right)}{3\left(a^2-b^2\right)d\left(a+b\sin\left(c+dx^3\right)\right)}$$

Mathematica [A] time = 0.205422, size = 91, normalized size = 0.97

$$\frac{\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}\left(c+dx^3\right)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b\cos\left(c+dx^3\right)}{a+b\sin\left(c+dx^3\right)}}{3d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(c + d*x^3)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (b*Cos[c + d*x^3])/(a + b*Sin[c + d*x^3]))/(3*(a - b)*(a + b)*d)
```

Maple [A] time = 0.015, size = 167, normalized size = 1.8

$$\frac{2\,b^2}{3\,da\left(a^2-b^2\right)}\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)\left(\left(\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)^2a+2\,\tan\left(1/2\,dx^3+c/2\right)b+a\right)^{-1}+\frac{2\,b}{3\,d\left(a^2-b^2\right)}\left(\left(\tan\left(\frac{dx^3}{2}+\frac{c}{2}\right)\right)^2a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] 2/3/d/(tan(1/2*d*x^3+1/2*c)^2*a+2*tan(1/2*d*x^3+1/2*c)*b+a)*b^2/a/(a^2-b^2)
*tan(1/2*d*x^3+1/2*c)+2/3/d/(tan(1/2*d*x^3+1/2*c)^2*a+2*tan(1/2*d*x^3+1/2*c
)*b+a)/(a^2-b^2)*b+2/3/d*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x^3+1/
2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.88947, size = 798, normalized size = 8.49

$$\left[\frac{\left(ab\sin\left(dx^3+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(dx^3+c\right)^2-2ab\sin\left(dx^3+c\right)-a^2-b^2-2\left(a\cos\left(dx^3+c\right)\sin\left(dx^3+c\right)+b\cos\left(dx^3+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx^3+c\right)^2-2ab\sin\left(dx^3+c\right)-a^2-b^2}\right.}{6\left(\left(a^4b-2a^2b^3+b^5\right)d\sin\left(dx^3+c\right)+\left(a^5-2a^3b^2+ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*sin(d*x^3 + c) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d
*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2 - 2*(a*cos(d*x^3 + c)*sin(d*
x^3 + c) + b*cos(d*x^3 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x^3 + c)^2 - 2*a*
b*sin(d*x^3 + c) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x^3 + c))/((a^4*b -
2*a^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/3*((a*
b*sin(d*x^3 + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x^3 + c) + b)/(sqr
t(a^2 - b^2)*cos(d*x^3 + c))) - (a^2*b - b^3)*cos(d*x^3 + c))/((a^4*b - 2*a
^2*b^3 + b^5)*d*sin(d*x^3 + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1237, size = 197, normalized size = 2.1

$$\frac{2\left(\pi\left\lfloor\frac{dx^3+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{3\left(a^2d-b^2d\right)\sqrt{a^2-b^2}}+\frac{2\left(b^2\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+ab\right)}{3\left(a^3d-ab^2d\right)\left(a\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx^3+\frac{1}{2}c\right)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(pi*floor(1/2*(d*x^3 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x^3 +
1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^2*d - b^2*d)*sqrt(a^2 - b^2)) + 2/3*(b^
2*tan(1/2*d*x^3 + 1/2*c) + a*b)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^3 + 1/2*c
)^2 + 2*b*tan(1/2*d*x^3 + 1/2*c) + a))
```

3.91 $\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [A] time = 0.0246425, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*x^3])^2),x]

[Out] Defer[Int][1/(x*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 10.0254, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*x^3])^2),x]

[Out] Integrate[1/(x*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A] time = 0.647, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(dx^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x\cos\left(dx^3+c\right)^2-2\,abx\sin\left(dx^3+c\right)-\left(a^2+b^2\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x*cos(d*x^3 + c)^2 - 2*a*b*x*sin(d*x^3 + c) - (a^2 + b^2)*
x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x*(a + b*sin(c + d*x**3))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x), x)
```

3.92 $\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [A] time = 0.0251098, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Defer[Int][1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2} dx = \int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Mathematica [A] time = 12.2686, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Integrate[1/(x^4*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A] time = 0.727, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+b\sin\left(dx^3+c\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x^4/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x^4\cos\left(dx^3+c\right)^2-2\,abx^4\sin\left(dx^3+c\right)-\left(a^2+b^2\right)x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^4*cos(d*x^3 + c)^2 - 2*a*b*x^4*sin(d*x^3 + c) - (a^2 + b
^2)*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^4), x)
```

3.93 $\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[x/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi [A] time = 0.014517, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Defer[Int][x/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 6.84591, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Integrate[x/(a + b*Sin[c + d*x^3])^2, x]
```

Maple [A] time = 0.501, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\sin\left(dx^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(x/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^
2)*sin(c)^2)*d*x*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^
4)*sin(2*c)^2)*d*x*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x*cos(c)*sin(d*x^3)
 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x*sin(d*x^3)^2
 + 4*(a^3*b - a*b^3)*d*x*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x + 2*(2*((a
^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x*cos(d*
x^3) - (a^2*b^2 - b^4)*d*x*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) +
(a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*
b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x*cos(d*x^3
) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d
*x*sin(d*x^3) + (a^2*b^2 - b^4)*d*x*sin(2*c))*sin(2*d*x^3))*integrate(-2/3*
(2*a*b*cos(d*x^3)*cos(c) + b^2*cos(2*c)*sin(2*d*x^3) + b^2*cos(2*d*x^3)*sin
(2*c) - 2*a*b*sin(d*x^3)*sin(c) - (a*b - (3*a*b*d*x^3*sin(2*c) + a*b*cos(2*
c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c) - a^2*sin(c))*cos(d*x^3) - (3*a*b*
d*x^3*cos(2*c) - a*b*sin(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + a^2*c
os(c))*sin(d*x^3))*cos(d*x^3 + c) + (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) -
a*b*sin(2*c))*cos(2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + a^2*cos(c))*cos(d*x^3)
 + (3*a*b*d*x^3*sin(2*c) + a*b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(
c) - a^2*sin(c))*sin(d*x^3))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 +
 (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(
c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(
2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^
3)*d*x^2*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*
sin(c)^2)*d*x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) +
(a^2*b^2 - b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*
b^3)*cos(2*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2
*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*
sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b -
 a*b^3)*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2
*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d
*x^2*sin(2*c))*sin(2*d*x^3)), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c
)*sin(2*d*x^3) + a*b*cos(2*d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d
*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x*c
os(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x
*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d
*x*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x*cos(c)*sin(d*x^3) + 4*((a^4 - a^2
*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x*sin(d*x^3)^2 + 4*(a^3*b - a*
b^3)*d*x*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x + 2*(2*((a^3*b - a*b^3)*co
s(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x*cos(d*x^3) - (a^2*b^2
- b^4)*d*x*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*
sin(2*c)*sin(c))*d*x*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2
*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x*cos(d*x^3) + 2*((a^3*b -
a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x*sin(d*x^3) +
```

```
(a^2*b^2 - b^4)*d*x*sin(2*c))*sin(2*d*x^3))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{b^2\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-x/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(x/(a + b*sin(c + d*x**3))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*sin(d*x^3 + c) + a)^2, x)
```

3.94 $\int \frac{1}{x^2(a+b\sin(c+dx^3))^2} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [A] time = 0.0242327, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2} dx=\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Mathematica [A] time = 10.6589, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A] time = 0.884, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(dx^3+c\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x^2/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^4*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^4*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^4*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^4*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^4*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^4*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^4 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^4*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^4*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^4*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^4*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^4*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^4*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(8*a*b*cos(d*x^3)*cos(c) + 4*b^2*cos(2*c)*sin(2*d
*x^3) + 4*b^2*cos(2*d*x^3)*sin(2*c) - 8*a*b*sin(d*x^3)*sin(c) - (4*a*b - (3
*a*b*d*x^3*sin(2*c) + 4*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c)
- 4*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 4*a*b*sin(2*c))*sin(2*
d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 4*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c) +
 (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 4*a*b*sin(2*c))*cos(2*d*x^3) + 2*(3
*a^2*d*x^3*sin(c) + 4*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 4*a*
b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 4*a^2*sin(c))*sin(d*x^3)
)*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2
)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(
c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*si
n(2*c)^2)*d*x^5*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(c)*sin(d*x^3)
+ 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*sin(d*x^3)^
2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^5 + 2*(
2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5
*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^5*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*
cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*sin(d*x^3))*cos(2*d*x^3) +
2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*
x^5*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2
*c)*sin(c))*d*x^5*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin(2*d*x^3)
), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(2*
d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4
)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^4*cos(2*d*x^3)^2 + 4*((a^4 -
 a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^4*cos(d*x^3)^2 + ((a^2*b
^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^4*sin(2*d*x^3)^2 + 4
*(a^3*b - a*b^3)*d*x^4*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a
^4 - a^2*b^2)*sin(c)^2)*d*x^4*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^4*cos(d*
x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^4 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
 - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^4*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^
4*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*
sin(c))*d*x^4*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos
(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^4*cos(d*x^3) + 2*((a^3*b - a*b^3
```

```
)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^4*sin(d*x^3) + (a^
2*b^2 - b^4)*d*x^4*sin(2*c))*sin(2*d*x^3))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x^2\cos\left(dx^3+c\right)^2-2\,abx^2\sin\left(dx^3+c\right)-\left(a^2+b^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos(d*x^3 + c)^2 - 2*a*b*x^2*sin(d*x^3 + c) - (a^2 + b
^2)*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*sin(c + d*x**3))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^2), x)
```

3.95 $\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x^3])^(-2), x]
```

Rubi [A] time = 0.005102, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x^3])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x^3])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 8.22203, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x^3])^(-2),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x^3])^(-2), x]
```

Maple [A] time = 0.593, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(dx^3+c\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^2*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^2*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d
*x^3) + 2*b^2*cos(2*d*x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) - (2*a*b - (3
*a*b*d*x^3*sin(2*c) + 2*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c)
- 2*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 2*a*b*sin(2*c))*sin(2*
d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c) +
 (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 2*a*b*sin(2*c))*cos(2*d*x^3) + 2*(3
*a^2*d*x^3*sin(c) + 2*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 2*a*
b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 2*a^2*sin(c))*sin(d*x^3)
)*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2
)*d*x^3*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(
c)^2)*d*x^3*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*si
n(2*c)^2)*d*x^3*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(c)*sin(d*x^3)
+ 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^3*sin(d*x^3)^
2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^3 + 2*(
2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^3
*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^3*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*
cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^3*sin(d*x^3))*cos(2*d*x^3) +
2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*
x^3*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2
*c)*sin(c))*d*x^3*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^3*sin(2*c))*sin(2*d*x^3)
), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(2*
d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b^4
)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*cos(2*d*x^3)^2 + 4*((a^4 -
 a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^2*cos(d*x^3)^2 + ((a^2*b
^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^2*sin(2*d*x^3)^2 + 4
*(a^3*b - a*b^3)*d*x^2*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a
^4 - a^2*b^2)*sin(c)^2)*d*x^2*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^2*cos(d*
x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^2 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c)
 - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^
2*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*
sin(c))*d*x^2*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos
(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^2*cos(d*x^3) + 2*((a^3*b - a*b^3
```

```
)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^2*sin(d*x^3) + (a^
2*b^2 - b^4)*d*x^2*sin(2*c))*sin(2*d*x^3))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x**3))**(-2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^(-2), x)
```

3.96 $\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi [A] time = 0.0246991, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 11.6172, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2),x]
```

```
[Out] Integrate[1/(x^3*(a + b*Sin[c + d*x^3])^2), x]
```

Maple [A] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+b\sin\left(dx^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int(1/x^3/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(4*a*b*cos(d*x^3)*cos(c) + 2*b^2*cos(2*c)*sin(2*d*x^3) + 2*b^2*cos(2*d*
x^3)*sin(2*c) - 4*a*b*sin(d*x^3)*sin(c) + 2*(a*b*cos(2*d*x^3)*cos(2*c) - 2*
a^2*cos(c)*sin(d*x^3) - a*b*sin(2*d*x^3)*sin(2*c) - 2*a^2*cos(d*x^3)*sin(c)
 - a*b)*cos(d*x^3 + c) - 3*(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*s
in(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*
b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2
- b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(c)*si
n(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*si
n(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*
x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(
c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^5*cos(2*c) - 2*((a^3*b - a*b^3)*
cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*sin(d*x^3))*cos(2*
d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*s
in(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b
^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin
(2*d*x^3))*integrate(-2/3*(10*a*b*cos(d*x^3)*cos(c) + 5*b^2*cos(2*c)*sin(2*
d*x^3) + 5*b^2*cos(2*d*x^3)*sin(2*c) - 10*a*b*sin(d*x^3)*sin(c) - (5*a*b -
(3*a*b*d*x^3*sin(2*c) + 5*a*b*cos(2*c))*cos(2*d*x^3) - 2*(3*a^2*d*x^3*cos(c
) - 5*a^2*sin(c))*cos(d*x^3) - (3*a*b*d*x^3*cos(2*c) - 5*a*b*sin(2*c))*sin(
2*d*x^3) + 2*(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*sin(d*x^3))*cos(d*x^3 + c)
 + (3*a*b*d*x^3 - (3*a*b*d*x^3*cos(2*c) - 5*a*b*sin(2*c))*cos(2*d*x^3) + 2*
(3*a^2*d*x^3*sin(c) + 5*a^2*cos(c))*cos(d*x^3) + (3*a*b*d*x^3*sin(2*c) + 5*
a*b*cos(2*c))*sin(2*d*x^3) + 2*(3*a^2*d*x^3*cos(c) - 5*a^2*sin(c))*sin(d*x^
3))*sin(d*x^3 + c))/(((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)
^2)*d*x^6*cos(2*d*x^3)^2 + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*si
n(c)^2)*d*x^6*cos(d*x^3)^2 + ((a^2*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*
sin(2*c)^2)*d*x^6*sin(2*d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(c)*sin(d*x^3
) + 4*((a^4 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^6*sin(d*x^3
)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^6 + 2
*(2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x
^6*cos(d*x^3) - (a^2*b^2 - b^4)*d*x^6*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c
)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^6*sin(d*x^3))*cos(2*d*x^3)
+ 2*(2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*
d*x^6*cos(d*x^3) + 2*((a^3*b - a*b^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos
(2*c)*sin(c))*d*x^6*sin(d*x^3) + (a^2*b^2 - b^4)*d*x^6*sin(2*c))*sin(2*d*x^
3)), x) + 2*(2*a^2*cos(d*x^3)*cos(c) + a*b*cos(2*c)*sin(2*d*x^3) + a*b*cos(
2*d*x^3)*sin(2*c) - 2*a^2*sin(d*x^3)*sin(c))*sin(d*x^3 + c))/(((a^2*b^2 - b
^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^5*cos(2*d*x^3)^2 + 4*((a^4
 - a^2*b^2)*cos(c)^2 + (a^4 - a^2*b^2)*sin(c)^2)*d*x^5*cos(d*x^3)^2 + ((a^2
*b^2 - b^4)*cos(2*c)^2 + (a^2*b^2 - b^4)*sin(2*c)^2)*d*x^5*sin(2*d*x^3)^2 +
 4*(a^3*b - a*b^3)*d*x^5*cos(c)*sin(d*x^3) + 4*((a^4 - a^2*b^2)*cos(c)^2 +
(a^4 - a^2*b^2)*sin(c)^2)*d*x^5*sin(d*x^3)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(
d*x^3)*sin(c) + (a^2*b^2 - b^4)*d*x^5 + 2*(2*((a^3*b - a*b^3)*cos(c)*sin(2*
c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*cos(d*x^3) - (a^2*b^2 - b^4)*d*
x^5*cos(2*c) - 2*((a^3*b - a*b^3)*cos(2*c)*cos(c) + (a^3*b - a*b^3)*sin(2*c
)*sin(c))*d*x^5*sin(d*x^3))*cos(2*d*x^3) + 2*(2*((a^3*b - a*b^3)*cos(2*c)*c
os(c) + (a^3*b - a*b^3)*sin(2*c)*sin(c))*d*x^5*cos(d*x^3) + 2*((a^3*b - a*b
```

```
^3)*cos(c)*sin(2*c) - (a^3*b - a*b^3)*cos(2*c)*sin(c))*d*x^5*sin(d*x^3) + (
a^2*b^2 - b^4)*d*x^5*sin(2*c))*sin(2*d*x^3))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x^3\cos\left(dx^3+c\right)^2-2\,abx^3\sin\left(dx^3+c\right)-\left(a^2+b^2\right)x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^3*cos(d*x^3 + c)^2 - 2*a*b*x^3*sin(d*x^3 + c) - (a^2 + b
^2)*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(dx^3+c\right)+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(d*x^3 + c) + a)^2*x^3), x)
```

3.97 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Rubi [A] time = 0.0247014, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx = \int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Mathematica [A] time = 0.863865, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^p, x]
```

Maple [A] time = 0.678, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^3+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**p,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(b\sin\left(dx^3+c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^3 + c) + a)^p, x)
```

3.98 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3 \, dx$

Optimal. Leaf size=442

$$\frac{ibe^{ic}\left(4a^2+b^2\right)\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{8e}-\frac{ibe^{-ic}\left(4a^2+b^2\right)\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},\right.}{8e}$$

```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/8)*b*(4*a^2 + b^2)*E^
(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])
/e - ((I/8)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 +
 m)/3, I*d*x^3])/(e*E^(I*c)) + (2^(-7/3 - m/3)*a*b^2*E^((2*I)*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3])/e + (2^(-7/3
- m/3)*a*b^2*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*
x^3])/(e*E^((2*I)*c)) - ((I/8)*3^(-4/3 - m/3)*b^3*E^((3*I)*c)*(e*x)^(1 + m)
*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3])/e + ((I/8)*3^(-4
/3 - m/3)*b^3*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (3*I)*d
*x^3])/(e*E^((3*I)*c))
```

Rubi [A] time = 0.414381, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3403, 6, 3390, 2218, 3389}

$$\frac{ibe^{ic}\left(4a^2+b^2\right)\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{8e}-\frac{ibe^{-ic}\left(4a^2+b^2\right)\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},\right.}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```


```
[Out] (a*(2*a^2 + 3*b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/8)*b*(4*a^2 + b^2)*E^
(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])
/e - ((I/8)*b*(4*a^2 + b^2)*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 +
 m)/3, I*d*x^3])/(e*E^(I*c)) + (2^(-7/3 - m/3)*a*b^2*E^((2*I)*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3])/e + (2^(-7/3
- m/3)*a*b^2*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*
x^3])/(e*E^((2*I)*c)) - ((I/8)*3^(-4/3 - m/3)*b^3*E^((3*I)*c)*(e*x)^(1 + m)
*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3])/e + ((I/8)*3^(-4
/3 - m/3)*b^3*(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (3*I)*d
*x^3])/(e*E^((3*I)*c))
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^3 dx = \int \left(a^3(ex)^m + \frac{3}{2}ab^2(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right) + 3a^2b(ex)^m\sin\left(c+dx^3\right) + \frac{3}{4}\right.$$

$$= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right) + 3a^2b(ex)^m\sin\left(c+dx^3\right) + \frac{3}{4}b^3(\right.$$

$$= \int \left(\left(a^3+\frac{3ab^2}{2}\right)(ex)^m - \frac{3}{2}ab^2(ex)^m\cos\left(2c+2dx^3\right) + \left(3a^2b+\frac{3b^3}{4}\right)(ex)^m\sin\left(c+dx\right.\right.$$

$$= \frac{a\left(2a^2+3b^2\right)(ex)^{1+m}}{2e(1+m)} - \frac{1}{2}\left(3ab^2\right)\int (ex)^m\cos\left(2c+2dx^3\right)dx - \frac{1}{4}b^3\int (ex)^m\sin\left(3c+\right.$$

$$= \frac{a\left(2a^2+3b^2\right)(ex)^{1+m}}{2e(1+m)} - \frac{1}{4}\left(3ab^2\right)\int e^{-2ic-2idx^3}(ex)^m\,dx - \frac{1}{4}\left(3ab^2\right)\int e^{2ic+2idx^3}(ex)^m\,d$$

$$= \frac{a\left(2a^2+3b^2\right)(ex)^{1+m}}{2e(1+m)} + \frac{ib\left(4a^2+b^2\right)e^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{8e} - \frac{ib\,(}{}$$

Mathematica [A] time = 12.4702, size = 373, normalized size = 0.84

$$\frac{1}{24}ix(ex)^m\left(3be^{ic}\left(4a^2+b^2\right)\left(-idx^3\right)^{-\frac{m}{3}-\frac{1}{3}}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right) - 3be^{-ic}\left(4a^2+b^2\right)\left(idx^3\right)^{-\frac{m}{3}-\frac{1}{3}}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^3,x]
```

```
[Out] (I/24)*x*(e*x)^m*(((-12*I)*a*(2*a^2 + 3*b^2))/(1 + m) + 3*b*(4*a^2 + b^2)*E
^(I*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (-I)*d*x^3] - (3*b*(4*a^2
 + b^2)*(I*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, I*d*x^3])/E^(I*c) - (3*I)*2
^(2/3 - m/3)*a*b^2*E^((2*I)*c)*((-I)*d*x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (
-2*I)*d*x^3] - ((3*I)*2^(2/3 - m/3)*a*b^2*(I*d*x^3)^(-1/3 - m/3)*Gamma[(1 +
 m)/3, (2*I)*d*x^3])/E^((2*I)*c) - 3^(-1/3 - m/3)*b^3*E^((3*I)*c)*((-I)*d*x
^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (-3*I)*d*x^3] + (3^(-1/3 - m/3)*b^3*(I*d*
x^3)^(-1/3 - m/3)*Gamma[(1 + m)/3, (3*I)*d*x^3])/E^((3*I)*c))
```

Maple [F] time = 0.389, size = 0, normalized size = 0.

$$\int (ex)^m\left(a+b\sin\left(dx^3+c\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00216, size = 926, normalized size = 2.1

$$36\left(2a^3+3ab^2\right)(ex)^m\,dx+\left(b^3e^2m+b^3e^2\right)e^{\left(-\frac{1}{3}(m-2)\log\left(\frac{3id}{e^3}\right)-3ic\right)}\Gamma\left(\frac{1}{3}m+\frac{1}{3},3idx^3\right)+\left(-9iab^2e^2m-9iab^2e^2\right)e^{\left(-\frac{1}{3}(m-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*(2*a^3 + 3*a*b^2)*(e*x)^m*d*x + (b^3*e^2*m + b^3*e^2)*e^(-1/3*(m -
 2)*log(3*I*d/e^3) - 3*I*c)*gamma(1/3*m + 1/3, 3*I*d*x^3) + (-9*I*a*b^2*e^2
*m - 9*I*a*b^2*e^2)*e^(-1/3*(m - 2)*log(2*I*d/e^3) - 2*I*c)*gamma(1/3*m + 1
/3, 2*I*d*x^3) - 9*((4*a^2*b + b^3)*e^2*m + (4*a^2*b + b^3)*e^2)*e^(-1/3*(m
 - 2)*log(I*d/e^3) - I*c)*gamma(1/3*m + 1/3, I*d*x^3) - 9*((4*a^2*b + b^3)*
e^2*m + (4*a^2*b + b^3)*e^2)*e^(-1/3*(m - 2)*log(-I*d/e^3) + I*c)*gamma(1/3
*m + 1/3, -I*d*x^3) + (9*I*a*b^2*e^2*m + 9*I*a*b^2*e^2)*e^(-1/3*(m - 2)*log
(-2*I*d/e^3) + 2*I*c)*gamma(1/3*m + 1/3, -2*I*d*x^3) + (b^3*e^2*m + b^3*e^2
)*e^(-1/3*(m - 2)*log(-3*I*d/e^3) + 3*I*c)*gamma(1/3*m + 1/3, -3*I*d*x^3))/
(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m\left(a+b\sin\left(c+dx^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**3,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3+c\right)+a\right)^3(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^3*(e*x)^m, x)
```

3.99 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 \, dx$

Optimal. Leaf size=285

$$\frac{iabe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{3e}-\frac{iabe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{3e}+\frac{b^2e^{2ic}2^{-\frac{m}{3}-}}{}$$

```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/3)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/3)*a*b*
(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
 + (2^(-7/3 - m/3)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*
Gamma[(1 + m)/3, (-2*I)*d*x^3])/(3*e) + (2^(-7/3 - m/3)*b^2*(e*x)^(1 + m)*(
I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3])/(3*e*E^((2*I)*c))
```

Rubi [A] time = 0.232444, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3403, 6, 3390, 2218, 3389}

$$\frac{iabe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{3e}-\frac{iabe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{3e}+\frac{b^2e^{2ic}2^{-\frac{m}{3}-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```


```
[Out] ((2*a^2 + b^2)*(e*x)^(1 + m))/(2*e*(1 + m)) + ((I/3)*a*b*E^(I*c)*(e*x)^(1 +
 m)*((-I)*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/3)*a*b*
(e*x)^(1 + m)*(I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
 + (2^(-7/3 - m/3)*b^2*E^((2*I)*c)*(e*x)^(1 + m)*((-I)*d*x^3)^((-1 - m)/3)*
Gamma[(1 + m)/3, (-2*I)*d*x^3])/(3*e) + (2^(-7/3 - m/3)*b^2*(e*x)^(1 + m)*(
I*d*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3])/(3*e*E^((2*I)*c))
```


Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2 dx = \int \left(a^2(ex)^m + \frac{1}{2}b^2(ex)^m - \frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right) + 2ab(ex)^m\sin\left(c+dx^3\right)\right) dx$$

$$= \int \left(\left(a^2+\frac{b^2}{2}\right)(ex)^m - \frac{1}{2}b^2(ex)^m\cos\left(2c+2dx^3\right) + 2ab(ex)^m\sin\left(c+dx^3\right)\right) dx$$

$$= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + (2ab)\int (ex)^m\sin\left(c+dx^3\right) dx - \frac{1}{2}b^2\int (ex)^m\cos\left(2c+2dx^3\right) d$$

$$= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + (iab)\int e^{-ic-idx^3}(ex)^m dx - (iab)\int e^{ic+idx^3}(ex)^m dx - \frac{1}{4}b^2\int e^{-2i}$$

$$= \frac{\left(2a^2+b^2\right)(ex)^{1+m}}{2e(1+m)} + \frac{iabe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{3e} - \frac{iabe^{-ic}(ex)^{1+m}\left(i\right.}{}$$

Mathematica [A] time = 6.8193, size = 556, normalized size = 1.95

$$\frac{2^{\frac{1}{3}(-m-7)}x\left(d^2x^6\right)^{\frac{1}{3}(-m-1)}(ex)^m\left(-iab2^{\frac{m+7}{3}}(m+1)(\cos(c)-i\sin(c))\left(-idx^3\right)^{\frac{m+1}{3}}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)+iab2^{\frac{m+7}{3}}(m+1)(\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] (2^((-7 - m)/3)*x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(3*2^((7 + m)/3)*a^2*(d^2*
x^6)^((1 + m)/3) + 3*2^((4 + m)/3)*b^2*(d^2*x^6)^((1 + m)/3) + b^2*(I*d*x^3
)^((1 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*m*(I*d*x^3)^((1
 + m)/3)*Cos[2*c]*Gamma[(1 + m)/3, (-2*I)*d*x^3] + b^2*((-I)*d*x^3)^((1 + m
)/3)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] + b^2*m*((-I)*d*x^3)^((1 + m)/3
)*Cos[2*c]*Gamma[(1 + m)/3, (2*I)*d*x^3] - I*2^((7 + m)/3)*a*b*(1 + m)*((-I
)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*2^((
7 + m)/3)*a*b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(C
os[c] + I*Sin[c]) + I*b^2*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x
^3]*Sin[2*c] + I*b^2*m*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-2*I)*d*x^3]
*Sin[2*c] - I*b^2*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3]*Si
n[2*c] - I*b^2*m*((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (2*I)*d*x^3]*Sin
[2*c]))/(3*(1 + m))
```

Maple [F] time = 0.381, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^3+c\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90572, size = 578, normalized size = 2.03

$$12\left(2\,a^2+b^2\right)(ex)^m\,dx+\left(-i\,b^2e^2m-i\,b^2e^2\right)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{2i\,d}{e^3}\right)-2i\,c\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},2i\,dx^3\right)-8\left(abe^2m+abe^2\right)e^{\left(-\frac{1}{3}\,(m-2)\log\left(\frac{i\,d}{e^3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(2*a^2 + b^2)*(e*x)^m*d*x + (-I*b^2*e^2*m - I*b^2*e^2)*e^(-1/3*(m
- 2)*log(2*I*d/e^3) - 2*I*c)*gamma(1/3*m + 1/3, 2*I*d*x^3) - 8*(a*b*e^2*m +
 a*b*e^2)*e^(-1/3*(m - 2)*log(I*d/e^3) - I*c)*gamma(1/3*m + 1/3, I*d*x^3) -
 8*(a*b*e^2*m + a*b*e^2)*e^(-1/3*(m - 2)*log(-I*d/e^3) + I*c)*gamma(1/3*m +
 1/3, -I*d*x^3) + (I*b^2*e^2*m + I*b^2*e^2)*e^(-1/3*(m - 2)*log(-2*I*d/e^3)
 + 2*I*c)*gamma(1/3*m + 1/3, -2*I*d*x^3))/(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3+c\right)+a\right)^2 (ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)^2*(e*x)^m, x)
```

3.100 $\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right)\,dx$

Optimal. Leaf size=134

$$\frac{ibe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{6e}+\frac{a(ex)^{m+1}}{e(m+1)}$$

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/6)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)
^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/6)*b*(e*x)^(1 + m)*(I*d
*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
```

Rubi [A] time = 0.0995589, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {14, 3389, 2218}

$$\frac{ibe^{ic}\left(-idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},-idx^3\right)}{6e}-\frac{ibe^{-ic}\left(idx^3\right)^{\frac{1}{3}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)}{6e}+\frac{a(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (a*(e*x)^(1 + m))/(e*(1 + m)) + ((I/6)*b*E^(I*c)*(e*x)^(1 + m)*((-I)*d*x^3)
^((-1 - m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3])/e - ((I/6)*b*(e*x)^(1 + m)*(I*d
*x^3)^((-1 - m)/3)*Gamma[(1 + m)/3, I*d*x^3])/(e*E^(I*c))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+dx^3\right)\right) dx = \int \left(a(ex)^m + b(ex)^m \sin\left(c+dx^3\right)\right) dx$$
$$= \frac{a(ex)^{1+m}}{e(1+m)} + b\int (ex)^m \sin\left(c+dx^3\right) dx$$
$$= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{1}{2}(ib)\int e^{-ic-idx^3}(ex)^m dx - \frac{1}{2}(ib)\int e^{ic+idx^3}(ex)^m dx$$
$$= \frac{a(ex)^{1+m}}{e(1+m)} + \frac{ibe^{ic}(ex)^{1+m}\left(-idx^3\right)^{\frac{1}{3}(-1-m)}\Gamma\left(\frac{1+m}{3},-idx^3\right)}{6e} - \frac{ibe^{-ic}(ex)^{1+m}\left(idx^3\right)^{\frac{1}{3}(-1-m)}\cdots}{6e}$$

Mathematica [A] time = 1.60107, size = 149, normalized size = 1.11

$$\frac{x\left(d^2x^6\right)^{\frac{1}{3}(-m-1)}(ex)^m\left(-ib(m+1)(\cos(c)-i\sin(c))\left(-idx^3\right)^{\frac{m+1}{3}}\text{Gamma}\left(\frac{m+1}{3},idx^3\right)+ib(m+1)(\cos(c)+i\sin(c))\left(id\cdots\right.\right.}{6(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^3]),x]
```

```
[Out] (x*(e*x)^m*(d^2*x^6)^((-1 - m)/3)*(6*a*(d^2*x^6)^((1 + m)/3) - I*b*(1 + m)*
((-I)*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, I*d*x^3]*(Cos[c] - I*Sin[c]) + I*
b*(1 + m)*(I*d*x^3)^((1 + m)/3)*Gamma[(1 + m)/3, (-I)*d*x^3]*(Cos[c] + I*Si
n[c])))/(6*(1 + m))
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(dx^3+c\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

```
[Out] int((e*x)^m*(a+b*sin(d*x^3+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74211, size = 274, normalized size = 2.04

$$\frac{6(ex)^m adx - \left(be^2m+be^2\right)e^{\left(-\frac{1}{3}(m-2)\log\left(\frac{id}{e^3}\right)-ic\right)}\Gamma\left(\frac{1}{3}m+\frac{1}{3},idx^3\right) - \left(be^2m+be^2\right)e^{\left(-\frac{1}{3}(m-2)\log\left(-\frac{id}{e^3}\right)+ic\right)}\Gamma\left(\frac{1}{3}m+\frac{1}{3},-idx^3\cdots\right.}{6(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(e*x)^m*a*d*x - (b*e^2*m + b*e^2)*e^(-1/3*(m - 2)*log(I*d/e^3) - I*c
)*gamma(1/3*m + 1/3, I*d*x^3) - (b*e^2*m + b*e^2)*e^(-1/3*(m - 2)*log(-I*d/
e^3) + I*c)*gamma(1/3*m + 1/3, -I*d*x^3))/(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a + b\sin\left(c + dx^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral((e*x)**m*(a + b*sin(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx^3 + c\right) + a\right)(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x^3 + c) + a)*(e*x)^m, x)
```

3.101 $\int \frac{(ex)^m}{a+b\sin(c+dx^3)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(ex)^m}{a+b\sin\left(c+dx^3\right)},x\right)$$

```
[Out] Unintegrable[(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Rubi [A] time = 0.0264576, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Rubi steps

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx=\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Mathematica [A] time = 0.406698, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]),x]
```

```
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3]), x]
```

Maple [A] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{a+b\sin\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

```
[Out] int((e*x)^m/(a+b*sin(d*x^3+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{b\sin\left(dx^3+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{a+b\sin\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**3+c)),x)
```

```
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{b\sin\left(dx^3+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^3 + c) + a), x)
```

3.102 $\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi [A] time = 0.0246136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Defer[Int][(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx=\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 0.720583, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2,x]
```

```
[Out] Integrate[(e*x)^m/(a + b*Sin[c + d*x^3])^2, x]
```

Maple [A] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(a+b\sin\left(dx^3+c\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

```
[Out] int((e*x)^m/(a+b*sin(d*x^3+c))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(ex)^m}{b^2\cos\left(dx^3+c\right)^2-2\,ab\sin\left(dx^3+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e*x)^m/(b^2*cos(d*x^3 + c)^2 - 2*a*b*sin(d*x^3 + c) - a^2 - b^2)
, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex)^m}{\left(a+b\sin\left(c+dx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*sin(d*x**3+c))**2,x)
```

```
[Out] Integral((e*x)**m/(a + b*sin(c + d*x**3))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex)^m}{\left(b\sin\left(dx^3+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*sin(d*x^3+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(b*sin(d*x^3 + c) + a)^2, x)
```

3.103 $\int x^2 \sin\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=78

$$\frac{1}{6}b^3\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)$$

```
[Out] (b*x^2*Cos[a + b/x])/6 + (b^3*Cos[a]*CosIntegral[b/x])/6 - (b^2*x*Sin[a + b
/x])/6 + (x^3*Sin[a + b/x])/3 - (b^3*Sin[a]*SinIntegral[b/x])/6
```

Rubi [A] time = 0.131339, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3379, 3297, 3303, 3299, 3302}

$$\frac{1}{6}b^3\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b/x],x]
```

```
[Out] (b*x^2*Cos[a + b/x])/6 + (b^3*Cos[a]*CosIntegral[b/x])/6 - (b^2*x*Sin[a + b
/x])/6 + (x^3*Sin[a + b/x])/3 - (b^3*Sin[a]*SinIntegral[b/x])/6
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin\left(a+\frac{b}{x}\right)dx &= -\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x^4}dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{3}b\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{x^3}dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}b^2\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x^2}dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}b^3\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{x}dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)+\frac{1}{6}\left(b^3\cos(a)\right)\operatorname{Subst}\left(\int \frac{\cos(bx)}{x}dx, x,\right.\\
&= \frac{1}{6}bx^2\cos\left(a+\frac{b}{x}\right)+\frac{1}{6}b^3\cos(a)\operatorname{Ci}\left(\frac{b}{x}\right)-\frac{1}{6}b^2x\sin\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sin\left(a+\frac{b}{x}\right)-\frac{1}{6}b^3\sin(a)\operatorname{Si}\left(\frac{b}{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.073134, size = 70, normalized size = 0.9

$$\frac{1}{6}\left(b^3\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)-b^3\sin(a)\text{Si}\left(\frac{b}{x}\right)+x\left(b^2\left(-\sin\left(a+\frac{b}{x}\right)\right)+2x^2\sin\left(a+\frac{b}{x}\right)+bx\cos\left(a+\frac{b}{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b/x],x]
```

```
[Out] (b^3*Cos[a]*CosIntegral[b/x] + x*(b*x*Cos[a + b/x] - b^2*Sin[a + b/x] + 2*x
^2*Sin[a + b/x]) - b^3*Sin[a]*SinIntegral[b/x])/6
```

Maple [A] time = 0.012, size = 73, normalized size = 0.9

$$-b^3\left(-\frac{x^3}{3\,b^3}\sin\left(a+\frac{b}{x}\right)-\frac{x^2}{6\,b^2}\cos\left(a+\frac{b}{x}\right)+\frac{x}{6\,b}\sin\left(a+\frac{b}{x}\right)+\frac{\sin(a)}{6}\operatorname{Si}\left(\frac{b}{x}\right)-\frac{\cos(a)}{6}\operatorname{Ci}\left(\frac{b}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b/x),x)
```

```
[Out] -b^3*(-1/3*sin(a+b/x)*x^3/b^3-1/6*cos(a+b/x)*x^2/b^2+1/6*sin(a+b/x)*x/b+1/6
*Si(b/x)*sin(a)-1/6*Ci(b/x)*cos(a))
```

Maxima [C] time = 1.14623, size = 116, normalized size = 1.49

$$\frac{1}{12}\left(\left(\operatorname{Ei}\left(\frac{ib}{x}\right)+\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a)+\left(i\operatorname{Ei}\left(\frac{ib}{x}\right)-i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)\right)b^3+\frac{1}{6}bx^2\cos\left(\frac{ax+b}{x}\right)-\frac{1}{6}\left(b^2x-2x^3\right)\sin\left(\frac{ax+b}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/12*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) + (I*Ei(I*b/x) - I*Ei(-I*b/x))*sin(a)
)*b^3 + 1/6*b*x^2*cos((a*x + b)/x) - 1/6*(b^2*x - 2*x^3)*sin((a*x + b)/x)
```

Fricas [A] time = 2.04289, size = 224, normalized size = 2.87

$$-\frac{1}{6}\,b^3\sin\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)+\frac{1}{6}\,bx^2\cos\left(\frac{ax+b}{x}\right)+\frac{1}{12}\left(b^3\operatorname{Ci}\left(\frac{b}{x}\right)+b^3\operatorname{Ci}\left(-\frac{b}{x}\right)\right)\cos\left(a\right)-\frac{1}{6}\left(b^2x-2\,x^3\right)\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="fricas")
```

```
[Out] -1/6*b^3*sin(a)*sin_integral(b/x) + 1/6*b*x^2*cos((a*x + b)/x) + 1/12*(b^3*
cos_integral(b/x) + b^3*cos_integral(-b/x))*cos(a) - 1/6*(b^2*x - 2*x^3)*si
n((a*x + b)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b/x),x)
```

```
[Out] Integral(x**2*sin(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(a + b/x), x)
```

3.104 $\int x \sin\left(a + \frac{b}{x}\right) dx$

Optimal. Leaf size=60

$$\frac{1}{2}b^2 \sin(a)\text{CosIntegral}\left(\frac{b}{x}\right) + \frac{1}{2}b^2 \cos(a)\text{Si}\left(\frac{b}{x}\right) + \frac{1}{2}x^2 \sin\left(a + \frac{b}{x}\right) + \frac{1}{2}bx \cos\left(a + \frac{b}{x}\right)$$

```
[Out] (b*x*Cos[a + b/x])/2 + (b^2*CosIntegral[b/x]*Sin[a])/2 + (x^2*Sin[a + b/x])
/2 + (b^2*Cos[a]*SinIntegral[b/x])/2
```

Rubi [A] time = 0.0976248, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3379, 3297, 3303, 3299, 3302}

$$\frac{1}{2}b^2 \sin(a)\text{CosIntegral}\left(\frac{b}{x}\right) + \frac{1}{2}b^2 \cos(a)\text{Si}\left(\frac{b}{x}\right) + \frac{1}{2}x^2 \sin\left(a + \frac{b}{x}\right) + \frac{1}{2}bx \cos\left(a + \frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b/x],x]
```

```
[Out] (b*x*Cos[a + b/x])/2 + (b^2*CosIntegral[b/x]*Sin[a])/2 + (x^2*Sin[a + b/x])
/2 + (b^2*Cos[a]*SinIntegral[b/x])/2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin\left(a+\frac{b}{x}\right)dx &= -\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)-\frac{1}{2}b\operatorname{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}\left(b^2\cos(a)\right)\operatorname{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,\frac{1}{x}\right)+\frac{1}{2}\left(b^2\sin(a)\right)\\ &= \frac{1}{2}bx\cos\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\operatorname{Ci}\left(\frac{b}{x}\right)\sin(a)+\frac{1}{2}x^2\sin\left(a+\frac{b}{x}\right)+\frac{1}{2}b^2\cos(a)\operatorname{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0517267, size = 52, normalized size = 0.87

$$\frac{1}{2}\left(b^2\sin(a)\text{CosIntegral}\left(\frac{b}{x}\right)+b^2\cos(a)\text{Si}\left(\frac{b}{x}\right)+x\left(x\sin\left(a+\frac{b}{x}\right)+b\cos\left(a+\frac{b}{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b/x],x]
```

```
[Out] (b^2*CosIntegral[b/x]*Sin[a] + x*(b*Cos[a + b/x] + x*Sin[a + b/x]) + b^2*Co
s[a]*SinIntegral[b/x])/2
```

Maple [A] time = 0.01, size = 57, normalized size = 1.

$$-b^2\left(-\frac{x^2}{2\,b^2}\sin\left(a+\frac{b}{x}\right)-\frac{x}{2\,b}\cos\left(a+\frac{b}{x}\right)-\frac{\cos(a)}{2}\operatorname{Si}\left(\frac{b}{x}\right)-\frac{\sin(a)}{2}\operatorname{Ci}\left(\frac{b}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b/x),x)
```

```
[Out] -b^2*(-1/2*sin(a+b/x)*x^2/b^2-1/2*cos(a+b/x)*x/b-1/2*cos(a)*Si(b/x)-1/2*Ci(
b/x)*sin(a))
```

Maxima [C] time = 1.1412, size = 103, normalized size = 1.72

$$\frac{1}{4}\left(\left(-i\operatorname{Ei}\left(\frac{ib}{x}\right)+i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a)+\left(\operatorname{Ei}\left(\frac{ib}{x}\right)+\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)\right)b^2+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/4*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a)
)*b^2 + 1/2*b*x*cos((a*x + b)/x) + 1/2*x^2*sin((a*x + b)/x)
```

Fricas [A] time = 2.00243, size = 203, normalized size = 3.38

$$\frac{1}{2}\,b^2\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)+\frac{1}{2}\,bx\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\,x^2\sin\left(\frac{ax+b}{x}\right)+\frac{1}{4}\left(b^2\operatorname{Ci}\left(\frac{b}{x}\right)+b^2\operatorname{Ci}\left(-\frac{b}{x}\right)\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/2*b^2*cos(a)*sin_integral(b/x) + 1/2*b*x*cos((a*x + b)/x) + 1/2*x^2*sin((
a*x + b)/x) + 1/4*(b^2*cos_integral(b/x) + b^2*cos_integral(-b/x))*sin(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x)
```

```
[Out] Integral(x*sin(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(x*sin(a + b/x), x)
```

3.105 $\int \sin\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=32

$$-b\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)+b\sin(a)\text{Si}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)$$

```
[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Rubi [A] time = 0.0722976, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3361, 3297, 3303, 3299, 3302}

$$-b\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)+b\sin(a)\text{Si}\left(\frac{b}{x}\right)+x\sin\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x],x]
```

```
[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+\frac{b}{x}\right)dx &= -\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x^2}dx, x, \frac{1}{x}\right)\\ &= x\sin\left(a+\frac{b}{x}\right) - b\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{x}dx, x, \frac{1}{x}\right)\\ &= x\sin\left(a+\frac{b}{x}\right) - (b\cos(a))\operatorname{Subst}\left(\int \frac{\cos(bx)}{x}dx, x, \frac{1}{x}\right) + (b\sin(a))\operatorname{Subst}\left(\int \frac{\sin(bx)}{x}dx, x, \frac{1}{x}\right)\\ &= -b\cos(a)\operatorname{Ci}\left(\frac{b}{x}\right) + x\sin\left(a+\frac{b}{x}\right) + b\sin(a)\operatorname{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0229493, size = 32, normalized size = 1.

$$-b\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right) + b\sin(a)\text{Si}\left(\frac{b}{x}\right) + x\sin\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x],x]

[Out] -(b*Cos[a]*CosIntegral[b/x]) + x*Sin[a + b/x] + b*Sin[a]*SinIntegral[b/x]
```

Maple [A] time = 0.011, size = 38, normalized size = 1.2

$$-b\left(-\frac{x}{b}\sin\left(a+\frac{b}{x}\right) - \operatorname{Si}\left(\frac{b}{x}\right)\sin(a) + \operatorname{Ci}\left(\frac{b}{x}\right)\cos(a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x),x)

[Out] -b*(-sin(a+b/x)*x/b-Si(b/x)*sin(a)+Ci(b/x)*cos(a))
```

Maxima [C] time = 1.14259, size = 78, normalized size = 2.44

$$-\frac{1}{2}\left(\left(\operatorname{Ei}\left(\frac{ib}{x}\right) + \operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a) - \left(-i\operatorname{Ei}\left(\frac{ib}{x}\right) + i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)\right)b + x\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x, algorithm="maxima")

[Out] -1/2*((Ei(I*b/x) + Ei(-I*b/x))*cos(a) - (-I*Ei(I*b/x) + I*Ei(-I*b/x))*sin(a
))*b + x*sin((a*x + b)/x)
```

Fricas [A] time = 1.93744, size = 144, normalized size = 4.5

$$b\sin(a)\operatorname{Si}\left(\frac{b}{x}\right) - \frac{1}{2}\left(b\operatorname{Ci}\left(\frac{b}{x}\right) + b\operatorname{Ci}\left(-\frac{b}{x}\right)\right)\cos(a) + x\sin\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x, algorithm="fricas")
```

```
[Out] b*sin(a)*sin_integral(b/x) - 1/2*(b*cos_integral(b/x) + b*cos_integral(-b/x
))*cos(a) + x*sin((a*x + b)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x)
```

```
[Out] Integral(sin(a + b/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x), x)
```

3.106 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal. Leaf size=21

$$\sin(a)\left(-\text{CosIntegral}\left(\frac{b}{x}\right)\right)-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Rubi [A] time = 0.0282507, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3377, 3376, 3375}

$$\sin(a)\left(-\text{CosIntegral}\left(\frac{b}{x}\right)\right)-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x,x]
```

```
[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx &= \cos(a)\int \frac{\sin\left(\frac{b}{x}\right)}{x}\,dx+\sin(a)\int \frac{\cos\left(\frac{b}{x}\right)}{x}\,dx\\ &= -\text{Ci}\left(\frac{b}{x}\right)\sin(a)-\cos(a)\text{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0469815, size = 21, normalized size = 1.

$$\sin(a)\left(-\text{CosIntegral}\left(\frac{b}{x}\right)\right)-\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x,x]
```

```
[Out] -(CosIntegral[b/x]*Sin[a]) - Cos[a]*SinIntegral[b/x]
```

Maple [A] time = 0.009, size = 22, normalized size = 1.1

$$-\cos\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)-\mathrm{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x,x)
```

```
[Out] -cos(a)*Si(b/x)-Ci(b/x)*sin(a)
```

Maxima [C] time = 1.14411, size = 58, normalized size = 2.76

$$\frac{1}{2}\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos\left(a\right)-\frac{1}{2}\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*cos(a) - 1/2*(Ei(I*b/x) + Ei(-I*b/x))*sin(
a)
```

Fricas [A] time = 1.92223, size = 109, normalized size = 5.19

$$-\frac{1}{2}\left(\mathrm{Ci}\left(\frac{b}{x}\right)+\mathrm{Ci}\left(-\frac{b}{x}\right)\right)\sin\left(a\right)-\cos\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(cos_integral(b/x) + cos_integral(-b/x))*sin(a) - cos(a)*sin_integral(
b/x)
```

Sympy [A] time = 1.33868, size = 17, normalized size = 0.81

$$-\sin\left(a\right)\mathrm{Ci}\left(\frac{b}{x}\right)-\cos\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x)
```

```
[Out] -sin(a)*Ci(b/x) - cos(a)*Si(b/x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)/x, x)
```

3.107 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=12

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] Cos[a + b/x]/b
```

Rubi [A] time = 0.0138755, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3379, 2638}

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^2,x]
```

```
[Out] Cos[a + b/x]/b
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\operatorname{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Mathematica [A] time = 0.01326, size = 12, normalized size = 1.

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^2,x]
```

```
[Out] Cos[a + b/x]/b
```

Maple [A] time = 0.004, size = 13, normalized size = 1.1

$$\frac{1}{b}\cos\left(a+\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^2,x)
```

```
[Out] cos(a+b/x)/b
```

Maxima [A] time = 0.956441, size = 16, normalized size = 1.33

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] cos(a + b/x)/b
```

Fricas [A] time = 1.69086, size = 27, normalized size = 2.25

$$\frac{\cos\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] cos((a*x + b)/x)/b
```

Sympy [A] time = 1.2605, size = 14, normalized size = 1.17

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**2,x)
```

```
[Out] Piecewise((cos(a + b/x)/b, Ne(b, 0)), (-sin(a)/x, True))
```

Giac [A] time = 1.10533, size = 16, normalized size = 1.33

$$\frac{\cos\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^2,x, algorithm="giac")
```

```
[Out] cos(a + b/x)/b
```

3.108 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=29

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

```
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Rubi [A] time = 0.0246476, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 3296, 2637}

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^3,x]
```

```
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx &= -\text{Subst}\left(\int x\sin(a+bx)\,dx,x,\frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\text{Subst}\left(\int \cos(a+bx)\,dx,x,\frac{1}{x}\right)}{b}\\ &= \frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0040076, size = 29, normalized size = 1.

$$\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^3,x]
```

```
[Out] Cos[a + b/x]/(b*x) - Sin[a + b/x]/b^2
```

Maple [A] time = 0.007, size = 42, normalized size = 1.5

$$-\frac{1}{b^2}\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)+a\cos\left(a+\frac{b}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^3,x)
```

```
[Out] -1/b^2*(sin(a+b/x)-(a+b/x)*cos(a+b/x)+a*cos(a+b/x))
```

Maxima [C] time = 1.13393, size = 68, normalized size = 2.34

$$-\frac{\left(i\,\Gamma\left(2,\frac{ib}{x}\right)-i\,\Gamma\left(2,-\frac{ib}{x}\right)\right)\cos\left(a\right)+\left(\Gamma\left(2,\frac{ib}{x}\right)+\Gamma\left(2,-\frac{ib}{x}\right)\right)\sin\left(a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((I*gamma(2, I*b/x) - I*gamma(2, -I*b/x))*cos(a) + (gamma(2, I*b/x) +
gamma(2, -I*b/x))*sin(a))/b^2
```

Fricas [A] time = 1.56024, size = 69, normalized size = 2.38

$$\frac{b\cos\left(\frac{ax+b}{x}\right)-x\sin\left(\frac{ax+b}{x}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] (b*cos((a*x + b)/x) - x*sin((a*x + b)/x))/(b^2*x)
```

Sympy [A] time = 2.42722, size = 29, normalized size = 1.

$$\begin{cases}\frac{\cos\left(a+\frac{b}{x}\right)}{bx}-\frac{\sin\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0\\ -\frac{\sin\left(a\right)}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**3,x)
```

```
[Out] Piecewise((cos(a + b/x)/(b*x) - sin(a + b/x)/b**2, Ne(b, 0)), (-sin(a)/(2*x
**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)/x^3, x)
```

3.109 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[Out] (-2*Cos[a + b/x])/b^3 + Cos[a + b/x]/(b*x^2) - (2*Sin[a + b/x])/(b^2*x)
```

Rubi [A] time = 0.0462511, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 3296, 2638}

$$-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^4,x]
```

```
[Out] (-2*Cos[a + b/x])/b^3 + Cos[a + b/x]/(b*x^2) - (2*Sin[a + b/x])/(b^2*x)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\,\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x} + \frac{2\,\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}$$
$$= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} + \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2} - \frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}$$

Mathematica [A] time = 0.0522672, size = 38, normalized size = 0.84

$$\frac{\left(b^2-2x^2\right)\cos\left(a+\frac{b}{x}\right)-2bx\sin\left(a+\frac{b}{x}\right)}{b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^4,x]
```

```
[Out] ((b^2 - 2*x^2)*Cos[a + b/x] - 2*b*x*Sin[a + b/x])/(b^3*x^2)
```

Maple [B] time = 0.009, size = 95, normalized size = 2.1

$$-\frac{1}{b^3}\left(-\left(a+\frac{b}{x}\right)^2\cos\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)-2a\left(\sin\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)\right)-a^2\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^4,x)
```

```
[Out] -1/b^3*(-(a+b/x)^2*cos(a+b/x)+2*cos(a+b/x)+2*(a+b/x)*sin(a+b/x)-2*a*(sin(a+
b/x)-(a+b/x)*cos(a+b/x))-a^2*cos(a+b/x))
```

Maxima [C] time = 1.3807, size = 69, normalized size = 1.53

$$-\frac{\left(\Gamma\left(3,\frac{ib}{x}\right)+\Gamma\left(3,-\frac{ib}{x}\right)\right)\cos(a)-\left(i\Gamma\left(3,\frac{ib}{x}\right)-i\Gamma\left(3,-\frac{ib}{x}\right)\right)\sin(a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*((gamma(3, I*b/x) + gamma(3, -I*b/x))*cos(a) - (I*gamma(3, I*b/x) - I*
gamma(3, -I*b/x))*sin(a))/b^3
```

Fricas [A] time = 1.67475, size = 95, normalized size = 2.11

$$-\frac{2\,bx\sin\left(\frac{ax+b}{x}\right)-\left(b^2-2\,x^2\right)\cos\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] -(2*b*x*sin((a*x + b)/x) - (b^2 - 2*x^2)*cos((a*x + b)/x))/(b^3*x^2)
```

Sympy [A] time = 4.45554, size = 46, normalized size = 1.02

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\sin\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b\neq 0 \\ -\frac{\sin(a)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**4,x)
```

```
[Out] Piecewise((cos(a + b/x)/(b*x**2) - 2*sin(a + b/x)/(b**2*x) - 2*cos(a + b/x)
/b**3, Ne(b, 0)), (-sin(a)/(3*x**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)/x^4, x)
```

3.110 $\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5} dx$

Optimal. Leaf size=61

$$-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}$$

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Rubi [A] time = 0.0676861, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 3296, 2637}

$$-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{6\sin\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x}+\frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]/x^5,x]
```

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{x}\right)}{x^5}\,dx &= -\mathrm{Subst}\left(\int x^3\sin(a+bx)\,dx,x,\frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\,\mathrm{Subst}\left(\int x^2\cos(a+bx)\,dx,x,\frac{1}{x}\right)}{b}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\,\mathrm{Subst}\left(\int x\sin(a+bx)\,dx,x,\frac{1}{x}\right)}{b^2}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\,\mathrm{Subst}\left(\int \cos(a+bx)\,dx,x,\frac{1}{x}\right)}{b^3}\\
&= \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2}
\end{aligned}$$

Mathematica [A] time = 0.0045402, size = 61, normalized size = 1.

$$-\frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]/x^5,x]
```

```
[Out] Cos[a + b/x]/(b*x^3) - (6*Cos[a + b/x])/(b^3*x) + (6*Sin[a + b/x])/b^4 - (3
*Sin[a + b/x])/(b^2*x^2)
```

Maple [B] time = 0.008, size = 165, normalized size = 2.7

$$-\frac{1}{b^4}\left(-\left(a+\frac{b}{x}\right)^3\cos\left(a+\frac{b}{x}\right)+3\left(a+\frac{b}{x}\right)^2\sin\left(a+\frac{b}{x}\right)-6\sin\left(a+\frac{b}{x}\right)+6\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)-3\,a\left(-\left(a+\frac{b}{x}\right)^2\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)/x^5,x)
```

```
[Out] -1/b^4*(-(a+b/x)^3*cos(a+b/x)+3*(a+b/x)^2*sin(a+b/x)-6*sin(a+b/x)+6*(a+b/x)
*cos(a+b/x)-3*a*(-(a+b/x)^2*cos(a+b/x)+2*cos(a+b/x)+2*(a+b/x)*sin(a+b/x))+3
*a^2*(sin(a+b/x)-(a+b/x)*cos(a+b/x))+a^3*cos(a+b/x))
```

Maxima [C] time = 1.14778, size = 68, normalized size = 1.11

$$\frac{\left(i\,\Gamma\left(4,\frac{i\,b}{x}\right)-i\,\Gamma\left(4,-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(\Gamma\left(4,\frac{i\,b}{x}\right)+\Gamma\left(4,-\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((I*gamma(4, I*b/x) - I*gamma(4, -I*b/x))*cos(a) + (gamma(4, I*b/x) + g
amma(4, -I*b/x))*sin(a))/b^4
```

Fricas [A] time = 1.53806, size = 112, normalized size = 1.84

$$\frac{\left(b^3 - 6\,bx^2\right)\cos\left(\frac{ax+b}{x}\right) - 3\left(b^2x - 2\,x^3\right)\sin\left(\frac{ax+b}{x}\right)}{b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] ((b^3 - 6*b*x^2)*cos((a*x + b)/x) - 3*(b^2*x - 2*x^3)*sin((a*x + b)/x))/(b^
4*x^3)
```

Sympy [A] time = 7.66222, size = 61, normalized size = 1.

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x}\right)}{bx^3} - \frac{3\sin\left(a+\frac{b}{x}\right)}{b^2x^2} - \frac{6\cos\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sin\left(a+\frac{b}{x}\right)}{b^4} & \text{for}\ b \neq 0 \\ -\frac{\sin(a)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x**5,x)
```

```
[Out] Piecewise((cos(a + b/x)/(b*x**3) - 3*sin(a + b/x)/(b**2*x**2) - 6*cos(a + b
/x)/(b**3*x) + 6*sin(a + b/x)/b**4, Ne(b, 0)), (-sin(a)/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)/x^5, x)
```

3.111 $\int x^2 \sin^2\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=97

$$\frac{2}{3}b^3\sin(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{2}{3}b^3\cos(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a\right.\right.$$

```
[Out] x^3/6 + (b^2*x*Cos[2*(a + b/x)])/3 - (x^3*Cos[2*(a + b/x)])/6 + (2*b^3*CosI
ntegral[(2*b)/x]*Sin[2*a])/3 + (b*x^2*Sin[2*(a + b/x)])/6 + (2*b^3*Cos[2*a]
*SinIntegral[(2*b)/x])/3
```

Rubi [A] time = 0.169148, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3425, 3380, 3297, 3303, 3299, 3302}

$$\frac{2}{3}b^3\sin(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)+\frac{2}{3}b^3\cos(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b/x]^2,x]
```

```
[Out] x^3/6 + (b^2*x*Cos[2*(a + b/x)])/3 - (x^3*Cos[2*(a + b/x)])/6 + (2*b^3*CosI
ntegral[(2*b)/x]*Sin[2*a])/3 + (b*x^2*Sin[2*(a + b/x)])/6 + (2*b^3*Cos[2*a]
*SinIntegral[(2*b)/x])/3
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\sin^2\left(a+\frac{b}{x}\right)dx &= \int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+\frac{2b}{x}\right)\right)dx\\
&= \frac{x^3}{6}-\frac{1}{2}\int x^2\cos\left(2a+\frac{2b}{x}\right)dx\\
&= \frac{x^3}{6}+\frac{1}{2}\operatorname{Subst}\left(\int\frac{\cos(2a+2bx)}{x^4}\,dx,x,\frac{1}{x}\right)\\
&= \frac{x^3}{6}-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{3}b\operatorname{Subst}\left(\int\frac{\sin(2a+2bx)}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= \frac{x^3}{6}-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{3}b^2\operatorname{Subst}\left(\int\frac{\cos(2a+2bx)}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{3}\left(2b^3\right)\operatorname{Subst}\left(\int \ldots\right.\\
&= \frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)+\frac{1}{3}\left(2b^3\cos(2a)\right)\operatorname{Sub}\ldots\\
&= \frac{x^3}{6}+\frac{1}{3}b^2x\cos\left(2\left(a+\frac{b}{x}\right)\right)-\frac{1}{6}x^3\cos\left(2\left(a+\frac{b}{x}\right)\right)+\frac{2}{3}b^3\operatorname{Ci}\left(\frac{2b}{x}\right)\sin(2a)+\frac{1}{6}bx^2\sin\left(2\left(a+\frac{b}{x}\right)\right)\ldots
\end{aligned}$$

Mathematica [A] time = 0.169807, size = 86, normalized size = 0.89

$$\frac{1}{6}\left(4b^3\sin(2a)\operatorname{CosIntegral}\left(\frac{2b}{x}\right)+4b^3\cos(2a)\operatorname{Si}\left(\frac{2b}{x}\right)+x\left(2b^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-x^2\cos\left(2\left(a+\frac{b}{x}\right)\right)+bx\sin\left(2\left(a+\frac{b}{x}\right)\ldots\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b/x]^2,x]
```

```
[Out] (4*b^3*CosIntegral[(2*b)/x]*Sin[2*a] + x*(x^2 + 2*b^2*Cos[2*(a + b/x)] - x^
2*Cos[2*(a + b/x)] + b*x*Sin[2*(a + b/x)]) + 4*b^3*Cos[2*a]*SinIntegral[(2*
b)/x])/6
```

Maple [A] time = 0.014, size = 96, normalized size = 1.

$$-b^3\left(-\frac{x^3}{6\,b^3}+\frac{x^3}{6\,b^3}\cos\left(2\,a+2\,\frac{b}{x}\right)-\frac{x^2}{6\,b^2}\sin\left(2\,a+2\,\frac{b}{x}\right)-\frac{x}{3\,b}\cos\left(2\,a+2\,\frac{b}{x}\right)-\frac{2\,\cos(2\,a)}{3}\operatorname{Si}\left(2\,\frac{b}{x}\right)-\frac{2\,\sin(2\,a)}{3}\operatorname{Ci}\left(2\,\frac{b}{x}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b/x)^2,x)
```

```
[Out] -b^3*(-1/6*x^3/b^3+1/6*cos(2*a+2*b/x)*x^3/b^3-1/6*sin(2*a+2*b/x)*x^2/b^2-1/
3*cos(2*a+2*b/x)*x/b-2/3*Si(2*b/x)*cos(2*a)-2/3*Ci(2*b/x)*sin(2*a))
```

Maxima [C] time = 1.15476, size = 134, normalized size = 1.38

$$\frac{1}{6}\left(\left(-2i\,\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+2i\,\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos(2\,a)+2\left(\mathrm{Ei}\left(\frac{2i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin(2\,a)\right)b^3+\frac{1}{6}\,bx^2\sin\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((-2*I*Ei(2*I*b/x) + 2*I*Ei(-2*I*b/x))*cos(2*a) + 2*(Ei(2*I*b/x) + Ei(-
2*I*b/x))*sin(2*a))*b^3 + 1/6*b*x^2*sin(2*(a*x + b)/x) + 1/6*x^3 + 1/6*(2*b
^2*x - x^3)*cos(2*(a*x + b)/x)
```

Fricas [A] time = 1.44585, size = 290, normalized size = 2.99

$$\frac{1}{3}\,bx^2\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+\frac{2}{3}\,b^3\cos(2\,a)\,\mathrm{Si}\left(\frac{2\,b}{x}\right)-\frac{1}{3}\,b^2x+\frac{1}{3}\,x^3+\frac{1}{3}\left(2\,b^2x-x^3\right)\cos\left(\frac{ax+b}{x}\right)^2+\frac{1}{3}\left(b^3\,\mathrm{Ci}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^2*cos((a*x + b)/x)*sin((a*x + b)/x) + 2/3*b^3*cos(2*a)*sin_integral
(2*b/x) - 1/3*b^2*x + 1/3*x^3 + 1/3*(2*b^2*x - x^3)*cos((a*x + b)/x)^2 + 1/
3*(b^3*cos_integral(2*b/x) + b^3*cos_integral(-2*b/x))*sin(2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin^2\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b/x)**2,x)
```

```
[Out] Integral(x**2*sin(a + b/x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left(a+\frac{b}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(a + b/x)^2, x)
```

3.112 $\int x \sin^2\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=65

$$b^2(-\cos(2a))\text{CosIntegral}\left(\frac{2b}{x}\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)$$

```
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x^2*Sin[a + b/x]^2)/2 + (b*x*Sin[2*
(a + b/x)])/2 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Rubi [A] time = 0.104219, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3393, 4573, 3373, 3361, 3297, 3303, 3299, 3302}

$$b^2(-\cos(2a))\text{CosIntegral}\left(\frac{2b}{x}\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b/x]^2,x]
```

```
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x^2*Sin[a + b/x]^2)/2 + (b*x*Sin[2*
(a + b/x)])/2 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Rule 3393

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[(x^(m +
 1)*Sin[a + b*x^n]^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4573

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 3373

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\sin^2\left(a+\frac{b}{x}\right)dx &= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+b\int\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}b\int\sin\left(2\left(a+\frac{b}{x}\right)\right)dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}b\int\sin\left(2a+\frac{2b}{x}\right)dx\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)-\frac{1}{2}b\,\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x^2}dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)-b^2\,\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x}dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)-\left(b^2\cos(2a)\right)\text{Subst}\left(\int\frac{\cos(2bx)}{x}dx,x,\frac{1}{x}\right)+\left(b^2\sin\right.\\
&= -b^2\cos(2a)\text{Ci}\left(\frac{2b}{x}\right)+\frac{1}{2}x^2\sin^2\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\sin\left(2\left(a+\frac{b}{x}\right)\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.161048, size = 65, normalized size = 1.

$$b^2(-\cos(2a))\text{CosIntegral}\left(\frac{2b}{x}\right)+b^2\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{1}{4}x\left(2b\sin\left(2\left(a+\frac{b}{x}\right)\right)+x\left(-\cos\left(2\left(a+\frac{b}{x}\right)\right)\right)+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b/x]^2,x]
```

```
[Out] -(b^2*Cos[2*a]*CosIntegral[(2*b)/x]) + (x*(x - x*Cos[2*(a + b/x)] + 2*b*Sin
[2*(a + b/x)]))/4 + b^2*Sin[2*a]*SinIntegral[(2*b)/x]
```

Maple [A] time = 0.015, size = 76, normalized size = 1.2

$$-b^2\left(-\frac{x^2}{4\,b^2}+\frac{x^2}{4\,b^2}\cos\left(2\,a+2\,\frac{b}{x}\right)-\frac{x}{2\,b}\sin\left(2\,a+2\,\frac{b}{x}\right)-\text{Si}\left(2\,\frac{b}{x}\right)\sin(2\,a)+\text{Ci}\left(2\,\frac{b}{x}\right)\cos(2\,a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b/x)^2,x)
```

```
[Out] -b^2*(-1/4*x^2/b^2+1/4*cos(2*a+2*b/x)*x^2/b^2-1/2*sin(2*a+2*b/x)*x/b-Si(2*b
/x)*sin(2*a)+Ci(2*b/x)*cos(2*a))
```

Maxima [C] time = 1.14962, size = 120, normalized size = 1.85

$$-\frac{1}{4}\left(2\left(\text{Ei}\left(\frac{2i\,b}{x}\right)+\text{Ei}\left(-\frac{2i\,b}{x}\right)\right)\cos\left(2\,a\right)-\left(-2i\,\text{Ei}\left(\frac{2i\,b}{x}\right)+2i\,\text{Ei}\left(-\frac{2i\,b}{x}\right)\right)\sin\left(2\,a\right)\right)b^2-\frac{1}{4}\,x^2\cos\left(\frac{2\,(ax+b)}{x}\right)+\frac{1}{2}\,bx\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) - (-2*I*Ei(2*I*b/x) + 2*I*Ei(
-2*I*b/x))*sin(2*a))*b^2 - 1/4*x^2*cos(2*(a*x + b)/x) + 1/2*b*x*sin(2*(a*x
+ b)/x) + 1/4*x^2
```

Fricas [A] time = 1.54, size = 246, normalized size = 3.78

$$-\frac{1}{2}\,x^2\cos\left(\frac{ax+b}{x}\right)^2+bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)+b^2\sin\left(2\,a\right)\text{Si}\left(\frac{2\,b}{x}\right)+\frac{1}{2}\,x^2-\frac{1}{2}\left(b^2\,\text{Ci}\left(\frac{2\,b}{x}\right)+b^2\,\text{Ci}\left(-\frac{2\,b}{x}\right)\right)\cos\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos((a*x + b)/x)^2 + b*x*cos((a*x + b)/x)*sin((a*x + b)/x) + b^2*s
in(2*a)*sin_integral(2*b/x) + 1/2*x^2 - 1/2*(b^2*cos_integral(2*b/x) + b^2*
cos_integral(-2*b/x))*cos(2*a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin^2\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)**2,x)
```

```
[Out] Integral(x*sin(a + b/x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(a+\frac{b}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(a + b/x)^2, x)
```

3.113 $\int \sin^2\left(a+\frac{b}{x}\right)dx$

Optimal. Leaf size=41

$$-b\sin(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)-b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)+x\sin^2\left(a+\frac{b}{x}\right)$$

```
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Rubi [A] time = 0.0905833, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3361, 3313, 12, 3303, 3299, 3302}

$$-b\sin(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)-b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)+x\sin^2\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2,x]
```

```
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin^2\left(a+\frac{b}{x}\right)dx &= -\text{Subst}\left(\int \frac{\sin^2(a+bx)}{x^2}dx, x, \frac{1}{x}\right)\\
&= x\sin^2\left(a+\frac{b}{x}\right) - (2b)\,\text{Subst}\left(\int \frac{\sin(2a+2bx)}{2x}dx, x, \frac{1}{x}\right)\\
&= x\sin^2\left(a+\frac{b}{x}\right) - b\,\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}dx, x, \frac{1}{x}\right)\\
&= x\sin^2\left(a+\frac{b}{x}\right) - (b\cos(2a))\,\text{Subst}\left(\int \frac{\sin(2bx)}{x}dx, x, \frac{1}{x}\right) - (b\sin(2a))\,\text{Subst}\left(\int \frac{\cos(2bx)}{x}dx, x\right.\\
&= -b\text{Ci}\left(\frac{2b}{x}\right)\sin(2a) + x\sin^2\left(a+\frac{b}{x}\right) - b\cos(2a)\text{Si}\left(\frac{2b}{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0913828, size = 41, normalized size = 1.

$$
-b\sin(2a)\text{CosIntegral}\left(\frac{2b}{x}\right) - b\cos(2a)\text{Si}\left(\frac{2b}{x}\right) + x\sin^2\left(a+\frac{b}{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2,x]
```

```
[Out] -(b*CosIntegral[(2*b)/x]*Sin[2*a]) + x*Sin[a + b/x]^2 - b*Cos[2*a]*SinInteg
ral[(2*b)/x]
```

Maple [A] time = 0.014, size = 52, normalized size = 1.3

$$
-b\left(-\frac{x}{2b} + \frac{x}{2b}\cos\left(2a+2\frac{b}{x}\right) + \text{Si}\left(2\frac{b}{x}\right)\cos(2a) + \text{Ci}\left(2\frac{b}{x}\right)\sin(2a)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2,x)
```

```
[Out] -b*(-1/2*x/b+1/2*cos(2*a+2*b/x)*x/b+Si(2*b/x)*cos(2*a)+Ci(2*b/x)*sin(2*a))
```

Maxima [C] time = 1.13638, size = 89, normalized size = 2.17

$$
-\frac{1}{2}\left(\left(-i\,\text{Ei}\left(\frac{2ib}{x}\right) + i\,\text{Ei}\left(-\frac{2ib}{x}\right)\right)\cos(2a) + \left(\text{Ei}\left(\frac{2ib}{x}\right) + \text{Ei}\left(-\frac{2ib}{x}\right)\right)\sin(2a)\right)b - \frac{1}{2}x\cos\left(\frac{2(ax+b)}{x}\right) + \frac{1}{2}x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((-I*Ei(2*I*b/x) + I*Ei(-2*I*b/x))*cos(2*a) + (Ei(2*I*b/x) + Ei(-2*I*b
/x))*sin(2*a))*b - 1/2*x*cos(2*(a*x + b)/x) + 1/2*x
```

Fricas [A] time = 1.45208, size = 167, normalized size = 4.07

$$-x\cos\left(\frac{ax+b}{x}\right)^2 - b\cos\left(2\,a\right)\operatorname{Si}\left(\frac{2\,b}{x}\right) - \frac{1}{2}\left(b\operatorname{Ci}\left(\frac{2\,b}{x}\right) + b\operatorname{Ci}\left(-\frac{2\,b}{x}\right)\right)\sin\left(2\,a\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] -x*cos((a*x + b)/x)^2 - b*cos(2*a)*sin_integral(2*b/x) - 1/2*(b*cos_integra
l(2*b/x) + b*cos_integral(-2*b/x))*sin(2*a) + x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^2\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2,x)
```

```
[Out] Integral(sin(a + b/x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)^2, x)
```

3.114 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x} \, dx$

Optimal. Leaf size=37

$$\frac{1}{2}\cos(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}$$

```
[Out] (Cos[2*a]*CosIntegral[(2*b)/x])/2 + Log[x]/2 - (Sin[2*a]*SinIntegral[(2*b)/
x])/2
```

Rubi [A] time = 0.049541, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3425, 3378, 3376, 3375}

$$\frac{1}{2}\cos(2a)\text{CosIntegral}\left(\frac{2b}{x}\right)-\frac{1}{2}\sin(2a)\text{Si}\left(\frac{2b}{x}\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[(2*b)/x])/2 + Log[x]/2 - (Sin[2*a]*SinIntegral[(2*b)/
x])/2
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x}\,dx = \int \left(\frac{1}{2x} - \frac{\cos\left(2a+\frac{2b}{x}\right)}{2x}\right)dx$$
$$= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos\left(2a+\frac{2b}{x}\right)}{x}\,dx$$
$$= \frac{\log(x)}{2} - \frac{1}{2}\cos(2a)\int \frac{\cos\left(\frac{2b}{x}\right)}{x}\,dx + \frac{1}{2}\sin(2a)\int \frac{\sin\left(\frac{2b}{x}\right)}{x}\,dx$$
$$= \frac{1}{2}\cos(2a)\mathrm{Ci}\left(\frac{2b}{x}\right) + \frac{\log(x)}{2} - \frac{1}{2}\sin(2a)\mathrm{Si}\left(\frac{2b}{x}\right)$$

Mathematica [A] time = 0.0586643, size = 32, normalized size = 0.86

$$\frac{1}{2}\left(\cos(2a)\mathrm{CosIntegral}\left(\frac{2b}{x}\right) - \sin(2a)\mathrm{Si}\left(\frac{2b}{x}\right) + \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[(2*b)/x] + Log[x] - Sin[2*a]*SinIntegral[(2*b)/x])/2
```

Maple [A] time = 0.011, size = 36, normalized size = 1.

$$-\frac{1}{2}\ln\left(\frac{b}{x}\right) - \frac{\sin(2a)}{2}\mathrm{Si}\left(2\frac{b}{x}\right) + \frac{\cos(2a)}{2}\mathrm{Ci}\left(2\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x,x)
```

```
[Out] -1/2*ln(b/x)-1/2*Si(2*b/x)*sin(2*a)+1/2*Ci(2*b/x)*cos(2*a)
```

Maxima [C] time = 1.12719, size = 69, normalized size = 1.86

$$\frac{1}{4}\left(\mathrm{Ei}\left(\frac{2ib}{x}\right) + \mathrm{Ei}\left(-\frac{2ib}{x}\right)\right)\cos(2a) + \frac{1}{4}\left(i\,\mathrm{Ei}\left(\frac{2ib}{x}\right) - i\,\mathrm{Ei}\left(-\frac{2ib}{x}\right)\right)\sin(2a) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*(Ei(2*I*b/x) + Ei(-2*I*b/x))*cos(2*a) + 1/4*(I*Ei(2*I*b/x) - I*Ei(-2*I*
b/x))*sin(2*a) + 1/2*log(x)
```

Fricas [A] time = 1.43462, size = 144, normalized size = 3.89

$$\frac{1}{4}\left(\mathrm{Ci}\left(\frac{2b}{x}\right) + \mathrm{Ci}\left(-\frac{2b}{x}\right)\right)\cos(2a) - \frac{1}{2}\sin(2a)\mathrm{Si}\left(\frac{2b}{x}\right) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*(cos_integral(2*b/x) + cos_integral(-2*b/x))*cos(2*a) - 1/2*sin(2*a)*si
n_integral(2*b/x) + 1/2*log(x)
```

Sympy [A] time = 4.6788, size = 31, normalized size = 0.84

$$\frac{\log(x)}{2} - \frac{\sin(2a)\,\mathrm{Si}\left(\frac{2b}{x}\right)}{2} + \frac{\cos(2a)\,\mathrm{Ci}\left(\frac{2b}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x,x)
```

```
[Out] log(x)/2 - sin(2*a)*Si(2*b/x)/2 + cos(2*a)*Ci(2*b/x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)^2/x, x)
```

3.115 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} dx$

Optimal. Leaf size=31

$$\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}-\frac{1}{2x}$$

```
[Out] -1/(2*x) + (Cos[a + b/x]*Sin[a + b/x])/(2*b)
```

Rubi [A] time = 0.0268488, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3379, 2635, 8}

$$\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2b}-\frac{1}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^2,x]
```

```
[Out] -1/(2*x) + (Cos[a + b/x]*Sin[a + b/x])/(2*b)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}-\frac{1}{2}\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2x}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0555031, size = 32, normalized size = 1.03

$$\frac{\sin\left(2\left(a+\frac{b}{x}\right)\right)}{4b}-\frac{a+\frac{b}{x}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^2,x]
```

```
[Out] -(a + b/x)/(2*b) + Sin[2*(a + b/x)]/(4*b)
```

Maple [A] time = 0.007, size = 34, normalized size = 1.1

$$-\frac{1}{b}\left(-\frac{1}{2}\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)+\frac{a}{2}+\frac{b}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^2,x)
```

```
[Out] -1/b*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)
```

Maxima [A] time = 0.962725, size = 34, normalized size = 1.1

$$\frac{x\sin\left(\frac{2(ax+b)}{x}\right)-2b}{4bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x*sin(2*(a*x + b)/x) - 2*b)/(b*x)
```

Fricas [A] time = 1.21303, size = 72, normalized size = 2.32

$$\frac{x\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-b}{2bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos((a*x + b)/x)*sin((a*x + b)/x) - b)/(b*x)
```

Sympy [A] time = 4.39074, size = 262, normalized size = 8.45

$$\begin{cases} -\frac{b\tan^{4}\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^{4}\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx}-\frac{2b\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)}{2bx\tan^{4}\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx}-\frac{b}{2bx\tan^{4}\left(\frac{a}{2}+\frac{b}{2x}\right)+4bx\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)+2bx}-\frac{2x \cdots}{2bx\tan^{4}\left(\frac{a}{2}+\cdots\right.} \\ -\frac{\sin^{2}(a)}{x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**2,x)
```

```
[Out] Piecewise((-b*tan(a/2 + b/(2*x))**4/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*ta
n(a/2 + b/(2*x))**2 + 2*b*x) - 2*b*tan(a/2 + b/(2*x))**2/(2*b*x*tan(a/2 + b
/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - b/(2*b*x*tan(a/2 + b/(2
*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) - 2*x*tan(a/2 + b/(2*x))**3/
(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**2 + 2*b*x) + 2*x*t
an(a/2 + b/(2*x))/(2*b*x*tan(a/2 + b/(2*x))**4 + 4*b*x*tan(a/2 + b/(2*x))**
2 + 2*b*x), Ne(b, 0)), (-sin(a)**2/x, True))
```

Giac [A] time = 1.09892, size = 30, normalized size = 0.97

$$\frac{\sin\left(2\,a+\frac{2\,b}{x}\right)}{4\,b}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/4*sin(2*a + 2*b/x)/b - 1/2/x
```

3.116 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3} dx$

Optimal. Leaf size=51

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}-\frac{1}{4x^2}$$

```
[Out] -1/(4*x^2) + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x) - Sin[a + b/x]^2/(4*b^2)
```

Rubi [A] time = 0.039533, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3379, 3310, 30}

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx}-\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^3,x]

[Out] -1/(4*x^2) + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x) - Sin[a + b/x]^2/(4*b^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^3} dx &= -\text{Subst}\left(\int x\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}-\frac{1}{2}\text{Subst}\left(\int x\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{4x^2}+\frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx}-\frac{\sin^2\left(a+\frac{b}{x}\right)}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.0793681, size = 43, normalized size = 0.84

$$\frac{x^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-2b\left(b-x\sin\left(2\left(a+\frac{b}{x}\right)\right)\right)}{8b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^3,x]
```

```
[Out] (x^2*Cos[2*(a + b/x)] - 2*b*(b - x*Sin[2*(a + b/x)]))/(8*b^2*x^2)
```

Maple [B] time = 0.011, size = 97, normalized size = 1.9

$$-\frac{1}{b^2}\left(\left(a+\frac{b}{x}\right)\left(-\frac{1}{2}\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)+\frac{a}{2}+\frac{b}{2x}\right)-\frac{1}{4}\left(a+\frac{b}{x}\right)^2+\frac{1}{4}\left(\sin\left(a+\frac{b}{x}\right)\right)^2-a\left(-\frac{1}{2}\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^3,x)
```

```
[Out] -1/b^2*((a+b/x)*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/4*(a+b/x)^2+1/
4*sin(a+b/x)^2-a*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x))
```

Maxima [C] time = 1.12844, size = 92, normalized size = 1.8

$$\frac{\left(\left(\Gamma\left(2,\frac{2ib}{x}\right)+\Gamma\left(2,-\frac{2ib}{x}\right)\right)\cos\left(2a\right)-\left(i\Gamma\left(2,\frac{2ib}{x}\right)-i\Gamma\left(2,-\frac{2ib}{x}\right)\right)\sin\left(2a\right)\right)x^2-4b^2}{16b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(((gamma(2, 2*I*b/x) + gamma(2, -2*I*b/x))*cos(2*a) - (I*gamma(2, 2*I*
b/x) - I*gamma(2, -2*I*b/x))*sin(2*a))*x^2 - 4*b^2)/(b^2*x^2)
```

Fricas [A] time = 1.32934, size = 132, normalized size = 2.59

$$\frac{2x^2\cos\left(\frac{ax+b}{x}\right)^2+4bx\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)-2b^2-x^2}{8b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^2*cos((a*x + b)/x)^2 + 4*b*x*cos((a*x + b)/x)*sin((a*x + b)/x) - 2
*b^2 - x^2)/(b^2*x^2)
```

Sympy [A] time = 6.38814, size = 445, normalized size = 8.73

$$\begin{cases} -\frac{b^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2} - \frac{2b^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2} - \frac{b^2}{4b^2x^2\tan^4\left(\frac{a}{2}+\frac{b}{2x}\right)+8b^2x^2\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)+4b^2x^2} \\ -\frac{\sin^2(a)}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**3,x)
```

```
[Out] Piecewise((-b**2*tan(a/2 + b/(2*x))**4/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 +
 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 2*b**2*tan(a/2 + b/(2*x
))**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**
2 + 4*b**2*x**2) - b**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*ta
n(a/2 + b/(2*x))**2 + 4*b**2*x**2) - 4*b*x*tan(a/2 + b/(2*x))**3/(4*b**2*x*
*2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2)
 + 4*b*x*tan(a/2 + b/(2*x))/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**
2*tan(a/2 + b/(2*x))**2 + 4*b**2*x**2) + 2*x**2*tan(a/2 + b/(2*x))**4/(4*b*
*2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b/(2*x))**2 + 4*b**2*
x**2) + 2*x**2/(4*b**2*x**2*tan(a/2 + b/(2*x))**4 + 8*b**2*x**2*tan(a/2 + b
/(2*x))**2 + 4*b**2*x**2), Ne(b, 0)), (-sin(a)**2/(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)^2/x^3, x)
```

3.117 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=87

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}+\frac{1}{4b^2x}-\frac{1}{6x^3}$$

```
[Out] -1/(6*x^3) + 1/(4*b^2*x) - (Cos[a + b/x]*Sin[a + b/x])/(4*b^3) + (Cos[a + b
/x]*Sin[a + b/x])/(2*b*x^2) - Sin[a + b/x]^2/(2*b^2*x)
```

Rubi [A] time = 0.0653922, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3379, 3311, 30, 2635, 8}

$$-\frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}-\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^2}+\frac{1}{4b^2x}-\frac{1}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^4,x]
```

```
[Out] -1/(6*x^3) + 1/(4*b^2*x) - (Cos[a + b/x]*Sin[a + b/x])/(4*b^3) + (Cos[a + b
/x]*Sin[a + b/x])/(2*b*x^2) - Sin[a + b/x]^2/(2*b^2*x)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\text{Subst}\left(\int x^2\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x} - \frac{1}{2}\text{Subst}\left(\int x^2\,dx, x, \frac{1}{x}\right) + \frac{\text{Subst}\left(\int \sin^2(a+bx)\,dx, x\right.}{2b^2}$$
$$= -\frac{1}{6x^3} - \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x} + \frac{\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)}{4b^2}$$
$$= -\frac{1}{6x^3} + \frac{1}{4b^2x} - \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^2} - \frac{\sin^2\left(a+\frac{b}{x}\right)}{2b^2x}$$

Mathematica [A] time = 0.126866, size = 54, normalized size = 0.62

$$\frac{-3\left(x^3-2b^2x\right)\sin\left(2\left(a+\frac{b}{x}\right)\right)+6bx^2\cos\left(2\left(a+\frac{b}{x}\right)\right)-4b^3}{24b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^4,x]
```

```
[Out] (-4*b^3 + 6*b*x^2*Cos[2*(a + b/x)] - 3*(-2*b^2*x + x^3)*Sin[2*(a + b/x)])/(
24*b^3*x^3)
```

Maple [B] time = 0.013, size = 197, normalized size = 2.3

$$-\frac{1}{b^3}\left(\left(a+\frac{b}{x}\right)^2\left(-\frac{1}{2}\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)+\frac{a}{2}+\frac{b}{2x}\right)-\frac{1}{2}\left(a+\frac{b}{x}\right)\left(\cos\left(a+\frac{b}{x}\right)\right)^2+\frac{1}{4}\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)+\frac{b}{4.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^4,x)
```

```
[Out] -1/b^3*((a+b/x)^2*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/2*(a+b/x)*co
s(a+b/x)^2+1/4*cos(a+b/x)*sin(a+b/x)+1/4*b/x+1/4*a-1/3*(a+b/x)^3-2*a*((a+b/
x)*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-1/4*(a+b/x)^2+1/4*sin(a+b/x)^
2)+a^2*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x))
```

Maxima [C] time = 1.13232, size = 92, normalized size = 1.06

$$-\frac{\left(\left(3i\,\Gamma\left(3,\frac{2ib}{x}\right)-3i\,\Gamma\left(3,-\frac{2ib}{x}\right)\right)\cos\left(2a\right)+3\left(\Gamma\left(3,\frac{2ib}{x}\right)+\Gamma\left(3,-\frac{2ib}{x}\right)\right)\sin\left(2a\right)\right)x^3+16\,b^3}{96\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(((3*I*gamma(3, 2*I*b/x) - 3*I*gamma(3, -2*I*b/x))*cos(2*a) + 3*(gamm
a(3, 2*I*b/x) + gamma(3, -2*I*b/x))*sin(2*a))*x^3 + 16*b^3)/(b^3*x^3)
```

Fricas [A] time = 1.28512, size = 158, normalized size = 1.82

$$\frac{6\,bx^2\cos\left(\frac{ax+b}{x}\right)^2 - 2\,b^3 - 3\,bx^2 + 3\left(2\,b^2x - x^3\right)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{12\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*x^2*cos((a*x + b)/x)^2 - 2*b^3 - 3*b*x^2 + 3*(2*b^2*x - x^3)*cos(
(a*x + b)/x)*sin((a*x + b)/x))/(b^3*x^3)
```

Sympy [A] time = 9.59835, size = 654, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**4,x)
```

```
[Out] Piecewise((-2*b**3*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**
4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 4*b**3*tan(a/2 + b
/(2*x))**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(
2*x))**2 + 12*b**3*x**3) - 2*b**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 12*b**2*x*tan(a/2 + b/(2*
x))**3/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x)
)**2 + 12*b**3*x**3) + 12*b**2*x*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 +
 b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 3*b*x**
2*tan(a/2 + b/(2*x))**4/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*
tan(a/2 + b/(2*x))**2 + 12*b**3*x**3) - 18*b*x**2*tan(a/2 + b/(2*x))**2/(12
*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*
b**3*x**3) + 3*b*x**2/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*ta
n(a/2 + b/(2*x))**2 + 12*b**3*x**3) + 6*x**3*tan(a/2 + b/(2*x))**3/(12*b**3
*x**3*tan(a/2 + b/(2*x))**4 + 24*b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*
x**3) - 6*x**3*tan(a/2 + b/(2*x))/(12*b**3*x**3*tan(a/2 + b/(2*x))**4 + 24*
b**3*x**3*tan(a/2 + b/(2*x))**2 + 12*b**3*x**3), Ne(b, 0)), (-sin(a)**2/(3*
x**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{x}\right)^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)^2/x^4, x)
```

3.118 $\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5} dx$

Optimal. Leaf size=107

$$-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4}-\frac{3\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}+\frac{3}{8b^2x^2}-\frac{1}{8x^4}$$

```
[Out] -1/(8*x^4) + 3/(8*b^2*x^2) + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x^3) - (3*Cos
[a + b/x]*Sin[a + b/x])/(4*b^3*x) + (3*Sin[a + b/x]^2)/(8*b^4) - (3*Sin[a +
 b/x]^2)/(4*b^2*x^2)
```

Rubi [A] time = 0.0807238, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3379, 3311, 30, 3310}

$$-\frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2}+\frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4}-\frac{3\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{4b^3x}+\frac{\sin\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{2bx^3}+\frac{3}{8b^2x^2}-\frac{1}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x]^2/x^5,x]
```

```
[Out] -1/(8*x^4) + 3/(8*b^2*x^2) + (Cos[a + b/x]*Sin[a + b/x])/(2*b*x^3) - (3*Cos
[a + b/x]*Sin[a + b/x])/(4*b^3*x) + (3*Sin[a + b/x]^2)/(8*b^4) - (3*Sin[a +
 b/x]^2)/(4*b^2*x^2)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\int \frac{\sin^2\left(a+\frac{b}{x}\right)}{x^5}\,dx = -\text{Subst}\left(\int x^3\sin^2(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2} - \frac{1}{2}\text{Subst}\left(\int x^3\,dx, x, \frac{1}{x}\right) + \frac{3\,\text{Subst}\left(\int x\sin^2(a+bx)\,dx\right.}{2b^2}$$
$$= -\frac{1}{8x^4} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x} + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin^2\left(a+\frac{b}{x}\right)}{4b^2x^2} + \cdots$$
$$= -\frac{1}{8x^4} + \frac{3}{8b^2x^2} + \frac{\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{2bx^3} - \frac{3\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)}{4b^3x} + \frac{3\sin^2\left(a+\frac{b}{x}\right)}{8b^4} - \frac{3\sin^2\left(a+\right.}{4b^2x^2}$$

Mathematica [A] time = 0.179913, size = 65, normalized size = 0.61

$$-\frac{2b\left(\left(3x^3-2b^2x\right)\sin\left(2\left(a+\frac{b}{x}\right)\right)+b^3\right)+3\left(x^4-2b^2x^2\right)\cos\left(2\left(a+\frac{b}{x}\right)\right)}{16b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x]^2/x^5,x]
```

```
[Out] -(3*(-2*b^2*x^2 + x^4)*Cos[2*(a + b/x)] + 2*b*(b^3 + (-2*b^2*x + 3*x^3)*Sin
[2*(a + b/x)]))/(16*b^4*x^4)
```

Maple [B] time = 0.012, size = 334, normalized size = 3.1

$$-\frac{1}{b^4}\left(\left(a+\frac{b}{x}\right)^3\left(-\frac{1}{2}\cos\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)+\frac{a}{2}+\frac{b}{2x}\right)-\frac{3}{4}\left(a+\frac{b}{x}\right)^2\left(\cos\left(a+\frac{b}{x}\right)\right)^2+\frac{3}{2}\left(a+\frac{b}{x}\right)\left(\frac{1}{2}\cos\left(a+\frac{b}{x}\right)\sin\left(a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x)^2/x^5,x)
```

```
[Out] -1/b^4*((a+b/x)^3*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1/2*b/x)-3/4*(a+b/x)^2*
cos(a+b/x)^2+3/2*(a+b/x)*(1/2*cos(a+b/x)*sin(a+b/x)+1/2*b/x+1/2*a)-3/8*(a+b
/x)^2-3/8*sin(a+b/x)^2-3/8*(a+b/x)^4-3*a*((a+b/x)^2*(-1/2*cos(a+b/x)*sin(a+
b/x)+1/2*a+1/2*b/x)-1/2*(a+b/x)*cos(a+b/x)^2+1/4*cos(a+b/x)*sin(a+b/x)+1/4*
b/x+1/4*a-1/3*(a+b/x)^3)+3*a^2*((a+b/x)*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2*a+1
/2*b/x)-1/4*(a+b/x)^2+1/4*sin(a+b/x)^2)-a^3*(-1/2*cos(a+b/x)*sin(a+b/x)+1/2
*a+1/2*b/x))
```

Maxima [C] time = 1.13318, size = 92, normalized size = 0.86

$$-\frac{\left(\left(\Gamma\left(4,\frac{2ib}{x}\right)+\Gamma\left(4,-\frac{2ib}{x}\right)\right)\cos(2a)-\left(i\Gamma\left(4,\frac{2ib}{x}\right)-i\Gamma\left(4,-\frac{2ib}{x}\right)\right)\sin(2a)\right)x^4+8b^4}{64b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/64*(((gamma(4, 2*I*b/x) + gamma(4, -2*I*b/x))*cos(2*a) - (I*gamma(4, 2*I
*b/x) - I*gamma(4, -2*I*b/x))*sin(2*a))*x^4 + 8*b^4)/(b^4*x^4)
```

Fricas [A] time = 1.56288, size = 194, normalized size = 1.81

$$-\frac{2\,b^4+6\,b^2x^2-3\,x^4-6\left(2\,b^2x^2-x^4\right)\cos\left(\frac{ax+b}{x}\right)^2-4\left(2\,b^3x-3\,bx^3\right)\cos\left(\frac{ax+b}{x}\right)\sin\left(\frac{ax+b}{x}\right)}{16\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4 + 6*b^2*x^2 - 3*x^4 - 6*(2*b^2*x^2 - x^4)*cos((a*x + b)/x)^2 -
 4*(2*b^3*x - 3*b*x^3)*cos((a*x + b)/x)*sin((a*x + b)/x))/(b^4*x^4)
```

Sympy [A] time = 16.8054, size = 726, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)**2/x**5,x)
```

```
[Out] Piecewise((-b**4*tan(a/2 + b/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 +
 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 2*b**4*tan(a/2 + b/(2*
x))**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))
**2 + 8*b**4*x**4) - b**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4
*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 8*b**3*x*tan(a/2 + b/(2*x))**3/(8*b
**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**
4*x**4) + 8*b**3*x*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 +
16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3*b**2*x**2*tan(a/2 + b
/(2*x))**4/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4) - 18*b**2*x**2*tan(a/2 + b/(2*x))**2/(8*b**4*x**4*ta
n(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 3
*b**2*x**2/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2
*x))**2 + 8*b**4*x**4) + 12*b*x**3*tan(a/2 + b/(2*x))**3/(8*b**4*x**4*tan(a
/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) - 12*b
*x**3*tan(a/2 + b/(2*x))/(8*b**4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*
tan(a/2 + b/(2*x))**2 + 8*b**4*x**4) + 12*x**4*tan(a/2 + b/(2*x))**2/(8*b**
4*x**4*tan(a/2 + b/(2*x))**4 + 16*b**4*x**4*tan(a/2 + b/(2*x))**2 + 8*b**4*
x**4), Ne(b, 0)), (-sin(a)**2/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{x}\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x)^2/x^5, x)
```

3.119 $\int \sin\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=80

$$\sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)+\sqrt{2\pi}\sqrt{b}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+x\sin\left(a+\frac{b}{x^2}\right)$$

```
[Out] -(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]) + Sqrt[b]*Sqr
t[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a] + x*Sin[a + b/x^2]
```

Rubi [A] time = 0.0577005, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3359, 3387, 3354, 3352, 3351}

$$\sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)+\sqrt{2\pi}\sqrt{b}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+x\sin\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2],x]
```

```
[Out] -(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]) + Sqrt[b]*Sqr
t[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a] + x*Sin[a + b/x^2]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{x^2}\right)\,dx = -\text{Subst}\left(\int \frac{\sin\left(a+bx^2\right)}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$= x\sin\left(a+\frac{b}{x^2}\right) - (2b)\,\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)$$
$$= x\sin\left(a+\frac{b}{x^2}\right) - (2b\cos(a))\,\text{Subst}\left(\int \cos\left(bx^2\right)\,dx,x,\frac{1}{x}\right) + (2b\sin(a))\,\text{Subst}\left(\int \sin\left(bx^2\right)\,dx,x,\frac{1}{x}\right)$$
$$= -\sqrt{b}\sqrt{2\pi}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a) + x\sin\left(a+\frac{b}{x^2}\right)$$

Mathematica [A] time = 0.134061, size = 81, normalized size = 1.01

$$-\sqrt{2\pi}\sqrt{b}\left(\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right) - \sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\right) + x\sin(a)\cos\left(\frac{b}{x^2}\right) + x\cos(a)\sin\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2],x]
```

```
[Out] x*Cos[b/x^2]*Sin[a] - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/x] - FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) + x*Cos[a]*Sin[b/x^2]
```

Maple [A] time = 0.01, size = 59, normalized size = 0.7

$$x\sin\left(a+\frac{b}{x^2}\right) - \sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}x}\sqrt{b}\right) - \sin(a)\,\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}x}\sqrt{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2),x)
```

```
[Out] x*sin(a+b/x^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Maxima [C] time = 1.41088, size = 539, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2*sqrt(abs(b)/x^2)*sin((a*x^2 + b)/x^2) - (((sqrt(pi)*(erf(sqrt(I*
b/x^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(1/4*pi + 1/2*arctan2
(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b/x^2)
) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) - (I*sqrt(pi)*(erf(sqrt(I*b/x^2))
- 1) - I*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)
```

```
) - (-I*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b/x^2))
 - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - ((I*sqrt(pi)*(erf(sqrt(I*
b/x^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(1/4*pi + 1/2*arcta
n2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*
b/x^2)) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/x^
2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(1/4*pi + 1/2*arctan2(0,
b)) - (sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b/x^2)) -
1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b)/(x*sqrt(abs(b)/x^2))
```

Fricas [A] time = 1.72826, size = 209, normalized size = 2.61

$$-\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)+x\sin\left(\frac{ax^2+b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) + sqrt(2)*p
i*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) + x*sin((a*x^2 + b)/x
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2),x)
```

```
[Out] Integral(sin(a + b/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2), x)
```

3.120 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\sin(a)\text{CosIntegral}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

```
[Out] -(CosIntegral[b/x^2]*Sin[a])/2 - (Cos[a]*SinIntegral[b/x^2])/2
```

Rubi [A] time = 0.0291302, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3377, 3376, 3375}

$$-\frac{1}{2}\sin(a)\text{CosIntegral}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x,x]
```

```
[Out] -(CosIntegral[b/x^2]*Sin[a])/2 - (Cos[a]*SinIntegral[b/x^2])/2
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx &= \cos(a)\int \frac{\sin\left(\frac{b}{x^2}\right)}{x}\,dx+\sin(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}\,dx\\ &= -\frac{1}{2}\text{Ci}\left(\frac{b}{x^2}\right)\sin(a)-\frac{1}{2}\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0487176, size = 25, normalized size = 1.

$$\frac{1}{2}\left(\sin(a)\left(-\text{CosIntegral}\left(\frac{b}{x^2}\right)\right)-\cos(a)\text{Si}\left(\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x,x]
```

```
[Out] (-(CosIntegral[b/x^2]*Sin[a]) - Cos[a]*SinIntegral[b/x^2])/2
```

Maple [A] time = 0.011, size = 22, normalized size = 0.9

$$-\frac{\cos(a)}{2}\operatorname{Si}\left(\frac{b}{x^2}\right)-\frac{\sin(a)}{2}\operatorname{Ci}\left(\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x,x)
```

```
[Out] -1/2*cos(a)*Si(b/x^2)-1/2*Ci(b/x^2)*sin(a)
```

Maxima [C] time = 1.12279, size = 58, normalized size = 2.32

$$\frac{1}{4}\left(i\operatorname{Ei}\left(\frac{ib}{x^2}\right)-i\operatorname{Ei}\left(-\frac{ib}{x^2}\right)\right)\cos(a)-\frac{1}{4}\left(\operatorname{Ei}\left(\frac{ib}{x^2}\right)+\operatorname{Ei}\left(-\frac{ib}{x^2}\right)\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x^2))*cos(a) - 1/4*(Ei(I*b/x^2) + Ei(-I*b/x^
2))*sin(a)
```

Fricas [A] time = 1.55555, size = 123, normalized size = 4.92

$$-\frac{1}{4}\left(\operatorname{Ci}\left(\frac{b}{x^2}\right)+\operatorname{Ci}\left(-\frac{b}{x^2}\right)\right)\sin(a)-\frac{1}{2}\cos(a)\operatorname{Si}\left(\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(b/x^2) + cos_integral(-b/x^2))*sin(a) - 1/2*cos(a)*sin_i
ntegral(b/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x,x)
```

```
[Out] Integral(sin(a + b/x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x, x)
```

3.121 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

```
[Out] -((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) - (Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Rubi [A] time = 0.030979, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3383, 3353, 3352, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) - (Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Rule 3383

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\text{Subst}\left(\int \sin\left(a+bx^2\right)dx, x, \frac{1}{x}\right)$$
$$= -\left(\cos(a)\,\text{Subst}\left(\int \sin\left(bx^2\right)dx, x, \frac{1}{x}\right)\right) - \sin(a)\,\text{Subst}\left(\int \cos\left(bx^2\right)dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}$$

Mathematica [A] time = 0.10151, size = 61, normalized size = 0.81

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)+\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])
```

Maple [A] time = 0.009, size = 47, normalized size = 0.6

$$-\frac{\sqrt{2}\sqrt{\pi}}{2}\left(\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}x}\sqrt{b}\right)+\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}x}\sqrt{b}\right)\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x^2,x)
```

```
[Out] -1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)+
sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Maxima [C] time = 1.17396, size = 495, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*(((I*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/x^2
)) - 1))*cos(1/4*pi + 1/2*arctan2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/x^2))
- 1) - I*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b
)) + (sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1
))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + s
qrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a)
 + ((sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1)
```

```
)*cos(1/4*pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + sq
rt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (-I*sq
rt(pi)*(erf(sqrt(I*b/x^2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin
(1/4*pi + 1/2*arctan2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - I*sqr
t(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))/
(x*sqrt(abs(b)/x^2))
```

Fricas [A] time = 1.70581, size = 186, normalized size = 2.48

$$-\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqrt
(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x**2,x)
```

```
[Out] Integral(sin(a + b/x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x^2, x)
```

3.122 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} dx$

Optimal. Leaf size=15

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] Cos[a + b/x^2]/(2*b)
```

Rubi [A] time = 0.0157894, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3379, 2638}

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x^3,x]

[Out] Cos[a + b/x^2]/(2*b)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3} dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0140532, size = 15, normalized size = 1.

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x^3,x]
```

```
[Out] Cos[a + b/x^2]/(2*b)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{1}{2b}\cos\left(a+\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x^3,x)
```

```
[Out] 1/2*cos(a+b/x^2)/b
```

Maxima [A] time = 0.942449, size = 18, normalized size = 1.2

$$\frac{\cos\left(a+\frac{b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*cos(a + b/x^2)/b
```

Fricas [A] time = 1.67289, size = 38, normalized size = 2.53

$$\frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos((a*x^2 + b)/x^2)/b
```

Sympy [A] time = 4.29878, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\cos\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\sin(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x**3,x)
```

```
[Out] Piecewise((cos(a + b/x**2)/(2*b), Ne(b, 0)), (-sin(a)/(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x^3, x)
```

3.123 $\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[Out] Cos[a + b/x^2]/(2*b*x) - (Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x
])/(2*b^(3/2)) + (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(2*b^
(3/2))
```

Rubi [A] time = 0.0585725, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3409, 3385, 3354, 3352, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/x^2]/x^4,x]
```

```
[Out] Cos[a + b/x^2]/(2*b*x) - (Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x
])/(2*b^(3/2)) + (Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(2*b^
(3/2))
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)dx, x, \frac{1}{x}\right)\\ &= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\text{Subst}\left(\int\cos\left(a+bx^2\right)dx, x, \frac{1}{x}\right)}{2b}\\ &= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\cos(a)\,\text{Subst}\left(\int\cos\left(bx^2\right)dx, x, \frac{1}{x}\right)}{2b} + \frac{\sin(a)\,\text{Subst}\left(\int\sin\left(bx^2\right)dx, x, \frac{1}{x}\right)}{2b}\\ &= \frac{\cos\left(a+\frac{b}{x^2}\right)}{2bx} - \frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.155992, size = 89, normalized size = 0.92

$$\frac{-\sqrt{2\pi}x\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)+\sqrt{2\pi}x\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+2\sqrt{b}\cos\left(a+\frac{b}{x^2}\right)}{4b^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/x^2]/x^4,x]
```

```
[Out] (2*Sqrt[b]*Cos[a + b/x^2] - Sqrt[2*Pi]*x*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
])/x] + Sqrt[2*Pi]*x*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(4*b^(3/2)*x)
```

Maple [A] time = 0.009, size = 65, normalized size = 0.7

$$\frac{1}{2\,bx}\cos\left(a+\frac{b}{x^2}\right)-\frac{\sqrt{2}\sqrt{\pi}}{4}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}x}\sqrt{b}\right)-\sin(a)\,\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}x}\sqrt{b}\right)\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/x^2)/x^4,x)
```

```
[Out] 1/2*cos(a+b/x^2)/b/x-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*
2^(1/2)/Pi^(1/2)/x)-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Maxima [C] time = 1.16273, size = 359, normalized size = 3.7

$$-\frac{\left(\left(-i\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)+i\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\cos\left(\frac{3}{4}\pi+\frac{3}{2}\arctan(0,b)\right)+\left(-i\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)+i\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\cos\left(-\frac{3}{4}\pi+\frac{3}{2}\arctan(0,b)\right)-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/8*(((-I*gamma(3/2, I*b/x^2) + I*gamma(3/2, -I*b/x^2))*cos(3/4*pi + 3/2*a
rctan2(0, b)) + (-I*gamma(3/2, I*b/x^2) + I*gamma(3/2, -I*b/x^2))*cos(-3/4*
pi + 3/2*arctan2(0, b)) - (gamma(3/2, I*b/x^2) + gamma(3/2, -I*b/x^2))*sin(
3/4*pi + 3/2*arctan2(0, b)) + (gamma(3/2, I*b/x^2) + gamma(3/2, -I*b/x^2))*
sin(-3/4*pi + 3/2*arctan2(0, b)))*cos(a) - ((gamma(3/2, I*b/x^2) + gamma(3/
2, -I*b/x^2))*cos(3/4*pi + 3/2*arctan2(0, b)) + (gamma(3/2, I*b/x^2) + gamm
a(3/2, -I*b/x^2))*cos(-3/4*pi + 3/2*arctan2(0, b)) - (I*gamma(3/2, I*b/x^2)
 - I*gamma(3/2, -I*b/x^2))*sin(3/4*pi + 3/2*arctan2(0, b)) - (-I*gamma(3/2,
 I*b/x^2) + I*gamma(3/2, -I*b/x^2))*sin(-3/4*pi + 3/2*arctan2(0, b)))*sin(a
))/(x^3*(abs(b)/x^2)^(3/2))
```

Fricas [A] time = 1.79547, size = 236, normalized size = 2.43

$$-\frac{\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)-\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)-2\,b\cos\left(\frac{ax^2+b}{x^2}\right)}{4\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) - sq
rt(2)*pi*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a) - 2*b*cos((a
*x^2 + b)/x^2))/(b^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x**2)/x**4,x)
```

```
[Out] Integral(sin(a + b/x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/x^2)/x^4, x)
```

3.124 $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\cos\left(\sqrt{x}\right)$$

[Out] -2*Cos[Sqrt[x]]

Rubi [A] time = 0.009238, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3379, 2638}

$$-2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[x]]/Sqrt[x],x]

[Out] -2*Cos[Sqrt[x]]

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= -2\cos\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0102346, size = 8, normalized size = 1.

$$-2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Sqrt[x]]/Sqrt[x],x]

[Out] -2*Cos[Sqrt[x]]

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*cos(x^(1/2))
```

Maxima [A] time = 0.938496, size = 8, normalized size = 1.

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*cos(sqrt(x))
```

Fricas [A] time = 1.62489, size = 23, normalized size = 2.88

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*cos(sqrt(x))
```

Sympy [A] time = 0.306216, size = 8, normalized size = 1.

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2))/x**(1/2),x)
```

```
[Out] -2*cos(sqrt(x))
```

Giac [A] time = 1.1002, size = 8, normalized size = 1.

$$-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*cos(sqrt(x))
```

3.125 $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\cos^3\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)$$

```
[Out] -2*Cos[Sqrt[x]] + (2*Cos[Sqrt[x]]^3)/3
```

Rubi [A] time = 0.0204478, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3379, 2633}

$$\frac{2}{3}\cos^3\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]]^3/Sqrt[x],x]

[Out] -2*Cos[Sqrt[x]] + (2*Cos[Sqrt[x]]^3)/3
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \sin^3(x)\,dx, x, \sqrt{x}\right)\\ &= -\left(2\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos\left(\sqrt{x}\right)\right)\right)\\ &= -2\cos\left(\sqrt{x}\right)+\frac{2}{3}\cos^3\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0231795, size = 23, normalized size = 1.1

$$\frac{1}{6}\cos\left(3\sqrt{x}\right)-\frac{3\cos\left(\sqrt{x}\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]]^3/Sqrt[x],x]
```

```
[Out] (-3*Cos[Sqrt[x]])/2 + Cos[3*Sqrt[x]]/6
```

Maple [A] time = 0.009, size = 15, normalized size = 0.7

$$-\frac{2}{3}\left(2+\left(\sin\left(\sqrt{x}\right)\right)^{2}\right)\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2))^3/x^(1/2),x)
```

```
[Out] -2/3*(2+sin(x^(1/2))^2)*cos(x^(1/2))
```

Maxima [A] time = 0.940897, size = 20, normalized size = 0.95

$$\frac{2}{3}\cos\left(\sqrt{x}\right)^{3}-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Fricas [A] time = 1.65846, size = 50, normalized size = 2.38

$$\frac{2}{3}\cos\left(\sqrt{x}\right)^{3}-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

Sympy [A] time = 0.902545, size = 29, normalized size = 1.38

$$-2\sin^{2}\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)-\frac{4\cos^{3}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2))**3/x**(1/2),x)
```

```
[Out] -2*sin(sqrt(x))**2*cos(sqrt(x)) - 4*cos(sqrt(x))**3/3
```

Giac [A] time = 1.09923, size = 20, normalized size = 0.95

$$\frac{2}{3}\cos\left(\sqrt{x}\right)^{3}-2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*cos(sqrt(x))^3 - 2*cos(sqrt(x))
```

3.126 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0118392, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3361, 3296, 2637}

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin\left(\sqrt{x}\right)\,dx &= 2\,\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0146252, size = 22, normalized size = 1.

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] time = 0.006, size = 17, normalized size = 0.8

$$2 \sin\left(\sqrt{x}\right) - 2 \cos\left(\sqrt{x}\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)

[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.948776, size = 22, normalized size = 1.

$$-2 \sqrt{x} \cos\left(\sqrt{x}\right) + 2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")

[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A] time = 1.62754, size = 57, normalized size = 2.59

$$-2 \sqrt{x} \cos\left(\sqrt{x}\right) + 2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")

[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] time = 0.303348, size = 20, normalized size = 0.91

$$-2\sqrt{x} \cos\left(\sqrt{x}\right) + 2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)

[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] time = 1.09883, size = 22, normalized size = 1.

$$-2 \sqrt{x} \cos\left(\sqrt{x}\right) + 2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.127 $\int \sin^2\left(\sqrt[3]{x}\right) dx$

Optimal. Leaf size=69

$$-\frac{3}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{2}-\frac{3\sqrt[3]{x}}{4}+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)+\frac{3}{4}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)$$

```
[Out] (-3*x^(1/3))/4 + x/2 + (3*Cos[x^(1/3)]*Sin[x^(1/3)])/4 - (3*x^(2/3)*Cos[x^(
1/3)]*Sin[x^(1/3)])/2 + (3*x^(1/3)*Sin[x^(1/3)]^2)/2
```

Rubi [A] time = 0.043028, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3361, 3311, 30, 2635, 8}

$$-\frac{3}{2}x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{x}{2}-\frac{3\sqrt[3]{x}}{4}+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)+\frac{3}{4}\sin\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(1/3)]^2,x]
```

```
[Out] (-3*x^(1/3))/4 + x/2 + (3*Cos[x^(1/3)]*Sin[x^(1/3)])/4 - (3*x^(2/3)*Cos[x^(
1/3)]*Sin[x^(1/3)])/2 + (3*x^(1/3)*Sin[x^(1/3)]^2)/2
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2\left(\sqrt[3]{x}\right)dx &= 3\,\text{Subst}\left(\int x^2\sin^2(x)\,dx,x,\sqrt[3]{x}\right)\\ &= -\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)+\frac{3}{2}\text{Subst}\left(\int x^2\,dx,x,\sqrt[3]{x}\right)-\frac{3}{2}\text{Subst}\left(\int \sin^2(x)\,dx,\right.\\ &= \frac{x}{2}+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)-\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)-\frac{3}{4}\text{Subst}\left(\int 1\,dx,x,\sqrt[3]{x}\right)\\ &= -\frac{3\sqrt[3]{x}}{4}+\frac{x}{2}+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)-\frac{3}{2}x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{2}\sqrt[3]{x}\sin^2\left(\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.049048, size = 41, normalized size = 0.59

$$\frac{1}{8}\left(\left(3-6x^{2/3}\right)\sin\left(2\sqrt[3]{x}\right)+4x-6\sqrt[3]{x}\cos\left(2\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(1/3)]^2,x]
```

```
[Out] (4*x - 6*x^(1/3)*Cos[2*x^(1/3)] + (3 - 6*x^(2/3))*Sin[2*x^(1/3)])/8
```

Maple [A] time = 0.01, size = 52, normalized size = 0.8

$$3\,x^{2/3}\left(-1/2\,\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+1/2\,\sqrt[3]{x}\right)-\frac{3}{2}\sqrt[3]{x}\left(\cos\left(\sqrt[3]{x}\right)\right)^2+\frac{3}{4}\cos\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{3}{4}\sqrt[3]{x}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3))^2,x)
```

```
[Out] 3*x^(2/3)*(-1/2*cos(x^(1/3))*sin(x^(1/3))+1/2*x^(1/3))-3/2*x^(1/3)*cos(x^(1
/3))^2+3/4*cos(x^(1/3))*sin(x^(1/3))+3/4*x^(1/3)-x
```

Maxima [A] time = 0.950069, size = 41, normalized size = 0.59

$$-\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] -3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

Fricas [A] time = 1.62251, size = 134, normalized size = 1.94

$$-\frac{3}{4}\left(2\,x^{\frac{2}{3}}-1\right)\cos\left(x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)-\frac{3}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^2+\frac{1}{2}\,x+\frac{3}{4}\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -3/4*(2*x^(2/3) - 1)*cos(x^(1/3))*sin(x^(1/3)) - 3/2*x^(1/3)*cos(x^(1/3))^2
 + 1/2*x + 3/4*x^(1/3)
```

Sympy [B] time = 1.51308, size = 379, normalized size = 5.49

$$\frac{12x^{\frac{2}{3}}\tan^{3}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}-\frac{12x^{\frac{2}{3}}\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}-\frac{3\sqrt[3]{x}\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan^{2}\left(\frac{\sqrt[3]{x}}{2}\right)+4}+\frac{18\sqrt[3]{x}\tan^{2}}{4\tan^{4}\left(\frac{\sqrt[3]{x}}{2}\right)+8\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/3))**2,x)
```

```
[Out] 12*x**(2/3)*tan(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2
 + 4) - 12*x**(2/3)*tan(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/
2)**2 + 4) - 3*x**(1/3)*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(x*
*(1/3)/2)**2 + 4) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 +
8*tan(x**(1/3)/2)**2 + 4) - 3*x**(1/3)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 2*x*tan(x**(1/3)/2)**4/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 4*x*tan(x**(1/3)/2)**2/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/
3)/2)**2 + 4) + 2*x/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4) - 6*t
an(x**(1/3)/2)**3/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4) + 6*tan
(x**(1/3)/2)/(4*tan(x**(1/3)/2)**4 + 8*tan(x**(1/3)/2)**2 + 4)
```

Giac [A] time = 1.09981, size = 41, normalized size = 0.59

$$-\frac{3}{8}\left(2\,x^{\frac{2}{3}}-1\right)\sin\left(2\,x^{\frac{1}{3}}\right)-\frac{3}{4}\,x^{\frac{1}{3}}\cos\left(2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^2,x, algorithm="giac")
```

```
[Out] -3/8*(2*x^(2/3) - 1)*sin(2*x^(1/3)) - 3/4*x^(1/3)*cos(2*x^(1/3)) + 1/2*x
```

3.128 $\int \sin^3\left(\sqrt[3]{x}\right) dx$

Optimal. Leaf size=87

$$-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+\frac{14}{3}\cos\left(\sqrt[3]{x}\right)$$

```
[Out] (14*Cos[x^(1/3)])/3 - 2*x^(2/3)*Cos[x^(1/3)] - (2*Cos[x^(1/3)]^3)/9 + 4*x^(
1/3)*Sin[x^(1/3)] - x^(2/3)*Cos[x^(1/3)]*Sin[x^(1/3)]^2 + (2*x^(1/3)*Sin[x^
(1/3)]^3)/3
```

Rubi [A] time = 0.0636996, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3361, 3311, 3296, 2638, 2633}

$$-2x^{2/3}\cos\left(\sqrt[3]{x}\right)-x^{2/3}\sin^2\left(\sqrt[3]{x}\right)\cos\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-\frac{2}{9}\cos^3\left(\sqrt[3]{x}\right)+\frac{14}{3}\cos\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(1/3)]^3,x]
```

```
[Out] (14*Cos[x^(1/3)])/3 - 2*x^(2/3)*Cos[x^(1/3)] - (2*Cos[x^(1/3)]^3)/9 + 4*x^(
1/3)*Sin[x^(1/3)] - x^(2/3)*Cos[x^(1/3)]*Sin[x^(1/3)]^2 + (2*x^(1/3)*Sin[x^
(1/3)]^3)/3
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
 - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sin^3\left(\sqrt[3]{x}\right) dx = 3\,\text{Subst}\left(\int x^2 \sin^3(x)\, dx, x, \sqrt[3]{x}\right)$$
$$= -x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right) - \frac{2}{3}\text{Subst}\left(\int \sin^3(x)\, dx, x, \sqrt[3]{x}\right) + 2\,\text{Subst}\left(\int x^2 \sin(x)\, d\right.$$
$$= -2x^{2/3}\cos\left(\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{x}\right) + \frac{2}{3}\text{Subst}\left(\int \left(1 - x^2\right) dx, x, \cos\left(\sqrt[3]{x}\right)\right)$$
$$= \frac{2}{3}\cos\left(\sqrt[3]{x}\right) - 2x^{2/3}\cos\left(\sqrt[3]{x}\right) - \frac{2}{9}\cos^3\left(\sqrt[3]{x}\right) + 4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\sqrt[3]{\right.$$
$$= \frac{14}{3}\cos\left(\sqrt[3]{x}\right) - 2x^{2/3}\cos\left(\sqrt[3]{x}\right) - \frac{2}{9}\cos^3\left(\sqrt[3]{x}\right) + 4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\sin^2\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\sin^3\left(\right.$$

Mathematica [A] time = 0.0550805, size = 62, normalized size = 0.71

$$\frac{1}{36}\left(-81\left(x^{2/3}-2\right)\cos\left(\sqrt[3]{x}\right)+\left(9x^{2/3}-2\right)\cos\left(3\sqrt[3]{x}\right)-6\sqrt[3]{x}\left(\sin\left(3\sqrt[3]{x}\right)-27\sin\left(\sqrt[3]{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(1/3)]^3,x]
```

```
[Out] (-81*(-2 + x^(2/3))*Cos[x^(1/3)] + (-2 + 9*x^(2/3))*Cos[3*x^(1/3)] - 6*x^(1
/3)*(-27*Sin[x^(1/3)] + Sin[3*x^(1/3)]))/36
```

Maple [A] time = 0.01, size = 59, normalized size = 0.7

$$-x^{\frac{2}{3}}\left(2+\left(\sin\left(\sqrt[3]{x}\right)\right)^2\right)\cos\left(\sqrt[3]{x}\right)+4\cos\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\left(\sin\left(\sqrt[3]{x}\right)\right)^3+\frac{2}{9}\left(2+\left(\sin\left(\sqrt[3]{x}\right)\right)^2\right)\cos\left(\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3))^3,x)
```

```
[Out] -x^(2/3)*(2+sin(x^(1/3))^2)*cos(x^(1/3))+4*cos(x^(1/3))+4*x^(1/3)*sin(x^(1/
3))+2/3*x^(1/3)*sin(x^(1/3))^3+2/9*(2+sin(x^(1/3))^2)*cos(x^(1/3))
```

Maxima [A] time = 0.972562, size = 63, normalized size = 0.72

$$\frac{1}{36}\left(9x^{\frac{2}{3}}-2\right)\cos\left(3x^{\frac{1}{3}}\right)-\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{1}{6}x^{\frac{1}{3}}\sin\left(3x^{\frac{1}{3}}\right)+\frac{9}{2}x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6*
x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

Fricas [A] time = 1.67498, size = 173, normalized size = 1.99

$$\frac{1}{9}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)^{3}-\frac{1}{3}\left(9\,x^{\frac{2}{3}}-14\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{2}{3}\left(x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^{2}-7\,x^{\frac{1}{3}}\right)\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/9*(9*x^(2/3) - 2)*cos(x^(1/3))^3 - 1/3*(9*x^(2/3) - 14)*cos(x^(1/3)) - 2/
3*(x^(1/3)*cos(x^(1/3))^2 - 7*x^(1/3))*sin(x^(1/3))
```

Sympy [A] time = 8.2981, size = 80, normalized size = 0.92

$$-\frac{9x^{\frac{2}{3}}\cos\left(\sqrt[3]{x}\right)}{4}+\frac{x^{\frac{2}{3}}\cos\left(3\sqrt[3]{x}\right)}{4}+\frac{9\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)}{2}-\frac{\sqrt[3]{x}\sin\left(3\sqrt[3]{x}\right)}{6}+\frac{9\cos\left(\sqrt[3]{x}\right)}{2}-\frac{\cos\left(3\sqrt[3]{x}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/3))**3,x)
```

```
[Out] -9*x**(2/3)*cos(x**(1/3))/4 + x**(2/3)*cos(3*x**(1/3))/4 + 9*x**(1/3)*sin(x
**(1/3))/2 - x**(1/3)*sin(3*x**(1/3))/6 + 9*cos(x**(1/3))/2 - cos(3*x**(1/3
))/18
```

Giac [A] time = 1.09136, size = 63, normalized size = 0.72

$$\frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(3\,x^{\frac{1}{3}}\right)-\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)-\frac{1}{6}\,x^{\frac{1}{3}}\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*cos(3*x^(1/3)) - 9/4*(x^(2/3) - 2)*cos(x^(1/3)) - 1/6*
x^(1/3)*sin(3*x^(1/3)) + 9/2*x^(1/3)*sin(x^(1/3))
```

3.129 $\int (ex)^m (b\sin(c+dx^n))^p \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((ex)^m (b\sin(c+dx^n))^p, x\right)$$

```
[Out] Unintegrable[(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Rubi [A] time = 0.0197795, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx = \int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Mathematica [A] time = 1.01865, size = 0, normalized size = 0.

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(b*Sin[c + d*x^n])^p, x]
```

Maple [A] time = 0.985, size = 0, normalized size = 0.

$$\int (ex)^m (b\sin(c+dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(b*sin(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m (b\sin(dx^n+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m\,(b\sin(dx^n+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(c+dx^n))^p\,(ex)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*sin(c+d*x**n))**p,x)
```

```
[Out] Integral((b*sin(c + d*x**n))**p*(e*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m\,(b\sin(dx^n+c))^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^n + c))^p, x)
```

3.130 $\int (ex)^m (a + b\sin(c + dx^n))^p \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((ex)^m (a + b\sin(c + dx^n))^p, x\right)$$

```
[Out] Unintegrable[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Rubi [A] time = 0.0234132, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx = \int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

Mathematica [A] time = 1.39225, size = 0, normalized size = 0.

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*x^n])^p, x]
```

Maple [A] time = 0.85, size = 0, normalized size = 0.

$$\int (ex)^m (a + b\sin(c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(a+b*sin(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m (b\sin(dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m(b\sin(dx^n+c)+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m(b\sin(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sin(d*x^n + c) + a)^p, x)
```

3.131 $\int (ex)^{-1+n} (b\sin (c+dx^n))^p \, dx$

Optimal. Leaf size=92

$$\frac{x^{-n}(ex)^n \cos (c+dx^n) (b\sin (c+dx^n))^{p+1} {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\sin^2 (dx^n+c)\right)}{bden(p+1)\sqrt{\cos^2 (c+dx^n)}}$$

```
[Out] ((e*x)^n*Cos[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[c
+ d*x^n]^2]*(b*Sin[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cos[c + d
*x^n]^2])
```

Rubi [A] time = 0.101912, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3381, 3379, 2643}

$$\frac{x^{-n}(ex)^n \cos (c+dx^n) (b\sin (c+dx^n))^{p+1} {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\sin^2 (dx^n+c)\right)}{bden(p+1)\sqrt{\cos^2 (c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*Cos[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[c
+ d*x^n]^2]*(b*Sin[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cos[c + d
*x^n]^2])
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx = \frac{(x^{-n}(ex)^n)\int x^{-1+n}(b\sin(c+dx^n))^p\,dx}{e}$$
$$= \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int (b\sin(c+dx))^p\,dx, x, x^n\right)}{en}$$
$$= \frac{x^{-n}(ex)^n\cos(c+dx^n)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2(c+dx^n)\right)(b\sin(c+dx^n))^{1+p}}{bden(1+p)\sqrt{\cos^2(c+dx^n)}}$$

Mathematica [A] time = 0.163942, size = 88, normalized size = 0.96

$$\frac{x^{1-n}(ex)^{n-1}\sqrt{\cos^2(c+dx^n)}\tan(c+dx^n)(b\sin(c+dx^n))^p\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\sin^2(dx^n+c)\right)}{dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] (x^(1 - n)*(e*x)^(-1 + n)*Sqrt[Cos[c + d*x^n]^2]*Hypergeometric2F1[1/2, (1
+ p)/2, (3 + p)/2, Sin[c + d*x^n]^2]*(b*Sin[c + d*x^n])^p*Tan[c + d*x^n])/(
d*n*(1 + p))
```

Maple [F] time = 1.123, size = 0, normalized size = 0.

$$\int (ex)^{-1+n}(b\sin(c+dx^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1}(b\sin(dx^n+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((ex)^{n-1}(b\sin(dx^n+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1} (b \sin (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c))^p, x)
```

3.132 $\int (ex)^{-1+2n} (b \sin (c + dx^n))^p \, dx$

Optimal. Leaf size=38

$$\frac{x^{-2n}(ex)^{2n}\text{Unintegrable}\left(x^{2n-1} \left(b \sin \left(c + dx^n\right)\right)^p, x\right)}{e}$$

```
[Out] ((e*x)^(2*n)*Unintegrable[x^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x])/(e*x^(2*n)
)
```

Rubi [A] time = 0.0505211, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^{-1+2n} \left(b \sin \left(c + dx^n\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\int (ex)^{-1+2n} \left(b \sin \left(c + dx^n\right)\right)^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right) \int x^{-1+2n} \left(b \sin \left(c + dx^n\right)\right)^p \, dx}{e}$$

Mathematica [A] time = 0.929364, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} \left(b \sin \left(c + dx^n\right)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Sin[c + d*x^n])^p, x]
```

Maple [A] time = 0.948, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} \left(b \sin \left(c + dx^n\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sin (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^{2n-1} (b \sin (dx^n + c))^p , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sin (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c))^p, x)
```

3.133 $\int (ex)^{-1+n} (a + b\sin(c + dx^n))^p \, dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{2}x^{-n}(ex)^n \cos(c + dx^n)(a + b\sin(c + dx^n))^p \left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; \frac{1}{2}(1 - \sin(dx^n + c)), \frac{b(1-\sin(dx^n+c))}{a+b}\right)}{den\sqrt{\sin(c + dx^n) + 1}}$$

```
[Out] -((Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Sin[c + d*x^n])/2, (b*(
1 - Sin[c + d*x^n]))/(a + b)]*Cos[c + d*x^n]*(a + b*Sin[c + d*x^n])^p)/(d*e
*n*x^n*Sqrt[1 + Sin[c + d*x^n]]*((a + b*Sin[c + d*x^n])/(a + b))^p))
```

Rubi [A] time = 0.190298, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3381, 3379, 2665, 139, 138}

$$-\frac{\sqrt{2}x^{-n}(ex)^n \cos(c + dx^n)(a + b\sin(c + dx^n))^p \left(\frac{a+b\sin(c+dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; \frac{1}{2}(1 - \sin(dx^n + c)), \frac{b(1-\sin(dx^n+c))}{a+b}\right)}{den\sqrt{\sin(c + dx^n) + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] -((Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Sin[c + d*x^n])/2, (b*(
1 - Sin[c + d*x^n]))/(a + b)]*Cos[c + d*x^n]*(a + b*Sin[c + d*x^n])^p)/(d*e
*n*x^n*Sqrt[1 + Sin[c + d*x^n]]*((a + b*Sin[c + d*x^n])/(a + b))^p))
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int (ex)^{-1+n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx &= \frac{\left(x^{-n}(ex)^n\right)\int x^{-1+n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx}{e}\\ &= \frac{\left(x^{-n}(ex)^n\right)\text{Subst}\left(\int (a+b\sin(c+dx))^p\,dx,x,x^n\right)}{en}\\ &= \frac{\left(x^{-n}(ex)^n\cos\left(c+dx^n\right)\right)\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sin\left(c+dx^n\right)\right)}{den\sqrt{1-\sin\left(c+dx^n\right)}\sqrt{1+\sin\left(c+dx^n\right)}}\\ &= \frac{\left(x^{-n}(ex)^n\cos\left(c+dx^n\right)\left(a+b\sin\left(c+dx^n\right)\right)^p\left(-\frac{a+b\sin(c+dx^n)}{-a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\right.}{\sqrt{1-x}\sqrt{1}}\right.}{den\sqrt{1-\sin\left(c+dx^n\right)}\sqrt{1+\sin\left(c+dx^n\right)}}\\ &= -\frac{\sqrt{2}x^{-n}(ex)^nF_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}\left(1-\sin\left(c+dx^n\right)\right),\frac{b(1-\sin(c+dx^n))}{a+b}\right)\cos\left(c+dx^n\right)(a+}{den\sqrt{1+\sin\left(c+dx^n\right)}}\end{aligned}$$

Mathematica [A] time = 0.44854, size = 148, normalized size = 1.12

$$\frac{x^{-n}(ex)^n\sec\left(c+dx^n\right)\sqrt{-\frac{b(\sin(c+dx^n)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx^n)+1)}{b-a}}\left(a+b\sin\left(c+dx^n\right)\right)^{p+1}F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{a+b\sin(dx^n+c)}{a-b},\frac{a+b\sin(dx}{a+b}\right.}{bden(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sin[c + d*x^n])/(a - b), (
a + b*Sin[c + d*x^n])/(a + b)]*Sec[c + d*x^n]*Sqrt[-((b*(-1 + Sin[c + d*x^n
]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x^n]))/(-a + b)]*(a + b*Sin[c + d*x^n]
)^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

Maple [F] time = 0.967, size = 0, normalized size = 0.

$$\int (ex)^{-1+n}\left(a+b\sin\left(c+dx^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1} (b\sin(dx^n + c) + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^{n-1} (b\sin(dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{n-1} (b\sin(dx^n + c) + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

3.134 $\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx$

Optimal. Leaf size=40

$$\frac{x^{-2n}(ex)^{2n}\text{Unintegrable}\left(x^{2n-1}(a+b\sin(c+dx^n))^p,x\right)}{e}$$

```
[Out] ((e*x)^(2*n)*Unintegrable[x^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x])/(e*x^(
2*n))
```

Rubi [A] time = 0.0587778, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x])/(e*x^(2*
n))
```

Rubi steps

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n} (a+b\sin(c+dx^n))^p \, dx}{e}$$

Mathematica [A] time = 1.26904, size = 0, normalized size = 0.

$$\int (ex)^{-1+2n} (a+b\sin(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sin[c + d*x^n])^p, x]
```

Maple [A] time = 0.765, size = 0, normalized size = 0.

$$\int (ex)^{-1+2\,n} (a+b\sin(c+dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sin (dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^{2n-1} (b \sin (dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sin(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{2n-1} (b \sin (dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sin(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*sin(d*x^n + c) + a)^p, x)
```

3.135 $\int \frac{\sin(a+bx^n)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\sin(a)\text{CosIntegral}\left(bx^n\right)}{n}+\frac{\cos(a)\text{Si}\left(bx^n\right)}{n}$$

```
[Out] (CosIntegral[b*x^n]*Sin[a])/n + (Cos[a]*SinIntegral[b*x^n])/n
```

Rubi [A] time = 0.038134, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3377, 3376, 3375}

$$\frac{\sin(a)\text{CosIntegral}\left(bx^n\right)}{n}+\frac{\cos(a)\text{Si}\left(bx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]/x,x]

[Out] (CosIntegral[b*x^n]*Sin[a])/n + (Cos[a]*SinIntegral[b*x^n])/n
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+bx^n\right)}{x}\,dx &= \cos(a)\int \frac{\sin\left(bx^n\right)}{x}\,dx+\sin(a)\int \frac{\cos\left(bx^n\right)}{x}\,dx\\ &= \frac{\text{Ci}\left(bx^n\right)\sin(a)}{n}+\frac{\cos(a)\text{Si}\left(bx^n\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0612245, size = 23, normalized size = 0.92

$$\frac{\sin(a)\text{CosIntegral}\left(bx^n\right)+\cos(a)\text{Si}\left(bx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]/x,x]
```

```
[Out] (CosIntegral[b*x^n]*Sin[a] + Cos[a]*SinIntegral[b*x^n])/n
```

Maple [A] time = 0.006, size = 24, normalized size = 1.

$$\frac{\mathrm{Si}\left(bx^{n}\right)\cos\left(a\right)+\mathrm{Ci}\left(bx^{n}\right)\sin\left(a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)/x,x)
```

```
[Out] 1/n*(Si(b*x^n)*cos(a)+Ci(b*x^n)*sin(a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 1.7626, size = 131, normalized size = 5.24

$$\frac{\mathrm{Ci}\left(bx^{n}\right)\sin\left(a\right)+\mathrm{Ci}\left(-bx^{n}\right)\sin\left(a\right)+2\,\cos\left(a\right)\mathrm{Si}\left(bx^{n}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(cos_integral(b*x^n)*sin(a) + cos_integral(-b*x^n)*sin(a) + 2*cos(a)*si
n_integral(b*x^n))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)/x,x)
```

```
[Out] Integral(sin(a + b*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(bx^{n}+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)/x, x)
```

3.136 $\int \frac{\sin^2(a+bx^n)}{x} dx$

Optimal. Leaf size=43

$$-\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}+\frac{\log(x)}{2}$$

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + Log[x]/2 + (Sin[2*a]*SinIntegral[2
*b*x^n])/(2*n)
```

Rubi [A] time = 0.0614448, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3425, 3378, 3376, 3375}

$$-\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^2/x,x]
```

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + Log[x]/2 + (Sin[2*a]*SinIntegral[2
*b*x^n])/(2*n)
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx^n)}{x}\,dx &= \int \left(\frac{1}{2x}-\frac{\cos(2a+2bx^n)}{2x}\right)dx\\ &= \frac{\log(x)}{2}-\frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx\\ &= \frac{\log(x)}{2}-\frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx+\frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\,dx\\ &= -\frac{\cos(2a)\text{Ci}(2bx^n)}{2n}+\frac{\log(x)}{2}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}\end{aligned}$$

Mathematica [A] time = 0.0767523, size = 37, normalized size = 0.86

$$\frac{-\cos(2a)\text{CosIntegral}(2bx^n)+\sin(2a)\text{Si}(2bx^n)+n\log(x)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^2/x,x]
```

```
[Out] (-(Cos[2*a]*CosIntegral[2*b*x^n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n
])/(2*n)
```

Maple [A] time = 0.007, size = 45, normalized size = 1.1

$$\frac{\ln(bx^n)}{2n}+\frac{\text{Si}(2bx^n)\sin(2a)}{2n}-\frac{\text{Ci}(2bx^n)\cos(2a)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^2/x,x)
```

```
[Out] 1/2/n*ln(b*x^n)+1/2*Si(2*b*x^n)*sin(2*a)/n-1/2*Ci(2*b*x^n)*cos(2*a)/n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 1.71182, size = 166, normalized size = 3.86

$$-\frac{\cos(2a)\,\text{Ci}(2bx^n)+\cos(2a)\,\text{Ci}(-2bx^n)-2n\log(x)-2\sin(2a)\,\text{Si}(2bx^n)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(2*a)*cos_integral(2*b*x^n) + cos(2*a)*cos_integral(-2*b*x^n) - 2*
n*log(x) - 2*sin(2*a)*sin_integral(2*b*x^n))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a + bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**2/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx^n + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^2/x, x)
```

3.137 $\int \frac{\sin^3(a+bx^n)}{x}\,dx$

Optimal. Leaf size=67

$$\frac{3\sin(a)\text{CosIntegral}(bx^n)}{4n}-\frac{\sin(3a)\text{CosIntegral}(3bx^n)}{4n}+\frac{3\cos(a)\text{Si}(bx^n)}{4n}-\frac{\cos(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] (3*CosIntegral[b*x^n]*Sin[a])/(4*n) - (CosIntegral[3*b*x^n]*Sin[3*a])/(4*n)
 + (3*Cos[a]*SinIntegral[b*x^n])/(4*n) - (Cos[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rubi [A] time = 0.0922735, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3425, 3377, 3376, 3375}

$$\frac{3\sin(a)\text{CosIntegral}(bx^n)}{4n}-\frac{\sin(3a)\text{CosIntegral}(3bx^n)}{4n}+\frac{3\cos(a)\text{Si}(bx^n)}{4n}-\frac{\cos(3a)\text{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^n]*Sin[a])/(4*n) - (CosIntegral[3*b*x^n]*Sin[3*a])/(4*n)
 + (3*Cos[a]*SinIntegral[b*x^n])/(4*n) - (Cos[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(a+bx^n)}{x}\,dx &= \int \left(\frac{3\sin(a+bx^n)}{4x} - \frac{\sin(3a+3bx^n)}{4x}\right) dx \\
&= -\left(\frac{1}{4}\int \frac{\sin(3a+3bx^n)}{x}\,dx\right) + \frac{3}{4}\int \frac{\sin(a+bx^n)}{x}\,dx \\
&= \frac{1}{4}(3\cos(a))\int \frac{\sin(bx^n)}{x}\,dx - \frac{1}{4}\cos(3a)\int \frac{\sin(3bx^n)}{x}\,dx + \frac{1}{4}(3\sin(a))\int \frac{\cos(bx^n)}{x}\,dx - \frac{1}{4}\sin(\\
&= \frac{3\mathrm{Ci}(bx^n)\sin(a)}{4n} - \frac{\mathrm{Ci}(3bx^n)\sin(3a)}{4n} + \frac{3\cos(a)\mathrm{Si}(bx^n)}{4n} - \frac{\cos(3a)\mathrm{Si}(3bx^n)}{4n}
\end{aligned}
$$

Mathematica [A] time = 0.105854, size = 54, normalized size = 0.81

$$
\frac{3\sin(a)\mathrm{CosIntegral}(bx^n) - \sin(3a)\mathrm{CosIntegral}(3bx^n) + 3\cos(a)\mathrm{Si}(bx^n) - \cos(3a)\mathrm{Si}(3bx^n)}{4n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^3/x,x]
```

```
[Out] (3*CosIntegral[b*x^n]*Sin[a] - CosIntegral[3*b*x^n]*Sin[3*a] + 3*Cos[a]*Sin
Integral[b*x^n] - Cos[3*a]*SinIntegral[3*b*x^n])/(4*n)
```

Maple [A] time = 0.012, size = 52, normalized size = 0.8

$$
\frac{1}{n}\left(-\frac{\mathrm{Si}(3\,bx^n)\cos(3\,a)}{4} - \frac{\mathrm{Ci}(3\,bx^n)\sin(3\,a)}{4} + \frac{3\,\mathrm{Si}(bx^n)\cos(a)}{4} + \frac{3\,\mathrm{Ci}(bx^n)\sin(a)}{4}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^3/x,x)
```

```
[Out] 1/n*(-1/4*Si(3*b*x^n)*cos(3*a)-1/4*Ci(3*b*x^n)*sin(3*a)+3/4*Si(b*x^n)*cos(a
)+3/4*Ci(b*x^n)*sin(a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 1.72273, size = 275, normalized size = 4.1

$$
-\frac{\mathrm{Ci}(3\,bx^n)\sin(3\,a) + \mathrm{Ci}(-3\,bx^n)\sin(3\,a) - 3\,\mathrm{Ci}(bx^n)\sin(a) - 3\,\mathrm{Ci}(-bx^n)\sin(a) + 2\,\cos(3\,a)\,\mathrm{Si}(3\,bx^n) - 6\,\cos(a}{8\,n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] -1/8*(cos_integral(3*b*x^n)*sin(3*a) + cos_integral(-3*b*x^n)*sin(3*a) - 3*
cos_integral(b*x^n)*sin(a) - 3*cos_integral(-b*x^n)*sin(a) + 2*cos(3*a)*sin
_integral(3*b*x^n) - 6*cos(a)*sin_integral(b*x^n))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**3/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx^n+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^3/x, x)
```

3.138 $\int \frac{\sin^4(a+bx^n)}{x} dx$

Optimal. Leaf size=79

$$-\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n}+\frac{\cos(4a)\text{CosIntegral}(4bx^n)}{8n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}-\frac{\sin(4a)\text{Si}(4bx^n)}{8n}+\frac{3\log(x)}{8}$$

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + (Cos[4*a]*CosIntegral[4*b*x^n])/(8
*n) + (3*Log[x])/8 + (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*SinI
ntegral[4*b*x^n])/(8*n)
```

Rubi [A] time = 0.100595, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3425, 3378, 3376, 3375}

$$-\frac{\cos(2a)\text{CosIntegral}(2bx^n)}{2n}+\frac{\cos(4a)\text{CosIntegral}(4bx^n)}{8n}+\frac{\sin(2a)\text{Si}(2bx^n)}{2n}-\frac{\sin(4a)\text{Si}(4bx^n)}{8n}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^4/x,x]
```

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + (Cos[4*a]*CosIntegral[4*b*x^n])/(8
*n) + (3*Log[x])/8 + (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*SinI
ntegral[4*b*x^n])/(8*n)
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx^n)}{x}\,dx = \int \left(\frac{3}{8x} - \frac{\cos(2a+2bx^n)}{2x} + \frac{\cos(4a+4bx^n)}{8x}\right) dx$$
$$= \frac{3\log(x)}{8} + \frac{1}{8}\int \frac{\cos(4a+4bx^n)}{x}\,dx - \frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx$$
$$= \frac{3\log(x)}{8} - \frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx + \frac{1}{8}\cos(4a)\int \frac{\cos(4bx^n)}{x}\,dx + \frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\,dx\ \cdot$$
$$= -\frac{\cos(2a)\text{Ci}(2bx^n)}{2n} + \frac{\cos(4a)\text{Ci}(4bx^n)}{8n} + \frac{3\log(x)}{8} + \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n}$$

Mathematica [A] time = 0.105067, size = 66, normalized size = 0.84

$$\frac{-4\cos(2a)\text{CosIntegral}(2bx^n) + \cos(4a)\text{CosIntegral}(4bx^n) + 4\sin(2a)\text{Si}(2bx^n) - \sin(4a)\text{Si}(4bx^n)}{8n} + \frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (-4*Cos[2*a]*CosIntegral[2*b*x^n] + Cos[4*a]*CosIntegral[4*b
*x^n] + 4*Sin[2*a]*SinIntegral[2*b*x^n] - Sin[4*a]*SinIntegral[4*b*x^n])/(8
*n)
```

Maple [A] time = 0.013, size = 77, normalized size = 1.

$$\frac{3\ln(bx^n)}{8n} - \frac{\text{Si}(4bx^n)\sin(4a)}{8n} + \frac{\text{Ci}(4bx^n)\cos(4a)}{8n} + \frac{\text{Si}(2bx^n)\sin(2a)}{2n} - \frac{\text{Ci}(2bx^n)\cos(2a)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^4/x,x)
```

```
[Out] 3/8/n*ln(b*x^n)-1/8*Si(4*b*x^n)*sin(4*a)/n+1/8*Ci(4*b*x^n)*cos(4*a)/n+1/2*S
i(2*b*x^n)*sin(2*a)/n-1/2*Ci(2*b*x^n)*cos(2*a)/n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 1.7933, size = 309, normalized size = 3.91

$$\frac{\cos(4a)\,\text{Ci}(4bx^n) - 4\cos(2a)\,\text{Ci}(2bx^n) - 4\cos(2a)\,\text{Ci}(-2bx^n) + \cos(4a)\,\text{Ci}(-4bx^n) + 6n\log(x) - 2\sin(4a)\,\text{Si}(4}{16n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(4*a)*cos_integral(4*b*x^n) - 4*cos(2*a)*cos_integral(2*b*x^n) - 4
*cos(2*a)*cos_integral(-2*b*x^n) + cos(4*a)*cos_integral(-4*b*x^n) + 6*n*lo
g(x) - 2*sin(4*a)*sin_integral(4*b*x^n) + 8*sin(2*a)*sin_integral(2*b*x^n))
/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^4(a + bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**4/x,x)
```

```
[Out] Integral(sin(a + b*x**n)**4/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx^n + a)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^4/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^4/x, x)
```

3.139 $\int \sin(a+bx^n)\,dx$

Optimal. Leaf size=87

$$\frac{ie^{ia}x\left(-ibx^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x\left(ibx^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[Out] ((I/2)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - ((I/2
)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rubi [A] time = 0.0270594, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3365, 2208}

$$\frac{ie^{ia}x\left(-ibx^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x\left(ibx^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - ((I/2
)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx^n)\,dx &= \frac{1}{2}i\int e^{-ia-ibx^n}\,dx-\frac{1}{2}i\int e^{ia+ibx^n}\,dx\\ &= \frac{ie^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A] time = 0.08404, size = 95, normalized size = 1.09

$$\frac{ix\left(b^2x^{2n}\right)^{-1/n}\left((\cos(a)+i\sin(a))\left(ibx^n\right)^{\frac{1}{n}}\operatorname{Gamma}\left(\frac{1}{n},-ibx^n\right)-(\cos(a)-i\sin(a))\left(-ibx^n\right)^{\frac{1}{n}}\operatorname{Gamma}\left(\frac{1}{n},ibx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*x*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[a] - I*Sin[a]))
 + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Sin[a])))/(n*(b^2
*x^(2*n))^n^(-1))
```

Maple [C] time = 0.077, size = 74, normalized size = 0.9

$$x_1F_2\left(\frac{1}{2n};\frac{1}{2},1+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\sin(a)+\frac{bx^{1+n}\cos(a)}{1+n}\,{}_1F_2\left(\frac{1}{2}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n),x)
```

```
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)+b/(1+n)*x^(1+n)*
hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx^n + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin(bx^n + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n),x)
```

```
[Out] Integral(sin(a + b*x**n), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(bx^n + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a), x)
```

3.140 $\int \sin^2(a+bx^n)\,dx$

Optimal. Leaf size=100

$$\frac{e^{2ia}2^{-\frac{1}{n}-2}x\left(-ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x\left(ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

```
[Out] x/2 + (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)*
b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)*a
)*n*(I*b*x^n)^n^(-1))
```

Rubi [A] time = 0.0721438, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3367, 3366, 2208}

$$\frac{e^{2ia}2^{-\frac{1}{n}-2}x\left(-ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x\left(ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^2,x]
```

```
[Out] x/2 + (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)*
b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)*a
)*n*(I*b*x^n)^n^(-1))
```

Rule 3367

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx^n)\,dx &= \int \left(\frac{1}{2}-\frac{1}{2}\cos(2a+2bx^n)\right)dx\\
&= \frac{x}{2}-\frac{1}{2}\int \cos(2a+2bx^n)\,dx\\
&= \frac{x}{2}-\frac{1}{4}\int e^{-2ia-2ibx^n}\,dx-\frac{1}{4}\int e^{2ia+2ibx^n}\,dx\\
&= \frac{x}{2}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}+\frac{2^{-2-\frac{1}{n}}e^{-2ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.223789, size = 94, normalized size = 0.94

$$\frac{x\left(e^{2ia}2^{-1/n}\left(-ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2ibx^n\right)+e^{-2ia}2^{-1/n}\left(ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2ibx^n\right)+2n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^2,x]
```

```
[Out] (x*(2*n + (E^((2*I)*a)*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b*x^n)^
n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^n^(-1)
)))/(4*n)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\sin(a+bx^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^2,x)
```

```
[Out] int(sin(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x - \frac{1}{2}\int \cos(2\,bx^n + 2\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*b*x^n + 2*a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\cos(bx^n+a)^2+1, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(-cos(b*x^n + a)^2 + 1, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^2(a+bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**2,x)
```

```
[Out] Integral(sin(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin{(bx^n + a)}^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^2, x)
```

3.141 $\int \sin^3(a+bx^n)\,dx$

Optimal. Leaf size=187

$$\frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right.}{8n}$$

```
[Out] (((3*I)/8)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - (
((3*I)/8)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1)) - ((I/8)*E
^((3*I)*a)*x*Gamma[n^(-1), (-3*I)*b*x^n])/(3^n^(-1)*n*((-I)*b*x^n)^n^(-1))
+ ((I/8)*x*Gamma[n^(-1), (3*I)*b*x^n])/(3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rubi [A] time = 0.0887636, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3367, 3365, 2208}

$$\frac{3ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x^n]^3,x]
```

```
[Out] (((3*I)/8)*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - (
((3*I)/8)*x*Gamma[n^(-1), I*b*x^n])/(E^(I*a)*n*(I*b*x^n)^n^(-1)) - ((I/8)*E
^((3*I)*a)*x*Gamma[n^(-1), (-3*I)*b*x^n])/(3^n^(-1)*n*((-I)*b*x^n)^n^(-1))
+ ((I/8)*x*Gamma[n^(-1), (3*I)*b*x^n])/(3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rule 3367

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\int \sin^3\left(a+bx^n\right)\,dx = \int \left(\frac{3}{4}\sin\left(a+bx^n\right)-\frac{1}{4}\sin\left(3a+3bx^n\right)\right)dx$$
$$= -\left(\frac{1}{4}\int \sin\left(3a+3bx^n\right)\,dx\right)+\frac{3}{4}\int \sin\left(a+bx^n\right)\,dx$$
$$= -\left(\frac{1}{8}i\int e^{-3ia-3ibx^n}\,dx\right)+\frac{1}{8}i\int e^{3ia+3ibx^n}\,dx+\frac{3}{8}i\int e^{-ia-ibx^n}\,dx-\frac{3}{8}i\int e^{ia+ibx^n}\,dx$$
$$= \frac{3ie^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}-\frac{i3^{-1/n}e^{3ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-3\right.}{8n}$$

Mathematica [A] time = 0.281482, size = 177, normalized size = 0.95

$$\frac{ie^{-3ia}3^{-1/n}x\left(b^2x^{2n}\right)^{-1/n}\left(e^{2ia}\left(-3^{\frac{1}{n}+1}\right)\left(-ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},ibx^n\right)+e^{4ia}3^{\frac{1}{n}+1}\left(ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},-ibx^n\right)-e^{6ia}\left(ibx^n\right)^{\frac{1}{n}}\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x^n]^3,x]
```

```
[Out] ((I/8)*x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*
x^n] - 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n
] - E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x^n)
^n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*n))^
n^(-1))
```

Maple [F] time = 0.287, size = 0, normalized size = 0.

$$\int \left(\sin\left(a+bx^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*x^n)^3,x)
```

```
[Out] int(sin(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(bx^n+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x^n + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(bx^n+a\right)^2-1\right)\sin\left(bx^n+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^3\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x**n)**3,x)
```

```
[Out] Integral(sin(a + b*x**n)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(bx^n+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x^n + a)^3, x)
```

3.142 $\int x^m \sin(a+bx^n)\,dx$

Optimal. Leaf size=109

$$\frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^((1
+ m)/n)) - ((I/2)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)
^((1 + m)/n))
```

Rubi [A] time = 0.0785224, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3423, 2218}

$$\frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^((1
+ m)/n)) - ((I/2)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(I*b*x^n)
^((1 + m)/n))
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\sin(a+bx^n)\,dx&=\frac{1}{2}i\int e^{-ia-ibx^n}x^m\,dx-\frac{1}{2}i\int e^{ia+ibx^n}x^m\,dx\\&=\frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n}-\frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A] time = 0.202344, size = 118, normalized size = 1.08

$$\frac{ix^{m+1}\left(b^2x^{2n}\right)^{-\frac{m+1}{n}}\left((\cos(a)+i\sin(a))(ibx^n)^{\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)-(\cos(a)-i\sin(a))(-ibx^n)^{\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},\right.\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x^n],x]
```

```
[Out] ((I/2)*x^(1 + m)*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n]*(Cos
[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n]*(Cos[
a] + I*Sin[a])))/(n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [C] time = 0.111, size = 110, normalized size = 1.

$$\frac{x^{1+m}\sin(a)}{1+m}\,{}_1F_2\left(\frac{m}{2n}+\frac{1}{2n};\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)+\frac{bx^{m+n+1}\cos(a)}{m+n+1}\,{}_1F_2\left(\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+b*x^n),x)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],-1/4*x^(2*n
)*b^2)*sin(a)+b/(m+n+1)*x^(m+n+1)*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+1/
2/n*m+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx^n + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \sin(bx^n + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*sin(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+b*x**n),x)
```

```
[Out] Integral(x**m*sin(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx^n + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x^n + a), x)
```

3.143 $\int x^m \sin^2(a + bx^n)\, dx$

Optimal. Leaf size=139

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},2ibx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

```
[Out] x^(1 + m)/(2*(1 + m)) + (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^
n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1 + m)*Gamma[(1
+ m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rubi [A] time = 0.168747, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3425, 3424, 2218}

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},2ibx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x^n]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^
n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1 + m)*Gamma[(1
+ m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^m \sin^2(a+bx^n)\,dx = \int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx^n)\right)dx$$
$$= \frac{x^{1+m}}{2(1+m)}-\frac{1}{2}\int x^m\cos(2a+2bx^n)\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}-\frac{1}{4}\int e^{-2ia-2ibx^n}x^m\,dx-\frac{1}{4}\int e^{2ia+2ibx^n}x^m\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)}{n}+\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma}{n}$$

Mathematica [A] time = 0.518604, size = 129, normalized size = 0.93

$$\frac{x^{m+1}\left(e^{2ia}(m+1)2^{-\frac{m+1}{n}}(-ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)+e^{-2ia}(m+1)2^{-\frac{m+1}{n}}(ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2ibx^n\right)+2\right.}{4(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x^n]^2,x]
```

```
[Out] (x^(1 + m)*(2*n + (E^((2*I)*a)*(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^n])/(2^(
(1 + m)/n)*((-I)*b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, (2*I)*b*x^
n])/(2^((1 + m)/n)*E^((2*I)*a)*(I*b*x^n)^((1 + m)/n))))/(4*(1 + m)*n)
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int x^m(\sin(a+bx^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+b*x^n)^2,x)
```

```
[Out] int(x^m*sin(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{xx^m-(m+1)\int x^m\cos(2\,bx^n+2\,a)\,dx}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))/(m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-x^m\cos(bx^n+a)^2+x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(-x^m*cos(b*x^n + a)^2 + x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^2(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+b*x**n)**2,x)
```

```
[Out] Integral(x**m*sin(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx^n + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x^n + a)^2, x)
```

3.144 $\int x^m \sin^3(a+bx^n)\,dx$

Optimal. Leaf size=237

$$\frac{3ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-}}{}$$

```
[Out] (((3*I)/8)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^
((1 + m)/n)) - (((3*I)/8)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(
I*b*x^n)^((1 + m)/n)) - ((I/8)*E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-3*I
)*b*x^n])/(3^((1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + ((I/8)*x^(1 + m)*Gam
ma[(1 + m)/n, (3*I)*b*x^n])/(3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rubi [A] time = 0.23821, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3425, 3423, 2218}

$$\frac{3ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{8n}-\frac{3ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{8n}-\frac{ie^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + b*x^n]^3,x]
```

```
[Out] (((3*I)/8)*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^
((1 + m)/n)) - (((3*I)/8)*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(E^(I*a)*n*(
I*b*x^n)^((1 + m)/n)) - ((I/8)*E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-3*I
)*b*x^n])/(3^((1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + ((I/8)*x^(1 + m)*Gam
ma[(1 + m)/n, (3*I)*b*x^n])/(3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^m \sin^3(a+bx^n)\,dx = \int \left(\frac{3}{4}x^m \sin(a+bx^n) - \frac{1}{4}x^m \sin(3a+3bx^n)\right) dx$$
$$= -\left(\frac{1}{4}\int x^m \sin(3a+3bx^n)\,dx\right) + \frac{3}{4}\int x^m \sin(a+bx^n)\,dx$$
$$= -\left(\frac{1}{8}i\int e^{-3ia-3ibx^n}x^m\,dx\right) + \frac{1}{8}i\int e^{3ia+3ibx^n}x^m\,dx + \frac{3}{8}i\int e^{-ia-ibx^n}x^m\,dx - \frac{3}{8}i\int e^{ia+ibx^n}x^m\,dx$$
$$= \frac{3ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n} - \frac{3ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n} - \frac{i3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}(}{}$$

Mathematica [A] time = 0.579187, size = 225, normalized size = 0.95

$$\frac{ie^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}\left(b^2x^{2n}\right)^{-\frac{m+1}{n}}\left(e^{2ia}\left(-3^{\frac{m+n+1}{n}}\right)(-ibx^n)^{\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)+e^{4ia}3^{\frac{m+n+1}{n}}(ibx^n)^{\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right.\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sin[a + b*x^n]^3,x]
```

```
[Out] ((I/8)*x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma
[(1 + m)/n, (-I)*b*x^n] - 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1 +
m)/n)*Gamma[(1 + m)/n, I*b*x^n] - E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(
1 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*I)*b
*x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int x^m (\sin(a+bx^n))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sin(a+b*x^n)^3,x)
```

```
[Out] int(x^m*sin(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx^n+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sin(b*x^n + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^m\cos(bx^n+a)^2 - x^m\right)\sin(bx^n+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(-(x^m*cos(b*x^n + a)^2 - x^m)*sin(b*x^n + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^3(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+b*x**n)**3,x)
```

```
[Out] Integral(x**m*sin(a + b*x**n)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sin(bx^n + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sin(b*x^n + a)^3, x)
```

3.145 $\int x^{-1+2n}\sin(a+bx^n)\,dx$

Optimal. Leaf size=35

$$\frac{\sin(a+bx^n)}{b^2n}-\frac{x^n\cos(a+bx^n)}{bn}$$

```
[Out] -((x^n*Cos[a + b*x^n])/(b*n)) + Sin[a + b*x^n]/(b^2*n)
```

Rubi [A] time = 0.0323856, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3379, 3296, 2637}

$$\frac{\sin(a+bx^n)}{b^2n}-\frac{x^n\cos(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -((x^n*Cos[a + b*x^n])/(b*n)) + Sin[a + b*x^n]/(b^2*n)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\sin(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int x\sin(a+bx)\,dx,x,x^n\right)}{n}\\ &= -\frac{x^n\cos(a+bx^n)}{bn}+\frac{\text{Subst}\left(\int\cos(a+bx)\,dx,x,x^n\right)}{bn}\\ &= -\frac{x^n\cos(a+bx^n)}{bn}+\frac{\sin(a+bx^n)}{b^2n}\end{aligned}$$

Mathematica [A] time = 0.0709262, size = 30, normalized size = 0.86

$$\frac{\sin(a+bx^n)-bx^n\cos(a+bx^n)}{b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sin[a + b*x^n],x]
```

```
[Out] (-(b*x^n*Cos[a + b*x^n]) + Sin[a + b*x^n])/(b^2*n)
```

Maple [A] time = 0.006, size = 44, normalized size = 1.3

$$\frac{\sin(a+bx^n)-(a+bx^n)\cos(a+bx^n)+a\cos(a+bx^n)}{nb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*sin(a+b*x^n),x)
```

```
[Out] 1/n/b^2*(sin(a+b*x^n)-(a+b*x^n)*cos(a+b*x^n)+a*cos(a+b*x^n))
```

Maxima [A] time = 1.01317, size = 43, normalized size = 1.23

$$-\frac{bx^n\cos(bx^n+a)-\sin(bx^n+a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Fricas [A] time = 1.72748, size = 68, normalized size = 1.94

$$-\frac{bx^n\cos(bx^n+a)-\sin(bx^n+a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -(b*x^n*cos(b*x^n + a) - sin(b*x^n + a))/(b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*sin(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2n-1} \sin\left(bx^n + a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)*sin(b*x^n + a), x)
```

3.146 $\int x^{-1+2n} \cos(a + bx^n) \, dx$

Optimal. Leaf size=34

$$\frac{\cos(a+bx^n)}{b^2n} + \frac{x^n \sin(a+bx^n)}{bn}$$

```
[Out] Cos[a + b*x^n]/(b^2*n) + (x^n*Sin[a + b*x^n])/(b*n)
```

Rubi [A] time = 0.030392, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3380, 3296, 2638}

$$\frac{\cos(a+bx^n)}{b^2n} + \frac{x^n \sin(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Cos[a + b*x^n],x]
```

```
[Out] Cos[a + b*x^n]/(b^2*n) + (x^n*Sin[a + b*x^n])/(b*n)
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+2n} \cos(a+bx^n) \, dx &= \frac{\text{Subst}\left(\int x \cos(a+bx) \, dx, x, x^n\right)}{n} \\
&= \frac{x^n \sin(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \sin(a+bx) \, dx, x, x^n\right)}{bn} \\
&= \frac{\cos(a+bx^n)}{b^2n} + \frac{x^n \sin(a+bx^n)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.0628097, size = 29, normalized size = 0.85

$$\frac{bx^n \sin(a+bx^n) + \cos(a+bx^n)}{b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Cos[a + b*x^n],x]
```

```
[Out] (Cos[a + b*x^n] + b*x^n*Sin[a + b*x^n])/(b^2*n)
```

Maple [A] time = 0.013, size = 44, normalized size = 1.3

$$\frac{\cos(a+bx^n)+(a+bx^n)\sin(a+bx^n)-a\sin(a+bx^n)}{nb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*cos(a+b*x^n),x)
```

```
[Out] 1/n/b^2*(cos(a+b*x^n)+(a+b*x^n)*sin(a+b*x^n)-a*sin(a+b*x^n))
```

Maxima [A] time = 1.00682, size = 39, normalized size = 1.15

$$\frac{bx^n\sin(bx^n+a)+\cos(bx^n+a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Fricas [A] time = 1.84286, size = 66, normalized size = 1.94

$$\frac{bx^n\sin(bx^n+a)+\cos(bx^n+a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(b^2*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*cos(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2n-1}\cos\left(bx^{n}+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)*cos(b*x^n + a), x)
```

3.147 $\int x^{-1-n} \sin(a+bx^n)\,dx$

Optimal. Leaf size=46

$$\frac{b\cos(a)\text{CosIntegral}(bx^n)}{n}-\frac{b\sin(a)\text{Si}(bx^n)}{n}-\frac{x^{-n}\sin(a+bx^n)}{n}$$

```
[Out] (b*Cos[a]*CosIntegral[b*x^n])/n - Sin[a + b*x^n]/(n*x^n) - (b*Sin[a]*SinInt
egral[b*x^n])/n
```

Rubi [A] time = 0.0887158, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3379, 3297, 3303, 3299, 3302}

$$\frac{b\cos(a)\text{CosIntegral}(bx^n)}{n}-\frac{b\sin(a)\text{Si}(bx^n)}{n}-\frac{x^{-n}\sin(a+bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n],x]
```

```
[Out] (b*Cos[a]*CosIntegral[b*x^n])/n - Sin[a + b*x^n]/(n*x^n) - (b*Sin[a]*SinInt
egral[b*x^n])/n
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int x^{-1-n}\sin\left(a+bx^n\right)dx=\frac{\operatorname{Subst}\left(\int\frac{\sin(a+bx)}{x^2}dx,x,x^n\right)}{n}$$
$$=-\frac{x^{-n}\sin\left(a+bx^n\right)}{n}+\frac{b\operatorname{Subst}\left(\int\frac{\cos(a+bx)}{x}dx,x,x^n\right)}{n}$$
$$=-\frac{x^{-n}\sin\left(a+bx^n\right)}{n}+\frac{(b\cos(a))\operatorname{Subst}\left(\int\frac{\cos(bx)}{x}dx,x,x^n\right)}{n}-\frac{(b\sin(a))\operatorname{Subst}\left(\int\frac{\sin(bx)}{x}dx,x,x^n\right)}{n}$$
$$=\frac{b\cos(a)\operatorname{Ci}\left(bx^n\right)}{n}-\frac{x^{-n}\sin\left(a+bx^n\right)}{n}-\frac{b\sin(a)\operatorname{Si}\left(bx^n\right)}{n}$$

Mathematica [A] time = 0.0743961, size = 47, normalized size = 1.02

$$\frac{x^{-n}\left(b\cos(a)x^n\operatorname{CosIntegral}\left(bx^n\right)-b\sin(a)x^n\operatorname{Si}\left(bx^n\right)-\sin\left(a+bx^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n],x]
```

```
[Out] (b*x^n*Cos[a]*CosIntegral[b*x^n] - Sin[a + b*x^n] - b*x^n*Sin[a]*SinIntegra
l[b*x^n])/(n*x^n)
```

Maple [A] time = 0.006, size = 44, normalized size = 1.

$$\frac{b}{n}\left(-\frac{\sin\left(a+bx^n\right)}{bx^n}-\operatorname{Si}\left(bx^n\right)\sin(a)+\operatorname{Ci}\left(bx^n\right)\cos(a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n-1)*sin(a+b*x^n),x)
```

```
[Out] 1/n*b*(-sin(a+b*x^n)/(x^n)/b-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1}\sin\left(bx^n+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

Fricas [A] time = 2.06855, size = 189, normalized size = 4.11

$$\frac{bx^n\cos(a)\operatorname{Ci}\left(bx^n\right)+bx^n\cos(a)\operatorname{Ci}\left(-bx^n\right)-2\,bx^n\sin(a)\operatorname{Si}\left(bx^n\right)-2\,\sin\left(bx^n+a\right)}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^n*cos(a)*cos_integral(b*x^n) + b*x^n*cos(a)*cos_integral(-b*x^n) -
 2*b*x^n*sin(a)*sin_integral(b*x^n) - 2*sin(b*x^n + a))/(n*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sin(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1}\sin\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a), x)
```

3.148 $\int x^{-1-n}\sin^2(a+bx^n)\,dx$

Optimal. Leaf size=67

$$\frac{b\sin(2a)\text{CosIntegral}(2bx^n)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

```
[Out] -1/(2*n*x^n) + Cos[2*(a + b*x^n)]/(2*n*x^n) + (b*CosIntegral[2*b*x^n]*Sin[2
*a])/n + (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Rubi [A] time = 0.120285, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3425, 3380, 3297, 3303, 3299, 3302}

$$\frac{b\sin(2a)\text{CosIntegral}(2bx^n)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

```
[Out] -1/(2*n*x^n) + Cos[2*(a + b*x^n)]/(2*n*x^n) + (b*CosIntegral[2*b*x^n]*Sin[2
*a])/n + (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int x^{-1-n}\sin^2(a+bx^n)\,dx = \int\left(\frac{x^{-1-n}}{2}-\frac{1}{2}x^{-1-n}\cos(2a+2bx^n)\right)dx$$
$$= -\frac{x^{-n}}{2n}-\frac{1}{2}\int x^{-1-n}\cos(2a+2bx^n)\,dx$$
$$= -\frac{x^{-n}}{2n}-\frac{\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x^2}dx,x,x^n\right)}{2n}$$
$$= -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{b\,\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x}dx,x,x^n\right)}{n}$$
$$= -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{(b\cos(2a))\,\text{Subst}\left(\int\frac{\sin(2bx)}{x}dx,x,x^n\right)}{n}+\frac{(b\sin(2a))\,\text{Subst}\left(\right.}{}$$
$$= -\frac{x^{-n}}{2n}+\frac{x^{-n}\cos(2(a+bx^n))}{2n}+\frac{b\text{Ci}(2bx^n)\sin(2a)}{n}+\frac{b\cos(2a)\text{Si}(2bx^n)}{n}$$

Mathematica [A] time = 0.130551, size = 58, normalized size = 0.87

$$\frac{x^{-n}\left(2b\sin(2a)x^n\text{CosIntegral}(2bx^n)+2b\cos(2a)x^n\text{Si}(2bx^n)+\cos(2(a+bx^n))-1\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n]^2,x]
```

```
[Out] (-1 + Cos[2*(a + b*x^n)] + 2*b*x^n*CosIntegral[2*b*x^n]*Sin[2*a] + 2*b*x^n*
Cos[2*a]*SinIntegral[2*b*x^n])/(2*n*x^n)
```

Maple [A] time = 0.021, size = 66, normalized size = 1.

$$-\frac{1}{2\,nx^n}-\frac{b}{n}\left(-\frac{\cos(2\,a+2\,bx^n)}{2\,bx^n}-\text{Si}(2\,bx^n)\cos(2\,a)-\text{Ci}(2\,bx^n)\sin(2\,a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n-1)*sin(a+b*x^n)^2,x)
```

```
[Out] -1/2/n/(x^n)-1/n*b*(-1/2*cos(2*a+2*b*x^n)/(x^n)/b-Si(2*b*x^n)*cos(2*a)-Ci(2
*b*x^n)*sin(2*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{nx^n\int\frac{\cos(2\,bx^n+2\,a)}{xx^n}dx+1}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(n*x^n*integrate(cos(2*b*x^n + 2*a)/(x*x^n), x) + 1)/(n*x^n)
```

Fricas [A] time = 2.14758, size = 213, normalized size = 3.18

$$\frac{bx^n \operatorname{Ci}(2\,bx^n)\sin(2\,a) + bx^n \operatorname{Ci}(-2\,bx^n)\sin(2\,a) + 2\,bx^n\cos(2\,a)\operatorname{Si}(2\,bx^n) + 2\,\cos(bx^n + a)^2 - 2}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^n*cos_integral(2*b*x^n)*sin(2*a) + b*x^n*cos_integral(-2*b*x^n)*si
n(2*a) + 2*b*x^n*cos(2*a)*sin_integral(2*b*x^n) + 2*cos(b*x^n + a)^2 - 2)/(
n*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sin(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1} \sin(bx^n + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^2, x)
```

3.149 $\int x^{-1-n}\sin^3(a+bx^n)\,dx$

Optimal. Leaf size=113

$$\frac{3b\cos(a)\text{CosIntegral}(bx^n)}{4n}-\frac{3b\cos(3a)\text{CosIntegral}(3bx^n)}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\text{Si}(3bx^n)}{4n}-\frac{3x^{-n}\sin(a+}{4n}$$

```
[Out] (3*b*Cos[a]*CosIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*CosIntegral[3*b*x^n])
/(4*n) - (3*Sin[a + b*x^n])/(4*n*x^n) + Sin[3*(a + b*x^n)]/(4*n*x^n) - (3*b
*Sin[a]*SinIntegral[b*x^n])/(4*n) + (3*b*Sin[3*a]*SinIntegral[3*b*x^n])/(4*
n)
```

Rubi [A] time = 0.214816, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3425, 3379, 3297, 3303, 3299, 3302}

$$\frac{3b\cos(a)\text{CosIntegral}(bx^n)}{4n}-\frac{3b\cos(3a)\text{CosIntegral}(3bx^n)}{4n}-\frac{3b\sin(a)\text{Si}(bx^n)}{4n}+\frac{3b\sin(3a)\text{Si}(3bx^n)}{4n}-\frac{3x^{-n}\sin(a+}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```



```
[Out] (3*b*Cos[a]*CosIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*CosIntegral[3*b*x^n])
/(4*n) - (3*Sin[a + b*x^n])/(4*n*x^n) + Sin[3*(a + b*x^n)]/(4*n*x^n) - (3*b
*Sin[a]*SinIntegral[b*x^n])/(4*n) + (3*b*Sin[3*a]*SinIntegral[3*b*x^n])/(4*
n)
```


Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\sin^3\left(a+bx^n\right)dx &= \int\left(\frac{3}{4}x^{-1-n}\sin\left(a+bx^n\right)-\frac{1}{4}x^{-1-n}\sin\left(3a+3bx^n\right)\right)dx\\ &= -\left(\frac{1}{4}\int x^{-1-n}\sin\left(3a+3bx^n\right)dx\right)+\frac{3}{4}\int x^{-1-n}\sin\left(a+bx^n\right)dx\\ &= -\frac{\text{Subst}\left(\int\frac{\sin(3a+3bx)}{x^2}dx,x,x^n\right)}{4n}+\frac{3\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}dx,x,x^n\right)}{4n}\\ &= -\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3\left(a+bx^n\right)\right)}{4n}+\frac{(3b)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x}dx,x,x^n\right)}{4n}-\frac{(3b)\,\text{Su}}{}\\ &= -\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3\left(a+bx^n\right)\right)}{4n}+\frac{(3b\cos(a))\,\text{Subst}\left(\int\frac{\cos(bx)}{x}dx,x,x^n\right)}{4n}-\frac{(3}{}\\ &= \frac{3b\cos(a)\text{Ci}\left(bx^n\right)}{4n}-\frac{3b\cos(3a)\text{Ci}\left(3bx^n\right)}{4n}-\frac{3x^{-n}\sin\left(a+bx^n\right)}{4n}+\frac{x^{-n}\sin\left(3\left(a+bx^n\right)\right)}{4n}-\end{aligned}$$

Mathematica [A] time = 0.189532, size = 95, normalized size = 0.84

$$\frac{x^{-n}\left(3b\cos(a)x^n\text{CosIntegral}\left(bx^n\right)-3b\cos(3a)x^n\text{CosIntegral}\left(3bx^n\right)-3b\sin(a)x^n\text{Si}\left(bx^n\right)+3b\sin(3a)x^n\text{Si}\left(3bx^n\right)\right.}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (3*b*x^n*Cos[a]*CosIntegral[b*x^n] - 3*b*x^n*Cos[3*a]*CosIntegral[3*b*x^n]
- 3*Sin[a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b*x^n*Sin[a]*SinIntegral[b*x^n]
 + 3*b*x^n*Sin[3*a]*SinIntegral[3*b*x^n])/(4*n*x^n)
```

Maple [A] time = 0.018, size = 99, normalized size = 0.9

$$\frac{3\,b}{4\,n}\left(-\frac{\sin\left(a+bx^n\right)}{bx^n}-\text{Si}\left(bx^n\right)\sin\left(a\right)+\text{Ci}\left(bx^n\right)\cos\left(a\right)\right)-\frac{3\,b}{4\,n}\left(-\frac{\sin\left(3\,a+3\,bx^n\right)}{3\,bx^n}-\text{Si}\left(3\,bx^n\right)\sin\left(3\,a\right)+\text{Ci}\left(3\,bx^n\right)\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n-1)*sin(a+b*x^n)^3,x)
```

```
[Out] 3/4/n*b*(-sin(a+b*x^n)/(x^n)/b-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))-3/4/n*b*(
-1/3*sin(3*a+3*b*x^n)/(x^n)/b-Si(3*b*x^n)*sin(3*a)+Ci(3*b*x^n)*cos(3*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1} \sin\left(bx^n + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

Fricas [A] time = 2.16657, size = 394, normalized size = 3.49

$$-\frac{3\,bx^n\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^n\right) - 3\,bx^n\cos\left(a\right)\mathrm{Ci}\left(bx^n\right) - 3\,bx^n\cos\left(a\right)\mathrm{Ci}\left(-bx^n\right) + 3\,bx^n\cos\left(3\,a\right)\mathrm{Ci}\left(-3\,bx^n\right) - 6\,bx^n\sin\left(3\,a\right)\mathrm{S}}{8\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*b*x^n*cos(3*a)*cos_integral(3*b*x^n) - 3*b*x^n*cos(a)*cos_integral(
b*x^n) - 3*b*x^n*cos(a)*cos_integral(-b*x^n) + 3*b*x^n*cos(3*a)*cos_integra
l(-3*b*x^n) - 6*b*x^n*sin(3*a)*sin_integral(3*b*x^n) + 6*b*x^n*sin(a)*sin_i
ntegral(b*x^n) - 8*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(n*x^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sin(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-n-1} \sin\left(bx^n + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sin(b*x^n + a)^3, x)
```

3.150 $\int x^{-1-2n} \sin(a+bx^n)\,dx$

Optimal. Leaf size=78

$$-\frac{b^2\sin(a)\mathrm{CosIntegral}\left(bx^n\right)}{2n}-\frac{b^2\cos(a)\mathrm{Si}\left(bx^n\right)}{2n}-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}-\frac{bx^{-n}\cos\left(a+bx^n\right)}{2n}$$

```
[Out] -(b*Cos[a + b*x^n])/(2*n*x^n) - (b^2*CosIntegral[b*x^n]*Sin[a])/(2*n) - Sin
[a + b*x^n]/(2*n*x^(2*n)) - (b^2*Cos[a]*SinIntegral[b*x^n])/(2*n)
```

Rubi [A] time = 0.110696, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3379, 3297, 3303, 3299, 3302}

$$-\frac{b^2\sin(a)\mathrm{CosIntegral}\left(bx^n\right)}{2n}-\frac{b^2\cos(a)\mathrm{Si}\left(bx^n\right)}{2n}-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}-\frac{bx^{-n}\cos\left(a+bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -(b*Cos[a + b*x^n])/(2*n*x^n) - (b^2*CosIntegral[b*x^n]*Sin[a])/(2*n) - Sin
[a + b*x^n]/(2*n*x^(2*n)) - (b^2*Cos[a]*SinIntegral[b*x^n])/(2*n)
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int x^{-1-2n}\sin\left(a+bx^n\right)\,dx=\frac{\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}\,dx,x,x^n\right)}{n}$$
$$=-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}+\frac{b\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}\,dx,x,x^n\right)}{2n}$$
$$=-\frac{bx^{-n}\cos\left(a+bx^n\right)}{2n}-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}-\frac{b^2\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}\,dx,x,x^n\right)}{2n}$$
$$=-\frac{bx^{-n}\cos\left(a+bx^n\right)}{2n}-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}-\frac{\left(b^2\cos(a)\right)\text{Subst}\left(\int\frac{\sin(bx)}{x}\,dx,x,x^n\right)}{2n}-\frac{\left(b^2\sin(a\right.}{}$$
$$=-\frac{bx^{-n}\cos\left(a+bx^n\right)}{2n}-\frac{b^2\text{Ci}\left(bx^n\right)\sin(a)}{2n}-\frac{x^{-2n}\sin\left(a+bx^n\right)}{2n}-\frac{b^2\cos(a)\text{Si}\left(bx^n\right)}{2n}$$

Mathematica [A] time = 0.122716, size = 68, normalized size = 0.87

$$-\frac{x^{-2n}\left(b^2\sin(a)x^{2n}\text{CosIntegral}\left(bx^n\right)+b^2\cos(a)x^{2n}\text{Si}\left(bx^n\right)+\sin\left(a+bx^n\right)+bx^n\cos\left(a+bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n],x]
```

```
[Out] -(b*x^n*Cos[a + b*x^n] + b^2*x^(2*n)*CosIntegral[b*x^n]*Sin[a] + Sin[a + b*
x^n] + b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n])/(2*n*x^(2*n))
```

Maple [A] time = 0.007, size = 65, normalized size = 0.8

$$\frac{b^2}{n}\left(-\frac{\sin\left(a+bx^n\right)}{2\left(x^n\right)^2b^2}-\frac{\cos\left(a+bx^n\right)}{2\,bx^n}-\frac{\text{Si}\left(bx^n\right)\cos\left(a\right)}{2}-\frac{\text{Ci}\left(bx^n\right)\sin\left(a\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*sin(a+b*x^n),x)
```

```
[Out] 1/n*b^2*(-1/2*sin(a+b*x^n)/(x^n)^2/b^2-1/2*cos(a+b*x^n)/(x^n)/b-1/2*Si(b*x^
n)*cos(a)-1/2*Ci(b*x^n)*sin(a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-2\,n-1}\sin\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

Fricas [A] time = 1.99264, size = 254, normalized size = 3.26

$$-\frac{b^2x^{2n}\,\mathrm{Ci}\,(bx^n)\sin\,(a)+b^2x^{2n}\,\mathrm{Ci}\,(-bx^n)\sin\,(a)+2\,b^2x^{2n}\cos\,(a)\,\mathrm{Si}\,(bx^n)+2\,bx^n\cos\,(bx^n+a)+2\,\sin\,(bx^n+a)}{4\,nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*x^(2*n)*cos_integral(b*x^n)*sin(a) + b^2*x^(2*n)*cos_integral(-b*
x^n)*sin(a) + 2*b^2*x^(2*n)*cos(a)*sin_integral(b*x^n) + 2*b*x^n*cos(b*x^n
+ a) + 2*sin(b*x^n + a))/(n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-2n-1}\sin\,(bx^n+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a), x)
```

3.151 $\int x^{-1-2n}\sin^2(a+bx^n)\,dx$

Optimal. Leaf size=95

$$\frac{b^2\cos(2a)\text{CosIntegral}(2bx^n)}{n}-\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{x^{-2n}}{4n}$$

```
[Out] -1/(4*n*x^(2*n)) + Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) + (b^2*Cos[2*a]*CosInte
gral[2*b*x^n])/n - (b*Sin[2*(a + b*x^n)])/(2*n*x^n) - (b^2*Sin[2*a]*SinInte
gral[2*b*x^n])/n
```

Rubi [A] time = 0.15066, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3425, 3380, 3297, 3303, 3299, 3302}

$$\frac{b^2\cos(2a)\text{CosIntegral}(2bx^n)}{n}-\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{x^{-2n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```


```
[Out] -1/(4*n*x^(2*n)) + Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) + (b^2*Cos[2*a]*CosInte
gral[2*b*x^n])/n - (b*Sin[2*(a + b*x^n)])/(2*n*x^n) - (b^2*Sin[2*a]*SinInte
gral[2*b*x^n])/n
```


Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-2n}\sin^2(a+bx^n)\,dx &= \int\left(\frac{1}{2}x^{-1-2n}-\frac{1}{2}x^{-1-2n}\cos(2a+2bx^n)\right)dx\\
&= -\frac{x^{-2n}}{4n}-\frac{1}{2}\int x^{-1-2n}\cos(2a+2bx^n)\,dx\\
&= -\frac{x^{-2n}}{4n}-\frac{\operatorname{Subst}\left(\int\frac{\cos(2a+2bx)}{x^3}\,dx,x,x^n\right)}{2n}\\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}+\frac{b\operatorname{Subst}\left(\int\frac{\sin(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}\\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+\frac{b^2\operatorname{Subst}\left(\int\frac{\cos(2a+2bx)}{x}\,dx,x,x\right.}{n}\\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+\frac{\left(b^2\cos(2a)\right)\operatorname{Subst}\left(\int\frac{\cos(2bx)}{x}\right.}{n}\\
&= -\frac{x^{-2n}}{4n}+\frac{x^{-2n}\cos(2(a+bx^n))}{4n}+\frac{b^2\cos(2a)\operatorname{Ci}(2bx^n)}{n}-\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{b^2\sin(}{}
\end{aligned}$$

Mathematica [A] time = 0.172302, size = 82, normalized size = 0.86

$$\frac{x^{-2n}\left(4b^2\cos(2a)x^{2n}\text{CosIntegral}(2bx^n)-4b^2\sin(2a)x^{2n}\text{Si}(2bx^n)-2bx^n\sin(2(a+bx^n))+\cos(2(a+bx^n))-1\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^2,x]
```

```
[Out] (-1 + Cos[2*(a + b*x^n)] + 4*b^2*x^(2*n)*Cos[2*a]*CosIntegral[2*b*x^n] - 2*
b*x^n*Sin[2*(a + b*x^n)] - 4*b^2*x^(2*n)*Sin[2*a]*SinIntegral[2*b*x^n])/(4*
n*x^(2*n))
```

Maple [A] time = 0.019, size = 89, normalized size = 0.9

$$-\frac{1}{4\,(x^n)^2\,n}-2\,\frac{b^2}{n}\left(-1/8\,\frac{\cos(2\,a+2\,bx^n)}{(x^n)^2\,b^2}+1/4\,\frac{\sin(2\,a+2\,bx^n)}{bx^n}+1/2\,\text{Si}(2\,bx^n)\sin(2\,a)-1/2\,\text{Ci}(2\,bx^n)\cos(2\,a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*sin(a+b*x^n)^2,x)
```

```
[Out] -1/4/(x^n)^2/n-2/n*b^2*(-1/8/(x^n)^2/b^2*cos(2*a+2*b*x^n)+1/4*sin(2*a+2*b*x
^n)/(x^n)/b+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.93317, size = 294, normalized size = 3.09

$$\frac{b^2x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^n\right)+b^2x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,bx^n\right)-2\,b^2x^{2\,n}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^n\right)-2\,bx^n\cos\left(bx^n+a\right)\sin\left(bx^n+a\right)+c}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^(2*n)*cos(2*a)*cos_integral(2*b*x^n) + b^2*x^(2*n)*cos(2*a)*cos_
integral(-2*b*x^n) - 2*b^2*x^(2*n)*sin(2*a)*sin_integral(2*b*x^n) - 2*b*x^n
*cos(b*x^n + a)*sin(b*x^n + a) + cos(b*x^n + a)^2 - 1)/(n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-2\,n-1}\sin\left(bx^n+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^2, x)
```

3.152 $\int x^{-1-2n}\sin^3(a+bx^n)\,dx$

Optimal. Leaf size=165

$$-\frac{3b^2\sin(a)\text{CosIntegral}(bx^n)}{8n}+\frac{9b^2\sin(3a)\text{CosIntegral}(3bx^n)}{8n}-\frac{3b^2\cos(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n}-\frac{3x^{-2}}{}$$

```
[Out] (-3*b*Cos[a + b*x^n])/(8*n*x^n) + (3*b*Cos[3*(a + b*x^n)])/(8*n*x^n) - (3*b
^2*CosIntegral[b*x^n]*Sin[a])/(8*n) + (9*b^2*CosIntegral[3*b*x^n]*Sin[3*a])
/(8*n) - (3*Sin[a + b*x^n])/(8*n*x^(2*n)) + Sin[3*(a + b*x^n)]/(8*n*x^(2*n)
) - (3*b^2*Cos[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Cos[3*a]*SinIntegral[3
*b*x^n])/(8*n)
```

Rubi [A] time = 0.263945, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3425, 3379, 3297, 3303, 3299, 3302}

$$-\frac{3b^2\sin(a)\text{CosIntegral}(bx^n)}{8n}+\frac{9b^2\sin(3a)\text{CosIntegral}(3bx^n)}{8n}-\frac{3b^2\cos(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\cos(3a)\text{Si}(3bx^n)}{8n}-\frac{3x^{-2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```


```
[Out] (-3*b*Cos[a + b*x^n])/(8*n*x^n) + (3*b*Cos[3*(a + b*x^n)])/(8*n*x^n) - (3*b
^2*CosIntegral[b*x^n]*Sin[a])/(8*n) + (9*b^2*CosIntegral[3*b*x^n]*Sin[3*a])
/(8*n) - (3*Sin[a + b*x^n])/(8*n*x^(2*n)) + Sin[3*(a + b*x^n)]/(8*n*x^(2*n)
) - (3*b^2*Cos[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Cos[3*a]*SinIntegral[3
*b*x^n])/(8*n)
```


Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-2n}\sin^3(a+bx^n)\,dx &= \int\left(\frac{3}{4}x^{-1-2n}\sin(a+bx^n)-\frac{1}{4}x^{-1-2n}\sin(3a+3bx^n)\right)dx\\
&= -\left(\frac{1}{4}\int x^{-1-2n}\sin(3a+3bx^n)\,dx\right)+\frac{3}{4}\int x^{-1-2n}\sin(a+bx^n)\,dx\\
&= -\frac{\text{Subst}\left(\int\frac{\sin(3a+3bx)}{x^3}dx,x,x^n\right)}{4n}+\frac{3\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^3}dx,x,x^n\right)}{4n}\\
&= -\frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3(a+bx^n))}{8n}+\frac{(3b)\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}dx,x,x^n\right)}{8n}-\frac{(3b)\,\text{Su}}{}\\
&= -\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n}-\frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3(a+bx}{8n}\\
&= -\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n}-\frac{3x^{-2n}\sin(a+bx^n)}{8n}+\frac{x^{-2n}\sin(3(a+bx}{8n}\\
&= -\frac{3bx^{-n}\cos(a+bx^n)}{8n}+\frac{3bx^{-n}\cos(3(a+bx^n))}{8n}-\frac{3b^2\text{Ci}(bx^n)\sin(a)}{8n}+\frac{9b^2\text{Ci}(3bx^n)\sin(3a}{8n}
\end{aligned}$$

Mathematica [A] time = 0.284787, size = 141, normalized size = 0.85

$$\frac{x^{-2n}\left(-3b^2\sin(a)x^{2n}\text{CosIntegral}(bx^n)+9b^2\sin(3a)x^{2n}\text{CosIntegral}(3bx^n)-3b^2\cos(a)x^{2n}\text{Si}(bx^n)+9b^2\cos(3a)x^{2n}\text{Si}\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sin[a + b*x^n]^3,x]
```

```
[Out] (-3*b*x^n*Cos[a + b*x^n] + 3*b*x^n*Cos[3*(a + b*x^n)] - 3*b^2*x^(2*n)*CosIn
tegral[b*x^n]*Sin[a] + 9*b^2*x^(2*n)*CosIntegral[3*b*x^n]*Sin[3*a] - 3*Sin[
a + b*x^n] + Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Cos[a]*SinIntegral[b*x^n] +
 9*b^2*x^(2*n)*Cos[3*a]*SinIntegral[3*b*x^n])/(8*n*x^(2*n))
```

Maple [A] time = 0.014, size = 144, normalized size = 0.9

$$\frac{3b^2}{4n}\left(-\frac{\sin(a+bx^n)}{2(x^n)^2b^2}-\frac{\cos(a+bx^n)}{2bx^n}-\frac{\text{Si}(bx^n)\cos(a)}{2}-\frac{\text{Ci}(bx^n)\sin(a)}{2}\right)-\frac{9b^2}{4n}\left(-\frac{\sin(3a+3bx^n)}{18(x^n)^2b^2}-\frac{\cos(3a+3bx^n)}{6bx^n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*sin(a+b*x^n)^3,x)
```

```
[Out] 3/4/n*b^2*(-1/2*sin(a+b*x^n)/(x^n)^2/b^2-1/2*cos(a+b*x^n)/(x^n)/b-1/2*Si(b*
x^n)*cos(a)-1/2*Ci(b*x^n)*sin(a))-9/4/n*b^2*(-1/18*sin(3*a+3*b*x^n)/(x^n)^2
/b^2-1/6*cos(3*a+3*b*x^n)/(x^n)/b-1/2*Si(3*b*x^n)*cos(3*a)-1/2*Ci(3*b*x^n)*
sin(3*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-2n-1} \sin(bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

Fricas [A] time = 1.9991, size = 522, normalized size = 3.16

$$24\,bx^n \cos(bx^n + a)^3 + 9\,b^2x^{2\,n}\,\mathrm{Ci}(3\,bx^n)\sin(3\,a) + 9\,b^2x^{2\,n}\,\mathrm{Ci}(-3\,bx^n)\sin(3\,a) - 3\,b^2x^{2\,n}\,\mathrm{Ci}(bx^n)\sin(a) - 3\,b^2x^{2\,n}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*b*x^n*cos(b*x^n + a)^3 + 9*b^2*x^(2*n)*cos_integral(3*b*x^n)*sin(3
*a) + 9*b^2*x^(2*n)*cos_integral(-3*b*x^n)*sin(3*a) - 3*b^2*x^(2*n)*cos_int
egral(b*x^n)*sin(a) - 3*b^2*x^(2*n)*cos_integral(-b*x^n)*sin(a) + 18*b^2*x^
(2*n)*cos(3*a)*sin_integral(3*b*x^n) - 6*b^2*x^(2*n)*cos(a)*sin_integral(b*
x^n) - 24*b*x^n*cos(b*x^n + a) + 8*(cos(b*x^n + a)^2 - 1)*sin(b*x^n + a))/(
n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*sin(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{-2n-1} \sin(bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*sin(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*sin(b*x^n + a)^3, x)
```

3.153 $\int (e+fx)^3 \sin \left(b(c+dx)^2\right) dx$

Optimal. Leaf size=223

$$\frac{3\sqrt{\frac{\pi}{2}} f^2(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(b(c+dx)^2\right)}{2b^2d^4}-\frac{3f^2(c+dx)(de-cf)\cos\left(b(c+dx)^2\right)}{2bd^4}+\frac{\sqrt{\frac{\pi}{2}}(de-}{}$$

```
[Out] (-3*f*(d*e - c*f)^2*Cos[b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*(c +
 d*x)*Cos[b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[b*(c + d*x)^2])/
(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(Sqrt[b]*d^4) + (f^3*Sin[b*(c + d*x)^2])/(2*b^2*d^4)
```

Rubi [A] time = 0.311852, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3433, 3351, 3379, 2638, 3385, 3352, 3296, 2637}

$$\frac{3\sqrt{\frac{\pi}{2}} f^2(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(b(c+dx)^2\right)}{2b^2d^4}-\frac{3f^2(c+dx)(de-cf)\cos\left(b(c+dx)^2\right)}{2bd^4}+\frac{\sqrt{\frac{\pi}{2}}(de-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```


```
[Out] (-3*f*(d*e - c*f)^2*Cos[b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*(c +
 d*x)*Cos[b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[b*(c + d*x)^2])/
(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(Sqrt[b]*d^4) + (f^3*Sin[b*(c + d*x)^2])/(2*b^2*d^4)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(b(c+dx)^2\right) dx = \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf\left(3d^2e^2-3cdef+c^2f^2\right)}{d^3e^3}\right)\sin\left(bx^2\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\sin\left(bx^2\right)\right.}{d^4}$$

$$= \frac{f^3\,\text{Subst}\left(\int x^3\sin\left(bx^2\right)dx,x,c+dx\right)}{d^4}+\frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x^2\sin\left(bx^2\right)dx,x,c\right.}{d^4}$$

$$= -\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4}+\frac{(de-cf)^3\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^4}+\frac{f^3\,\text{Subs}}{}$$

$$= -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\,c}{2}$$

$$= -\frac{3f(de-cf)^2\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{3f^2(de-cf)(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^4}-\frac{f^3(c+dx)^2\,c}{2}$$

Mathematica [A] time = 1.02722, size = 173, normalized size = 0.78

$$\frac{4\sqrt{2\pi}b^{3/2}(de-cf)^3S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)-4bf\cos\left(b(c+dx)^2\right)\left(c^2f^2-cdf(3e+fx)+d^2\left(3e^2+3efx+f^2x^2\right)\right)-6\sqrt{2\pi}\sqrt{b}}{8b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos
[b*(c + d*x)^2] - 6*Sqrt[b]*f^2*(-(d*e) + c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)] + 4*b^(3/2)*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelS[Sqrt[b]
*Sqrt[2/Pi]*(c + d*x)] + 4*f^3*Sin[b*(c + d*x)^2])/(8*b^2*d^4)
```

Maple [B] time = 0.014, size = 586, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin((d*x+c)^2*b),x)
```

```
[Out] -1/2*f^3/d^2/b*x^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^3*c/d*(-1/2/d^2/b*x*cos
(b*d^2*x^2+2*b*c*d*x+b*c^2)-c/d*(-1/2/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-
1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1
/2)*(b*d^2*x+b*c*d)))+1/4/d^2/b*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelC(2^(
1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))+f^3/d^2/b*(1/2/d^2/b*sin(b*d^
2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelC(2^(1
/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))-3/2*e*f^2/d^2/b*x*cos(b*d^2*x^
2+2*b*c*d*x+b*c^2)-3*e*f^2*c/d*(-1/2/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1
/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/
2)*(b*d^2*x+b*c*d)))+3/4*e*f^2/d^2/b*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-3/2*e^2*f/d^2/b*cos(b*d^2
*x^2+2*b*c*d*x+b*c^2)-3/2*e^2*f*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS
(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*Pi^(1/2)/(d^2*
b)^(1/2)*e^3*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))
```

Maxima [C] time = 4.5003, size = 3780, normalized size = 16.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*((I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arct
an2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0,
 b)))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I*cos(1/4*pi + 1/2*arctan2(0, b))
 + I*cos(-1/4*pi + 1/2*arctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + s
in(-1/4*pi + 1/2*arctan2(0, b)))*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))*e^3/(d*
sqrt(abs(b))) - 3/4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2) - (((
-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sq
rt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c*d*x + (-2
*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt
(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2)*cos(1/2*
arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) - 2*((sqrt(pi)*(erf(sqrt(I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c*d*x + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 +
2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*
x - I*b*c^2)) - 1))*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^
2, 0)))*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*e^2*f/(b*d^2*abs(4*b*
d^2*x^2 + 8*b*c*d*x + 4*b*c^2)) + 3/32*(16*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)^
2*c*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*
x - I*b*c^2)) - (((-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^
2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1)
)*b*c^2*d*x + (-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
- 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*
c^3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8
*b*c*d*x + 4*b*c^2, 0)) - ((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x +
```

```
I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
- 1))*b*c^2*d*x + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
 - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c
^3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*
b*c*d*x + 4*b*c^2, 0)) + (b*d^3*x^3*(4*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d
*x + I*b*c^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b*c
*d^2*x^2*(12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 12*I*gamma
(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b*c^2*d*x*(12*I*gamma(3/2, I
*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 12*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*
c*d*x - I*b*c^2)) + b*c^3*(4*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c
^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*cos(3/2*arctan
2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + 4*(b*d^3*x^3*(gamma(3/2, I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2)) + 3*b*c*d^2*x^2*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + g
amma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 3*b*c^2*d*x*(gamma(3/2,
I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*
x - I*b*c^2)) + b*c^3*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + ga
mma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*sin(3/2*arctan2(4*b*d^2*x^
2 + 8*b*c*d*x + 4*b*c^2, 0)))*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))
*e*f^2/((b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*b*d^3) - 1/64*(16*(b*d^2*x^2 + 2*
b*c*d*x + b*c^2)^2*(3*b*c^2*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-
I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - I*gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2) + I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - (((-2*I*
sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt(pi
)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b^2*c^3*d*x + (-2*
I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt(
pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b^2*c^4)*abs(4*b
*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4
*b*c^2, 0)) - 2*((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
- 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b^2*
c^3*d*x + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) +
sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b^2*c^4)*ab
s(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*
x + 4*b*c^2, 0)) + (b^2*c*d^3*x^3*(24*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2) - 24*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b^2
*c^2*d^2*x^2*(72*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 72*I*g
amma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b^2*c^3*d*x*(72*I*gamma(
3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 72*I*gamma(3/2, -I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2)) + b^2*c^4*(24*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2) - 24*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*cos(
3/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + 24*(b^2*c*d^3*x^3*(gam
ma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2)) + 3*b^2*c^2*d^2*x^2*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c
*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 3*b^2
*c^3*d*x*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b
*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b^2*c^4*(gamma(3/2, I*b*d^2*x^2 + 2*I*
b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*sin
(3/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)))*sqrt(abs(4*b*d^2*x^2 +
 8*b*c*d*x + 4*b*c^2)))*f^3/((b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*b^2*d^4)
```

Fricas [A] time = 1.67144, size = 582, normalized size = 2.61

$$\frac{2\,df^3\sin\left(bd^2x^2+2\,bcdx+bc^2\right)+3\,\sqrt{2}\pi\left(def^2-cf^3\right)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+2\,\sqrt{2}\pi\left(bd^3e^3-3\,bcd^2e^2f+3\,bc^2def^2-b\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2) + 3*sqrt(2)*pi*(d*e*f^2 - c
*f^3)*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) + 2*sq
rt(2)*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt(b
*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3*x^2
 + 3*b*d^3*e^2*f - 3*b*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2
*f^3)*x)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^3 \sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] time = 1.24218, size = 1381, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^3/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt
(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x +
 c/d))*e^3/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4*(-3*I*sqrt(2)*s
qrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c
/d))*e^2/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 3*f*e^(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2 + 2)/(b*d))/d - 1/4*(3*I*sqrt(2)*sqrt(pi)*c*f*erf(-1/
2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^2/(sqrt(b*d
^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) + 3*f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*
c^2 + 2)/(b*d))/d - 1/8*(I*sqrt(2)*sqrt(pi)*(6*b*c^2*f^2 - 3*I*f^2)*erf(-1/
2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e/(sqrt(b*d^2)
*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*I*(d*f^2*(-3*I*x - 3*I*c/d) + 6*I*c*f^2
)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 + 1)/(b*d))/d^2 - 1/8*(-I*sqrt(2)
*sqrt(pi)*(6*b*c^2*f^2 + 3*I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sq
rt(b^2*d^4) + 1)*(x + c/d))*e/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b)
+ 2*I*(d*f^2*(-3*I*x - 3*I*c/d) + 6*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x +
 I*b*c^2 + 1)/(b*d))/d^2 + 1/8*(sqrt(2)*sqrt(pi)*(2*I*b*c^3*f^3 + 3*c*f^3)*
erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b
*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)^2 - 3*b*c*d*f
^3*(x + c/d) + 3*b*c^2*f^3 - I*f^3)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
)/(b^2*d))/d^3 + 1/8*(sqrt(2)*sqrt(pi)*(-2*I*b*c^3*f^3 + 3*c*f^3)*erf(-1/2*
sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(-
I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)^2 - 3*b*c*d*f^3*(x +
 c/d) + 3*b*c^2*f^3 + I*f^3)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b^2*d
))/d^3
```

3.154 $\int (e+fx)^2 \sin\left(b(c+dx)^2\right) dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\frac{\pi}{2}}f^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^3}+\frac{\sqrt{\frac{\pi}{2}}(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}-\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(b(\right.}{2bd^3}$$

```
[Out] -((f*(d*e - c*f)*Cos[b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[b*(c + d
*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])
/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)])/(Sqrt[b]*d^3)
```

Rubi [A] time = 0.163992, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3433, 3351, 3379, 2638, 3385, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}f^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^3}+\frac{\sqrt{\frac{\pi}{2}}(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3}-\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3}-\frac{f^2(c+dx)\cos\left(b(\right.}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```


```
[Out] -((f*(d*e - c*f)*Cos[b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[b*(c + d
*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])
/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)])/(Sqrt[b]*d^3)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(b(c+dx)^2\right)\,dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(bx^2\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(bx^2\right)+f^2x^2\sin\left(bx\right.\right.\right.}{d^3}$$

$$= \frac{f^2\,\text{Subst}\left(\int x^2\sin\left(bx^2\right)\,dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\,\text{Subst}\left(\int x\sin\left(bx^2\right)\,dx,x,c\right.}{d^3}$$

$$= -\frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3} + \frac{(de-cf)^2\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^3} + \frac{f^2\,\text{Subst}\left(\int \cos\right.}{}$$

$$= -\frac{f(de-cf)\cos\left(b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(b(c+dx)^2\right)}{2bd^3} + \frac{f^2\sqrt{\frac{\pi}{2}}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+\right.}{2b^{3/2}d^3}$$

Mathematica [A] time = 0.618085, size = 117, normalized size = 0.78

$$\frac{2\sqrt{2\pi}b(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)-2\sqrt{b}f\cos\left(b(c+dx)^2\right)(-cf+2de+dfx)+\sqrt{2\pi}f^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{4b^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-2*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[b*(c + d*x)^2] + f^2*Sqrt[2*Pi]*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + 2*b*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(4*b^(3/2)*d^3)
```

Maple [B] time = 0.005, size = 291, normalized size = 1.9

$$-\frac{f^2x\cos\left(d^2x^2b+2\,cdxb+c^2b\right)}{2\,d^2b} - \frac{f^2c}{d}\left(-\frac{\cos\left(d^2x^2b+2\,cdxb+c^2b\right)}{2\,d^2b} - \frac{c\sqrt{2}\sqrt{\pi}}{2\,d}\text{FresnelS}\left(\frac{\sqrt{2}\left(bd^2x+bcd\right)}{\sqrt{\pi}}\frac{1}{\sqrt{d^2b}}\right) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin((d*x+c)^2*b),x)
```

```
[Out] -1/2*f^2/d^2/b*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-f^2*c/d*(-1/2/d^2/b*cos(b*d
^2*x^2+2*b*c*d*x+b*c^2)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(
1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/4*f^2/d^2/b*2^(1/2)*Pi^(1/2
)/(d^2*b)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-e*
f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-e*f*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/
2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1/2)*Pi^
```

```
(1/2)/(d^2*b)^(1/2)*e^2*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*
c*d))
```

Maxima [C] time = 3.69617, size = 2245, normalized size = 14.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*((I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arct
an2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0,
 b)))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I*cos(1/4*pi + 1/2*arctan2(0, b))
 + I*cos(-1/4*pi + 1/2*arctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + s
in(-1/4*pi + 1/2*arctan2(0, b)))*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))*e^2/(d*
sqrt(abs(b))) - 1/2*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2) - (((
-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sq
rt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c*d*x + (-2
*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt
(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2)*cos(1/2*
arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) - 2*((sqrt(pi)*(erf(sqrt(I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c*d*x + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 +
2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*
x - I*b*c^2)) - 1))*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^
2, 0)))*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*e*f/(b*d^2*abs(4*b*d^
2*x^2 + 8*b*c*d*x + 4*b*c^2)) + 1/32*(16*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*
c*(e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x
- I*b*c^2)) - (((-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)
) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*
b*c^2*d*x + (-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) -
1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^
3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8*b
*c*d*x + 4*b*c^2, 0)) - ((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
1))*b*c^2*d*x + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) -
 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^3
)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*
c*d*x + 4*b*c^2, 0)) + (b*d^3*x^3*(4*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x
 + I*b*c^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b*c*d
^2*x^2*(12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 12*I*gamma(3
/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + b*c^2*d*x*(12*I*gamma(3/2, I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 12*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*
d*x - I*b*c^2)) + b*c^3*(4*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2
) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*cos(3/2*arctan2(
4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + 4*(b*d^3*x^3*(gamma(3/2, I*b*d^2*x
^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c
^2)) + 3*b*c*d^2*x^2*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gam
ma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) + 3*b*c^2*d*x*(gamma(3/2, I*
b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x
- I*b*c^2)) + b*c^3*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamm
a(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)))*sin(3/2*arctan2(4*b*d^2*x^2
+ 8*b*c*d*x + 4*b*c^2, 0)))*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*f
^2/((b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*b*d^3)
```

Fricas [A] time = 1.61034, size = 392, normalized size = 2.61

$$\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}f^2\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+2\sqrt{2}\pi\left(bd^2e^2-2\,bcdef+bc^2f^2\right)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-2\left(bd^2f^2x+2\,bd^2ef-bcdf^2\cdots\right.}{4\,b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(b*d^2/pi)*f^2*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x
+ c)/d) + 2*sqrt(2)*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt(b*d^2/pi)
*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*f^2*x + 2*b*d^2
*e*f - b*c*d*f^2)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b^2*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e+fx)^2\sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] time = 1.19738, size = 903, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^2/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt
(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x +
 c/d))*e^2/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/2*(-I*sqrt(2)*sqr
t(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d
))*e/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*
c*d*x - I*b*c^2 + 1)/(b*d))/d - 1/2*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2
)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e/(sqrt(b*d^2)*(-I*b*
d^2/sqrt(b^2*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + 1)/(b*
d))/d - 1/8*(I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 - I*f^2)*erf(-1/2*sqrt(2)*sqrt
(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b
^2*d^4) + 1)*b) + 2*I*(d*f^2*(-I*x - I*c/d) + 2*I*c*f^2)*e^(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2)/(b*d))/d^2 - 1/8*(-I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 +
 I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d)
)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*I*(d*f^2*(-I*x - I*c/d)
+ 2*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d^2
```

3.155 $\int (e+fx)\sin\left(b(c+dx)^2\right)\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{\frac{\pi}{2}}(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}-\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}$$

```
[Out] -(f*Cos[b*(c + d*x)^2])/(2*b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2)
```

Rubi [A] time = 0.0726357, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3433, 3351, 3379, 2638}

$$\frac{\sqrt{\frac{\pi}{2}}(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}-\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

```
[Out] -(f*Cos[b*(c + d*x)^2])/(2*b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (e+fx)\sin\left(b(c+dx)^2\right)dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(bx^2\right)+fx\sin\left(bx^2\right)\right)dx, x, c+dx\right)}{d^2}$$
$$= \frac{f\,\text{Subst}\left(\int x\sin\left(bx^2\right)dx, x, c+dx\right)}{d^2} + \frac{(de-cf)\,\text{Subst}\left(\int \sin\left(bx^2\right)dx, x, c+dx\right)}{d^2}$$
$$= \frac{(de-cf)\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2} + \frac{f\,\text{Subst}\left(\int \sin(bx)\,dx, x, (c+dx)^2\right)}{2d^2}$$
$$= -\frac{f\cos\left(b(c+dx)^2\right)}{2bd^2} + \frac{(de-cf)\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}$$

Mathematica [A] time = 0.174191, size = 66, normalized size = 0.96

$$\frac{\sqrt{2\pi}\sqrt{b}(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)-f\cos\left(b(c+dx)^2\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[b*(c + d*x)^2],x]
```

```
[Out] (-(f*Cos[b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*
Sqrt[2/Pi]*(c + d*x)])/(2*b*d^2)
```

Maple [B] time = 0.006, size = 120, normalized size = 1.7

$$-\frac{f\cos\left(d^2x^2b+2\,cdxb+c^2b\right)}{2\,d^2b}-\frac{cf\sqrt{2}\sqrt{\pi}}{2\,d}\text{FresnelS}\left(\frac{\sqrt{2}\left(bd^2x+bcd\right)}{\sqrt{\pi}}\frac{1}{\sqrt{d^2b}}\right)\frac{1}{\sqrt{d^2b}}+\frac{\sqrt{2}\sqrt{\pi}e}{2}\text{FresnelS}\left(\frac{\sqrt{2}\left(bd^2x+\right.}{\sqrt{\pi}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin((d*x+c)^2*b),x)
```

```
[Out] -1/2*f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2)-1/2*f*c/d*2^(1/2)*Pi^(1/2)/(d^2
*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+1/2*2^(1
/2)*Pi^(1/2)/(d^2*b)^(1/2)*e*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2
*x+b*c*d))
```

Maxima [C] time = 2.48203, size = 829, normalized size = 12.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*((I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arct
an2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0,
 b)))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I*cos(1/4*pi + 1/2*arctan2(0, b))
 + I*cos(-1/4*pi + 1/2*arctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + s
```

```
in(-1/4*pi + 1/2*arctan2(0, b)))*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))*e/(d*sq
rt(abs(b))) - 1/4*((e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x
^2 - 2*I*b*c*d*x - I*b*c^2))*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2) - (((-2
*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt
(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c*d*x + (-2*I
*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt(p
i)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*b*c^2)*cos(1/2*ar
ctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) - 2*((sqrt(pi)*(erf(sqrt(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2
*I*b*c*d*x - I*b*c^2)) - 1))*b*c*d*x + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*
I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x
- I*b*c^2)) - 1))*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2,
 0)))*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*f/(b*d^2*abs(4*b*d^2*x^
2 + 8*b*c*d*x + 4*b*c^2))
```

Fricas [A] time = 1.65385, size = 192, normalized size = 2.78

$$\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\left(de-cf\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-df\cos\left(bd^2x^2+2\,bcdx+bc^2\right)}{2\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*fresnel_sin(sqrt(2)*sqrt(b*d^2/p
i)*(d*x + c)/d) - d*f*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2))/(b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)\sin\left(bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] time = 1.17714, size = 495, normalized size = 7.17

$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{4\sqrt{bd^2}\left(\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e}{4\sqrt{bd^2}\left(-\frac{i\,bd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}cf\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{b\cdots\right.}{\sqrt{bd^2}\left(\frac{ib\cdots}{\sqrt{b^{\cdots}}}\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt(2
)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c
/d))*e/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4*(-I*sqrt(2)*sqrt(pi
)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(
sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x
- I*b*c^2)/(b*d))/d - 1/4*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d
^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2
*d^4) + 1)) + f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)/(b*d))/d
```

3.156 $\int \sin\left(b(c+dx)^2\right) dx$

Optimal. Leaf size=39

$$\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

```
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Rubi [A] time = 0.0080351, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3351}

$$\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(b(c+dx)^2\right) dx = \frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

Mathematica [A] time = 0.0151665, size = 39, normalized size = 1.

$$\frac{\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d)
```

Maple [A] time = 0.007, size = 42, normalized size = 1.1

$$\frac{\sqrt{2}\sqrt{\pi}}{2}\text{FresnelS}\left(\frac{\sqrt{2}\left(bd^2x+bcd\right)}{\sqrt{\pi}}\frac{1}{\sqrt{d^2b}}\right)\frac{1}{\sqrt{d^2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((d*x+c)^2*b),x)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*
(b*d^2*x+b*c*d))
```

Maxima [C] time = 1.83639, size = 193, normalized size = 4.95

$$\sqrt{\pi}\left(\left(i\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,b)\right)+i\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,b)\right)+\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,b)\right)-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\,\mathrm{ar}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*((I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arct
an2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0,
 b)))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + (I*cos(1/4*pi + 1/2*arctan2(0, b))
 + I*cos(-1/4*pi + 1/2*arctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + s
in(-1/4*pi + 1/2*arctan2(0, b)))*erf((I*b*d*x + I*b*c)/sqrt(-I*b)))/(d*sqrt
(abs(b)))
```

Fricas [A] time = 1.5917, size = 117, normalized size = 3.

$$\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/
d)/(b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(b(c+dx)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)**2),x)
```

```
[Out] Integral(sin(b*(c + d*x)**2), x)
```

Giac [C] time = 1.10767, size = 193, normalized size = 4.95

$$-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 1/4*I*sqrt(2)*
sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d
))/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1))
```

3.157 $\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sin\left(b(c+dx)^2\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Rubi [A] time = 0.012457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[b*(c + d*x)^2]/(e + f*x),x]

[Out] Defer[Int][Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx = \int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Mathematica [A] time = 5.19824, size = 0, normalized size = 0.

$$\int \frac{\sin\left(b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[b*(c + d*x)^2]/(e + f*x),x]

[Out] Integrate[Sin[b*(c + d*x)^2]/(e + f*x), x]
```

Maple [A] time = 0.131, size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2 b\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((d*x+c)^2*b)/(f*x+e),x)

[Out] int(sin((d*x+c)^2*b)/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2 b\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(bd^2x^2+2\,bcdx+bc^2\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(bc^2+2bcdx+bd^2x^2\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)**2)/(f*x+e),x)
```

```
[Out] Integral(sin(b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2 b\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e), x)
```

3.158 $\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0121149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]

[Out] Defer[Int][Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 9.8352, size = 0, normalized size = 0.

$$\int \frac{\sin\left(b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2,x]

[Out] Integrate[Sin[b*(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A] time = 0.196, size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2 b\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((d*x+c)^2*b)/(f*x+e)^2,x)
```

```
[Out] int(sin((d*x+c)^2*b)/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2 b\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(bd^2x^2+2\,bcdx+bc^2\right)}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2 b\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^2*b)/(f*x + e)^2, x)
```

3.159 $\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=337

$$\frac{2\sqrt{2\pi}b^{3/2}f^2(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4}+\frac{b^2f^3\mathrm{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}-\frac{3bf(de-cf)^2\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4}+\frac{f^2(c+dx)^3(de-cf)\sin\left(\right.}{d^4}$$

```
[Out] (2*b*f^2*(d*e - c*f)*(c + d*x)*Cos[b/(c + d*x)^2])/d^4 + (b*f^3*(c + d*x)^2
*Cos[b/(c + d*x)^2])/(4*d^4) - (3*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)
^2])/(2*d^4) - (Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)])/d^4 + (2*b^(3/2)*f^2*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi])/(c + d*x)])/d^4 + ((d*e - c*f)^3*(c + d*x)*Sin[b/(c + d*x)^2
])/d^4 + (3*f*(d*e - c*f)^2*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^4) + (f^2*
(d*e - c*f)*(c + d*x)^3*Sin[b/(c + d*x)^2])/d^4 + (f^3*(c + d*x)^4*Sin[b/(c
 + d*x)^2])/(4*d^4) + (b^2*f^3*SinIntegral[b/(c + d*x)^2])/(4*d^4)
```

Rubi [A] time = 0.420275, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3433, 3359, 3387, 3352, 3379, 3297, 3302, 3409, 3388, 3351, 3299}

$$\frac{2\sqrt{2\pi}b^{3/2}f^2(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^4}+\frac{b^2f^3\mathrm{Si}\left(\frac{b}{(c+dx)^2}\right)}{4d^4}-\frac{3bf(de-cf)^2\mathrm{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^4}+\frac{f^2(c+dx)^3(de-cf)\sin\left(\right.}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*f^2*(d*e - c*f)*(c + d*x)*Cos[b/(c + d*x)^2])/d^4 + (b*f^3*(c + d*x)^2
*Cos[b/(c + d*x)^2])/(4*d^4) - (3*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)
^2])/(2*d^4) - (Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/P
i])/(c + d*x)])/d^4 + (2*b^(3/2)*f^2*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi])/(c + d*x)])/d^4 + ((d*e - c*f)^3*(c + d*x)*Sin[b/(c + d*x)^2
])/d^4 + (3*f*(d*e - c*f)^2*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^4) + (f^2*
(d*e - c*f)*(c + d*x)^3*Sin[b/(c + d*x)^2])/d^4 + (f^3*(c + d*x)^4*Sin[b/(c
 + d*x)^2])/(4*d^4) + (b^2*f^3*SinIntegral[b/(c + d*x)^2])/(4*d^4)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(\frac{b}{x^2}\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\sin\left(\frac{b}{x^2}\right)\right.\right.}{d^4}$$

$$= \frac{f^3 \text{Subst}\left(\int x^3 \sin\left(\frac{b}{x^2}\right) dx, x, c+dx\right)}{d^4} + \frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x^2 \sin\left(\frac{b}{x^2}\right) dx, x,\right.}{d^4}$$

$$= -\frac{f^3 \text{Subst}\left(\int \frac{\sin(bx)}{x^3} dx, x, \frac{1}{(c+dx)^2}\right)}{2d^4} - \frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int \frac{\sin(bx^2)}{x^4} dx, x, \frac{1}{c+dx}\right)}{d^4} -$$

$$= \frac{(de-cf)^3(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{3f(de-cf)^2(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^4} + \frac{f^2(de-cf)(}{}$$

$$= \frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4} - \frac{3bf(de-cf)^2\text{Ci}\left(\right.}{2d^4}$$

$$= \frac{2bf^2(de-cf)(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{d^4} + \frac{bf^3(c+dx)^2\cos\left(\frac{b}{(c+dx)^2}\right)}{4d^4} - \frac{3bf(de-cf)^2\text{Ci}\left(\right.}{2d^4}$$

Mathematica [A] time = 0.869559, size = 440, normalized size = 1.31

$$\frac{8\sqrt{2\pi}b^{3/2}def^2S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) - 8\sqrt{2\pi}b^{3/2}cf^3S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) + b^2f^3\text{Si}\left(\frac{b}{(c+dx)^2}\right) - 6c^2d^2e^2f\sin\left(\frac{b}{(c+dx)^2}\right) + 4c^3def^2\sin\left(\frac{b}{(c+dx)^2}\right) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*Sin[b/(c + d*x)^2],x]
```

```
[Out] (8*b*c*d*e*f^2*Cos[b/(c + d*x)^2] - 7*b*c^2*f^3*Cos[b/(c + d*x)^2] + 8*b*d^
2*e*f^2*x*Cos[b/(c + d*x)^2] - 6*b*c*d*f^3*x*Cos[b/(c + d*x)^2] + b*d^2*f^3
*x^2*Cos[b/(c + d*x)^2] - 6*b*f*(d*e - c*f)^2*CosIntegral[b/(c + d*x)^2] -
4*Sqrt[b]*(d*e - c*f)^3*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]
 + 8*b^(3/2)*d*e*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] -
8*b^(3/2)*c*f^3*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 4*c*d
^3*e^3*Sin[b/(c + d*x)^2] - 6*c^2*d^2*e^2*f*Sin[b/(c + d*x)^2] + 4*c^3*d*e*
f^2*Sin[b/(c + d*x)^2] - c^4*f^3*Sin[b/(c + d*x)^2] + 4*d^4*e^3*x*Sin[b/(c
+ d*x)^2] + 6*d^4*e^2*f*x^2*Sin[b/(c + d*x)^2] + 4*d^4*e*f^2*x^3*Sin[b/(c +
 d*x)^2] + d^4*f^3*x^4*Sin[b/(c + d*x)^2] + b^2*f^3*SinIntegral[b/(c + d*x)
^2])/(4*d^4)
```

Maple [A] time = 0.013, size = 365, normalized size = 1.1

$$\frac{1}{d^4}\left(-\left(c^3f^3 - 3c^2def^2 + 3cd^2e^2f - d^3e^3\right)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right) + \left(c^3f^3 - 3c^2def^2 + 3cd^2e^2f - d^3e^3\right)\sqrt{b}\sqrt{2}\sqrt{\pi}\text{Fre}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(b/(d*x+c)^2),x)
```

```
[Out] 1/d^4*(-(c^3*f^3-3*c^2*d*e*f^2+3*c*d^2*e^2*f-d^3*e^3)*(d*x+c)*sin(b/(d*x+c)
^2)+(c^3*f^3-3*c^2*d*e*f^2+3*c*d^2*e^2*f-d^3*e^3)*b^(1/2)*2^(1/2)*Pi^(1/2)*
```

```
FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-1/2*(-3*c^2*f^3+6*c*d*e*f^2-3*d^
2*e^2*f)*(d*x+c)^2*sin(b/(d*x+c)^2)+1/2*(-3*c^2*f^3+6*c*d*e*f^2-3*d^2*e^2*f
)*b*Ci(b/(d*x+c)^2)-1/3*(3*c*f^3-3*d*e*f^2)*(d*x+c)^3*sin(b/(d*x+c)^2)+2/3*
(3*c*f^3-3*d*e*f^2)*b*(-(d*x+c)*cos(b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*F
resnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/4*f^3*(d*x+c)^4*sin(b/(d*x+c)^
2)-1/2*f^3*b*(-1/2*(d*x+c)^2*cos(b/(d*x+c)^2)-1/2*b*Si(b/(d*x+c)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{-\left(\frac{4c^3ef^2\sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)}{d^3}-\frac{3c^4f^3\sin\left(\frac{b}{d^2x^2+2cdx+c^2}\right)}{d^4}-2\int-\frac{2\left(3\left(bd^3e^2f-2bcd^2ef^2+bc^2df^3\right)x^2+2\left(bd^3e^3-3bc^2def^2+2bc^3f^3\right)x\right)\cos\left(\frac{b}{d^2x^2+2cdx+c^2}\right)}{2\left(d^5x^3+3cd^4x^2+3c^2d^3x+c^3d^2\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*d^3*integrate(1/4*((4*b*c^3*d*e*f^2 - 3*b*c^4*f^3 - 6*(b*d^4*e^2*f
- 2*b*c*d^3*e*f^2 + b*c^2*d^2*f^3)*x^2 - 4*(b*d^4*e^3 - 3*b*c^2*d^2*e*f^2 +
 2*b*c^3*d*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b^2*d^2*f^3*x^2 + 2*
(4*b^2*d^2*e*f^2 - 3*b^2*c*d*f^3)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/(d^6
*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3), x) + 4*d^3*integrate(1/4*((4*b
*c^3*d*e*f^2 - 3*b*c^4*f^3 - 6*(b*d^4*e^2*f - 2*b*c*d^3*e*f^2 + b*c^2*d^2*f
^3)*x^2 - 4*(b*d^4*e^3 - 3*b*c^2*d^2*e*f^2 + 2*b*c^3*d*f^3)*x)*cos(b/(d^2*x
^2 + 2*c*d*x + c^2)) + (b^2*d^2*f^3*x^2 + 2*(4*b^2*d^2*e*f^2 - 3*b^2*c*d*f^
3)*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 + (d^6*x^3 + 3*c*d^5*x^2 +
 3*c^2*d^4*x + c^3*d^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) - (b*d*f^3*
x^2 + 2*(4*b*d*e*f^2 - 3*b*c*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - (d^
3*f^3*x^4 + 4*d^3*e*f^2*x^3 + 6*d^3*e^2*f*x^2 + 4*d^3*e^3*x)*sin(b/(d^2*x^2
 + 2*c*d*x + c^2)))/d^3
```

Fricas [A] time = 2.06195, size = 1023, normalized size = 3.04

$$\frac{b^2f^3\,\mathrm{Si}\left(\frac{b}{d^2x^2+2cdx+c^2}\right)-4\sqrt{2}\pi\left(d^4e^3-3cd^3e^2f+3c^2d^2ef^2-c^3df^3\right)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+8\sqrt{2}\pi\left(bd^2ef^2-bcdf^3\right)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*f^3*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - 4*sqrt(2)*pi*(d^4*
e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2 - c^3*d*f^3)*sqrt(b/(pi*d^2))*fresnel
_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 8*sqrt(2)*pi*(b*d^2*e*f^2 - b*
c*d*f^3)*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))
 + (b*d^2*f^3*x^2 + 8*b*c*d*e*f^2 - 7*b*c^2*f^3 + 2*(4*b*d^2*e*f^2 - 3*b*c*
d*f^3)*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*d^2*e^2*f - 2*b*c*d*e*f^2
 + b*c^2*f^3)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*d^2*e^2*f -
2*b*c*d*e*f^2 + b*c^2*f^3)*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2)) + (d^
4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 4*c*d^3*e^3 -
 6*c^2*d^2*e^2*f + 4*c^3*d*e*f^2 - c^4*f^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)
```

```
))/d^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(b/(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^3 \sin\left(\frac{b}{(dx+c)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin(b/(d*x + c)^2), x)
```

3.160 $\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=233

$$\frac{2\sqrt{2\pi}b^{3/2}f^2S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\sqrt{b}(de-cf)^2\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^3}+\frac{f(c+dx)^2(de-cf)}{d^3}$$

```
[Out] (2*b*f^2*(c + d*x)*Cos[b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*CosIntegr
al[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[b/(c + d*x)
^2])/d^3 + (f*(d*e - c*f)*(c + d*x)^2*Sin[b/(c + d*x)^2])/d^3 + (f^2*(c + d
*x)^3*Sin[b/(c + d*x)^2])/(3*d^3)
```

Rubi [A] time = 0.249208, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3433, 3359, 3387, 3352, 3379, 3297, 3302, 3409, 3388, 3351}

$$\frac{2\sqrt{2\pi}b^{3/2}f^2S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\sqrt{b}(de-cf)^2\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^3}+\frac{f(c+dx)^2(de-cf)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```


```
[Out] (2*b*f^2*(c + d*x)*Cos[b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*CosIntegr
al[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[b/(c + d*x)
^2])/d^3 + (f*(d*e - c*f)*(c + d*x)^2*Sin[b/(c + d*x)^2])/d^3 + (f^2*(c + d
*x)^3*Sin[b/(c + d*x)^2])/(3*d^3)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(\frac{b}{(c+dx)^2}\right) dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(\frac{b}{x^2}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(\frac{b}{x^2}\right)+f^2x^2\sin\left(\frac{b}{x^2}\right)\right) dx}{d^3}$$

$$= \frac{f^2\,\text{Subst}\left(\int x^2\sin\left(\frac{b}{x^2}\right) dx, x, c+dx\right)}{d^3} + \frac{(2f(de-cf))\,\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right) dx, x, c+dx\right.}{d^3}$$

$$= -\frac{f^2\,\text{Subst}\left(\int \frac{\sin(bx^2)}{x^4} dx, x, \frac{1}{c+dx}\right)}{d^3} - \frac{(f(de-cf))\,\text{Subst}\left(\int \frac{\sin(bx)}{x^2} dx, x, \frac{1}{(c+dx)^2}\right)}{d^3} - \frac{(de-}{}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{d^3} + \frac{f^2(c+dx)^3\sin\left(\right.}{3d^3}$$

$$= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} + \cdots$$

$$= \frac{2bf^2(c+dx)\cos\left(\frac{b}{(c+dx)^2}\right)}{3d^3} - \frac{bf(de-cf)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3} - \frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^3} + \cdots$$

Mathematica [A] time = 0.492441, size = 265, normalized size = 1.14

$$\frac{2\sqrt{2\pi}b^{3/2}f^2S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) - 3c^2def\sin\left(\frac{b}{(c+dx)^2}\right) + c^3f^2\sin\left(\frac{b}{(c+dx)^2}\right) + 3bf(cf-de)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right) + 3cd^2e^2\sin\left(\frac{b}{(c+dx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[b/(c + d*x)^2],x]
```

```
[Out] (2*b*c*f^2*Cos[b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[b/(c + d*x)^2] + 3*b*f*(-(d
*e) + c*f)*CosIntegral[b/(c + d*x)^2] - 3*Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*
FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*b^(3/2)*f^2*Sqrt[2*Pi]*Fresnel
S[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*c*d^2*e^2*Sin[b/(c + d*x)^2] - 3*c^2*
d*e*f*Sin[b/(c + d*x)^2] + c^3*f^2*Sin[b/(c + d*x)^2] + 3*d^3*e^2*x*Sin[b/(
c + d*x)^2] + 3*d^3*e*f*x^2*Sin[b/(c + d*x)^2] + d^3*f^2*x^3*Sin[b/(c + d*x
)^2])/(3*d^3)
```

Maple [A] time = 0.01, size = 225, normalized size = 1.

$$-\frac{1}{d^3}\left(-\left(c^2f^2-2\,cdef+d^2e^2\right)(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right)+\left(c^2f^2-2\,cdef+d^2e^2\right)\sqrt{b}\sqrt{2}\sqrt{\pi}\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\sqrt{b}\right)-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(b/(d*x+c)^2),x)
```

```
[Out] -1/d^3*(-(c^2*f^2-2*c*d*e*f+d^2*e^2)*(d*x+c)*sin(b/(d*x+c)^2)+(c^2*f^2-2*c*
d*e*f+d^2*e^2)*b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(
d*x+c))-1/2*(-2*c*f^2+2*d*e*f)*(d*x+c)^2*sin(b/(d*x+c)^2)+1/2*(-2*c*f^2+2*d
*e*f)*b*Ci(b/(d*x+c)^2)-1/3*f^2*(d*x+c)^3*sin(b/(d*x+c)^2)+2/3*f^2*b*(-(d*x
+c)*cos(b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(
```

```
1/2)/(d*x+c))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,bf^2x\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+\left(\frac{c^3f^2\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{d^3}-2\int\frac{2\,b^2f^2x\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)-3\left(\left(bd^2ef-bcdf^2\right)x^2+\left(bd^2e^2-bc^2f^2\right)x\right)\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right.}{2\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*b*f^2*x*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*d^2*integrate(1/3*(2*b^
2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*
c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos(b/(d^2*x^2 + 2*c*d*x +
c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2), x) - 3*d^2*integrat
e(1/3*(2*b^2*d*f^2*x*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d
^3*e*f - b*c*d^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos(b/(d^2*x^2 +
 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*cos(b/(d
^2*x^2 + 2*c*d*x + c^2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2
)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d^2*e*f*x^2 +
3*d^2*e^2*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Fricas [A] time = 1.85872, size = 713, normalized size = 3.06

$$4\sqrt{2}\pi bdf^2\sqrt{\frac{b}{\pi d^2}}\,S\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-6\sqrt{2}\pi\left(d^3e^2-2\,cd^2ef+c^2df^2\right)\sqrt{\frac{b}{\pi d^2}}\,C\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+4\left(bdf^2x+bcf^2\right)\cos\left(\frac{b}{d^2x^2+2\,cdx+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(2)*pi*b*d*f^2*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi
*d^2))/(d*x + c)) - 6*sqrt(2)*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sqrt(b
/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) + 4*(b*d*f^2*x
 + b*c*f^2)*cos(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*d*e*f - b*c*f^2)*cos_in
tegral(b/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(b*d*e*f - b*c*f^2)*cos_integral(-b
/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x
+ 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^
3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)^2\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \sin\left(\frac{b}{(dx+c)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(b/(d*x + c)^2), x)
```

3.161 $\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)\,dx$

Optimal. Leaf size=120

$$-\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{2\pi}\sqrt{b}(de-cf)\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^2}+\frac{(c+dx)(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

```
[Out] -(b*f*CosIntegral[b/(c + d*x)^2])/(2*d^2) - (Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + ((d*e - c*f)*(c + d*x)*Sin
[b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^2)
```

Rubi [A] time = 0.128448, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3433, 3359, 3387, 3352, 3379, 3297, 3302}

$$-\frac{bf\,\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{2\pi}\sqrt{b}(de-cf)\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^2}+\frac{(c+dx)(de-cf)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[b/(c + d*x)^2],x]
```


```
[Out] -(b*f*CosIntegral[b/(c + d*x)^2])/(2*d^2) - (Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]
*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + ((d*e - c*f)*(c + d*x)*Sin
[b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[b/(c + d*x)^2])/(2*d^2)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(\frac{b}{(c+dx)^2}\right)dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(\frac{b}{x^2}\right)+fx\sin\left(\frac{b}{x^2}\right)\right)dx,x,c+dx\right)}{d^2}$$
$$= \frac{f\,\text{Subst}\left(\int x\sin\left(\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\,\text{Subst}\left(\int \sin\left(\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^2}$$
$$= -\frac{f\,\text{Subst}\left(\int \frac{\sin(bx)}{x^2}dx,x,\frac{1}{(c+dx)^2}\right)}{2d^2}-\frac{(de-cf)\,\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}dx,x,\frac{1}{c+dx}\right)}{d^2}$$
$$= \frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+dx)^2\sin\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{(bf)\,\text{Subst}\left(\int \frac{\cos(bx)}{x}dx,x,\frac{}{(c\text{-}}\right.}{2d^2}$$
$$= -\frac{bf\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{b}(de-cf)\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}+\frac{(de-cf)(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d^2}+\frac{f(c+}{}$$

Mathematica [A] time = 0.283668, size = 95, normalized size = 0.79

$$-\frac{bf\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)+2\sqrt{2\pi}\sqrt{b}(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)+(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)(cf-2de-dfx)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[b/(c + d*x)^2],x]
```

```
[Out] -(b*f*CosIntegral[b/(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Fresnel
C[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + (c + d*x)*(-2*d*e + c*f - d*f*x)*Sin[b/
(c + d*x)^2])/(2*d^2)
```

Maple [A] time = 0.012, size = 101, normalized size = 0.8

$$\frac{1}{d^2}\left(-\left(cf-de\right)\left(dx+c\right)\sin\left(\frac{b}{\left(dx+c\right)^2}\right)+\left(cf-de\right)\sqrt{b}\sqrt{2}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}\left(dx+c\right)}\sqrt{b}\right)+\frac{f\left(dx+c\right)^2}{2}\sin\left(\frac{b}{\left(dx+c\right)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(b/(d*x+c)^2),x)
```

```
[Out] 1/d^2*(-(c*f-d*e)*(d*x+c)*sin(b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(1/
2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+1/2*f*(d*x+c)^2*sin(b/(d*x+c)
^2)-1/2*f*b*Ci(b/(d*x+c)^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(fx^2+2\,ex\right)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+\int\frac{\left(bdfx^2+2\,bdex\right)\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{2\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)}\,dx+\int\frac{}{2\left(\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(1/2*(b*d*f
*x^2 + 2*b*d*e*x)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos(b/(d^2*x^
2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x
^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d
^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Fricas [A] time = 1.73599, size = 383, normalized size = 3.19

$$-\frac{4\,\sqrt{2}\pi\left(d^2e-cdf\right)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)+bf\,\mathrm{Ci}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)+bf\,\mathrm{Ci}\left(-\frac{b}{d^2x^2+2\,cdx+c^2}\right)-2\left(d^2fx^2+2\,d^2ex+2\,cde-c^2\right.}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*s
qrt(b/(pi*d^2))/(d*x + c)) + b*f*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2))
+ b*f*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(d^2*f*x^2 + 2*d^2*e*x
 + 2*c*d*e - c^2*f)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)\sin\left(\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\sin\left(\frac{b}{(dx+c)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(b/(d*x + c)^2), x)
```

3.162 $\int \sin\left(\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=60

$$\frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}-\frac{\sqrt{2\pi}\sqrt{b}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d}$$

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
d*x)*Sin[b/(c + d*x)^2])/d
```

Rubi [A] time = 0.0335677, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3359, 3387, 3352}

$$\frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}-\frac{\sqrt{2\pi}\sqrt{b}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
d*x)*Sin[b/(c + d*x)^2])/d
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin(bx^2)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b)\,\text{Subst}\left(\int \cos\left(bx^2\right)dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= -\frac{\sqrt{b}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d}$$

Mathematica [A] time = 0.0328287, size = 60, normalized size = 1.

$$\frac{(c+dx)\sin\left(\frac{b}{(c+dx)^2}\right)}{d} - \frac{\sqrt{2\pi}\sqrt{b}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) + ((c +
d*x)*Sin[b/(c + d*x)^2])/d
```

Maple [A] time = 0.007, size = 52, normalized size = 0.9

$$-\frac{1}{d}\left(-(dx+c)\sin\left(\frac{b}{(dx+c)^2}\right) + \sqrt{b}\sqrt{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}(dx+c)}\sqrt{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(d*x+c)^2),x)
```

```
[Out] -1/d*(-(d*x+c)*sin(b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)/(d*x+c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$bd\int \frac{x\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\,dx + bd\int \frac{x\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\cos\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)^2 + \left(d^3x^3+3\,cd^2x^2+3\,c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3
*c^2*d*x + c^3), x) + b*d*integrate(x*cos(b/(d^2*x^2 + 2*c*d*x + c^2))/((d^
3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b/(d^2*x^2 + 2*c*d*x + c^2))^2 +
 (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
```

```
^2), x) + x*sin(b/(d^2*x^2 + 2*c*d*x + c^2))
```

Fricas [A] time = 1.69066, size = 177, normalized size = 2.95

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-(dx+c)\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x
 + c)) - (d*x + c)*sin(b/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{b}{(c+dx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)**2),x)
```

```
[Out] Integral(sin(b/(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(\frac{b}{(dx+c)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2), x)
```

3.163 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[b/(c + d*x)^2]/(e + f*x), x]
```

Rubi [A] time = 0.012773, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[b/(c + d*x)^2]/(e + f*x),x]

[Out] Defer[Int][Sin[b/(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Mathematica [A] time = 1.50788, size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[b/(c + d*x)^2]/(e + f*x),x]

[Out] Integrate[Sin[b/(c + d*x)^2]/(e + f*x), x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(\frac{b}{(dx+c)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

```
[Out] int(sin(b/(d*x+c)^2)/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)**2)/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e), x)
```

3.164 $\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0122059, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 18.378, size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[b/(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A] time = 0.425, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^2}\sin\left(\frac{b}{(dx+c)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

```
[Out] int(sin(b/(d*x+c)^2)/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right)}{f^2x^2+2\,efx+e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b/(d^2*x^2 + 2*c*d*x + c^2))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{b}{(dx+c)^2}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b/(d*x + c)^2)/(f*x + e)^2, x)
```

3.165 $\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right) dx$

Optimal. Leaf size=341

$$\frac{3\sqrt{\frac{\pi}{2}}f^2\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^4}-\frac{3\sqrt{\frac{\pi}{2}}f^2\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(a+b(c+dx)^2\right.}{2b^2d^4}$$

```
[Out] (-3*f*(d*e - c*f)^2*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*
(c + d*x)*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[a + b*(c
 + d*x)^2])/(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*Cos[a
]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^4) + ((d*e - c*f)^3*Sq
rt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^4) - (3*
f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(
2*b^(3/2)*d^4) + (f^3*Sin[a + b*(c + d*x)^2])/(2*b^2*d^4)
```

Rubi [A] time = 0.57157, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3433, 3353, 3352, 3351, 3379, 2638, 3385, 3354, 3296, 2637}

$$\frac{3\sqrt{\frac{\pi}{2}}f^2\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^4}-\frac{3\sqrt{\frac{\pi}{2}}f^2\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^4}+\frac{f^3\sin\left(a+b(c+dx)^2\right.}{2b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-3*f*(d*e - c*f)^2*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (3*f^2*(d*e - c*f)*
(c + d*x)*Cos[a + b*(c + d*x)^2])/(2*b*d^4) - (f^3*(c + d*x)^2*Cos[a + b*(c
 + d*x)^2])/(2*b*d^4) + (3*f^2*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)])/(2*b^(3/2)*d^4) + ((d*e - c*f)^3*Sqrt[Pi/2]*Cos[a
]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^4) + ((d*e - c*f)^3*Sq
rt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^4) - (3*
f^2*(d*e - c*f)*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(
2*b^(3/2)*d^4) + (f^3*Sin[a + b*(c + d*x)^2])/(2*b^2*d^4)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^2\right)\,dx = \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf\left(3d^2e^2-3cdef+c^2f^2\right)}{d^3e^3}\right)\sin\left(a+bx^2\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\right.\right.}{}$$

$$= \frac{f^3\,\text{Subst}\left(\int x^3\sin\left(a+bx^2\right)\,dx,x,c+dx\right)}{d^4} + \frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x^2\sin\left(a\right.\right.}{d^4}$$

$$= -\frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4} + \frac{f^3\,\text{Subst}\left(\int x\sin(a+bx)\,dx,x,(c\right.}{2d^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4}$$

$$= -\frac{3f(de-cf)^2\cos\left(a+b(c+dx)^2\right)}{2bd^4} - \frac{3f^2(de-cf)(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^4}$$

Mathematica [A] time = 2.94714, size = 218, normalized size = 0.64

$$\frac{-4bf\left(c^2f^2-cdf(3e+fx)+d^2\left(3e^2+3efx+f^2x^2\right)\right)\cos\left(a+b(c+dx)^2\right)+2\sqrt{2\pi}\sqrt{b}(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-4*b*f*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos
[a + b*(c + d*x)^2] + 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqr
t[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - 3*f^2*Sin[a]) + 2*Sqrt[b]*(d
*e - c*f)*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(3*f^2*Cos[a] +
 2*b*(d*e - c*f)^2*Sin[a]) + 4*f^3*Sin[a + b*(c + d*x)^2])/(8*b^2*d^4)
```

Maple [B] time = 0.013, size = 1248, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(a+(d*x+c)^2*b),x)
```

```
[Out] -1/2*f^3/d^2/b*x^2*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-f^3*c/d*(-1/2/d^2/b*x*c
os(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-c/d*(-1/2/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c
^2+a)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2
+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b
^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*
(b*d^2*x+b*c*d))))+1/4/d^2/b*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d
^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x
+b*c*d))+sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)
/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))))+f^3/d^2/b*(1/2/d^2/b*sin(b*d^2*x^2+2*b*c*
d*x+b*c^2+a)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b
*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))
+sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)
^(1/2)*(b*d^2*x+b*c*d))))-3/2*e*f^2/d^2/b*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a
)-3*e*f^2*c/d*(-1/2/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-1/2*c/d*2^(1/2)*
Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2
^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-d^2*b*(b*c^
2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))))+3/4
*e*f^2/d^2/b*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+
a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+sin((b^
2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(
b*d^2*x+b*c*d)))-3/2*e^2*f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-3/2*e^2*f
*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/
b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^
2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+
b*c*d)))+1/2*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*e^3*(cos((b^2*c^2*d^2-d^2*b*(b*
c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin
((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/
2)*(b*d^2*x+b*c*d)))
```

Maxima [C] time = 4.86107, size = 6666, normalized size = 19.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + sin(-1/4*pi + 1/2*arctan2
(0, b)))*cos(a) - (cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arct
an2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan
2(0, b)))*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + ((-I*cos(1/4*pi + 1/2*
arctan2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arct
an2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + (cos(1/4*pi + 1/2*a
rctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arcta
n2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((I*b*d*x + I*b*
c)/sqrt(-I*b)))*e^3/(d*sqrt(abs(b))) - 3/4*(((e^(I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (I*e^(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
))*sin(a))*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2) + ((((2*I*sqrt(pi)*(erf(s
qrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*
b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*
b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
- 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c*d*x + ((2*I*sqrt(pi)*(erf(sqrt(
I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*b*d^
2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8*
b*c*d*x + 4*b*c^2, 0)) + ((2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)
) - 1))*cos(a) + (-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c
^2)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
 1))*sin(a))*b*c*d*x + (2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
 1))*cos(a) + (-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)
) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1)
)*sin(a))*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)))*sq
rt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*e^2*f/(b*d^2*abs(4*b*d^2*x^2 +
8*b*c*d*x + 4*b*c^2)) + 3/32*(16*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*((e^(I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
))*cos(a) + (I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - I*e^(-I*b*d^2*x^2
- 2*I*b*c*d*x - I*b*c^2))*sin(a))*c + ((((I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*
c*d*x - I*b*c^2)) - 1))*cos(a) + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*
d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*
c^2)) - 1))*sin(a))*b*c^2*d*x + ((I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*
c*d*x + I*b*c^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x -
I*b*c^2)) - 1))*cos(a) + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
1))*sin(a))*b*c^3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*cos(1/2*arctan2(4
*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + (((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 +
 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d
*x - I*b*c^2)) - 1))*cos(a) + (-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*
d*x + I*b*c^2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2)) - 1))*sin(a))*b*c^2*d*x + ((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*
c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2)) - 1))*cos(a) + (-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
 - 1))*sin(a))*b*c^3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*sin(1/2*arctan
2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + (((-4*I*gamma(3/2, I*b*d^2*x^2 +
 2*I*b*c*d*x + I*b*c^2) + 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c
^2))*cos(a) - 4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/
```

```
2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*d^3*x^3 + ((-12*I*gamma
(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 12*I*gamma(3/2, -I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2))*cos(a) - 12*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*
d^2*x^2 + ((-12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 12*I*ga
mma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - 12*(gamma(3/2, I*b
*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x -
 I*b*c^2))*sin(a))*b*c^2*d*x + ((-4*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2) + 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) -
 4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x
^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^3)*cos(3/2*arctan2(4*b*d^2*x^2 + 8
*b*c*d*x + 4*b*c^2, 0)) - ((4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c
^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (4*I*gamma
(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2))*sin(a))*b*d^3*x^3 + (12*(gamma(3/2, I*b*d^2*x^2 + 2
*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*c
os(a) - (12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 12*I*gamma(
3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d^2*x^2 + (12*(gamm
a(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I
*b*c*d*x - I*b*c^2))*cos(a) - (12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) - 12*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b
*c^2*d*x + (4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2,
 -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (4*I*gamma(3/2, I*b*d^2*x^
2 + 2*I*b*c*d*x + I*b*c^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2))*sin(a))*b*c^3)*sin(3/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0
)))*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*e*f^2/((b*d^2*x^2 + 2*b*c
*d*x + b*c^2)^2*b*d^3) - 1/64*(16*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*((3*(e^
(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2))*cos(a) + (3*I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 3*I*e^(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^2 + (-I*gamma(2, I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2) + I*gamma(2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)
)*cos(a) - (gamma(2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(2, -I*b*d
^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a)) + ((((2*I*sqrt(pi)*(erf(sqrt(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2
 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c
*d*x - I*b*c^2)) - 1))*sin(a))*b^2*c^3*d*x + ((2*I*sqrt(pi)*(erf(sqrt(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2
 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c
*d*x - I*b*c^2)) - 1))*sin(a))*b^2*c^4)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c
^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + ((2*(sqrt(pi)*
(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-
I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + (-2*I*sqrt(pi)*(erf(sq
rt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b
*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b^2*c^3*d*x + (2*(sqrt(pi)
*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(
-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + (-2*I*sqrt(pi)*(erf(s
qrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*
b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b^2*c^4)*abs(4*b*d^2*x^2
+ 8*b*c*d*x + 4*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0
)) + (((-24*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 24*I*gamma(
3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - 24*(gamma(3/2, I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2))*sin(a))*b^2*c*d^3*x^3 + ((-72*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x
 + I*b*c^2) + 72*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a)
 - 72*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b^2*c^2*d^2*x^2 + ((-72*I*gamma(3/2
, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 72*I*gamma(3/2, -I*b*d^2*x^2 - 2*I
```

```
*b*c*d*x - I*b*c^2))*cos(a) - 72*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b^2*c^3*
d*x + ((-24*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 24*I*gamma(
3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - 24*(gamma(3/2, I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2))*sin(a))*b^2*c^4)*cos(3/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2,
0)) - ((24*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I
*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (24*I*gamma(3/2, I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2) - 24*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b
*c^2))*sin(a))*b^2*c*d^3*x^3 + (72*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (72*I
*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 72*I*gamma(3/2, -I*b*d^2
*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b^2*c^2*d^2*x^2 + (72*(gamma(3/2, I*
b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x
- I*b*c^2))*cos(a) - (72*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)
- 72*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b^2*c^3*d*
x + (24*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (24*I*gamma(3/2, I*b*d^2*x^2 + 2
*I*b*c*d*x + I*b*c^2) - 24*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^
2))*sin(a))*b^2*c^4)*sin(3/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0))
)*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*f^3/((b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)^2*b^2*d^4)
```

Fricas [A] time = 1.95853, size = 760, normalized size = 2.23

$$2\,df^3\sin\left(bd^2x^2 + 2\,bcdx + bc^2 + a\right) + \sqrt{2}\left(3\,\pi\left(def^2 - cf^3\right)\cos\left(a\right) + 2\,\pi\left(bd^3e^3 - 3\,bcd^2e^2f + 3\,bc^2def^2 - bc^3f^3\right)\sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^3*sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a) + sqrt(2)*(3*pi*(d*e*f^
2 - c*f^3)*cos(a) + 2*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b
*c^3*f^3)*sin(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x +
c)/d) + sqrt(2)*(2*pi*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^
3*f^3)*cos(a) - 3*pi*(d*e*f^2 - c*f^3)*sin(a))*sqrt(b*d^2/pi)*fresnel_sin(s
qrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^3*f^3*x^2 + 3*b*d^3*e^2*f - 3*b
*c*d^2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2*f^3)*x)*cos(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a))/(b^2*d^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^{3}\sin\left(a + bc^{2} + 2bcdx + bd^{2}x^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**3*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] time = 1.21802, size = 1449, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a + 3)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)*(x + c/d))*e^(-I*a + 3)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4
*(3*I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*
d^4) + 1)*(x + c/d))*e^(I*a + 2)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1))
 + 3*f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a + 2)/(b*d))/d - 1/4*(-3
*I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(-I*a + 2)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + 3
*f*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a + 2)/(b*d))/d - 1/8*(-I*sq
rt(2)*sqrt(pi)*(6*b*c^2*f^2 + 3*I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d
^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(I*a + 1)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^
2*d^4) + 1)*b) + 2*I*(d*f^2*(-3*I*x - 3*I*c/d) + 6*I*c*f^2)*e^(I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2 + I*a + 1)/(b*d))/d^2 - 1/8*(I*sqrt(2)*sqrt(pi)*(6*
b*c^2*f^2 - 3*I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
1)*(x + c/d))*e^(-I*a + 1)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*
I*(d*f^2*(-3*I*x - 3*I*c/d) + 6*I*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2 - I*a + 1)/(b*d))/d^2 + 1/8*(sqrt(2)*sqrt(pi)*(-2*I*b*c^3*f^3 + 3*c*f
^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^
(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x + c/d)
^2 - 3*b*c*d*f^3*(x + c/d) + 3*b*c^2*f^3 + I*f^3)*e^(I*b*d^2*x^2 + 2*I*b*c*
d*x + I*b*c^2 + I*a)/(b^2*d))/d^3 + 1/8*(sqrt(2)*sqrt(pi)*(2*I*b*c^3*f^3 +
3*c*f^3)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d)
)*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) - 2*(b*d^2*f^3*(x +
c/d)^2 - 3*b*c*d*f^3*(x + c/d) + 3*b*c^2*f^3 - I*f^3)*e^(-I*b*d^2*x^2 - 2*I
*b*c*d*x - I*b*c^2 - I*a)/(b^2*d))/d^3
```

3.166 $\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal. Leaf size=256

$$\frac{\sqrt{\frac{\pi}{2}}f^2\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^3}-\frac{\sqrt{\frac{\pi}{2}}f^2\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\right.}{\sqrt{b}d^3}$$

```
[Out] -((f*(d*e - c*f)*Cos[a + b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[a +
b*(c + d*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*F
resnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^3) - (f^2*Sqrt[Pi/
2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b^(3/2)*d^3)
```

Rubi [A] time = 0.339827, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3433, 3353, 3352, 3351, 3379, 2638, 3385, 3354}

$$\frac{\sqrt{\frac{\pi}{2}}f^2\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{2b^{3/2}d^3}-\frac{\sqrt{\frac{\pi}{2}}f^2\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{2b^{3/2}d^3}+\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\right.}{\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```


```
[Out] -((f*(d*e - c*f)*Cos[a + b*(c + d*x)^2])/(b*d^3)) - (f^2*(c + d*x)*Cos[a +
b*(c + d*x)^2])/(2*b*d^3) + (f^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/
Pi]*(c + d*x)])/(2*b^(3/2)*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelS
[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^3) + ((d*e - c*f)^2*Sqrt[Pi/2]*F
resnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^3) - (f^2*Sqrt[Pi/
2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b^(3/2)*d^3)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^2\right)\,dx = \frac{\text{Subst}\left(\int \left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+bx^2\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+bx^2\right)+f^2x\right.\right.}{d^3}$$

$$= \frac{f^2\,\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)dx,x,c+dx\right)}{d^3} + \frac{(2f(de-cf))\,\text{Subst}\left(\int x\sin\left(a+bx^2\right.\right.}{d^3}$$

$$= -\frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} + \frac{f^2\,\text{Subst}\left(\int\cos\left(a+bx^2\right)dx,x,c+dx\right)}{2bd^3} + \frac{(f(\ldots}{\ldots}$$

$$= -\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} + \frac{(de-cf)^2\sqrt{\ldots}}{\ldots}$$

$$= -\frac{f(de-cf)\cos\left(a+b(c+dx)^2\right)}{bd^3} - \frac{f^2(c+dx)\cos\left(a+b(c+dx)^2\right)}{2bd^3} + \frac{f^2\sqrt{\frac{\pi}{2}}\cos(a)\ldots}{\ldots}$$

Mathematica [A] time = 1.82488, size = 151, normalized size = 0.59

$$\frac{2\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)\left(2b\sin(a)(de-cf)^2+f^2\cos(a)\right)+2\sqrt{2\pi}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\left(2b\cos(a)(de-cf)^2-f^2\ldots\right.}{8b^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-4*Sqrt[b]*f*(2*d*e - c*f + d*f*x)*Cos[a + b*(c + d*x)^2] + 2*Sqrt[2*Pi]*F
resnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(2*b*(d*e - c*f)^2*Cos[a] - f^2*Sin[a
]) + 2*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*(f^2*Cos[a] + 2*b*
(d*e - c*f)^2*Sin[a]))/(8*b^(3/2)*d^3)
```

Maple [B] time = 0.008, size = 669, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+(d*x+c)^2*b),x)
```

```
[Out] -1/2*f^2/d^2/b*x*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-f^2*c/d*(-1/2/d^2/b*cos(b
*d^2*x^2+2*b*c*d*x+b*c^2+a)-1/2*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^
2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(
b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/P
i^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))))+1/4*f^2/d^2/b*2^(1/2)*Pi^(1/2)/(d^
2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1
/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))+sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)
*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))-e*f/d^2/b*cos(b*
d^2*x^2+2*b*c*d*x+b*c^2+a)-e*f*c/d*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2
*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b
*d^2*x+b*c*d))-sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi
^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))+1/2*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*e
^2*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2
*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*Fresnel
C(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))
```

Maxima [C] time = 3.63412, size = 3950, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + sin(-1/4*pi + 1/2*arctan2
(0, b)))*cos(a) - (cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arct
an2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan
2(0, b)))*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + ((-I*cos(1/4*pi + 1/2*
arctan2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arct
an2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + (cos(1/4*pi + 1/2*a
rctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arcta
n2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((I*b*d*x + I*b*
c)/sqrt(-I*b)))*e^2/(d*sqrt(abs(b))) - 1/2*(((e^(I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (I*e^(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
))*sin(a))*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2) + ((((2*I*sqrt(pi)*(erf(s
qrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*
b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*
b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2
- 2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c*d*x + ((2*I*sqrt(pi)*(erf(sqrt(
I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^
2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*b*d^
2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8*
b*c*d*x + 4*b*c^2, 0)) + ((2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x
+ I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)
```

```
) - 1))*cos(a) + (-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c
^2)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
 1))*sin(a))*b*c*d*x + (2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I
*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
 1))*cos(a) + (-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)
) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1)
)*sin(a))*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)))*sq
rt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*e*f/(b*d^2*abs(4*b*d^2*x^2 + 8*
b*c*d*x + 4*b*c^2)) + 1/32*(16*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)^2*((e^(I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
*cos(a) + (I*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - I*e^(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2))*sin(a))*c + ((((I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 +
2*I*b*c*d*x + I*b*c^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*
d*x - I*b*c^2)) - 1))*cos(a) + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^
2)) - 1))*sin(a))*b*c^2*d*x + ((I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*
d*x + I*b*c^2)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*
b*c^2)) - 1))*cos(a) + (sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*
c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1)
)*sin(a))*b*c^3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*cos(1/2*arctan2(4*b
*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + (((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2
*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x
 - I*b*c^2)) - 1))*cos(a) + (-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*
x + I*b*c^2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*
c^2)) - 1))*sin(a))*b*c^2*d*x + ((sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*
d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*
c^2)) - 1))*cos(a) + (-I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b
*c^2)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) -
 1))*sin(a))*b*c^3)*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)*sin(1/2*arctan2(
4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)) + (((-4*I*gamma(3/2, I*b*d^2*x^2 + 2
*I*b*c*d*x + I*b*c^2) + 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2
))*cos(a) - 4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2,
 -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*d^3*x^3 + ((-12*I*gamma(3
/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 12*I*gamma(3/2, -I*b*d^2*x^2 - 2
*I*b*c*d*x - I*b*c^2))*cos(a) - 12*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d^
2*x^2 + ((-12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + 12*I*gamm
a(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - 12*(gamma(3/2, I*b*d
^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I
*b*c^2))*sin(a))*b*c^2*d*x + ((-4*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - 4
*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2
 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c^3)*cos(3/2*arctan2(4*b*d^2*x^2 + 8*b
*c*d*x + 4*b*c^2, 0)) - ((4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2
) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (4*I*gamma(3
/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*
I*b*c*d*x - I*b*c^2))*sin(a))*b*d^3*x^3 + (12*(gamma(3/2, I*b*d^2*x^2 + 2*I
*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos
(a) - (12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) - 12*I*gamma(3/
2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c*d^2*x^2 + (12*(gamma(
3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -I*b*d^2*x^2 - 2*I*b
*c*d*x - I*b*c^2))*cos(a) - (12*I*gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2) - 12*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*sin(a))*b*c
^2*d*x + (4*(gamma(3/2, I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2) + gamma(3/2, -
I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) - (4*I*gamma(3/2, I*b*d^2*x^2
+ 2*I*b*c*d*x + I*b*c^2) - 4*I*gamma(3/2, -I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*
c^2))*sin(a))*b*c^3)*sin(3/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0))
)*sqrt(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*f^2/((b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)^2*b*d^3)
```

Fricas [A] time = 1.83911, size = 513, normalized size = 2.

$$\frac{\sqrt{2}\left(\pi f^2\cos(a)+2\,\pi\left(bd^2e^2-2\,bcdef+bc^2f^2\right)\sin(a)\right)\sqrt{\frac{bd^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)-\sqrt{2}\left(\pi f^2\sin(a)-2\,\pi\left(bd^2e^2-2\,bcdef\right.\right.}{4\,b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(pi*f^2*cos(a) + 2*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*si
n(a))*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - sqrt
(2)*(pi*f^2*sin(a) - 2*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos(a))*sqr
t(b*d^2/pi)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d) - 2*(b*d^2*f^2*
x + 2*b*d^2*e*f - b*c*d*f^2)*cos(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^2*d
^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)^2\sin\left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] time = 1.23698, size = 952, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a + 2)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)*(x + c/d))*e^(-I*a + 2)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/2
*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^
4) + 1)*(x + c/d))*e^(I*a + 1)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) +
 f*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a + 1)/(b*d))/d - 1/2*(-I*sqr
t(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*
(x + c/d))*e^(-I*a + 1)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I
*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a + 1)/(b*d))/d - 1/8*(-I*sqrt(2)*sq
rt(pi)*(2*b*c^2*f^2 + I*f^2)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^
2*d^4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)*b)
 + 2*I*(d*f^2*(-I*x - I*c/d) + 2*I*c*f^2)*e^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*
b*c^2 + I*a)/(b*d))/d^2 - 1/8*(I*sqrt(2)*sqrt(pi)*(2*b*c^2*f^2 - I*f^2)*erf
```

```
(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d))*e^(-I*a)/(
sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*b) + 2*I*(d*f^2*(-I*x - I*c/d) + 2*
I*c*f^2)*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))/d^2
```

3.167 $\int (e+fx)\sin\left(a+b(c+dx)^2\right)\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{\sqrt{b}d^2}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}-\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}$$

```
[Out] -(f*Cos[a + b*(c + d*x)^2])/(2*b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*Cos[a]*Fres
nelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2) + ((d*e - c*f)*Sqrt[Pi/2]
*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^2)
```

Rubi [A] time = 0.17774, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3433, 3353, 3352, 3351, 3379, 2638}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{\sqrt{b}d^2}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}-\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] -(f*Cos[a + b*(c + d*x)^2])/(2*b*d^2) + ((d*e - c*f)*Sqrt[Pi/2]*Cos[a]*Fres
nelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d^2) + ((d*e - c*f)*Sqrt[Pi/2]
*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d^2)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b(c+dx)^2\right)dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+bx^2\right)+fx\sin\left(a+bx^2\right)\right)dx,x,c+dx\right)}{d^2}$$
$$=\frac{f\,\text{Subst}\left(\int x\sin\left(a+bx^2\right)dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\,\text{Subst}\left(\int \sin\left(a+bx^2\right)dx,x,c\right.}{d^2}$$
$$=\frac{f\,\text{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^2\right)}{2d^2}+\frac{((de-cf)\cos(a))\,\text{Subst}\left(\int \sin\left(bx^2\right)dx,\right.}{d^2}$$
$$=-\frac{f\cos\left(a+b(c+dx)^2\right)}{2bd^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d^2}+\frac{(de-cf)\sqrt{\frac{\pi}{2}}C}{}$$

Mathematica [A] time = 0.60075, size = 114, normalized size = 0.93

$$\frac{\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)+\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)-f\cos\left(a+b(c+dx)^2\right)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (-(f*Cos[a + b*(c + d*x)^2]) + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fresne
lS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi]*FresnelC[
Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(2*b*d^2)
```

Maple [B] time = 0.01, size = 309, normalized size = 2.5

$$-\frac{f\cos\left(d^2x^2b+2\,cdxb+c^2b+a\right)}{2\,d^2b}-\frac{cf\sqrt{2}\sqrt{\pi}}{2\,d}\left(\cos\left(\frac{b^2c^2d^2-d^2b\left(c^2b+a\right)}{d^2b}\right)\text{FresnelS}\left(\frac{\sqrt{2}\left(bd^2x+bcd\right)}{\sqrt{\pi}}\frac{1}{\sqrt{d^2b}}\right)-\sin\left(\frac{b}{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+(d*x+c)^2*b),x)
```

```
[Out] -1/2*f/d^2/b*cos(b*d^2*x^2+2*b*c*d*x+b*c^2+a)-1/2*f*c/d*2^(1/2)*Pi^(1/2)/(d
^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^(1/2)/Pi^(
1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b
)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))+1/2*2^(1/2)*Pi^
(1/2)/(d^2*b)^(1/2)*e*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b)*FresnelS(2^
(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2-d^2*b*(b*c^2
+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d)))
```

Maxima [C] time = 2.53045, size = 1435, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + sin(-1/4*pi + 1/2*arctan2
(0, b)))*cos(a) - (cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arct
an2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan
2(0, b)))*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + ((-I*cos(1/4*pi + 1/2*
arctan2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arct
an2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + (cos(1/4*pi + 1/2*a
rctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arcta
n2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((I*b*d*x + I*b*
c)/sqrt(-I*b)))*e/(d*sqrt(abs(b))) - 1/4*(((e^(I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2) + e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))*cos(a) + (I*e^(I*b*d^2
*x^2 + 2*I*b*c*d*x + I*b*c^2) - I*e^(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
*sin(a))*abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2) + ((((2*I*sqrt(pi)*(erf(sqr
t(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*b*
d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*b*
d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 -
2*I*b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c*d*x + ((2*I*sqrt(pi)*(erf(sqrt(I*
b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*
x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*cos(a) + 2*(sqrt(pi)*(erf(sqrt(I*b*d^2*
x^2 + 2*I*b*c*d*x + I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*
b*c*d*x - I*b*c^2)) - 1))*sin(a))*b*c^2)*cos(1/2*arctan2(4*b*d^2*x^2 + 8*b*
c*d*x + 4*b*c^2, 0)) + ((2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x +
I*b*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2))
- 1))*cos(a) + (-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2
)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1
))*sin(a))*b*c*d*x + (2*(sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b
*c^2)) - 1) + sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1
))*cos(a) + (-2*I*sqrt(pi)*(erf(sqrt(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2))
- 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)) - 1))*
sin(a))*b*c^2)*sin(1/2*arctan2(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2, 0)))*sqrt
(abs(4*b*d^2*x^2 + 8*b*c*d*x + 4*b*c^2)))*f/(b*d^2*abs(4*b*d^2*x^2 + 8*b*c*
d*x + 4*b*c^2))
```

Fricas [A] time = 1.82045, size = 335, normalized size = 2.75

$$\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\left(de-cf\right)\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\left(de-cf\right)\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)\sin\left(a\right)-df\cos\left(bd^2x^2+2\,bcdx+\right.}{2\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*cos(a)*fresnel_sin(sqrt(2)*sqrt(
b*d^2/pi)*(d*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*(d*e - c*f)*fresnel_cos(
sqrt(2)*sqrt(b*d^2/pi)*(d*x + c)/d)*sin(a) - d*f*cos(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + a))/(b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e+fx)\sin\left(a+bc^2+2bcdx+bd^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2), x)
```

Giac [C] time = 1.1967, size = 525, normalized size = 4.3

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia+1)}}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia+1)}}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}cf\operatorname{erf}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a + 1)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) +
 1)*(x + c/d))*e^(-I*a + 1)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4
*(I*sqrt(2)*sqrt(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^
4) + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e
^(I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2 + I*a)/(b*d))/d - 1/4*(-I*sqrt(2)*sqr
t(pi)*c*f*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*(x + c/d
))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)) + f*e^(-I*b*d^2*x^2 -
 2*I*b*c*d*x - I*b*c^2 - I*a)/(b*d))/d
```

3.168 $\int \sin\left(a+b(c+dx)^2\right)\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d) + (S
qrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d)
```

Rubi [A] time = 0.0424326, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3353, 3352, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)])/(Sqrt[b]*d) + (S
qrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a])/(Sqrt[b]*d)
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+b(c+dx)^2\right)\,dx&=\cos(a)\int \sin\left(b(c+dx)^2\right)\,dx+\sin(a)\int \cos\left(b(c+dx)^2\right)\,dx\\&=\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)}{\sqrt{b}d}+\frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\sin(a)}{\sqrt{b}d}\end{aligned}$$

Mathematica [A] time = 0.0708423, size = 67, normalized size = 0.81

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}(c+dx)\right)+\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}(c+dx)\right)\right)}{\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)] + FresnelC[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)]*Sin[a]))/(Sqrt[b]*d)
```

Maple [B] time = 0.009, size = 136, normalized size = 1.6

$$\frac{\sqrt{2}\sqrt{\pi}}{2}\left(\cos\left(\frac{b^2c^2d^2-d^2b\left(c^2b+a\right)}{d^2b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\left(bd^2x+bcd\right)}{\sqrt{\pi}}\frac{1}{\sqrt{d^2b}}\right)-\sin\left(\frac{b^2c^2d^2-d^2b\left(c^2b+a\right)}{d^2b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+(d*x+c)^2*b),x)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/(d^2*b)^(1/2)*(cos((b^2*c^2*d^2-d^2*b*(b*c^2+a))/d^2/b
)*FresnelS(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b*c*d))-sin((b^2*c^2*d^2
-d^2*b*(b*c^2+a))/d^2/b)*FresnelC(2^(1/2)/Pi^(1/2)/(d^2*b)^(1/2)*(b*d^2*x+b
*c*d)))
```

Maxima [C] time = 1.79814, size = 335, normalized size = 4.04

$$-\frac{\sqrt{\pi}\left(\left(\left(-i\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,b\right)\right)-i\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,b\right)\right)-\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,b\right)\right)+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\mathrm{ar}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, b)) - sin(1/4*pi + 1/2*arctan2(0, b)) + sin(-1/4*pi + 1/2*arctan2
(0, b)))*cos(a) - (cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arct
an2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan
2(0, b)))*sin(a))*erf((I*b*d*x + I*b*c)/sqrt(I*b)) + ((-I*cos(1/4*pi + 1/2*
arctan2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arct
an2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + (cos(1/4*pi + 1/2*a
rctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arcta
n2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((I*b*d*x + I*b*
c)/sqrt(-I*b)))/(d*sqrt(abs(b)))
```

Fricas [A] time = 1.64334, size = 242, normalized size = 2.92

$$\frac{\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)+\sqrt{2}\pi\sqrt{\frac{bd^2}{\pi}}\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{bd^2}{\pi}}(dx+c)}{d}\right)\sin\left(a\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b*d^2/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b*d^2/pi)*(d
*x + c)/d) + sqrt(2)*pi*sqrt(b*d^2/pi)*fresnel_cos(sqrt(2)*sqrt(b*d^2/pi)*(
d*x + c)/d)*sin(a))/(b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + b\left(c + dx\right)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**2),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**2), x)
```

Giac [C] time = 1.13234, size = 204, normalized size = 2.46

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(ia)}}{4\sqrt{bd^2}\left(-\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)} - \frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)\left(x+\frac{c}{d}\right)\right)e^{(-ia)}}{4\sqrt{bd^2}\left(\frac{ibd^2}{\sqrt{b^2d^4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4)
 + 1)*(x + c/d))*e^(I*a)/(sqrt(b*d^2)*(-I*b*d^2/sqrt(b^2*d^4) + 1)) - 1/4*I
*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1)*
(x + c/d))*e^(-I*a)/(sqrt(b*d^2)*(I*b*d^2/sqrt(b^2*d^4) + 1))
```

3.169 $\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^2\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Rubi [A] time = 0.0126257, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

Mathematica [A] time = 15.2501, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x), x]
```

Maple [A] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+(dx+c)^2 b\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+(d*x+c)^2*b)/(f*x+e),x)
```

```
[Out] int(sin(a+(d*x+c)^2*b)/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx + c)^2 b + a\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(bd^2x^2 + 2\,bcdx + bc^2 + a\right)}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + bc^2 + 2bcdx + bd^2x^2\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**2)/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx + c)^2 b + a\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e), x)
```

3.170 $\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0126098, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 25.9943, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^2\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+(dx+c)^2b\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+(d*x+c)^2*b)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+(d*x+c)^2*b)/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(bd^2x^2+2\,bcdx+bc^2+a\right)}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^2*x^2 + 2*e*f*x + e^2),
x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bc^2+2bcdx+bd^2x^2\right)}{\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(e + f*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^2b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

3.171 $\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right) dx$

Optimal. Leaf size=434

$$\frac{ie^{ia}f(c+dx)^2(de-cf)^2\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)^2\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de}{}$$

```
[Out] -((f^2*(d*e - c*f)*Cos[a + b*(c + d*x)^3])/(b*d^4)) - (f^3*(c + d*x)*Cos[a
+ b*(c + d*x)^3])/(3*b*d^4) - (E^(I*a)*f^3*(c + d*x)*Gamma[1/3, (-I)*b*(c +
 d*x)^3])/(18*b*d^4*((-I)*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*(d*e - c*f
)^3*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(1/
3)) - (f^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(18*b*d^4*E^(I*a)*(I*b*(c
 + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)
^3])/(d^4*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/2)*E^(I*a)*f*(d*e - c*f)^2
*(c + d*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(2/3
)) - ((I/2)*f*(d*e - c*f)^2*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^4*E
^(I*a)*(I*b*(c + d*x)^3)^(2/3))
```

Rubi [A] time = 0.444276, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3433, 3355, 2208, 3389, 2218, 3379, 2638, 3385, 3356}

$$\frac{ie^{ia}f(c+dx)^2(de-cf)^2\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{2d^4\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)^2\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{2d^4\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```


```
[Out] -((f^2*(d*e - c*f)*Cos[a + b*(c + d*x)^3])/(b*d^4)) - (f^3*(c + d*x)*Cos[a
+ b*(c + d*x)^3])/(3*b*d^4) - (E^(I*a)*f^3*(c + d*x)*Gamma[1/3, (-I)*b*(c +
 d*x)^3])/(18*b*d^4*((-I)*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*(d*e - c*f
)^3*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(1/
3)) - (f^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(18*b*d^4*E^(I*a)*(I*b*(c
 + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)^3*(c + d*x)*Gamma[1/3, I*b*(c + d*x)
^3])/(d^4*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/2)*E^(I*a)*f*(d*e - c*f)^2
*(c + d*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^4*((-I)*b*(c + d*x)^3)^(2/3
)) - ((I/2)*f*(d*e - c*f)^2*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^4*E
^(I*a)*(I*b*(c + d*x)^3)^(2/3))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rubi steps

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right) dx = \frac{\text{Subst}\left(\int \left(d^3e^3\left(1-\frac{cf(3d^2e^2-3cdef+c^2f^2)}{d^3e^3}\right)\sin\left(a+bx^3\right)+3d^2e^2f\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)x\,\text{s}\right.\right.}{d^{4}}$$

$$= \frac{f^3\,\text{Subst}\left(\int x^3\sin\left(a+bx^3\right)dx, x, c+dx\right)}{d^4} + \frac{\left(3f^2(de-cf)\right)\text{Subst}\left(\int x^2\sin\left(a+b\right.\right.}{d^4}$$

$$= -\frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} + \frac{f^3\,\text{Subst}\left(\int \cos\left(a+bx^3\right)dx, x, c+dx\right)}{3bd^4} + \frac{(f^2}{}$$

$$= -\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4} - \frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} + \frac{ie^{ia}(de-cf)}{6}$$

$$= -\frac{f^2(de-cf)\cos\left(a+b(c+dx)^3\right)}{bd^4} - \frac{f^3(c+dx)\cos\left(a+b(c+dx)^3\right)}{3bd^4} - \frac{e^{ia}f^3(c+dx}{18bd^4}$$

Mathematica [F] time = 102.679, size = 0, normalized size = 0.

$$\int (e+fx)^3 \sin\left(a+b(c+dx)^3\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] Integrate[(e + f*x)^3*Sin[a + b*(c + d*x)^3], x]
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{3}\sin\left(a+(dx+c)^{3}b\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sin(a+(d*x+c)^3*b),x)
```

```
[Out] int((f*x+e)^3*sin(a+(d*x+c)^3*b),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{3}\sin\left((dx+c)^{3}b+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

Fricas [A] time = 1.97094, size = 1007, normalized size = 2.32

$$-\frac{\left(3\,bd^3e^3 - 9\,bcd^2e^2f + 9\,bc^2def^2 - 3\,bc^3f^3 - i\,f^3\right)\left(i\,bd^3\right)^{\frac{2}{3}}e^{(-i\,a)}\Gamma\left(\frac{1}{3}, i\,bd^3x^3 + 3i\,bcd^2x^2 + 3i\,bc^2dx + i\,bc^3\right) + \left(3\,bd^3e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/18*((3*b*d^3*e^3 - 9*b*c*d^2*e^2*f + 9*b*c^2*d*e*f^2 - 3*b*c^3*f^3 - I*f
^3)*(I*b*d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I
*b*c^2*d*x + I*b*c^3) + (3*b*d^3*e^3 - 9*b*c*d^2*e^2*f + 9*b*c^2*d*e*f^2 -
3*b*c^3*f^3 + I*f^3)*(-I*b*d^3)^(2/3)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3 - 3*I
*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + 9*(b*d^3*e^2*f - 2*b*c*d^2*e*f^2
+ b*c^2*d*f^3)*(I*b*d^3)^(1/3)*e^(-I*a)*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^
2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + 9*(b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2
*d*f^3)*(-I*b*d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2
- 3*I*b*c^2*d*x - I*b*c^3) + 6*(b*d^3*f^3*x + 3*b*d^3*e*f^2 - 2*b*c*d^2*f^3
)*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(b^2*d^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^{3} \sin\left(a + bc^{3} + 3bc^{2}dx + 3bcd^{2}x^{2} + bd^{3}x^{3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)**3*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d
**3*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)^{3} \sin\left((dx + c)^{3}b + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sin((d*x + c)^3*b + a), x)
```

3.172 $\int (e+fx)^2 \sin\left(a+b(c+dx)^3\right)\,dx$

Optimal. Leaf size=280

$$\frac{ie^{ia}f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{3d^3\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{3d^3\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de-}{6a}$$

```
[Out] -(f^2*Cos[a + b*(c + d*x)^3])/(3*b*d^3) + ((I/6)*E^(I*a)*(d*e - c*f)^2*(c +
 d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(1/3)) - ((
I/6)*(d*e - c*f)^2*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I*b
*(c + d*x)^3)^(1/3)) + ((I/3)*E^(I*a)*f*(d*e - c*f)*(c + d*x)^2*Gamma[2/3,
(-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(2/3)) - ((I/3)*f*(d*e - c*f
)*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I*b*(c + d*x)^3)^(
2/3))
```

Rubi [A] time = 0.276133, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3433, 3355, 2208, 3389, 2218, 3379, 2638}

$$\frac{ie^{ia}f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^3\right)}{3d^3\left(-ib(c+dx)^3\right)^{2/3}}-\frac{ie^{-ia}f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^3\right)}{3d^3\left(ib(c+dx)^3\right)^{2/3}}+\frac{ie^{ia}(c+dx)(de-}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] -(f^2*Cos[a + b*(c + d*x)^3])/(3*b*d^3) + ((I/6)*E^(I*a)*(d*e - c*f)^2*(c +
 d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(1/3)) - ((
I/6)*(d*e - c*f)^2*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I*b
*(c + d*x)^3)^(1/3)) + ((I/3)*E^(I*a)*f*(d*e - c*f)*(c + d*x)^2*Gamma[2/3,
(-I)*b*(c + d*x)^3])/(d^3*((-I)*b*(c + d*x)^3)^(2/3)) - ((I/3)*f*(d*e - c*f
)*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^3*E^(I*a)*(I*b*(c + d*x)^3)^(
2/3))
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (e+fx)^2\sin\left(a+b(c+dx)^3\right)\,dx = \frac{\text{Subst}\left(\int\left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+bx^3\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+bx^3\right)+\right.\right.}{d^3}$$

$$=\frac{f^2\,\text{Subst}\left(\int x^2\sin\left(a+bx^3\right)\,dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\,\text{Subst}\left(\int x\sin\left(a+\right.\right.}{d^3}$$

$$=\frac{f^2\,\text{Subst}\left(\int \sin(a+bx)\,dx,x,(c+dx)^3\right)}{3d^3}+\frac{(if(de-cf))\,\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx,\right.}{d^3}$$

$$=-\frac{f^2\cos\left(a+b(c+dx)^3\right)}{3bd^3}+\frac{ie^{ia}(de-cf)^2(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^3\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(de-}{}$$

Mathematica [F] time = 40.1766, size = 0, normalized size = 0.

$$\int (e+fx)^2\sin\left(a+b(c+dx)^3\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^3],x]
```

```
[Out] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^3], x]
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{2}\sin\left(a+(dx+c)^{3}b\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+(d*x+c)^3*b),x)
```

```
[Out] int((f*x+e)^2*sin(a+(d*x+c)^3*b),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2 \sin\left((dx+c)^3b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

Fricas [A] time = 1.87395, size = 774, normalized size = 2.76

$$-\frac{2\,d^2 f^2 \cos\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)+\left(i\,bd^3\right)^{\frac{2}{3}}\left(d^2e^2-2\,cdef+c^2f^2\right)e^{(-i\,a)}\Gamma\left(\frac{1}{3}, i\,bd^3x^3+3i\,bcd^2x^2+3i\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*d^2*f^2*cos(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) +
(I*b*d^3)^(2/3)*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*e^(-I*a)*gamma(1/3, I*b*d^3
*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*(d^2*e
^2 - 2*c*d*e*f + c^2*f^2)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3 - 3*I*b*c*d^2*x^2
 - 3*I*b*c^2*d*x - I*b*c^3) + 2*(I*b*d^3)^(1/3)*(d^2*e*f - c*d*f^2)*e^(-I*a
)*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x + I*b*c^3) + 2*(
-I*b*d^3)^(1/3)*(d^2*e*f - c*d*f^2)*e^(I*a)*gamma(2/3, -I*b*d^3*x^3 - 3*I*b
*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^2 \sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d
**3*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2 \sin\left((dx+c)^3b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^3*b + a), x)
```

3.173 $\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$

Optimal. Leaf size=235

$$\frac{ie^{ia}(c+dx)(de-cf)\mathrm{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)(de-cf)\mathrm{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}+\frac{ie^{ia}f(c+dx)^2\mathrm{Gamma}}{6d^2\left(-ib(c+\right.}$$

```
[Out] ((I/6)*E^(I*a)*(d*e - c*f)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^2*(
(-I)*b*(c + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)*(c + d*x)*Gamma[1/3, I*b*(c
 + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*f*(c + d
*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^2*((-I)*b*(c + d*x)^3)^(2/3)) - ((
I/6)*f*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)
^3)^(2/3))
```

Rubi [A] time = 0.192315, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3433, 3355, 2208, 3389, 2218}

$$\frac{ie^{ia}(c+dx)(de-cf)\mathrm{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)(de-cf)\mathrm{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}+\frac{ie^{ia}f(c+dx)^2\mathrm{Gamma}}{6d^2\left(-ib(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^3],x]
```


```
[Out] ((I/6)*E^(I*a)*(d*e - c*f)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d^2*(
(-I)*b*(c + d*x)^3)^(1/3)) - ((I/6)*(d*e - c*f)*(c + d*x)*Gamma[1/3, I*b*(c
 + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)^3)^(1/3)) + ((I/6)*E^(I*a)*f*(c + d
*x)^2*Gamma[2/3, (-I)*b*(c + d*x)^3])/(d^2*((-I)*b*(c + d*x)^3)^(2/3)) - ((
I/6)*f*(c + d*x)^2*Gamma[2/3, I*b*(c + d*x)^3])/(d^2*E^(I*a)*(I*b*(c + d*x)
^3)^(2/3))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+bx^3\right)+fx\sin\left(a+bx^3\right)\right)dx,x,c+dx\right)}{d^2}$$
$$= \frac{f\,\text{Subst}\left(\int x\sin\left(a+bx^3\right)dx,x,c+dx\right)}{d^2}+\frac{(de-cf)\,\text{Subst}\left(\int \sin\left(a+bx^3\right)dx,\right.}{d^2}$$
$$= \frac{(if)\,\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx,x,c+dx\right)}{2d^2}-\frac{(if)\,\text{Subst}\left(\int e^{ia+ibx^3}x\,dx,x,c+dx\right)}{2d^2}+\frac{(i}{}$$
$$= \frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d^2\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(de-cf)(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d^2\sqrt[3]{ib(c+dx)^3}}$$

Mathematica [F] time = 76.3972, size = 0, normalized size = 0.

$$\int (e+fx)\sin\left(a+b(c+dx)^3\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^3],x]

[Out] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^3], x]
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \left(fx+e\right)\sin\left(a+(dx+c)^3 b\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+(d*x+c)^3*b),x)

[Out] int((f*x+e)*sin(a+(d*x+c)^3*b),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\sin\left((dx+c)^3 b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

Fricas [A] time = 1.76977, size = 578, normalized size = 2.46

$$-\frac{\left(i\,bd^3\right)^{\frac{1}{3}}\,dfe^{(-i\,a)}\Gamma\left(\frac{2}{3}, i\,bd^3x^3 + 3i\,bcd^2x^2 + 3i\,bc^2dx + i\,bc^3\right) + \left(-i\,bd^3\right)^{\frac{1}{3}}\,dfe^{(i\,a)}\Gamma\left(\frac{2}{3}, -i\,bd^3x^3 - 3i\,bcd^2x^2 - 3i\,bc^2dx - i\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*((I*b*d^3)^(1/3)*d*f*e^(-I*a)*gamma(2/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2
 + 3*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(1/3)*d*f*e^(I*a)*gamma(2/3, -I*b*
d^3*x^3 - 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3) + (I*b*d^3)^(2/3)*(d*e
 - c*f)*e^(-I*a)*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3*I*b*c^2*d*x +
 I*b*c^3) + (-I*b*d^3)^(2/3)*(d*e - c*f)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3 -
3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)\sin\left(a + bc^3 + 3bc^2dx + 3bcd^2x^2 + bd^3x^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3
*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\sin\left(\left(dx + c\right)^3 b + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin((d*x + c)^3*b + a), x)
```

3.174 $\int \sin\left(a+b(c+dx)^3\right) dx$

Optimal. Leaf size=107

$$\frac{ie^{ia}(c+dx)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

```
[Out] ((I/6)*E^(I*a)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d*((-I)*b*(c + d*
x)^3)^(1/3)) - ((I/6)*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d*E^(I*a)*(I*
b*(c + d*x)^3)^(1/3))
```

Rubi [A] time = 0.028851, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3355, 2208}

$$\frac{ie^{ia}(c+dx)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*E^(I*a)*(c + d*x)*Gamma[1/3, (-I)*b*(c + d*x)^3])/(d*((-I)*b*(c + d*
x)^3)^(1/3)) - ((I/6)*(c + d*x)*Gamma[1/3, I*b*(c + d*x)^3])/(d*E^(I*a)*(I*
b*(c + d*x)^3)^(1/3))
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+b(c+dx)^3\right) dx &= \frac{1}{2}i\int e^{-ia-ib(c+dx)^3}\,dx-\frac{1}{2}i\int e^{ia+ib(c+dx)^3}\,dx\\ &= \frac{ie^{ia}(c+dx)\Gamma\left(\frac{1}{3},-ib(c+dx)^3\right)}{6d\sqrt[3]{-ib(c+dx)^3}}-\frac{ie^{-ia}(c+dx)\Gamma\left(\frac{1}{3},ib(c+dx)^3\right)}{6d\sqrt[3]{ib(c+dx)^3}}\end{aligned}$$

Mathematica [A] time = 0.0168614, size = 115, normalized size = 1.07

$$\frac{i(c+dx)\left((\cos(a)+i\sin(a))\sqrt[3]{ib(c+dx)^3}\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^3\right)-(\cos(a)-i\sin(a))\sqrt[3]{-ib(c+dx)^3}\text{Gamma}\left(\frac{1}{3},ib(\right.\right.}{6d\sqrt[3]{b^2(c+dx)^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^3],x]
```

```
[Out] ((I/6)*(c + d*x)*(-(((-I)*b*(c + d*x)^3)^(1/3)*Gamma[1/3, I*b*(c + d*x)^3]*
(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^3)^(1/3)*Gamma[1/3, (-I)*b*(c + d*x)^
3]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^6)^(1/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \sin\left(a + (dx + c)^3 b\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+(d*x+c)^3*b),x)
```

```
[Out] int(sin(a+(d*x+c)^3*b),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left((dx + c)^3 b + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a), x)
```

Fricas [A] time = 1.80001, size = 277, normalized size = 2.59

$$-\frac{\left(i\,bd^3\right)^{\frac{2}{3}} e^{(-i\,a)}\Gamma\left(\frac{1}{3}, i\,bd^3x^3 + 3i\,bcd^2x^2 + 3i\,bc^2dx + i\,bc^3\right) + \left(-i\,bd^3\right)^{\frac{2}{3}} e^{(i\,a)}\Gamma\left(\frac{1}{3}, -i\,bd^3x^3 - 3i\,bcd^2x^2 - 3i\,bc^2dx - i\,bc^3\right)}{6\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*((I*b*d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b*d^3*x^3 + 3*I*b*c*d^2*x^2 + 3
*I*b*c^2*d*x + I*b*c^3) + (-I*b*d^3)^(2/3)*e^(I*a)*gamma(1/3, -I*b*d^3*x^3
- 3*I*b*c*d^2*x^2 - 3*I*b*c^2*d*x - I*b*c^3))/(b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + b\left(c + dx\right)^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left((dx+c)^3 b + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^3*b + a), x)
```

3.175 $\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Rubi [A] time = 0.012963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx = \int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Mathematica [A] time = 57.8878, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x), x]
```

Maple [A] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+(dx+c)^3 b\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+(d*x+c)^3*b)/(f*x+e),x)
```

```
[Out] int(sin(a+(d*x+c)^3*b)/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^3b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f*x + e)
, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**3)/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
+ f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^3b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e), x)
```

3.176 $\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0118501, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 121.895, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^3\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^3]/(e + f*x)^2, x]
```

Maple [A] time = 0.164, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+(dx+c)^3 b\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+(d*x+c)^3*b)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+(d*x+c)^3*b)/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^3b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(f^2*x^2
+ 2*e*f*x + e^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)}{\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**3)/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(e
+ f*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^3b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^3*b + a)/(f*x + e)^2, x)
```

3.177 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^2}\right) dx$

Optimal. Leaf size=371

$$\frac{2\sqrt{2\pi}b^{3/2}f^2\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{3d^3}+\frac{2\sqrt{2\pi}b^{3/2}f^2\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf\cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\cdots}{\cdots}$$

```
[Out] (2*b*f^2*(c + d*x)*Cos[a + b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*Cos[a
]*CosIntegral[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*Cos[a
]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]
*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + (2*b^(3/2)*f^2*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/(3*d^3) + (Sqrt
[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a
])/d^3 + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^3 + (f*(d*e - c
*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^2])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/(c
 + d*x)^2])/(3*d^3) + (b*f*(d*e - c*f)*Sin[a]*SinIntegral[b/(c + d*x)^2])/d
^3
```

Rubi [A] time = 0.479071, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {3433, 3359, 3387, 3354, 3352, 3351, 3379, 3297, 3303, 3299, 3302, 3409, 3388, 3353}

$$\frac{2\sqrt{2\pi}b^{3/2}f^2\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{3d^3}+\frac{2\sqrt{2\pi}b^{3/2}f^2\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{3d^3}-\frac{bf\cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{2\pi}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]
```


```
[Out] (2*b*f^2*(c + d*x)*Cos[a + b/(c + d*x)^2])/(3*d^3) - (b*f*(d*e - c*f)*Cos[a
]*CosIntegral[b/(c + d*x)^2])/d^3 - (Sqrt[b]*(d*e - c*f)^2*Sqrt[2*Pi]*Cos[a
]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^3 + (2*b^(3/2)*f^2*Sqrt[2*Pi]
*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/(3*d^3) + (2*b^(3/2)*f^2*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/(3*d^3) + (Sqrt
[b]*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a
])/d^3 + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^3 + (f*(d*e - c
*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^2])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/(c
 + d*x)^2])/(3*d^3) + (b*f*(d*e - c*f)*Sin[a]*SinIntegral[b/(c + d*x)^2])/d
^3
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
```

```
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)dx=\frac{\text{Subst}\left(\int\left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+\frac{b}{x^2}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+\frac{b}{x^2}\right)+f^2x^2 s\right.}{d^3}$$

$$=\frac{f^2\,\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\,\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^2}\right)\right.}{d^3}$$

$$=-\frac{f^2\,\text{Subst}\left(\int\frac{\sin(a+bx^2)}{x^4}dx,x,\frac{1}{c+dx}\right)}{d^3}-\frac{(f(de-cf))\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}dx,x,\frac{1}{(c+dx)^2}\right)}{d^3}$$

$$=\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f^2(c+}{}$$

$$=\frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}+\frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^3}+\frac{f(de-cf)(c}{}$$

$$=\frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}-\frac{bf(de-cf)\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}}{d}$$

$$=\frac{2bf^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^2}\right)}{3d^3}-\frac{bf(de-cf)\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{d^3}-\frac{\sqrt{b}(de-cf)^2\sqrt{2\pi}}{d}$$

Mathematica [A] time = 1.46365, size = 467, normalized size = 1.26

$$\frac{2\sqrt{2\pi}b^{3/2}f^2\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)-3c^2def\sin\left(a+\frac{b}{(c+dx)^2}\right)+3\sqrt{2\pi}\sqrt{b}c^2f^2\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)+c^3f^2\sin\left(a+\frac{b}{(c+dx)^2}\right)+3bf\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (2*b*c*f^2*Cos[a + b/(c + d*x)^2] + 2*b*d*f^2*x*Cos[a + b/(c + d*x)^2] + 3*
b*f*(-(d*e) + c*f)*Cos[a]*CosIntegral[b/(c + d*x)^2] + 2*b^(3/2)*f^2*Sqrt[2
*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 3*Sqrt[b]*d^2*e^2*Sq
```

```
rt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 6*Sqrt[b]*c*d*e*
f*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 3*Sqrt[b]*c^
2*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + Sqrt[b]*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*(-3*(d*e - c*f)^2*Cos[a
] + 2*b*f^2*Sin[a]) + 3*c*d^2*e^2*Sin[a + b/(c + d*x)^2] - 3*c^2*d*e*f*Sin[
a + b/(c + d*x)^2] + c^3*f^2*Sin[a + b/(c + d*x)^2] + 3*d^3*e^2*x*Sin[a + b
/(c + d*x)^2] + 3*d^3*e*f*x^2*Sin[a + b/(c + d*x)^2] + d^3*f^2*x^3*Sin[a +
b/(c + d*x)^2] + 3*b*d*e*f*Sin[a]*SinIntegral[b/(c + d*x)^2] - 3*b*c*f^2*Si
n[a]*SinIntegral[b/(c + d*x)^2])/(3*d^3)
```

Maple [A] time = 0.016, size = 302, normalized size = 0.8

$$-\frac{1}{d^3}\left(-\left(c^2f^2-2\,cdef+d^2e^2\right)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+\left(c^2f^2-2\,cdef+d^2e^2\right)\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\cdots}{\sqrt{\pi}\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^2),x)
```

```
[Out] -1/d^3*(-(c^2*f^2-2*c*d*e*f+d^2*e^2)*(d*x+c)*sin(a+b/(d*x+c)^2)+(c^2*f^2-2*
c*d*e*f+d^2*e^2)*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/
Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))-1/2*(-
2*c*f^2+2*d*e*f)*(d*x+c)^2*sin(a+b/(d*x+c)^2)+(-2*c*f^2+2*d*e*f)*b*(1/2*cos
(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2))-1/3*f^2*(d*x+c)^3*sin(a+b/(
d*x+c)^2)+2/3*f^2*b*(-(d*x+c)*cos(a+b/(d*x+c)^2)-b^(1/2)*2^(1/2)*Pi^(1/2)*(
cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))+sin(a)*FresnelC(b^(1/2)*2
^(1/2)/Pi^(1/2)/(d*x+c)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,bf^2x\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)+\left(\frac{c^3f^2\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{d^3}-2\int\frac{2\,b^2f^2x\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)-3\left(\left(bd^2ef-bcdf^2\right)x^2+\left(bd^2e^2-bc^2f^2\right)x\right)\cdots}{2\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*b*f^2*x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2)) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x + a*
c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*c*d^2*f
^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^
3*d^2), x) - 3*d^2*integrate(1/3*(2*b^2*d*f^2*x*sin((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)) + (b*c^3*f^2 - 3*(b*d^3*e*f - b*c*d
^2*f^2)*x^2 - 3*(b*d^3*e^2 - b*c^2*d*f^2)*x)*cos((a*d^2*x^2 + 2*a*c*d*x + a
*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/((d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x
 + c^3*d^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2))^2 + (d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)*sin((a*d^2*x^2 + 2*
a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + (d^2*f^2*x^3 + 3*d
^2*e*f*x^2 + 3*d^2*e^2*x)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
+ 2*c*d*x + c^2)))/d^2
```

Fricas [A] time = 2.07514, size = 1040, normalized size = 2.8

$$2\sqrt{2}\left(2\,\pi bdf^2\sin(a) - 3\,\pi\left(d^3e^2 - 2\,cd^2ef + c^2df^2\right)\cos(a)\right)\sqrt{\frac{b}{\pi d^2}}\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right) + 2\sqrt{2}\left(2\,\pi bdf^2\cos(a) + 3\,\pi\left(d^3e^2 - 2\,cd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(2)*(2*pi*b*d*f^2*sin(a) - 3*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f
^2)*cos(a))*sqrt(b/(pi*d^2))*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x +
c)) + 2*sqrt(2)*(2*pi*b*d*f^2*cos(a) + 3*pi*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*
f^2)*sin(a))*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x +
 c)) + 6*(b*d*e*f - b*c*f^2)*sin(a)*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2
)) - 3*((b*d*e*f - b*c*f^2)*cos_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*
d*e*f - b*c*f^2)*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2)))*cos(a) + 4*(b*
d*f^2*x + b*c*f^2)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d
*x + c^2)) + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2 - 3
*c^2*d*e*f + c^3*f^2)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*
c*d*x + c^2)))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \sin\left(a + \frac{b}{(dx+c)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^2), x)
```

3.178 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)\,dx$

Optimal. Leaf size=198

$$-\frac{bf\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^2}+\frac{\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}+$$

```
[Out] -(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2])/(2*d^2) - (Sqrt[b]*(d*e - c*f)*Sqr
t[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + (Sqrt[b]*(d*
e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d^2 +
((d*e - c*f)*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[a +
 b/(c + d*x)^2])/(2*d^2) + (b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/(2*d^2)
```

Rubi [A] time = 0.259283, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3433, 3359, 3387, 3354, 3352, 3351, 3379, 3297, 3303, 3299, 3302}

$$-\frac{bf\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right)}{2d^2}-\frac{\sqrt{2\pi}\sqrt{b}\cos(a)(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d^2}+\frac{\sqrt{2\pi}\sqrt{b}\sin(a)(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] -(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2])/(2*d^2) - (Sqrt[b]*(d*e - c*f)*Sqr
t[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d^2 + (Sqrt[b]*(d*
e - c*f)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d^2 +
((d*e - c*f)*(c + d*x)*Sin[a + b/(c + d*x)^2])/d^2 + (f*(c + d*x)^2*Sin[a +
 b/(c + d*x)^2])/(2*d^2) + (b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/(2*d^2)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^2}\right)dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+\frac{b}{x^2}\right)+fx\sin\left(a+\frac{b}{x^2}\right)\right)dx, x, c+dx\right)}{d^2}$$

$$= \frac{f\,\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^2}\right)dx, x, c+dx\right)}{d^2} + \frac{(de-cf)\,\text{Subst}\left(\int \sin\left(a+\frac{b}{x^2}\right)dx, x,\right.}{d^2}$$

$$= -\frac{f\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}dx, x, \frac{1}{(c+dx)^2}\right)}{2d^2} - \frac{(de-cf)\,\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{(bf)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{(bf\cos(a))\,\text{Su}}{}$$

$$= -\frac{bf\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^2}\right)}{2d^2} - \frac{\sqrt{b}(de-cf)\sqrt{2\pi}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d^2} + \frac{\sqrt{b}(de-cf)\sqrt{2\pi}S}{d^2}$$

Mathematica [A] time = 0.771596, size = 242, normalized size = 1.22

$$\frac{c^2(-f)\sin\left(a+\frac{b}{(c+dx)^2}\right) - bf\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^2}\right) + 2d^2ex\sin\left(a+\frac{b}{(c+dx)^2}\right) + d^2fx^2\sin\left(a+\frac{b}{(c+dx)^2}\right) - 2\sqrt{2\pi}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (-(b*f*Cos[a]*CosIntegral[b/(c + d*x)^2]) - 2*Sqrt[b]*(d*e - c*f)*Sqrt[2*Pi
]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)] + 2*Sqrt[b]*d*e*Sqrt[2*Pi
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] - 2*Sqrt[b]*c*f*Sqrt[2*Pi
]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + 2*c*d*e*Sin[a + b/(c +
d*x)^2] - c^2*f*Sin[a + b/(c + d*x)^2] + 2*d^2*e*x*Sin[a + b/(c + d*x)^2] +
 d^2*f*x^2*Sin[a + b/(c + d*x)^2] + b*f*Sin[a]*SinIntegral[b/(c + d*x)^2])/
(2*d^2)
```

Maple [A] time = 0.013, size = 150, normalized size = 0.8

$$\frac{1}{d^2}\left(-\left(cf-de\right)(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right)+\left(cf-de\right)\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}\,(dx+c)}\sqrt{b}\right)-\sin(a)\,\text{Fresn}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^2),x)
```

```
[Out] 1/d^2*(-(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^2)+(c*f-d*e)*b^(1/2)*2^(1/2)*Pi^(
1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)))+1/2*f*(d*x+c)^2*sin(a+b/(d*x+c)^2)-f*b*(1/2
*cos(a)*Ci(b/(d*x+c)^2)-1/2*sin(a)*Si(b/(d*x+c)^2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(fx^2+2\,ex\right)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)+\int\frac{\left(bdfx^2+2\,bdex\right)\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{2\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)}\,dx+\int\frac{}{2\left(\left(d^3x^3+3\,cd^2x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*
d*x + c^2)) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*
d*x + c^3), x) + integrate(1/2*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^2*x^2 + 2*a
*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 + 3*c*d^2*x^2 + 3*
c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((a*d^2*x^2 + 2*a*c
*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x)
```

Fricas [A] time = 1.98199, size = 670, normalized size = 3.38

$$-\frac{4\sqrt{2}\pi\left(d^2e-cdf\right)\sqrt{\frac{b}{\pi d^2}}\cos\left(a\right)C\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-4\sqrt{2}\pi\left(d^2e-cdf\right)\sqrt{\frac{b}{\pi d^2}}S\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)\sin\left(a\right)-2\,bf\sin\left(a\right)\mathrm{Si}\left(\frac{b}{d^2x^2+2\,cdx+c^2}\right.}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqrt
(2)*d*sqrt(b/(pi*d^2))/(d*x + c)) - 4*sqrt(2)*pi*(d^2*e - c*d*f)*sqrt(b/(pi
*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/(pi*d^2))/(d*x + c))*sin(a) - 2*b*f*sin
(a)*sin_integral(b/(d^2*x^2 + 2*c*d*x + c^2)) + (b*f*cos_integral(b/(d^2*x^
2 + 2*c*d*x + c^2)) + b*f*cos_integral(-b/(d^2*x^2 + 2*c*d*x + c^2)))*cos(a
) - 2*(d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*sin((a*d^2*x^2 + 2*a*c*d*x
+ a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)\sin\left(a+\frac{b}{c^2+2cdx+d^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(fx+e\right)\sin\left(a+\frac{b}{\left(dx+c\right)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^2), x)
```

3.179 $\int \sin\left(a+\frac{b}{(c+dx)^2}\right)dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d}+\frac{\sqrt{2\pi}\sqrt{b}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) +
 (Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d + (
(c + d*x)*Sin[a + b/(c + d*x)^2])/d
```

Rubi [A] time = 0.0684306, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3359, 3387, 3354, 3352, 3351}

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right)}{d}+\frac{\sqrt{2\pi}\sqrt{b}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^2],x]
```

```
[Out] -((Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)])/d) +
 (Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a])/d + (
(c + d*x)*Sin[a + b/(c + d*x)^2])/d
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b)\,\text{Subst}\left(\int \cos\left(a+bx^2\right)dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d} - \frac{(2b\cos(a))\,\text{Subst}\left(\int \cos\left(bx^2\right)dx, x, \frac{1}{c+dx}\right)}{d} + \frac{(2b\sin(a))\,\text{Sub}}{}$$
$$= -\frac{\sqrt{b}\sqrt{2\pi}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)}{d} + \frac{\sqrt{b}\sqrt{2\pi}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

Mathematica [A] time = 0.170412, size = 100, normalized size = 0.95

$$\frac{\sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{c+dx}\right) + \sqrt{2\pi}\sqrt{b}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{c+dx}\right) + (c+dx)\sin\left(a+\frac{b}{(c+dx)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^2],x]
```

```
[Out] (-(Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]) + Sq
rt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)]*Sin[a] + (c + d*x
)*Sin[a + b/(c + d*x)^2])/d
```

Maple [A] time = 0.009, size = 80, normalized size = 0.8

$$-\frac{1}{d}\left(-(dx+c)\sin\left(a+\frac{b}{(dx+c)^2}\right) + \sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}(dx+c)}\sqrt{b}\right) - \sin(a)\,\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}(dx+c)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^2),x)
```

```
[Out] -1/d*(-(d*x+c)*sin(a+b/(d*x+c)^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
/(d*x+c))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$bd\int \frac{x\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\,dx + bd\int \frac{x\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\cos\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)^2 + \left(d^3x^3+3\,cd^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*d*integrate(x*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
+ c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x) + b*d*integrate(x*cos
((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^
2*x^2 + 2*c*d*x + c^2))^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin((
a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))^2), x) + x*si
n((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
```

Fricas [A] time = 1.72837, size = 352, normalized size = 3.35

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d^2}}\,\mathrm{S}\left(\frac{\sqrt{2}d\sqrt{\frac{b}{\pi d^2}}}{dx+c}\right)\sin(a)-(dx+c)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*d*sqrt(b/(pi*d^2))*cos(a)*fresnel_cos(sqrt(2)*d*sqrt(b/(pi*d^2
))/(d*x + c)) - sqrt(2)*pi*d*sqrt(b/(pi*d^2))*fresnel_sin(sqrt(2)*d*sqrt(b/
(pi*d^2))/(d*x + c))*sin(a) - (d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2
+ b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{(c+dx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**2),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{(dx+c)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2), x)
```

3.180 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Rubi [A] time = 0.0126299, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Mathematica [A] time = 4.55574, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x), x]
```

Maple [A] time = 0.221, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(a+\frac{b}{(dx+c)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^2)/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^2}\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**2)/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^2}\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e), x)
```

3.181 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0123733, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 21.7072, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^2}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b/(c + d*x)^2]/(e + f*x)^2, x]
```

Maple [A] time = 0.309, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^{2}}\sin\left(a+\frac{b}{\left(dx+c\right)^{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^2}\right)}{\left(fx + e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}\right)}{f^2x^2 + 2\,efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**2)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^2}\right)}{\left(fx + e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^2)/(f*x + e)^2, x)
```

3.182 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal. Leaf size=330

$$-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+d}\right.}{3d^3}$$

```
[Out] -(b*f^2*Cos[a]*CosIntegral[b/(c + d*x)^3])/(3*d^3) - ((I/3)*E^(I*a)*f*(d*e
- c*f)*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I)*b)/(c + d
*x)^3])/d^3 + ((I/3)*f*(d*e - c*f)*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Ga
mma[-2/3, (I*b)/(c + d*x)^3])/(d^3*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)^2*
(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d
^3 + ((I/6)*(d*e - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (
I*b)/(c + d*x)^3])/(d^3*E^(I*a)) + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^3])
/(3*d^3) + (b*f^2*Sin[a]*SinIntegral[b/(c + d*x)^3])/(3*d^3)
```

Rubi [A] time = 0.300248, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3433, 3365, 2208, 3423, 2218, 3379, 3297, 3303, 3299, 3302}

$$-\frac{ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+d}\right.}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] -(b*f^2*Cos[a]*CosIntegral[b/(c + d*x)^3])/(3*d^3) - ((I/3)*E^(I*a)*f*(d*e
- c*f)*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I)*b)/(c + d
*x)^3])/d^3 + ((I/3)*f*(d*e - c*f)*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Ga
mma[-2/3, (I*b)/(c + d*x)^3])/(d^3*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)^2*
(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d
^3 + ((I/6)*(d*e - c*f)^2*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (
I*b)/(c + d*x)^3])/(d^3*E^(I*a)) + (f^2*(c + d*x)^3*Sin[a + b/(c + d*x)^3])
/(3*d^3) + (b*f^2*Sin[a]*SinIntegral[b/(c + d*x)^3])/(3*d^3)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
```

```
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)^2\sin\left(a+\frac{b}{(c+dx)^3}\right)dx = \frac{\text{Subst}\left(\int\left(d^2e^2\left(1+\frac{cf(-2de+cf)}{d^2e^2}\right)\sin\left(a+\frac{b}{x^3}\right)+2def\left(1-\frac{cf}{de}\right)x\sin\left(a+\frac{b}{x^3}\right)+f^2\right.\right.}{d^3}$$

$$=\frac{f^2\,\text{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^3}\right)dx,x,c+dx\right)}{d^3}+\frac{(2f(de-cf))\,\text{Subst}\left(\int x\sin\left(a+\frac{b}{x}\right.\right.}{d^3}$$

$$=-\frac{f^2\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}dx,x,\frac{1}{(c+dx)^3}\right)}{3d^3}+\frac{(if(de-cf))\,\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,c+\right.}{d^3}$$

$$=-\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)}{}$$

$$=-\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+\frac{ie^{-ia}f(de-cf)\left(\frac{ib}{(c+dx)^3}\right)}{}$$

$$=-\frac{bf^2\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^3}\right)}{3d^3}-\frac{ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{3d^3}+$$

Mathematica [A] time = 2.52395, size = 405, normalized size = 1.23

$$\frac{\frac{3bf(de-cf)\left((\cos(a)-i\sin(a))\sqrt[3]{-\frac{ib}{(c+dx)^3}}\text{Gamma}\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)+(\cos(a)+i\sin(a))\sqrt[3]{\frac{ib}{(c+dx)^3}}\text{Gamma}\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)\right)}{2(c+dx)\sqrt[3]{\frac{b^2}{(c+dx)^6}}}+\frac{3b(de-cf)^2\left((\cos(a)-i\sin(a))\left(-\frac{ib}{(c+dx)^3}\right)\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((3*b*f*(d*e - c*f)*((((-I)*b)/(c + d*x)^3)^(1/3)*Gamma[1/3, (I*b)/(c + d*x
)^3]*(Cos[a] - I*Sin[a]) + ((I*b)/(c + d*x)^3)^(1/3)*Gamma[1/3, ((-I)*b)/(c
 + d*x)^3]*(Cos[a] + I*Sin[a])))/(2*(b^2/(c + d*x)^6)^(1/3)*(c + d*x)) + (3
*b*(d*e - c*f)^2*((((-I)*b)/(c + d*x)^3)^(2/3)*Gamma[2/3, (I*b)/(c + d*x)^3
]*(Cos[a] - I*Sin[a]) + ((I*b)/(c + d*x)^3)^(2/3)*Gamma[2/3, ((-I)*b)/(c +
d*x)^3]*(Cos[a] + I*Sin[a])))/(2*(b^2/(c + d*x)^6)^(2/3)*(c + d*x)^2) + (c
+ d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Cos[
b/(c + d*x)^3]*Sin[a] + (c + d*x)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2
 + 3*e*f*x + f^2*x^2))*Cos[a]*Sin[b/(c + d*x)^3] - b*f^2*(Cos[a]*CosIntegra
l[b/(c + d*x)^3] - Sin[a]*SinIntegral[b/(c + d*x)^3]))/(3*d^3)
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int (fx+e)^2\sin\left(a+\frac{b}{(dx+c)^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

```
[Out] int((f*x+e)^2*sin(a+b/(d*x+c)^3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(f^2x^3+3\,efx^2+3\,e^2x\right)\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+\int\frac{\left(bdf^2x^3+3\,bdefx^2+3\,bde^2x\right)\cos\left(\frac{ad^3x^3+3\,acd}{d^3x^3+3\,c}\right.}{2\left(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^
2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(1
/2*(b*d*f^2*x^3 + 3*b*d*e*f*x^2 + 3*b*d*e^2*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x
^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d
^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + integrate(1/2
*(b*d*f^2*x^3 + 3*b*d*e*f*x^2 + 3*b*d*e^2*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2
 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((d^
4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a*d^3*x^3 + 3*a
*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*sin((a*
d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3))^2), x)
```

Fricas [A] time = 2.08244, size = 1116, normalized size = 3.38

$$-\frac{bf^2\mathrm{Ei}\left(\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)e^{(ia)}+bf^2\mathrm{Ei}\left(-\frac{ib}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)e^{(-ia)}-\left(-3i\,d^3ef+3i\,cd^2f^2\right)\left(\frac{ib}{d^3}\right)^{\frac{2}{3}}e^{(-ia)}\Gamma\left(\frac{1}{3},\frac{}{d^3x^3+3\,cd^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(b*f^2*Ei(I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*e^(I*a) + b*f
^2*Ei(-I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*e^(-I*a) - (-3*I*d^3*
e*f + 3*I*c*d^2*f^2)*(I*b/d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b/(d^3*x^3 + 3*c
*d^2*x^2 + 3*c^2*d*x + c^3)) - (3*I*d^3*e*f - 3*I*c*d^2*f^2)*(-I*b/d^3)^(2/
3)*e^(I*a)*gamma(1/3, -I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - (-3
*I*d^3*e^2 + 6*I*c*d^2*e*f - 3*I*c^2*d*f^2)*(I*b/d^3)^(1/3)*e^(-I*a)*gamma(
2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - (3*I*d^3*e^2 - 6*I*c*
d^2*e*f + 3*I*c^2*d*f^2)*(-I*b/d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b/(d^3*x^3
+ 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*
e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2
 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^3), x)
```

3.183 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)\,dx$

Optimal. Leaf size=235

$$-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}-\frac{ie^{ia}f(c}{}$$

```
[Out] ((-I/6)*E^(I*a)*f*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I
)*b)/(c + d*x)^3])/d^2 + ((I/6)*f*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gam
ma[-2/3, (I*b)/(c + d*x)^3])/(d^2*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)*(((
-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d^2
+ ((I/6)*(d*e - c*f)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (I*b)/
(c + d*x)^3])/(d^2*E^(I*a))
```

Rubi [A] time = 0.14581, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3433, 3365, 2208, 3423, 2218}

$$-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}(de-cf)\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2}+\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}(de-cf)\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d^2}-\frac{ie^{ia}f(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```


```
[Out] ((-I/6)*E^(I*a)*f*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gamma[-2/3, ((-I
)*b)/(c + d*x)^3])/d^2 + ((I/6)*f*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2*Gam
ma[-2/3, (I*b)/(c + d*x)^3])/(d^2*E^(I*a)) - ((I/6)*E^(I*a)*(d*e - c*f)*(((
-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)/(c + d*x)^3])/d^2
+ ((I/6)*(d*e - c*f)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (I*b)/
(c + d*x)^3])/(d^2*E^(I*a))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^3}\right)dx = \frac{\text{Subst}\left(\int \left(de\left(1-\frac{cf}{de}\right)\sin\left(a+\frac{b}{x^3}\right)+fx\sin\left(a+\frac{b}{x^3}\right)\right)dx,x,c+dx\right)}{d^2}$$

$$= \frac{f\,\text{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)dx,x,c+dx\right)}{d^2} + \frac{(de-cf)\,\text{Subst}\left(\int \sin\left(a+\frac{b}{x^3}\right)dx,x,\cdots\right.}{d^2}$$

$$= \frac{(if)\,\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{2d^2} - \frac{(if)\,\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx,x,c+dx\right)}{2d^2} + \frac{(i(d\cdots}{\cdots}$$

$$= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{6d^2} + \frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},\cdots\right.}{6d^2}$$

Mathematica [B] time = 2.27234, size = 700, normalized size = 2.98

$$\frac{3bf\left(\frac{1}{2}\cos(a)\left(\frac{\text{Gamma}\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}}+\frac{\text{Gamma}\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}}\right)+\frac{1}{2}i\sin(a)\left(\frac{\text{Gamma}\left(\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{3(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}}-\frac{\text{Gamma}\left(\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{3(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}}\right)\right)}{2d^2} - \frac{3bcf\left(\frac{1}{2}\cos(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^3],x]
```

```
[Out] (e*(c + d*x)*Cos[b/(c + d*x)^3]*Sin[a])/d + (f*(-c + d*x)*(c + d*x)*Cos[b/(
c + d*x)^3]*Sin[a])/(2*d^2) + (3*b*f*((Cos[a]*(Gamma[1/3, ((-I)*b)/(c + d*x
)^3]/(3*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)) + Gamma[1/3, (I*b)/(c + d*x
)^3]/(3*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x))))/2 + (I/2)*(Gamma[1/3, ((-I)*
b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)) - Gamma[1/3, (I*
b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)))*Sin[a]))/(2*d^2) +
 (3*b*e*((Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3
)^(2/3)*(c + d*x)^2) + Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)
^(2/3)*(c + d*x)^2)))/2 + (I/2)*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)
*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*
b)/(c + d*x)^3)^(2/3)*(c + d*x)^2))*Sin[a]))/d - (3*b*c*f*((Cos[a]*(Gamma[2
/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2) + Ga
mma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c + d*x)^2)))/2 +
 (I/2)*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(3*(((-I)*b)/(c + d*x)^3)^(2/3)*(c
 + d*x)^2) - Gamma[2/3, (I*b)/(c + d*x)^3]/(3*((I*b)/(c + d*x)^3)^(2/3)*(c
+ d*x)^2))*Sin[a]))/d^2 + (e*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^3])/d + (f*(-
c + d*x)*(c + d*x)*Cos[a]*Sin[b/(c + d*x)^3])/(2*d^2)
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int \left(fx+e\right) \sin\left(a+\frac{b}{\left(dx+c\right)^{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

```
[Out] int((f*x+e)*sin(a+b/(d*x+c)^3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(fx^{2}+2\,ex\right)\sin\left(\frac{ad^{3}x^{3}+3\,acd^{2}x^{2}+3\,ac^{2}dx+ac^{3}+b}{d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}}\right)+\int\frac{3\left(bdfx^{2}+2\,bdex\right)\cos\left(\frac{ad^{3}x^{3}+3\,acd^{2}x^{2}+3\,ac^{2}dx+ac^{3}+b}{d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}}\right)}{4\left(d^{4}x^{4}+4\,cd^{3}x^{3}+6\,c^{2}d^{2}x^{2}+4\,c^{3}dx+c^{4}\right)}\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*e*x)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 +
b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(3/4*(b*d*f*x^2 +
2*b*d*e*x)*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x
^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2
 + 4*c^3*d*x + c^4), x) + integrate(3/4*(b*d*f*x^2 + 2*b*d*e*x)*cos((a*d^3*
x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c
^2*d*x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*c
os((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2
*x^2 + 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3
*d*x + c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*
x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x)
```

Fricas [A] time = 2.0135, size = 761, normalized size = 3.24

$$\frac{-i\,d^{2}f\left(\frac{i\,b}{d^{3}}\right)^{\frac{2}{3}}e^{\left(-i\,a\right)}\Gamma\left(\frac{1}{3},\frac{i\,b}{d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}}\right)+i\,d^{2}f\left(-\frac{i\,b}{d^{3}}\right)^{\frac{2}{3}}e^{\left(i\,a\right)}\Gamma\left(\frac{1}{3},-\frac{i\,b}{d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}}\right)+\left(-2i\,d^{2}e+2i\,cdf\right)\left(\frac{i\,b}{d^{3}}\right)^{\frac{1}{3}}e^{(-i}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*d^2*f*(I*b/d^3)^(2/3)*e^(-I*a)*gamma(1/3, I*b/(d^3*x^3 + 3*c*d^2*x^
2 + 3*c^2*d*x + c^3)) + I*d^2*f*(-I*b/d^3)^(2/3)*e^(I*a)*gamma(1/3, -I*b/(d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + (-2*I*d^2*e + 2*I*c*d*f)*(I*b/d^
3)^(1/3)*e^(-I*a)*gamma(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
 + (2*I*d^2*e - 2*I*c*d*f)*(-I*b/d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b/(d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e -
c^2*f)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^3), x)
```

3.184 $\int \sin\left(a+\frac{b}{(c+dx)^3}\right)dx$

Optimal. Leaf size=107

$$\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}$$

```
[Out] ((-I/6)*E^(I*a)*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)
/(c + d*x)^3])/d + ((I/6)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (
I*b)/(c + d*x)^3])/(d*E^(I*a))
```

Rubi [A] time = 0.0267101, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3365, 2208}

$$\frac{ie^{-ia}(c+dx)\sqrt[3]{\frac{ib}{(c+dx)^3}}\text{Gamma}\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}-\frac{ie^{ia}(c+dx)\sqrt[3]{-\frac{ib}{(c+dx)^3}}\text{Gamma}\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^3],x]
```

```
[Out] ((-I/6)*E^(I*a)*(((-I)*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, ((-I)*b)
/(c + d*x)^3])/d + ((I/6)*((I*b)/(c + d*x)^3)^(1/3)*(c + d*x)*Gamma[-1/3, (
I*b)/(c + d*x)^3])/(d*E^(I*a))
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+\frac{b}{(c+dx)^3}\right)dx&=\frac{1}{2}i\int e^{-ia-\frac{ib}{(c+dx)^3}}\,dx-\frac{1}{2}i\int e^{ia+\frac{ib}{(c+dx)^3}}\,dx\\&=-\frac{ie^{ia}\sqrt[3]{-\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{ib}{(c+dx)^3}\right)}{6d}+\frac{ie^{-ia}\sqrt[3]{\frac{ib}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},\frac{ib}{(c+dx)^3}\right)}{6d}\end{aligned}$$

Mathematica [A] time = 0.487147, size = 203, normalized size = 1.9

$$\frac{b\cos(a)\left(\frac{\text{Gamma}\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}+\frac{\text{Gamma}\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)+ib\sin(a)\left(\frac{\text{Gamma}\left(\frac{2}{3},-\frac{ib}{(c+dx)^3}\right)}{\left(-\frac{ib}{(c+dx)^3}\right)^{2/3}}-\frac{\text{Gamma}\left(\frac{2}{3},\frac{ib}{(c+dx)^3}\right)}{\left(\frac{ib}{(c+dx)^3}\right)^{2/3}}\right)+2\sin(a)(c+dx)^3\cos\left(\right.}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^3],x]
```

```
[Out] (b*Cos[a]*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(c + d*x)^3)^(2/3) +
Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)) + 2*(c + d*x)^3*Co
s[b/(c + d*x)^3]*Sin[a] + I*b*(Gamma[2/3, ((-I)*b)/(c + d*x)^3]/(((-I)*b)/(
c + d*x)^3)^(2/3) - Gamma[2/3, (I*b)/(c + d*x)^3]/((I*b)/(c + d*x)^3)^(2/3)
)*Sin[a] + 2*(c + d*x)^3*Cos[a]*Sin[b/(c + d*x)^3])/(2*d*(c + d*x)^2)
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{(dx+c)^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3\,bd\int\frac{x\cos\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{2\left(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4\right)}\,dx+3\,bd\int\frac{\cdots}{2\left(\left(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4\right)\cos\left(\frac{ad^3x^3\cdots}{d^3\cdots}\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 3*b*d*integrate(1/2*x*cos((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3
+ b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*
c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + 3*b*d*integrate(1/2*x*cos((a*d^3*x^3 +
 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3))/((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*cos((a
*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
+ 3*c^2*d*x + c^3))^2 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
+ c^4)*sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3))^2), x) + x*sin((a*d^3*x^3 + 3*a*c*d^2*x^2
+ 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
```

Fricas [B] time = 1.86678, size = 408, normalized size = 3.81

$$\frac{-i\,d\left(\frac{i\,b}{d^3}\right)^{\frac{1}{3}}e^{(-i\,a)}\Gamma\left(\frac{2}{3},\frac{i\,b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+i\,d\left(-\frac{i\,b}{d^3}\right)^{\frac{1}{3}}e^{(i\,a)}\Gamma\left(\frac{2}{3},-\frac{i\,b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+2\,(dx+c)\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\cdots}{d^3x^3+3\,cd^2x^2+\cdots}\right.\cdots}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*(I*b/d^3)^(1/3)*e^(-I*a)*gamma(2/3, I*b/(d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3)) + I*d*(-I*b/d^3)^(1/3)*e^(I*a)*gamma(2/3, -I*b/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + 2*(d*x + c)*sin((a*d^3*x^3 + 3*a*c*d^2*x
^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{b}{(c + dx)^3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{b}{(dx + c)^3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3), x)
```

3.185 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Rubi [A] time = 0.0132455, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Mathematica [A] time = 5.15019, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x), x]
```

Maple [A] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(a+\frac{b}{(dx+c)^3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^3)/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**3)/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^3}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e), x)
```

3.186 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0134374, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]

[Out] Defer[Int][Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 26.6649, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^3}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2,x]

[Out] Integrate[Sin[a + b/(c + d*x)^3]/(e + f*x)^2, x]
```

Maple [A] time = 0.379, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^{2}}\sin\left(a+\frac{b}{\left(dx+c\right)^{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^3}\right)}{\left(fx + e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{f^2x^2 + 2\,efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**3)/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^3}\right)}{\left(fx + e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^3)/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^3)/(f*x + e)^2, x)
```

3.187 $\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=410

$$\frac{12f(c+dx)(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{24f\sqrt{}}{}$$

```
[Out] (-240*f^2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^5*d^3) + (24*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (2*(d*e - c*f)^2*S
qrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^3) + (40*f^2*(c + d*x)^(3/2)*Co
s[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (4*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^3) - (2*f^2*(c + d*x)^(5/2)*Cos[a + b*Sqrt[c + d*x
]])/(b*d^3) + (240*f^2*Sin[a + b*Sqrt[c + d*x]])/(b^6*d^3) - (24*f*(d*e - c
*f)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3) + (2*(d*e - c*f)^2*Sin[a + b*Sqrt[c
 + d*x]])/(b^2*d^3) - (120*f^2*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3
) + (12*f*(d*e - c*f)*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3) + (10*f
^2*(c + d*x)^2*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```

Rubi [A] time = 0.39884, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3431, 3296, 2637}

$$\frac{12f(c+dx)(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{2(de-cf)^2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{24f(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{24f\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```


```
[Out] (-240*f^2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^5*d^3) + (24*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (2*(d*e - c*f)^2*S
qrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^3) + (40*f^2*(c + d*x)^(3/2)*Co
s[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (4*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^3) - (2*f^2*(c + d*x)^(5/2)*Cos[a + b*Sqrt[c + d*x
]])/(b*d^3) + (240*f^2*Sin[a + b*Sqrt[c + d*x]])/(b^6*d^3) - (24*f*(d*e - c
*f)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3) + (2*(d*e - c*f)^2*Sin[a + b*Sqrt[c
 + d*x]])/(b^2*d^3) - (120*f^2*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^3
) + (12*f*(d*e - c*f)*(c + d*x)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3) + (10*f
^2*(c + d*x)^2*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b\sqrt{c+dx}\right) dx = \frac{2\,\text{Subst}\left(\int \left(\frac{(de-cf)^2 x \sin(a+bx)}{d^2} + \frac{2f(de-cf)x^3 \sin(a+bx)}{d^2} + \frac{f^2x^5 \sin(a+bx)}{d^2}\right) dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{\left(2f^2\right)\text{Subst}\left(\int x^5 \sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\,\text{Subst}\left(\int x^3 \sin(a+\right.}{d^3}$$
$$= -\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+d\right.}{bd^3}$$
$$= -\frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4f(de-cf)(c+dx)^{3/2}\cos\left(a+b\sqrt{c+d\right.}{bd^3}$$
$$= \frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}$$
$$= \frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{2(de-cf)^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^3}$$
$$= -\frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3} + \frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - 2$$
$$= -\frac{240f^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^5d^3} + \frac{24f(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - 2$$

Mathematica [A] time = 1.76851, size = 138, normalized size = 0.34

$$\frac{2\sin\left(a+b\sqrt{c+dx}\right)\left(b^4d(e+fx)(4cf+d(e+5fx))-12b^2f(4cf+d(e+5fx))+120f^2\right)-2b\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right.}{b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*(120*f^2 + b^4*d^2*(e + f*x)^2 - 4*b^2*f*(3*d*e + 2*c*f
 + 5*d*f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(120*f^2 - 12*b^2*f*(4*c*f + d*(e
 + 5*f*x)) + b^4*d*(e + f*x)*(4*c*f + d*(e + 5*f*x)))*Sin[a + b*Sqrt[c + d*
x]])/(b^6*d^3)
```

Maple [B] time = 0.013, size = 1246, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^3/b^2*(c^2*f^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x
+c)^(1/2)))-2*c*d*e*f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(
d*x+c)^(1/2)))+d^2*e^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*
(d*x+c)^(1/2)))+a*c^2*f^2*cos(a+b*(d*x+c)^(1/2))-2*a*c*d*e*f*cos(a+b*(d*x+c
)^(1/2))+a*d^2*e^2*cos(a+b*(d*x+c)^(1/2))-2/b^2*c*f^2*(-(a+b*(d*x+c)^(1/2))
^3*cos(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-6*
```

```
sin(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+2/b^2*
d*e*f*(-(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^
2*sin(a+b*(d*x+c)^(1/2))-6*sin(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos
(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*f^2*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^
(1/2))+2*cos(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)
))-6/b^2*a*d*e*f*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+2*cos(a+b*(
d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-6/b^2*a^2*c*f^2
*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+6/b^2*
a^2*d*e*f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)
))-2/b^2*a^3*c*f^2*cos(a+b*(d*x+c)^(1/2))+2/b^2*a^3*d*e*f*cos(a+b*(d*x+c)^(
1/2))+1/b^4*f^2*(-(a+b*(d*x+c)^(1/2))^5*cos(a+b*(d*x+c)^(1/2))+5*(a+b*(d*x+
c)^(1/2))^4*sin(a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)
^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))+120*sin(a+b*(d*x+c)
^(1/2))-120*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-5/b^4*a*f^2*(-(a+b*
(d*x+c)^(1/2))^4*cos(a+b*(d*x+c)^(1/2))+4*(a+b*(d*x+c)^(1/2))^3*sin(a+b*(d*
x+c)^(1/2))+12*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))-24*cos(a+b*(d*x
+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*f^2*(-
(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+
b*(d*x+c)^(1/2))-6*sin(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*
x+c)^(1/2)))-10/b^4*a^3*f^2*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+
2*cos(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+5/b^
4*a^4*f^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)
))+1/b^4*a^5*f^2*cos(a+b*(d*x+c)^(1/2)))
```

<u>Maxima</u> [B] time = 1.18474, size = 1486, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*e^2*cos(sqrt(d*x + c)*b + a) - 2*a*c*e*f*cos(sqrt(d*x + c)*b + a)/d +
a*c^2*f^2*cos(sqrt(d*x + c)*b + a)/d^2 - ((sqrt(d*x + c)*b + a)*cos(sqrt(d*
x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e^2 + 2*((sqrt(d*x + c)*b + a)*co
s(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c*e*f/d - ((sqrt(d*x + c
)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*c^2*f^2/d^2 +
 2*a^3*e*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) - 2*a^3*c*f^2*cos(sqrt(d*x + c)
*b + a)/(b^2*d^2) - 6*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin
(sqrt(d*x + c)*b + a))*a^2*e*f/(b^2*d) + 6*((sqrt(d*x + c)*b + a)*cos(sqrt(
d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*c*f^2/(b^2*d^2) + a^5*f^2*c
os(sqrt(d*x + c)*b + a)/(b^4*d^2) + 6*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sq
rt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a*e*
f/(b^2*d) - 5*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*
x + c)*b + a))*a^4*f^2/(b^4*d^2) - 6*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqr
t(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a*c*f
^2/(b^2*d^2) - 2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(s
qrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b +
 a))*e*f/(b^2*d) + 10*(((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b +
a) - 2*(sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a))*a^3*f^2/(b^4*d^2) +
2*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b
+ a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*c*f^2/(b^2
*d^2) - 10*(((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqrt(d*
x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*a
^2*f^2/(b^4*d^2) + 5*(((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2
 + 24)*cos(sqrt(d*x + c)*b + a) - 4*((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x +
 c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))*a*f^2/(b^4*d^2) - (((sqrt(d*x + c)*b
```

```
 + a)^5 - 20*(sqrt(d*x + c)*b + a)^3 + 120*sqrt(d*x + c)*b + 120*a)*cos(sqr
t(d*x + c)*b + a) - 5*((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2
 + 24)*sin(sqrt(d*x + c)*b + a))*f^2/(b^4*d^2))/(b^2*d)
```

Fricas [A] time = 1.75107, size = 431, normalized size = 1.05

$$-\frac{2\left(\left(b^5d^2f^2x^2+b^5d^2e^2-12\,b^3def-8\left(b^3c-15\,b\right)f^2+2\left(b^5d^2ef-10\,b^3df^2\right)x\right)\sqrt{dx+c}\cos\left(\sqrt{dx+c}b+a\right)-\left(5\,b^4d^2f^2x\right.}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b^5*d^2*f^2*x^2 + b^5*d^2*e^2 - 12*b^3*d*e*f - 8*(b^3*c - 15*b)*f^2 +
2*(b^5*d^2*e*f - 10*b^3*d*f^2)*x)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) -
(5*b^4*d^2*f^2*x^2 + b^4*d^2*e^2 + 4*(b^4*c - 3*b^2)*d*e*f - 24*(2*b^2*c -
5)*f^2 + 2*(3*b^4*d^2*e*f + 2*(b^4*c - 15*b^2)*d*f^2)*x)*sin(sqrt(d*x + c)*
b + a))/(b^6*d^3)
```

Sympy [A] time = 2.70185, size = 549, normalized size = 1.34

$$\begin{cases}\left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin\left(a\right) \\ \left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin\left(a+b\sqrt{c}\right) \\ \left(e^2x+efx^2+\frac{f^2x^3}{3}\right)\sin\left(a\right) \\ -\frac{2e^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{4efx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2f^2x^2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{8cef\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{8cf^2x\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2e^2\sin}{}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise(((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a), Eq(b, 0) & Eq(d, 0)),
((e**2*x + e*f*x**2 + f**2*x**3/3)*sin(a + b*sqrt(c)), Eq(d, 0)), ((e**2*x
+ e*f*x**2 + f**2*x**3/3)*sin(a), Eq(b, 0)), (-2*e**2*sqrt(c + d*x)*cos(a +
 b*sqrt(c + d*x))/(b*d) - 4*e*f*x*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b
*d) - 2*f**2*x**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) + 8*c*e*f*si
n(a + b*sqrt(c + d*x))/(b**2*d**2) + 8*c*f**2*x*sin(a + b*sqrt(c + d*x))/(b
**2*d**2) + 2*e**2*sin(a + b*sqrt(c + d*x))/(b**2*d) + 12*e*f*x*sin(a + b*s
qrt(c + d*x))/(b**2*d) + 10*f**2*x**2*sin(a + b*sqrt(c + d*x))/(b**2*d) + 1
6*c*f**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d**3) + 24*e*f*sqrt(c
 + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d**2) + 40*f**2*x*sqrt(c + d*x)*cos(
a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*f**2*sin(a + b*sqrt(c + d*x))/(b**4
*d**3) - 24*e*f*sin(a + b*sqrt(c + d*x))/(b**4*d**2) - 120*f**2*x*sin(a + b
*sqrt(c + d*x))/(b**4*d**2) - 240*f**2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x
))/(b**5*d**3) + 240*f**2*sin(a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

Giac [B] time = 1.6002, size = 1848, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(f^2*(((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b +
a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^
2*b^2*c + 2*a^3*b^2*c + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)^4
*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*(s
qrt(d*x + c)*b + a)*a^4 - a^5 + 12*(sqrt(d*x + c)*b + a)*b^2*c - 12*a*b^2*c
 - 20*(sqrt(d*x + c)*b + a)^3 + 60*(sqrt(d*x + c)*b + a)^2*a - 60*(sqrt(d*x
 + c)*b + a)*a^2 + 20*a^3 + 120*sqrt(d*x + c)*b)*cos(-(sqrt(d*x + c)*b + a)
*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b)
- a)/(b^3*d^2) + (b^4*c^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 6*(sqrt(d*x
+ c)*b + a)^2*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 12*(sqrt(d*x + c)*
b + a)*a*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 6*a^2*b^2*c*sgn((sqrt(d
*x + c)*b + a)*b - a*b) + 5*(sqrt(d*x + c)*b + a)^4*sgn((sqrt(d*x + c)*b +
a)*b - a*b) - 20*(sqrt(d*x + c)*b + a)^3*a*sgn((sqrt(d*x + c)*b + a)*b - a*
b) + 30*(sqrt(d*x + c)*b + a)^2*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 20
*(sqrt(d*x + c)*b + a)*a^3*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 5*a^4*sgn((
sqrt(d*x + c)*b + a)*b - a*b) + 12*b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b)
 - 60*(sqrt(d*x + c)*b + a)^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 120*(sqr
t(d*x + c)*b + a)*a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 60*a^2*sgn((sqrt(d
*x + c)*b + a)*b - a*b) + 120*sgn((sqrt(d*x + c)*b + a)*b - a*b))*sin(-(sqr
t(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)
*b + a)*b - a*b) - a)/(b^3*d^2))/b^2 + (((sqrt(d*x + c)*b + a)*b - a*b)*cos
(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*
x + c)*b + a)*b - a*b) - a) + b*sin(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x +
c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a)/sgn((sqrt(d*
x + c)*b + a)*b - a*b))*e^2/b^2 - 2*f*(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2
*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x +
c)*b + a)*a^2 + a^3 + 6*sqrt(d*x + c)*b)*cos(-(sqrt(d*x + c)*b + a)*sgn((sq
rt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a)/b +
 (b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3*(sqrt(d*x + c)*b + a)^2*sgn(
(sqrt(d*x + c)*b + a)*b - a*b) + 6*(sqrt(d*x + c)*b + a)*a*sgn((sqrt(d*x +
c)*b + a)*b - a*b) - 3*a^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 6*sgn((sqrt
(d*x + c)*b + a)*b - a*b))*sin(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b
+ a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a)/b)*e/(b^2*d))/(b*
d)
```

3.188 $\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right) dx$

Optimal. Leaf size=185

$$\frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+\right.}}{b^3d^2}$$

```
[Out] (12*f*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*(d*e - c*f)*Sq
rt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^2) - (2*f*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^2) - (12*f*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (
2*(d*e - c*f)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^2) + (6*f*(c + d*x)*Sin[a +
b*Sqrt[c + d*x]])/(b^2*d^2)
```

Rubi [A] time = 0.159242, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3431, 3296, 2637}

$$\frac{2(de-cf)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{6f(c+dx)\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12f\sin\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+\right.}}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (12*f*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*(d*e - c*f)*Sq
rt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d^2) - (2*f*(c + d*x)^(3/2)*Cos[a
+ b*Sqrt[c + d*x]])/(b*d^2) - (12*f*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (
2*(d*e - c*f)*Sin[a + b*Sqrt[c + d*x]])/(b^2*d^2) + (6*f*(c + d*x)*Sin[a +
b*Sqrt[c + d*x]])/(b^2*d^2)
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b\sqrt{c+dx}\right)dx = \frac{2\,\text{Subst}\left(\int\left(\frac{(de-cf)x\sin(a+bx)}{d}+\frac{fx^3\sin(a+bx)}{d}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{(2f)\,\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(2(de-cf))\,\text{Subst}\left(\int x\sin(a+b\right.}{d^2}$$
$$= -\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{(6}{}$$
$$= -\frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c+dx)^{3/2}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(}{}$$
$$= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c}{}$$
$$= \frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2(de-cf)\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2f(c}{}$$

Mathematica [A] time = 0.415749, size = 85, normalized size = 0.46

$$\frac{2\sin\left(a+b\sqrt{c+dx}\right)\left(b^2(2cf+d(e+3fx))-6f\right)-2b\sqrt{c+dx}\left(b^2d(e+fx)-6f\right)\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*(-6*f + b^2*d*(e + f*x))*Cos[a + b*Sqrt[c + d*x]] + 2*(
-6*f + b^2*(2*c*f + d*(e + 3*f*x)))*Sin[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [B] time = 0.008, size = 366, normalized size = 2.

$$2\frac{1}{d^2b^2}\left(-cf\left(\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)\right)+de\left(\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^2/b^2*(-c*f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)
^(1/2)))+d*e*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1
/2)))-a*c*f*cos(a+b*(d*x+c)^(1/2))+a*d*e*cos(a+b*(d*x+c)^(1/2))+1/b^2*f*(-(
a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*sin(a+b
*(d*x+c)^(1/2))-6*sin(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x
+c)^(1/2)))-3/b^2*a*f*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+2*cos(
a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+3/b^2*a^2*
f*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+1/b^2
*a^3*f*cos(a+b*(d*x+c)^(1/2)))
```

Maxima [B] time = 1.02153, size = 470, normalized size = 2.54

$$2\left(ae\cos\left(\sqrt{dx+c}b+a\right)-\frac{acf\cos\left(\sqrt{dx+c}b+a\right)}{d}-\left(\left(\sqrt{dx+c}b+a\right)\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)e+\frac{\left(\left(\sqrt{dx+c}b+a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*e*cos(sqrt(d*x + c)*b + a) - a*c*f*cos(sqrt(d*x + c)*b + a)/d - ((sqrt
(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*e + (
(sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*
c*f/d + a^3*f*cos(sqrt(d*x + c)*b + a)/(b^2*d) - 3*((sqrt(d*x + c)*b + a)*c
os(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))*a^2*f/(b^2*d) + 3*(((sq
rt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) - 2*(sqrt(d*x + c)*b + a
)*sin(sqrt(d*x + c)*b + a))*a*f/(b^2*d) - (((sqrt(d*x + c)*b + a)^3 - 6*sqr
t(d*x + c)*b - 6*a)*cos(sqrt(d*x + c)*b + a) - 3*((sqrt(d*x + c)*b + a)^2 -
 2)*sin(sqrt(d*x + c)*b + a))*f/(b^2*d))/(b^2*d)
```

Fricas [A] time = 1.68919, size = 208, normalized size = 1.12

$$-\frac{2\left(\left(b^3dfx+b^3de-6\,bf\right)\sqrt{dx+c}\cos\left(\sqrt{dx+c}b+a\right)-\left(3\,b^2dfx+b^2de+2\left(b^2c-3\right)f\right)\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b^3*d*f*x + b^3*d*e - 6*b*f)*sqrt(d*x + c)*cos(sqrt(d*x + c)*b + a) -
(3*b^2*d*f*x + b^2*d*e + 2*(b^2*c - 3)*f)*sin(sqrt(d*x + c)*b + a))/(b^4*d^
2)
```

Sympy [A] time = 0.741943, size = 231, normalized size = 1.25

$$\begin{cases}\left(ex+\frac{fx^2}{2}\right)\sin(a) \\ \left(ex+\frac{fx^2}{2}\right)\sin\left(a+b\sqrt{c}\right) \\ \left(ex+\frac{fx^2}{2}\right)\sin(a) \\ -\frac{2e\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2fx\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{4cf\sin\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2e\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{6fx\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{12f\sqrt{c+dx}\cos\left(a+b\sqrt{\right.}}{b^3d^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise(((e*x + f*x**2/2)*sin(a), Eq(b, 0) & Eq(d, 0)), ((e*x + f*x**2/2)
*sin(a + b*sqrt(c)), Eq(d, 0)), ((e*x + f*x**2/2)*sin(a), Eq(b, 0)), (-2*e*
sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) - 2*f*x*sqrt(c + d*x)*cos(a +
b*sqrt(c + d*x))/(b*d) + 4*c*f*sin(a + b*sqrt(c + d*x))/(b**2*d**2) + 2*e*s
in(a + b*sqrt(c + d*x))/(b**2*d) + 6*f*x*sin(a + b*sqrt(c + d*x))/(b**2*d)
+ 12*f*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*f*sin(a + b*
sqrt(c + d*x))/(b**4*d**2), True))
```

Giac [B] time = 1.299, size = 714, normalized size = 3.86

$$-\frac{2\left(\frac{\left(\left(\left(\sqrt{dx+c}\,b+a\right)b-ab\right)\cos\left(-\left(\sqrt{dx+c}\,b+a\right)\mathrm{sgn}\left(\left(\sqrt{dx+c}\,b+a\right)b-ab\right)+a\,\mathrm{sgn}\left(\left(\sqrt{dx+c}\,b+a\right)b-ab\right)-a\right)+\frac{b\sin\left(-\left(\sqrt{dx+c}\,b+a\right)\mathrm{sgn}\left(\left(\sqrt{dx+c}\,b+a\right)b-ab\right)+a\,\mathrm{sgn}\left(\left(\sqrt{dx+c}\,b+a\right)b-\right.\right.}{\mathrm{sgn}\left(\left(\sqrt{dx+c}\,b+a\right)b-ab\right)}}{b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*((((sqrt(d*x + c)*b + a)*b - a*b)*cos(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d
*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a) + b*sin
(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*
x + c)*b + a)*b - a*b) - a)/sgn((sqrt(d*x + c)*b + a)*b - a*b))*e/b^2 - f*(
((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(
d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 + 6*sqrt(d*x + c)*b
)*cos(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sq
rt(d*x + c)*b + a)*b - a*b) - a)/b + (b^2*c*sgn((sqrt(d*x + c)*b + a)*b - a
*b) - 3*(sqrt(d*x + c)*b + a)^2*sgn((sqrt(d*x + c)*b + a)*b - a*b) + 6*(sqr
t(d*x + c)*b + a)*a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - 3*a^2*sgn((sqrt(d*
x + c)*b + a)*b - a*b) + 6*sgn((sqrt(d*x + c)*b + a)*b - a*b))*sin(-(sqrt(d
*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b
+ a)*b - a*b) - a)/b)/(b^2*d))/(b*d)
```

3.189 $\int \sin\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=54

$$\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}-\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}$$

```
[Out] (-2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d) + (2*Sin[a + b*Sqrt[c + d
*x]])/(b^2*d)
```

Rubi [A] time = 0.0277695, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3361, 3296, 2637}

$$\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}-\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]])/(b*d) + (2*Sin[a + b*Sqrt[c + d
*x]])/(b^2*d)
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{2\,\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\,\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd}\\
&= -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] time = 0.0738688, size = 50, normalized size = 0.93

$$\frac{2\sin\left(a+b\sqrt{c+dx}\right)-2b\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[c + d*x]*Cos[a + b*Sqrt[c + d*x]] + 2*Sin[a + b*Sqrt[c + d*x]])/
(b^2*d)
```

Maple [A] time = 0.007, size = 61, normalized size = 1.1

$$2\,\frac{\sin\left(a+b\sqrt{dx+c}\right)-\left(a+b\sqrt{dx+c}\right)\cos\left(a+b\sqrt{dx+c}\right)+a\cos\left(a+b\sqrt{dx+c}\right)}{db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d/b^2*(sin(a+b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))+
a*cos(a+b*(d*x+c)^(1/2)))
```

Maxima [A] time = 0.950191, size = 84, normalized size = 1.56

$$-\frac{2\left(\left(\sqrt{dx+c}b+a\right)\cos\left(\sqrt{dx+c}b+a\right)-a\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) - a*cos(sqrt(d*x + c)*b
+ a) - sin(sqrt(d*x + c)*b + a))/(b^2*d)
```

Fricas [A] time = 1.64205, size = 111, normalized size = 2.06

$$-\frac{2\left(\sqrt{dx+c}b\cos\left(\sqrt{dx+c}b+a\right)-\sin\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(d*x + c)*b*cos(sqrt(d*x + c)*b + a) - sin(sqrt(d*x + c)*b + a))/(b
^2*d)
```

Sympy [A] time = 0.4912, size = 66, normalized size = 1.22

$$\begin{cases} x\sin(a) & \text{for } b = 0 \wedge d = 0 \\ x\sin\left(a + b\sqrt{c}\right) & \text{for } d = 0 \\ x\sin(a) & \text{for } b = 0 \\ -\frac{2\sqrt{c+dx}\cos\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{2\sin\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & Eq(d, 0)), (x*sin(a + b*sqrt(c)), Eq(d, 0))
, (x*sin(a), Eq(b, 0)), (-2*sqrt(c + d*x)*cos(a + b*sqrt(c + d*x))/(b*d) +
2*sin(a + b*sqrt(c + d*x))/(b**2*d), True))
```

Giac [B] time = 1.20166, size = 225, normalized size = 4.17

$$-\frac{2\left(\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)\cos\left(-\left(\sqrt{dx+c}b+a\right)\text{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)+a\text{sgn}\left(\left(\sqrt{dx+c}b+a\right)b-ab\right)-a\right)+\frac{b\sin}{}\right.}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(((sqrt(d*x + c)*b + a)*b - a*b)*cos(-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*
x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x + c)*b + a)*b - a*b) - a) + b*sin(
-(sqrt(d*x + c)*b + a)*sgn((sqrt(d*x + c)*b + a)*b - a*b) + a*sgn((sqrt(d*x
 + c)*b + a)*b - a*b) - a)/sgn((sqrt(d*x + c)*b + a)*b - a*b))/(b^3*d)
```

3.190 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$

Optimal. Leaf size=238

$$\frac{\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{f}+\frac{\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f}-\cos\left(a\right.$$

```
[Out] (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]]*Sin[a - (b*S
qrt[-(d*e) + c*f])/Sqrt[f]])/f + (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f
] - b*Sqrt[c + d*x]]*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]])/f - (Cos[a +
(b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f]
- b*Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegra
l[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/f
```

Rubi [A] time = 0.748664, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3431, 3303, 3299, 3302}

$$\frac{\sin\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{f}+\frac{\sin\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{f}-\cos\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]]*Sin[a - (b*S
qrt[-(d*e) + c*f])/Sqrt[f]])/f + (CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f
] - b*Sqrt[c + d*x]]*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]])/f - (Cos[a +
(b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f]
- b*Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegra
l[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/f
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx = \frac{2\,\mathrm{Subst}\left(\int\left(-\frac{d\sin(a+bx)}{2\sqrt{f}\left(\sqrt{-de+cf}-\sqrt{f}x\right)}+\frac{d\sin(a+bx)}{2\sqrt{f}\left(\sqrt{-de+cf}+\sqrt{f}x\right)}\right)dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{-de+cf}-\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{f}} + \frac{\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{-de+cf}+\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{f}}$$

$$= \frac{\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{\sqrt{f}} + \frac{\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\mathrm{Subst}\left(\int -\ldots\right.}{\sqrt{\ldots}}$$

$$= \frac{\mathrm{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c+dx}\right)\sin\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f} + \frac{\mathrm{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)\sin\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)}{f}$$

Mathematica [C] time = 1.46982, size = 238, normalized size = 1.

$$\frac{ie^{-i\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)}\left(-e^{2i\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)}\mathrm{Ei}\left(ib\left(\sqrt{c+dx}-\frac{\sqrt{cf-de}}{\sqrt{f}}\right)\right)-e^{2ia}\mathrm{Ei}\left(ib\left(\frac{\sqrt{cf-de}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)+\mathrm{Ei}\left(-ib\left(\sqrt{c+dx}-\frac{\sqrt{cf-de}}{\sqrt{f}}\right)\right)+\ldots}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] ((I/2)*(ExpIntegralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x]
)] - E^((2*I)*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*ExpIntegralEi[I*b*(-(Sq
rt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])] + E^(((2*I)*b*Sqrt[-(d*e) + c*f
])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x
])] - E^((2*I)*a)*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
d*x])]))/(E^(I*(a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]))*f)
```

Maple [B] time = 0.021, size = 785, normalized size = 3.3

$$2\,\frac{1}{b^2}\left(1/2\,\frac{b^2\left(af+\sqrt{b^2cf^2-b^2def}\right)}{f^2}\left(\mathrm{Si}\left(b\sqrt{dx+c}+a-\frac{af+\sqrt{b^2cf^2-b^2def}}{f}\right)\cos\left(\frac{af+\sqrt{b^2cf^2-b^2def}}{f}\right)+\mathrm{Ci}\left(b\sqrt{dx\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x)
```

```
[Out] 2/b^2*(1/2*b^2*(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/((a*f+(b^2*c*f^2-b^2*d
*e*f)^(1/2))/f-a)*(Si(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f
)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-1/2*b^2
*(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f^2/(-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2)
```

```
)/f-a)*(Si(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*
f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2
*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))-a*b^2*(1/2/((a
*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f-a)/f*(Si(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c
)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/f))+1/2/(-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f-a)/f*(Si(b*(d*x+c)
^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^2*c*f^2-b^2*d*e
*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*si
n((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

Fricas [C] time = 1.83603, size = 524, normalized size = 2.2

$$\frac{-i\,\mathrm{Ei}\left(i\sqrt{dx+c}b-\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia+\sqrt{\frac{b^2de-b^2cf}{f}}\right)}-i\,\mathrm{Ei}\left(i\sqrt{dx+c}b+\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia-\sqrt{\frac{b^2de-b^2cf}{f}}\right)}+i\,\mathrm{Ei}\left(-i\sqrt{dx+c}b-\ldots\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*Ei(I*sqrt(d*x + c)*b - sqrt((b^2*d*e - b^2*c*f)/f))*e^(I*a + sqrt((
b^2*d*e - b^2*c*f)/f)) - I*Ei(I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/
f))*e^(I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + I*Ei(-I*sqrt(d*x + c)*b - sqrt(
(b^2*d*e - b^2*c*f)/f))*e^(-I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + I*Ei(-I*sq
rt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/f))*e^(-I*a - sqrt((b^2*d*e - b^2*
c*f)/f)))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*sqrt(c + d*x))/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e), x)
```

3.191 $\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=339

$$\frac{bd\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}-\frac{bd\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}+$$

```
[Out] (b*d*Cos[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c
*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - (b*d*Cos[
a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt
[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - Sin[a + b*Sqrt[c +
 d*x]]/(f*(e + f*x)) + (b*d*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinInte
gral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d
*e) + c*f]) + (b*d*Sin[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*S
qrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f
])
```

Rubi [A] time = 0.982629, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3431, 3341, 3334, 3303, 3299, 3302}

$$\frac{bd\cos\left(a+\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}-\frac{bd\cos\left(a-\frac{b\sqrt{cf-de}}{\sqrt{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt{cf-de}}{\sqrt{f}}+b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{cf-de}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```


```
[Out] (b*d*Cos[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c
*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - (b*d*Cos[
a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*CosIntegral[(b*Sqrt[-(d*e) + c*f])/Sqrt
[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f]) - Sin[a + b*Sqrt[c +
 d*x]]/(f*(e + f*x)) + (b*d*Sin[a + (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinInte
gral[(b*Sqrt[-(d*e) + c*f])/Sqrt[f] - b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d
*e) + c*f]) + (b*d*Sin[a - (b*Sqrt[-(d*e) + c*f])/Sqrt[f]]*SinIntegral[(b*S
qrt[-(d*e) + c*f])/Sqrt[f] + b*Sqrt[c + d*x]])/(2*f^(3/2)*Sqrt[-(d*e) + c*f
])
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{(e+fx)^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x\sin(a+bx)}{\left(e-\frac{cf}{d}+\frac{fx^2}{d}\right)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{b\,\text{Subst}\left(\int \frac{\cos(a+bx)}{e-\frac{cf}{d}+\frac{fx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} + \frac{b\,\text{Subst}\left(\int \left(\frac{\sqrt{-de+cf}\cos(a+bx)}{2\left(e-\frac{cf}{d}\right)\left(\sqrt{-de+cf}-\sqrt{f}x\right)} + \frac{\sqrt{-de+cf}\cos(a+bx)}{2\left(e-\frac{cf}{d}\right)\left(\sqrt{-de+cf}+\sqrt{f}x\right)}\right)dx, x, \sqrt{c+
}{f}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{-de+cf}-\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{2f\sqrt{-de+cf}} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{-de+cf}+
}{2f\sqrt{-de
}$$

$$= -\frac{\sin\left(a+b\sqrt{c+dx}\right)}{f(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+bx\right)}{\sqrt{-de+cf}+\sqrt{f}x}\,dx, x, \sqrt{c+dx}\right)}{2f\sqrt{-de+cf}} -
$$

$$= \frac{bd\cos\left(a+\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}-b\sqrt{c+dx}\right)}{2f^{3/2}\sqrt{-de+cf}} - \frac{bd\cos\left(a-\frac{b\sqrt{-de+cf}}{\sqrt{f}}\right)\text{Ci}\left(\frac{b\sqrt{-de+cf}}{\sqrt{f}}+b\sqrt{c
}{2f^{3/2}\sqrt{-de+cf}}$$

Mathematica [C] time = 3.57704, size = 397, normalized size = 1.17

$$\frac{ie^{-ia}d\left(e^{2ia}\left(-\frac{ibe^{\frac{ib\sqrt{cf-de}}{\sqrt{f}}}\text{Ei}\left(ib\left(\sqrt{c+dx}-\frac{\sqrt{cf-de}}{\sqrt{f}}\right)\right)}{\sqrt{cf-de}} + \frac{ibe^{-\frac{ib\sqrt{cf-de}}{\sqrt{f}}}\text{Ei}\left(ib\left(\frac{\sqrt{cf-de}}{\sqrt{f}}+\sqrt{c+dx}\right)\right)}{\sqrt{cf-de}} + \frac{2\sqrt{f}e^{ib\sqrt{c+dx}}}{de+dfx}\right) - \frac{ibe^{-\frac{ib\sqrt{cf-de}}{\sqrt{f}}}\text{Ei}\left(-ib\left(\sqrt{c+dx}-\frac{\sqrt{cf-de}}{\sqrt{f}}\right)\right)}{\sqrt{cf-de}} +
}{4f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] ((I/4)*d*((-2*Sqrt[f])/(E^(I*b*Sqrt[c + d*x])*(d*e + d*f*x)) - (I*b*ExpInte
gralEi[(-I)*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/(E^((I*b*Sq
rt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]) + (I*b*E^((I*b*Sqrt[-(d*e) +
 c*f])/Sqrt[f])*ExpIntegralEi[(-I)*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c +
 d*x])])/Sqrt[-(d*e) + c*f] + E^((2*I)*a)*((2*E^(I*b*Sqrt[c + d*x])*Sqrt[f]
)/(d*e + d*f*x) - (I*b*E^((I*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*ExpIntegralEi[I
*b*(-(Sqrt[-(d*e) + c*f]/Sqrt[f]) + Sqrt[c + d*x])])/Sqrt[-(d*e) + c*f] + (
I*b*ExpIntegralEi[I*b*(Sqrt[-(d*e) + c*f]/Sqrt[f] + Sqrt[c + d*x])])/(E^((I
*b*Sqrt[-(d*e) + c*f])/Sqrt[f])*Sqrt[-(d*e) + c*f]))))/(E^(I*a)*f^(3/2))
```

Maple [B] time = 0.048, size = 1817, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x)
```

```
[Out] 2*d/b^2*(sin(a+b*(d*x+c)^(1/2))*(-1/2*a*b^2/(c*f-d*e)*(a+b*(d*x+c)^(1/2))+1
/2*b^2*(-b^2*c*f+b^2*d*e+a^2*f)/(c*f-d*e)/f)/(-c*f*b^2+d*e*b^2+(a+b*(d*x+c)
^(1/2))^2*f-2*(a+b*(d*x+c)^(1/2))*a*f+a^2*f)-1/4*a*b^2/(c*f-d*e)/f/((a*f+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/f-a)*(Si(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d
*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)
+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2
))/f))-1/4*a*b^2/(c*f-d*e)/f/(-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f-a)*(Si(
b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^2*c*f^
2-b^2*d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4*b^2*(c*f*b^2-d*e*b^2+
(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))*a-a^2*f)/(c*f-d*e)/f^2/((a*f+(b^2*c*f^2-b
^2*d*e*f)^(1/2))/f-a)*(-Si(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(a*f+(
b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/
4*b^2*(c*f*b^2-d*e*b^2-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))*a-a^2*f)/(c*f-d*e
)/f^2/(-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f-a)*(Si(b*(d*x+c)^(1/2)+a+(-a*f
+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+
Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f))-a*b^4*(sin(a+b*(d*x+c)^(1/2))*(-1/2/b^2/(c*f-d*e
)*(a+b*(d*x+c)^(1/2))+1/2*a/b^2/(c*f-d*e))/(-c*f*b^2+d*e*b^2+(a+b*(d*x+c)^(
1/2))^2*f-2*(a+b*(d*x+c)^(1/2))*a*f+a^2*f)-1/4/b^2/(c*f-d*e)/f/((a*f+(b^2*c
*f^2-b^2*d*e*f)^(1/2))/f-a)*(Si(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f
)^(1/2))/f)*cos((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(
a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f
))-1/4/b^2/(c*f-d*e)/f/(-(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f-a)*(Si(b*(d*x
+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos((-a*f+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/f)-Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)
*sin((-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4/f/b^2/(c*f-d*e)*(-Si(b*(d*x
+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((a*f+(b^2*c*f^2-b^2*d*
e*f)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a-(a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*co
s((a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))+1/4/f/b^2/(c*f-d*e)*(Si(b*(d*x+c)^(
1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*sin((-a*f+(b^2*c*f^2-b^2*d*e*f
)^(1/2))/f)+Ci(b*(d*x+c)^(1/2)+a+(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f)*cos(
(-a*f+(b^2*c*f^2-b^2*d*e*f)^(1/2))/f))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

Fricas [C] time = 1.96785, size = 873, normalized size = 2.58

$$-\frac{\left(-idfx-ide\right)\sqrt{\frac{b^2de-b^2cf}{f}}\,\mathrm{Ei}\left(i\sqrt{dx+c}b-\sqrt{\frac{b^2de-b^2cf}{f}}\right)e^{\left(ia+\sqrt{\frac{b^2de-b^2cf}{f}}\right)}+\left(idfx+ide\right)\sqrt{\frac{b^2de-b^2cf}{f}}\,\mathrm{Ei}\left(i\sqrt{dx+c}b+\sqrt{\frac{b^2de}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((-I*d*f*x - I*d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(I*sqrt(d*x + c)*b -
 sqrt((b^2*d*e - b^2*c*f)/f))*e^(I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + (I*d*
f*x + I*d*e)*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(I*sqrt(d*x + c)*b + sqrt((b^2*d
*e - b^2*c*f)/f))*e^(I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + (I*d*f*x + I*d*e)
*sqrt((b^2*d*e - b^2*c*f)/f)*Ei(-I*sqrt(d*x + c)*b - sqrt((b^2*d*e - b^2*c*
f)/f))*e^(-I*a + sqrt((b^2*d*e - b^2*c*f)/f)) + (-I*d*f*x - I*d*e)*sqrt((b^
2*d*e - b^2*c*f)/f)*Ei(-I*sqrt(d*x + c)*b + sqrt((b^2*d*e - b^2*c*f)/f))*e^
(-I*a - sqrt((b^2*d*e - b^2*c*f)/f)) + 4*(d*e - c*f)*sin(sqrt(d*x + c)*b +
a))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\sqrt{c+dx}\right)}{\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/2))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*sqrt(c + d*x))/(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\sqrt{dx+c}b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(sqrt(d*x + c)*b + a)/(f*x + e)^2, x)
```

3.192 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal. Leaf size=382

$$-\frac{2e^{ia}f\sqrt{c+dx}(de-cf)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{2e^{-ia}f\sqrt{c+dx}(de-cf)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}}+\frac{ie^{ia}(c+dx)(d}{}$$

```
[Out] (-4*f*(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*
f^2*(c + d*x)^(3/2)*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*E^(I*a)*f*(d
*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^3*((-I)*
b*(c + d*x)^(3/2))^(1/3)) - (2*f*(d*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, I*b*(
c + d*x)^(3/2)])/(9*b*d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(
I*a)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^3*((-I)
*b*(c + d*x)^(3/2))^(2/3)) - ((I/3)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, I*b*
(c + d*x)^(3/2)])/(d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3)) + (2*f^2*Sin[a
+ b*(c + d*x)^(3/2)])/(3*b^2*d^3)
```

Rubi [A] time = 0.305561, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {3433, 3389, 2218, 3385, 3356, 2208, 3379, 3296, 2637}

$$-\frac{2e^{ia}f\sqrt{c+dx}(de-cf)\text{Gamma}\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{2e^{-ia}f\sqrt{c+dx}(de-cf)\text{Gamma}\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{9bd^3\sqrt[3]{ib(c+dx)^{3/2}}}+\frac{ie^{ia}(c+dx)(d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-4*f*(d*e - c*f)*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*
f^2*(c + d*x)^(3/2)*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^3) - (2*E^(I*a)*f*(d
*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^3*((-I)*
b*(c + d*x)^(3/2))^(1/3)) - (2*f*(d*e - c*f)*Sqrt[c + d*x]*Gamma[1/3, I*b*(
c + d*x)^(3/2)])/(9*b*d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(
I*a)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^3*((-I)
*b*(c + d*x)^(3/2))^(2/3)) - ((I/3)*(d*e - c*f)^2*(c + d*x)*Gamma[2/3, I*b*
(c + d*x)^(3/2)])/(d^3*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3)) + (2*f^2*Sin[a
+ b*(c + d*x)^(3/2)])/(3*b^2*d^3)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)^2\sin\left(a+b(c+dx)^{3/2}\right)dx = \frac{2\,\text{Subst}\left(\int\left((de-cf)^2x\sin\left(a+bx^3\right)-2f(-de+cf)x^3\sin\left(a+bx^3\right)+f^2x^5\sin\left(\right.\right.\right.}{d^3}$$
$$= \frac{\left(2f^2\right)\text{Subst}\left(\int x^5\sin\left(a+bx^3\right)dx,x,\sqrt{c+dx}\right)}{d^3}+\frac{(4f(de-cf))\,\text{Subst}\left(\int x^3\sin\right.}{d^3}$$
$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}+\frac{\left(2f^2\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,\right.}{3d^3}$$
$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}-\frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}$$
$$= -\frac{4f(de-cf)\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}-\frac{2f^2(c+dx)^{3/2}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^3}$$

Mathematica [A] time = 3.14633, size = 419, normalized size = 1.1

$$-\frac{i\left((\cos(a)+i\sin(a))\left(-\frac{2f(c+dx)^2(de-cf)\text{Gamma}\left(\frac{4}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{4/3}}-\frac{(c+dx)(de-cf)^2\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{\left(-ib(c+dx)^{3/2}\right)^{2/3}}+\frac{if^2\sin\left(b(c+dx)^{3/2}\right)}{b^2}+\frac{f^2\cos\left(b(c\right.}{b^2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((-I/3)*((Cos[a] + I*Sin[a])*((f^2*Cos[b*(c + d*x)^(3/2)])/b^2 - ((d*e - c*
f)^2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/((-I)*b*(c + d*x)^(3/2))
^(2/3) - (2*f*(d*e - c*f)*(c + d*x)^2*Gamma[4/3, (-I)*b*(c + d*x)^(3/2)])/(
(-I)*b*(c + d*x)^(3/2))^(4/3) + (I*f^2*Sin[b*(c + d*x)^(3/2)])/b^2 + (f^2*(
c + d*x)^(3/2)*((-I)*Cos[b*(c + d*x)^(3/2)] + Sin[b*(c + d*x)^(3/2)]))/b) -
 (Cos[a] - I*Sin[a])*((f^2*Cos[b*(c + d*x)^(3/2)])/b^2 - ((d*e - c*f)^2*(c
+ d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(I*b*(c + d*x)^(3/2))^(2/3) - (2*f*
(d*e - c*f)*(c + d*x)^2*Gamma[4/3, I*b*(c + d*x)^(3/2)])/(I*b*(c + d*x)^(3/
2))^(4/3) - (I*f^2*Sin[b*(c + d*x)^(3/2)])/b^2 + (f^2*(c + d*x)^(3/2)*(I*Co
s[b*(c + d*x)^(3/2)] + Sin[b*(c + d*x)^(3/2)]))/b)))/d^3
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int\left(fx+e\right)^2\sin\left(a+b\left(dx+c\right)^{\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x)
```

Maxima [B] time = 2.57191, size = 2504, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/18*(3*((d*x + c)^(3/2)*abs(b))^(1/3)*(((-I*gamma(2/3, I*(d*x + c)^(3/2)*b
) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (
-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*co
s(-1/3*pi + 2/3*arctan2(0, b)) - (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2
/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*
(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*ar
ctan2(0, b)))*cos(a) - ((gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d
*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*(d*x + c)
^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*arctan2(0,
b)) - (I*gamma(2/3, I*(d*x + c)^(3/2)*b) - I*gamma(2/3, -I*(d*x + c)^(3/2)*
b))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-I*gamma(2/3, I*(d*x + c)^(3/2)*b) +
 I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*sin(
a))*e^2/(sqrt(d*x + c)*abs(b)) - 6*((d*x + c)^(3/2)*abs(b))^(1/3)*(((-I*gam
ma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*
pi + 2/3*arctan2(0, b)) + (-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3
, -I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*arctan2(0, b)) - (gamma(2/3, I*(
d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arct
an2(0, b)) + (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/
2)*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*cos(a) - ((gamma(2/3, I*(d*x + c)^
(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)
) + (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*co
s(-1/3*pi + 2/3*arctan2(0, b)) - (I*gamma(2/3, I*(d*x + c)^(3/2)*b) - I*gam
ma(2/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-I*gamma(
2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi
 + 2/3*arctan2(0, b)))*sin(a))*c*e*f/(sqrt(d*x + c)*d*abs(b)) + 3*((d*x + c
)^(3/2)*abs(b))^(1/3)*(((-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3,
-I*(d*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (-I*gamma(2/3, I*(
d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*a
rctan2(0, b)) - (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^
(3/2)*b))*sin(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*(d*x + c)^(3/2)*b
) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*cos
(a) - ((gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))
*cos(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma
(2/3, -I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*arctan2(0, b)) - (I*gamma(2/
3, I*(d*x + c)^(3/2)*b) - I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi +
2/3*arctan2(0, b)) - (-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*
(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*sin(a))*c^2*f^2/(sqrt
(d*x + c)*d^2*abs(b)) - 2*(12*((d*x + c)^(3/2)*abs(b))^(1/3)*sqrt(d*x + c)*
cos((d*x + c)^(3/2)*b + a) + sqrt(d*x + c)*(((gamma(1/3, I*(d*x + c)^(3/2)*
b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(1/6*pi + 1/3*arctan2(0, b)) + (g
amma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(-1/6
*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*(d*x + c)^(3/2)*b) + I*gamma(1/
3, -I*(d*x + c)^(3/2)*b))*sin(1/6*pi + 1/3*arctan2(0, b)) + (I*gamma(1/3, I
*(d*x + c)^(3/2)*b) - I*gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(-1/6*pi + 1/3
*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3, I*(d*x + c)^(3/2)*b) + I*gamma(1/
3, -I*(d*x + c)^(3/2)*b))*cos(1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3,
I*(d*x + c)^(3/2)*b) + I*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(-1/6*pi + 1/
3*arctan2(0, b)) - (gamma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*(d*x +
c)^(3/2)*b))*sin(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*(d*x + c)^(3/2
)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(-1/6*pi + 1/3*arctan2(0, b)))*
sin(a)))*e*f/(((d*x + c)^(3/2)*abs(b))^(1/3)*b*d) + 2*(12*((d*x + c)^(3/2)*
abs(b))^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) + sqrt(d*x + c)*(((g
amma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(1/6*
pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*
(d*x + c)^(3/2)*b))*cos(-1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*(d*
x + c)^(3/2)*b) + I*gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(1/6*pi + 1/3*arct
an2(0, b)) + (I*gamma(1/3, I*(d*x + c)^(3/2)*b) - I*gamma(1/3, -I*(d*x + c)
```

```
^(3/2)*b))*sin(-1/6*pi + 1/3*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3, I*(d*
x + c)^(3/2)*b) + I*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(1/6*pi + 1/3*arct
an2(0, b)) + (-I*gamma(1/3, I*(d*x + c)^(3/2)*b) + I*gamma(1/3, -I*(d*x + c
)^(3/2)*b))*cos(-1/6*pi + 1/3*arctan2(0, b)) - (gamma(1/3, I*(d*x + c)^(3/2
)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(1/6*pi + 1/3*arctan2(0, b)) +
(gamma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(-1
/6*pi + 1/3*arctan2(0, b)))*sin(a))*c*f^2/(((d*x + c)^(3/2)*abs(b))^(1/3)*
b*d^2) - 12*((d*x + c)^(3/2)*b*cos((d*x + c)^(3/2)*b + a) - sin((d*x + c)^(
3/2)*b + a))*f^2/(b^2*d^2))/d
```

Fricas [A] time = 2.31503, size = 740, normalized size = 1.94

$$\frac{\left(2 i def - 2 i cf^2\right) (i b)^{\frac{2}{3}} e^{(-i a)} \Gamma\left(\frac{1}{3}, (i bdx + i bc)\sqrt{dx + c}\right) + \left(-2 i def + 2 i cf^2\right) (-i b)^{\frac{2}{3}} e^{(i a)} \Gamma\left(\frac{1}{3}, (-i bdx - i bc)\sqrt{dx + c}\right) - 3 \left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/9*((2*I*d*e*f - 2*I*c*f^2)*(I*b)^(2/3)*e^(-I*a)*gamma(1/3, (I*b*d*x + I*b
*c)*sqrt(d*x + c)) + (-2*I*d*e*f + 2*I*c*f^2)*(-I*b)^(2/3)*e^(I*a)*gamma(1/
3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f
^2)*(I*b)^(1/3)*e^(-I*a)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) - 3*(b
*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*(-I*b)^(1/3)*e^(I*a)*gamma(2/3, (-I*b*d
*x - I*b*c)*sqrt(d*x + c)) + 6*f^2*sin((b*d*x + b*c)*sqrt(d*x + c) + a) - 6
*(b*d*f^2*x + 2*b*d*e*f - b*c*f^2)*sqrt(d*x + c)*cos((b*d*x + b*c)*sqrt(d*x
 + c) + a))/(b^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^2 \sin\left(a + bc\sqrt{c + dx} + bdx\sqrt{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)^2 \sin\left((dx + c)^{\frac{3}{2}} b + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin((d*x + c)^(3/2)*b + a), x)
```

3.193 $\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)\,dx$

Optimal. Leaf size=291

$$\frac{ie^{ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}-\frac{e^{ia}f\sqrt{c+dx}\text{Ga}}{9bd^2\sqrt[3]{}}$$

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^2) - (E^(I*a)*f*Sqrt
[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^2*((-I)*b*(c + d*x)^(3
/2))^(1/3)) - (f*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(9*b*d^2*E^
(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(I*a)*(d*e - c*f)*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^2*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (
(I/3)*(d*e - c*f)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d^2*E^(I*a)*(
I*b*(c + d*x)^(3/2))^(2/3))
```

Rubi [A] time = 0.1999, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3433, 3389, 2218, 3385, 3356, 2208}

$$\frac{ie^{ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)(de-cf)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d^2\left(ib(c+dx)^{3/2}\right)^{2/3}}-\frac{e^{ia}f\sqrt{c+dx}\text{Ga}}{9bd^2\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a + b*(c + d*x)^(3/2)])/(3*b*d^2) - (E^(I*a)*f*Sqrt
[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(9*b*d^2*((-I)*b*(c + d*x)^(3
/2))^(1/3)) - (f*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(9*b*d^2*E^
(I*a)*(I*b*(c + d*x)^(3/2))^(1/3)) + ((I/3)*E^(I*a)*(d*e - c*f)*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d^2*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (
(I/3)*(d*e - c*f)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d^2*E^(I*a)*(
I*b*(c + d*x)^(3/2))^(2/3))
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^{3/2}\right)dx &= \frac{2\,\mathrm{Subst}\left(\int\left((de-cf)x\sin\left(a+bx^3\right)+fx^3\sin\left(a+bx^3\right)\right)dx,x,\sqrt{c+dx}\right)}{d^2}\\ &= \frac{(2f)\,\mathrm{Subst}\left(\int x^3\sin\left(a+bx^3\right)dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2(de-cf))\,\mathrm{Subst}\left(\int x\sin\left(a+\right.\right.}{d^2}\\ &= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}+\frac{(2f)\,\mathrm{Subst}\left(\int\cos\left(a+bx^3\right)dx,x,\sqrt{c+dx}\right)}{3bd^2}\\ &= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}+\frac{ie^{ia}(de-cf)(c+dx)\Gamma\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d^2\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie}{}\\ &= -\frac{2f\sqrt{c+dx}\cos\left(a+b(c+dx)^{3/2}\right)}{3bd^2}-\frac{e^{ia}f\sqrt{c+dx}\Gamma\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{9bd^2\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{e^{-ia}f\sqrt{c+}}{9bd}\end{aligned}$$

Mathematica [B] time = 2.61017, size = 705, normalized size = 2.42

$$\frac{f\cos(a)\left(-\frac{2\sqrt{c+dx}\mathrm{Gamma}\left(\frac{1}{3},-ib(c+dx)^{3/2}\right)}{3\sqrt[3]{-ib(c+dx)^{3/2}}}-\frac{2\sqrt{c+dx}\mathrm{Gamma}\left(\frac{1}{3},ib(c+dx)^{3/2}\right)}{3\sqrt[3]{ib(c+dx)^{3/2}}}\right)}{6bd^2}+\frac{icf\cos(a)\left(\frac{2(c+dx)\mathrm{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3\left(ib(c+dx)^{3/2}\right)^{2/3}}-\frac{2(c+dx)\mathrm{Gamma}(}{3(-ib(c+}\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] (-2*f*Sqrt[c + d*x]*Cos[a]*Cos[b*(c + d*x)^(3/2)])/(3*b*d^2) + (f*Cos[a]*((
-2*Sqrt[c + d*x]*Gamma[1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(
3/2))^(1/3)) - (2*Sqrt[c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c
 + d*x)^(3/2))^(1/3))))/(6*b*d^2) - ((I/2)*e*Cos[a]*((-2*(c + d*x)*Gamma[2/
3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*
x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d + (
(I/2)*c*f*Cos[a]*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I
)*b*(c + d*x)^(3/2))^(2/3)) + (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])
/(3*(I*b*(c + d*x)^(3/2))^(2/3))))/d^2 + ((I/6)*f*((-2*Sqrt[c + d*x]*Gamma[
1/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(1/3)) + (2*Sqrt[
c + d*x]*Gamma[1/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(1/3)))*
```

```
Sin[a])/(b*d^2) + (e*((-2*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*
((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/
2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))*Sin[a])/(2*d) - (c*f*((-2*(c + d*x)*G
amma[2/3, (-I)*b*(c + d*x)^(3/2)])/(3*((-I)*b*(c + d*x)^(3/2))^(2/3)) - (2*
(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(3*(I*b*(c + d*x)^(3/2))^(2/3)))
*Sin[a])/(2*d^2) + (2*f*Sqrt[c + d*x]*Sin[a]*Sin[b*(c + d*x)^(3/2)])/(3*b*d
^2)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(fx + e\right) \sin \left(a + b\left(dx + c\right)^{\frac{3}{2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x)
```

Maxima [B] time = 1.98336, size = 1449, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/18*(3*((d*x + c)^(3/2)*abs(b))^(1/3)*(((-I*gamma(2/3, I*(d*x + c)^(3/2)*b
) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (
-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*co
s(-1/3*pi + 2/3*arctan2(0, b)) - (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2
/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*
(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*ar
ctan2(0, b)))*cos(a) - ((gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d
*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*(d*x + c)
^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*arctan2(0,
b)) - (I*gamma(2/3, I*(d*x + c)^(3/2)*b) - I*gamma(2/3, -I*(d*x + c)^(3/2)*
b))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-I*gamma(2/3, I*(d*x + c)^(3/2)*b) +
 I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*sin(
a))*e/(sqrt(d*x + c)*abs(b)) - 3*((d*x + c)^(3/2)*abs(b))^(1/3)*(((-I*gamma
(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*pi
 + 2/3*arctan2(0, b)) + (-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3,
-I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*arctan2(0, b)) - (gamma(2/3, I*(d*
x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arctan
2(0, b)) + (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)
*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*cos(a) - ((gamma(2/3, I*(d*x + c)^(3
/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b))
+ (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(
-1/3*pi + 2/3*arctan2(0, b)) - (I*gamma(2/3, I*(d*x + c)^(3/2)*b) - I*gamma
(2/3, -I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-I*gamma(2/
3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi +
 2/3*arctan2(0, b)))*sin(a))*c*f/(sqrt(d*x + c)*d*abs(b)) - (12*((d*x + c)^
(3/2)*abs(b))^(1/3)*sqrt(d*x + c)*cos((d*x + c)^(3/2)*b + a) + sqrt(d*x + c
)*(((gamma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*co
s(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/
```

```
3, -I*(d*x + c)^(3/2)*b))*cos(-1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3,
 I*(d*x + c)^(3/2)*b) + I*gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(1/6*pi + 1/
3*arctan2(0, b)) + (I*gamma(1/3, I*(d*x + c)^(3/2)*b) - I*gamma(1/3, -I*(d*
x + c)^(3/2)*b))*sin(-1/6*pi + 1/3*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3,
 I*(d*x + c)^(3/2)*b) + I*gamma(1/3, -I*(d*x + c)^(3/2)*b))*cos(1/6*pi + 1/
3*arctan2(0, b)) + (-I*gamma(1/3, I*(d*x + c)^(3/2)*b) + I*gamma(1/3, -I*(d
*x + c)^(3/2)*b))*cos(-1/6*pi + 1/3*arctan2(0, b)) - (gamma(1/3, I*(d*x + c
)^(3/2)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*sin(1/6*pi + 1/3*arctan2(0,
b)) + (gamma(1/3, I*(d*x + c)^(3/2)*b) + gamma(1/3, -I*(d*x + c)^(3/2)*b))*
sin(-1/6*pi + 1/3*arctan2(0, b)))*sin(a))*f/(((d*x + c)^(3/2)*abs(b))^(1/3
)*b*d))/d
```

Fricas [A] time = 2.33272, size = 524, normalized size = 1.8

$$\frac{i\,(i\,b)^{\frac{2}{3}}\,fe^{(-i\,a)}\Gamma\left(\frac{1}{3},(i\,bdx+i\,bc)\sqrt{dx+c}\right)-i\,(-i\,b)^{\frac{2}{3}}\,fe^{(i\,a)}\Gamma\left(\frac{1}{3},(-i\,bdx-i\,bc)\sqrt{dx+c}\right)-6\,\sqrt{dx+cbf}\,\cos\left((bdx+bc)\sqrt{dx}\right.}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/9*(I*(I*b)^(2/3)*f*e^(-I*a)*gamma(1/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) -
 I*(-I*b)^(2/3)*f*e^(I*a)*gamma(1/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)) - 6*
sqrt(d*x + c)*b*f*cos((b*d*x + b*c)*sqrt(d*x + c) + a) - 3*(b*d*e - b*c*f)*
(I*b)^(1/3)*e^(-I*a)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) - 3*(b*d*e
 - b*c*f)*(-I*b)^(1/3)*e^(I*a)*gamma(2/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c))
)/(b^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)\sin\left(a+bc\sqrt{c+dx}+bdx\sqrt{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*c*sqrt(c + d*x) + b*d*x*sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin((d*x + c)^(3/2)*b + a), x)
```

3.194 $\int \sin\left(a+b(c+dx)^{3/2}\right) dx$

Optimal. Leaf size=115

$$\frac{ie^{ia}(c+dx)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

```
[Out] ((I/3)*E^(I*a)*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d*((-I)*b*(c
+ d*x)^(3/2))^(2/3)) - ((I/3)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d
*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3))
```

Rubi [A] time = 0.0812329, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3363, 3389, 2218}

$$\frac{ie^{ia}(c+dx)\text{Gamma}\left(\frac{2}{3},-ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}}-\frac{ie^{-ia}(c+dx)\text{Gamma}\left(\frac{2}{3},ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/3)*E^(I*a)*(c + d*x)*Gamma[2/3, (-I)*b*(c + d*x)^(3/2)])/(d*((-I)*b*(c
+ d*x)^(3/2))^(2/3)) - ((I/3)*(c + d*x)*Gamma[2/3, I*b*(c + d*x)^(3/2)])/(d
*E^(I*a)*(I*b*(c + d*x)^(3/2))^(2/3))
```

Rule 3363

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \sin\left(a+b(c+dx)^{3/2}\right) dx = \frac{2\,\text{Subst}\left(\int x\sin\left(a+bx^3\right) dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{i\,\text{Subst}\left(\int e^{-ia-ibx^3}x\,dx, x, \sqrt{c+dx}\right)}{d} - \frac{i\,\text{Subst}\left(\int e^{ia+ibx^3}x\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{ie^{ia}(c+dx)\Gamma\left(\frac{2}{3}, -ib(c+dx)^{3/2}\right)}{3d\left(-ib(c+dx)^{3/2}\right)^{2/3}} - \frac{ie^{-ia}(c+dx)\Gamma\left(\frac{2}{3}, ib(c+dx)^{3/2}\right)}{3d\left(ib(c+dx)^{3/2}\right)^{2/3}}$$

Mathematica [A] time = 0.149353, size = 123, normalized size = 1.07

$$\frac{i(c+dx)\left((\cos(a)+i\sin(a))\left(ib(c+dx)^{3/2}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}, -ib(c+dx)^{3/2}\right) - (\cos(a)-i\sin(a))\left(-ib(c+dx)^{3/2}\right)^{2/3}\text{Gamm}}{3d\left(b^2(c+dx)^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)],x]
```

```
[Out] ((I/3)*(c + d*x)*(-(((-I)*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, I*b*(c + d*x)
^(3/2)]*(Cos[a] - I*Sin[a])) + (I*b*(c + d*x)^(3/2))^(2/3)*Gamma[2/3, (-I)*
b*(c + d*x)^(3/2)]*(Cos[a] + I*Sin[a])))/(d*(b^2*(c + d*x)^3)^(2/3))
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(dx+c)^{\frac{3}{2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(3/2)),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2)),x)
```

Maxima [B] time = 1.3706, size = 466, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/6*((d*x + c)^(3/2)*abs(b))^(1/3)*(((-I*gamma(2/3, I*(d*x + c)^(3/2)*b) +
I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (-I*g
amma(2/3, I*(d*x + c)^(3/2)*b) + I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(-1
/3*pi + 2/3*arctan2(0, b)) - (gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3,
-I*(d*x + c)^(3/2)*b))*sin(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*(d*x
 + c)^(3/2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*arctan
2(0, b)))*cos(a) - ((gamma(2/3, I*(d*x + c)^(3/2)*b) + gamma(2/3, -I*(d*x +
 c)^(3/2)*b))*cos(1/3*pi + 2/3*arctan2(0, b)) + (gamma(2/3, I*(d*x + c)^(3/
2)*b) + gamma(2/3, -I*(d*x + c)^(3/2)*b))*cos(-1/3*pi + 2/3*arctan2(0, b))
```

```
- (I*gamma(2/3, I*(d*x + c)^(3/2)*b) - I*gamma(2/3, -I*(d*x + c)^(3/2)*b))*
sin(1/3*pi + 2/3*arctan2(0, b)) - (-I*gamma(2/3, I*(d*x + c)^(3/2)*b) + I*g
amma(2/3, -I*(d*x + c)^(3/2)*b))*sin(-1/3*pi + 2/3*arctan2(0, b)))*sin(a))/
(sqrt(d*x + c)*d*abs(b))
```

Fricas [A] time = 2.14939, size = 198, normalized size = 1.72

$$-\frac{(i\,b)^{\frac{1}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{2}{3},(i\,bdx+i\,bc)\sqrt{dx+c}\right)+(-i\,b)^{\frac{1}{3}}\,e^{(i\,a)}\Gamma\left(\frac{2}{3},(-i\,bdx-i\,bc)\sqrt{dx+c}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/3*((I*b)^(1/3)*e^(-I*a)*gamma(2/3, (I*b*d*x + I*b*c)*sqrt(d*x + c)) + (-
I*b)^(1/3)*e^(I*a)*gamma(2/3, (-I*b*d*x - I*b*c)*sqrt(d*x + c)))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(c+dx)^{\frac{3}{2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(3/2)),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left((dx+c)^{\frac{3}{2}}b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a), x)
```

3.195 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi [A] time = 0.0132693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

Mathematica [A] time = 10.7782, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x), x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(a+b\left(dx+c\right)^{\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left((bdx+bc)\sqrt{dx+c}+a\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{3}{2}}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e), x)
```

3.196 $\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0134022, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 13.406, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^{3/2}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Maple [A] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^{2}}\sin\left(a+b\left(dx+c\right)^{\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{3}{2}}b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left((bdx+bc)\sqrt{dx+c}+a\right)}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((b*d*x + b*c)*sqrt(d*x + c) + a)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(3/2))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{3}{2}}b+a\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(3/2)*b + a)/(f*x + e)^2, x)
```

3.197 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$

Optimal. Leaf size=611

$$-\frac{b^4 f \sin(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3}+\frac{b^2\sin(a)(de-cf)^2\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{b^6 f^2\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3}$$

```
[Out] (b^5*f^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(360*d^3) - (b^3*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(6*d^3) + (b*(d*e - c*f)^2*Sqr
t[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^3 - (b^3*f^2*(c + d*x)^(3/2)*Cos[a +
 b/Sqrt[c + d*x]])/(180*d^3) + (b*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a + b/S
qrt[c + d*x]])/(3*d^3) + (b*f^2*(c + d*x)^(5/2)*Cos[a + b/Sqrt[c + d*x]])/(
15*d^3) + (b^6*f^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(360*d^3) - (b^4*f*
(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(6*d^3) + (b^2*(d*e - c*f)
^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^3 + (b^4*f^2*(c + d*x)*Sin[a + b/
Sqrt[c + d*x]])/(360*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)*Sin[a + b/Sqrt[c +
 d*x]])/(6*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d^3 -
(b^2*f^2*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(60*d^3) + (f*(d*e - c*f)*(c
 + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/Sqrt[c
 + d*x]])/(3*d^3) + (b^6*f^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(360*d^3)
 - (b^4*f*(d*e - c*f)*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(6*d^3) + (b^2*(
d*e - c*f)^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d^3
```

Rubi [A] time = 0.790798, antiderivative size = 611, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3431, 3297, 3303, 3299, 3302}

$$-\frac{b^4 f \sin(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{6d^3}+\frac{b^2\sin(a)(de-cf)^2\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^3}+\frac{b^6 f^2\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{360d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```


```
[Out] (b^5*f^2*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(360*d^3) - (b^3*f*(d*e -
c*f)*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(6*d^3) + (b*(d*e - c*f)^2*Sqr
t[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^3 - (b^3*f^2*(c + d*x)^(3/2)*Cos[a +
 b/Sqrt[c + d*x]])/(180*d^3) + (b*f*(d*e - c*f)*(c + d*x)^(3/2)*Cos[a + b/S
qrt[c + d*x]])/(3*d^3) + (b*f^2*(c + d*x)^(5/2)*Cos[a + b/Sqrt[c + d*x]])/(
15*d^3) + (b^6*f^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(360*d^3) - (b^4*f*
(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(6*d^3) + (b^2*(d*e - c*f)
^2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^3 + (b^4*f^2*(c + d*x)*Sin[a + b/
Sqrt[c + d*x]])/(360*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)*Sin[a + b/Sqrt[c +
 d*x]])/(6*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d^3 -
(b^2*f^2*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(60*d^3) + (f*(d*e - c*f)*(c
 + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/d^3 + (f^2*(c + d*x)^3*Sin[a + b/Sqrt[c
 + d*x]])/(3*d^3) + (b^6*f^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(360*d^3)
 - (b^4*f*(d*e - c*f)*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(6*d^3) + (b^2*(
d*e - c*f)^2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d^3
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
```

```
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx = -\frac{2\,\text{Subst}\left(\int \left(\frac{f^2\sin(a+bx)}{d^2x^7}+\frac{2f(de-cf)\sin(a+bx)}{d^2x^5}+\frac{(de-cf)^2\sin(a+bx)}{d^2x^3}\right)dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{\left(2f^2\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^7}dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d^3} - \frac{(4f(de-cf))\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}dx, x, \dots\right.}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{f^2(c \dots}{}$$

$$= \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \dots$$

$$= \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} + \frac{bf(de-cf)(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{3d^3} + \dots$$

$$= -\frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \dots$$

$$= -\frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-cf)^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^3} - \dots$$

$$= \frac{b^5f^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-c \dots}{}$$

$$= \frac{b^5f^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-c \dots}{}$$

$$= \frac{b^5f^2\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{360d^3} - \frac{b^3f(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^3} + \frac{b(de-c \dots}{}$$

Mathematica [C] time = 2.20019, size = 557, normalized size = 0.91

$$ie^{-ia}\left(-e^{2ia}b^2\left(f^2\left(60b^2c+b^4+360c^2\right)-60def\left(b^2+12c\right)+360d^2e^2\right)\text{Ei}\left(\frac{ib}{\sqrt{c+dx}}\right)-\sqrt{c+dx}e^{i\left(2a+\frac{b}{\sqrt{c+dx}}\right)}\left(-2ib^3f(-29cf+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] ((I/720)*((Sqrt[c + d*x]*((-I)*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] + (2*I)*b^3*
f*(30*d*e - 29*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x
) + 120*Sqrt[c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x +
 f^2*x^2)) - (24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*
e*f*x + f^2*x^2))))/E^((I*b)/Sqrt[c + d*x]) - E^(I*(2*a + b/Sqrt[c + d*x]))
*Sqrt[c + d*x]*(I*b^5*f^2 + b^4*f^2*Sqrt[c + d*x] - (2*I)*b^3*f*(30*d*e - 2
9*c*f + d*f*x) - 6*b^2*f*Sqrt[c + d*x]*(10*d*e - 9*c*f + d*f*x) + 120*Sqrt[
c + d*x]*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2)) +
(24*I)*b*(11*c^2*f^2 - c*d*f*(25*e + 3*f*x) + d^2*(15*e^2 + 5*e*f*x + f^2*x
^2))) + b^2*(360*d^2*e^2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^
2)*f^2)*ExpIntegralEi[((-I)*b)/Sqrt[c + d*x]] - b^2*E^((2*I)*a)*(360*d^2*e^
2 - 60*(b^2 + 12*c)*d*e*f + (b^4 + 60*b^2*c + 360*c^2)*f^2)*ExpIntegralEi[(
I*b)/Sqrt[c + d*x]]))/(d^3*E^(I*a))
```

Maple [A] time = 0.059, size = 696, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x)
```

```
[Out] -2/d^3*b^2*(c^2*f^2*(-1/2*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2-1/2*cos(a+b/(d
*x+c)^(1/2))*(d*x+c)^(1/2)/b-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c
)^(1/2))*sin(a))-2*c*d*e*f*(-1/2*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2-1/2*cos
(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/b-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b
/(d*x+c)^(1/2))*sin(a))+d^2*e^2*(-1/2*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2-1/
2*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/b-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2
*Ci(b/(d*x+c)^(1/2))*sin(a))-2*b^2*c*f^2*(-1/4*sin(a+b/(d*x+c)^(1/2))*(d*x+
c)^2/b^4-1/12*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(3/2)/b^3+1/24*sin(a+b/(d*x+c)
^(1/2))*(d*x+c)/b^2+1/24*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/b+1/24*Si(b/(
d*x+c)^(1/2))*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(a))+2*b^2*d*e*f*(-1/4*sin
(a+b/(d*x+c)^(1/2))*(d*x+c)^2/b^4-1/12*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(3/2)
/b^3+1/24*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2+1/24*cos(a+b/(d*x+c)^(1/2))*(d
*x+c)^(1/2)/b+1/24*Si(b/(d*x+c)^(1/2))*cos(a)+1/24*Ci(b/(d*x+c)^(1/2))*sin(
a))+b^4*f^2*(-1/6*sin(a+b/(d*x+c)^(1/2))*(d*x+c)^3/b^6-1/30*cos(a+b/(d*x+c)
^(1/2))*(d*x+c)^(5/2)/b^5+1/120*sin(a+b/(d*x+c)^(1/2))*(d*x+c)^2/b^4+1/360*
cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(3/2)/b^3-1/720*sin(a+b/(d*x+c)^(1/2))*(d*x+
c)/b^2-1/720*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/b-1/720*Si(b/(d*x+c)^(1/2
))*cos(a)-1/720*Ci(b/(d*x+c)^(1/2))*sin(a)))
```

Maxima [C] time = 2.04818, size = 1184, normalized size = 1.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/720*(360*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) +
 (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x
+ c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x
 + c)*a + b)/sqrt(d*x + c)))*e^2 - 720*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-
I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)
))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c))
 + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*e*f/d + 360*(((-
I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d
*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqr
t(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a + b)/sq
rt(d*x + c)))*c^2*f^2/d^2 + 60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei(-I*b/sqrt(
d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a)
)*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(d*x + c)*a +
b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*sin((sqrt(d*x + c)*a
+ b)/sqrt(d*x + c)))*e*f/d - 60*(((I*Ei(I*b/sqrt(d*x + c)) - I*Ei(-I*b/sqrt
(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a
))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/2)*b)*cos((sqrt(d*x + c)*a +
 b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x + c)^2)*sin((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)))*c*f^2/d^2 + (((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/s
qrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*si
n(a))*b^6 + 2*(sqrt(d*x + c)*b^5 - 2*(d*x + c)^(3/2)*b^3 + 24*(d*x + c)^(5/
2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*((d*x + c)*b^4 - 6*(d*x
+ c)^2*b^2 + 120*(d*x + c)^3)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f^2
/d^2)/d
```

Fricas [A] time = 2.38604, size = 1119, normalized size = 1.83

$$\left(360\,b^2d^2e^2-60\left(b^4+12\,b^2c\right)def+\left(b^6+60\,b^4c+360\,b^2c^2\right)f^2\right)\operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+\left(360\,b^2d^2e^2-60\left(b^4+12\,b^2c\right)de$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/720*((360*b^2*d^2*e^2 - 60*(b^4 + 12*b^2*c)*d*e*f + (b^6 + 60*b^4*c + 360
*b^2*c^2)*f^2)*cos_integral(b/sqrt(d*x + c))*sin(a) + (360*b^2*d^2*e^2 - 60
*(b^4 + 12*b^2*c)*d*e*f + (b^6 + 60*b^4*c + 360*b^2*c^2)*f^2)*cos_integral(
-b/sqrt(d*x + c))*sin(a) + 2*(360*b^2*d^2*e^2 - 60*(b^4 + 12*b^2*c)*d*e*f +
 (b^6 + 60*b^4*c + 360*b^2*c^2)*f^2)*cos(a)*sin_integral(b/sqrt(d*x + c)) +
 2*(24*b*d^2*f^2*x^2 + 360*b*d^2*e^2 - 60*(b^3 + 10*b*c)*d*e*f + (b^5 + 58*
b^3*c + 264*b*c^2)*f^2 + 2*(60*b*d^2*e*f - (b^3 + 36*b*c)*d*f^2)*x)*sqrt(d*
x + c)*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)) + 2*(120*d^3*f^2*x^3
+ 360*c*d^2*e^2 - 60*(b^2*c + 6*c^2)*d*e*f + (b^4*c + 54*b^2*c^2 + 120*c^3)
*f^2 - 6*(b^2*d^2*f^2 - 60*d^3*e*f)*x^2 - (60*b^2*d^2*e*f - 360*d^3*e^2 - (
b^4 + 48*b^2*c)*d*f^2)*x)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/d
^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/2)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \sin\left(a+\frac{b}{\sqrt{dx+c}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/sqrt(d*x + c)), x)
```

3.198 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)\,dx$

Optimal. Leaf size=301

$$\frac{b^2\sin(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{b^4f\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b^2\cos(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{b^4f\cos(}{1}$$

```
[Out] -(b^3*f*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(12*d^2) + (b*(d*e - c*f)*S
qrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b*f*(c + d*x)^(3/2)*Cos[a + b
/Sqrt[c + d*x]])/(6*d^2) - (b^4*f*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(12*
d^2) + (b^2*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^2 - (b^2*f*(
c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(12*d^2) + ((d*e - c*f)*(c + d*x)*Sin[a
+ b/Sqrt[c + d*x]])/d^2 + (f*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(2*d^2)
- (b^4*f*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(12*d^2) + (b^2*(d*e - c*f)*C
os[a]*SinIntegral[b/Sqrt[c + d*x]])/d^2
```

Rubi [A] time = 0.393377, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3431, 3297, 3303, 3299, 3302}

$$\frac{b^2\sin(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{b^4f\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{12d^2}+\frac{b^2\cos(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d^2}-\frac{b^4f\cos(}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] -(b^3*f*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/(12*d^2) + (b*(d*e - c*f)*S
qrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d^2 + (b*f*(c + d*x)^(3/2)*Cos[a + b
/Sqrt[c + d*x]])/(6*d^2) - (b^4*f*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/(12*
d^2) + (b^2*(d*e - c*f)*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/d^2 - (b^2*f*(
c + d*x)*Sin[a + b/Sqrt[c + d*x]])/(12*d^2) + ((d*e - c*f)*(c + d*x)*Sin[a
+ b/Sqrt[c + d*x]])/d^2 + (f*(c + d*x)^2*Sin[a + b/Sqrt[c + d*x]])/(2*d^2)
- (b^4*f*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/(12*d^2) + (b^2*(d*e - c*f)*C
os[a]*SinIntegral[b/Sqrt[c + d*x]])/d^2
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx = -\frac{2\,\mathrm{Subst}\left(\int\left(\frac{f\sin(a+bx)}{dx^5}+\frac{(de-cf)\sin(a+bx)}{dx^3}\right)dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{(2f)\,\mathrm{Subst}\left(\int\frac{\sin(a+bx)}{x^5}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^2} - \frac{(2(de-cf))\,\mathrm{Subst}\left(\int\frac{\sin(a+bx)}{x^3}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d^2}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{2d^2} - \frac{(bf)\,\mathrm{Subst}\left(\int\frac{\cos(a}{x^4}\right.}{2d^2}$$

$$= \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2} + \frac{(de-cf)(c+d}{}$$

$$= \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{6d^2} - \frac{b^2f(c+dx)\sin}{12d}$$

$$= -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}c}{6}$$

$$= -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}c}{6}$$

$$= -\frac{b^3f\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{12d^2} + \frac{b(de-cf)\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d^2} + \frac{bf(c+dx)^{3/2}c}{6}$$

Mathematica [A] time = 0.620483, size = 367, normalized size = 1.22

$$-\frac{b^2f\left(b^2+12c\right)\left(\sin(a)\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)+\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right)}{12d^2} + \frac{b^2e\left(\sin(a)\mathrm{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)+\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (e*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(b*Cos[a] + Sqrt[c + d*x]*Sin[a]))/d
+ (f*Sqrt[c + d*x]*Cos[b/Sqrt[c + d*x]]*(-(b^3*Cos[a]) - 12*b*c*Cos[a] + 2*
b*(c + d*x)*Cos[a] - b^2*Sqrt[c + d*x]*Sin[a] - 12*c*Sqrt[c + d*x]*Sin[a] +
 6*(c + d*x)^(3/2)*Sin[a]))/(12*d^2) + (e*Sqrt[c + d*x]*(Sqrt[c + d*x]*Cos[
a] - b*Sin[a])*Sin[b/Sqrt[c + d*x]])/d + (f*Sqrt[c + d*x]*(-(b^2*Sqrt[c + d
*x]*Cos[a]) - 12*c*Sqrt[c + d*x]*Cos[a] + 6*(c + d*x)^(3/2)*Cos[a] + b^3*Si
n[a] + 12*b*c*Sin[a] - 2*b*(c + d*x)*Sin[a])*Sin[b/Sqrt[c + d*x]])/(12*d^2)
 + (b^2*e*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] + Cos[a]*SinIntegral[b/Sqrt[
```

```
c + d*x]]))/d - (b^2*(b^2 + 12*c)*f*(CosIntegral[b/Sqrt[c + d*x]]*Sin[a] +
Cos[a]*SinIntegral[b/Sqrt[c + d*x]]))/(12*d^2)
```

Maple [A] time = 0.028, size = 295, normalized size = 1.

$$-2\,{\frac{{b}^{2}}{{d}^{2}}}\left(-cf\left(-1/2\,{\frac{dx+c}{{b}^{2}}\sin\left(a+{\frac{b}{\sqrt{dx+c}}}\right)}-1/2\,{\frac{\sqrt{dx+c}}{b}\cos\left(a+{\frac{b}{\sqrt{dx+c}}}\right)}-1/2\,{\rm Si}\left({\frac{b}{\sqrt{dx+c}}}\right)\cos\left(a\right)-1/2\,{\rm Ci}\left(\frac{}{\sqrt{d}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x)
```

```
[Out] -2/d^2*b^2*(-c*f*(-1/2*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2-1/2*cos(a+b/(d*x+
c)^(1/2))*(d*x+c)^(1/2)/b-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(
1/2))*sin(a))+d*e*(-1/2*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2-1/2*cos(a+b/(d*x
+c)^(1/2))*(d*x+c)^(1/2)/b-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^
(1/2))*sin(a))+b^2*f*(-1/4*sin(a+b/(d*x+c)^(1/2))*(d*x+c)^2/b^4-1/12*cos(a+
b/(d*x+c)^(1/2))*(d*x+c)^(3/2)/b^3+1/24*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2+
1/24*cos(a+b/(d*x+c)^(1/2))*(d*x+c)^(1/2)/b+1/24*Si(b/(d*x+c)^(1/2))*cos(a)
+1/24*Ci(b/(d*x+c)^(1/2))*sin(a)))
```

Maxima [C] time = 1.55219, size = 549, normalized size = 1.82

$$\frac{12\left(\left(\left(-i\,{\rm Ei}\left(\frac{i\,b}{\sqrt{dx+c}}\right)+i\,{\rm Ei}\left(-\frac{i\,b}{\sqrt{dx+c}}\right)\right)\cos\left(a\right)+\left({\rm Ei}\left(\frac{i\,b}{\sqrt{dx+c}}\right)+{\rm Ei}\left(-\frac{i\,b}{\sqrt{dx+c}}\right)\right)\sin\left(a\right)\right)b^{2}+2\,\sqrt{dx+c}b\cos\left(\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right)+2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (
Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x +
c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x +
 c)*a + b)/sqrt(d*x + c)))*e - 12*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/s
qrt(d*x + c)))*cos(a) + (Ei(I*b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*si
n(a))*b^2 + 2*sqrt(d*x + c)*b*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*
(d*x + c)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*c*f/d + (((I*Ei(I*b/sqr
t(d*x + c)) - I*Ei(-I*b/sqrt(d*x + c)))*cos(a) - (Ei(I*b/sqrt(d*x + c)) + E
i(-I*b/sqrt(d*x + c)))*sin(a))*b^4 - 2*(sqrt(d*x + c)*b^3 - 2*(d*x + c)^(3/
2)*b)*cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) - 2*((d*x + c)*b^2 - 6*(d*x
+ c)^2)*sin((sqrt(d*x + c)*a + b)/sqrt(d*x + c)))*f/d)/d
```

Fricas [A] time = 2.28389, size = 621, normalized size = 2.06

$$\frac{\left(12\,b^{2}de-\left(b^{4}+12\,b^{2}c\right)f\right){\rm Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+\left(12\,b^{2}de-\left(b^{4}+12\,b^{2}c\right)f\right){\rm Ci}\left(-\frac{b}{\sqrt{dx+c}}\right)\sin\left(a\right)+2\left(12\,b^{2}de-\left(b^{4}+12\,b\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/24*((12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos_integral(b/sqrt(d*x + c))*sin(a
) + (12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos_integral(-b/sqrt(d*x + c))*sin(a)
 + 2*(12*b^2*d*e - (b^4 + 12*b^2*c)*f)*cos(a)*sin_integral(b/sqrt(d*x + c))
 + 2*(2*b*d*f*x + 12*b*d*e - (b^3 + 10*b*c)*f)*sqrt(d*x + c)*cos((a*d*x + a
*c + sqrt(d*x + c)*b)/(d*x + c)) + 2*(6*d^2*f*x^2 + 12*c*d*e - (b^2*c + 6*c
^2)*f - (b^2*d*f - 12*d^2*e)*x)*sin((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x +
c)))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(1/2)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/sqrt(d*x + c)), x)
```

3.199 $\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right) dx$

Optimal. Leaf size=94

$$\frac{b^2\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d + (b^2*CosIntegral[b/Sqrt[c +
d*x]]*Sin[a])/d + ((c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d + (b^2*Cos[a]*SinI
ntegral[b/Sqrt[c + d*x]])/d
```

Rubi [A] time = 0.118033, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3361, 3297, 3303, 3299, 3302}

$$\frac{b^2\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}+\frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]])/d + (b^2*CosIntegral[b/Sqrt[c +
d*x]]*Sin[a])/d + ((c + d*x)*Sin[a + b/Sqrt[c + d*x]])/d + (b^2*Cos[a]*SinI
ntegral[b/Sqrt[c + d*x]])/d
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{\sqrt{c+dx}}\right)dx = -\frac{2\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$
$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$
$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{\left(b^2\cos(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{\sqrt{c+}}\right.}{d}$$
$$= \frac{b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\text{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d} + \frac{b^2\cos(a)\text{Si}\left(\frac{}{\sqrt{}}\right.}{d}$$

Mathematica [A] time = 0.0762403, size = 99, normalized size = 1.05

$$\frac{b^2\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt{c+dx}}\right) + b^2\cos(a)\text{Si}\left(\frac{b}{\sqrt{c+dx}}\right) + c\sin\left(a+\frac{b}{\sqrt{c+dx}}\right) + dx\sin\left(a+\frac{b}{\sqrt{c+dx}}\right) + b\sqrt{c+dx}\cos\left(a+\frac{b}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]],x]
```

```
[Out] (b*Sqrt[c + d*x]*Cos[a + b/Sqrt[c + d*x]] + b^2*CosIntegral[b/Sqrt[c + d*x]
]*Sin[a] + c*Sin[a + b/Sqrt[c + d*x]] + d*x*Sin[a + b/Sqrt[c + d*x]] + b^2*
Cos[a]*SinIntegral[b/Sqrt[c + d*x]])/d
```

Maple [A] time = 0.015, size = 84, normalized size = 0.9

$$-2\,\frac{b^2}{d}\left(-1/2\,\frac{dx+c}{b^2}\sin\left(a+\frac{b}{\sqrt{dx+c}}\right) - 1/2\,\frac{\sqrt{dx+c}}{b}\cos\left(a+\frac{b}{\sqrt{dx+c}}\right) - 1/2\,\text{Si}\left(\frac{b}{\sqrt{dx+c}}\right)\cos(a) - 1/2\,\text{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\text{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/2)),x)
```

```
[Out] -2/d*b^2*(-1/2*sin(a+b/(d*x+c)^(1/2))*(d*x+c)/b^2-1/2*cos(a+b/(d*x+c)^(1/2)
)*(d*x+c)^(1/2)/b-1/2*Si(b/(d*x+c)^(1/2))*cos(a)-1/2*Ci(b/(d*x+c)^(1/2))*si
n(a))
```

Maxima [C] time = 1.20623, size = 167, normalized size = 1.78

$$\frac{\left(\left(-i\,\text{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right) + i\,\text{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)\right)\cos(a) + \left(\text{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right) + \text{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)\right)\sin(a)\right)b^2 + 2\sqrt{dx+c}\,b\cos\left(\frac{\sqrt{dx+c}a+b}{\sqrt{dx+c}}\right) + 2\,(dx+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((-I*Ei(I*b/sqrt(d*x + c)) + I*Ei(-I*b/sqrt(d*x + c)))*cos(a) + (Ei(I*
b/sqrt(d*x + c)) + Ei(-I*b/sqrt(d*x + c)))*sin(a))*b^2 + 2*sqrt(d*x + c)*b*
cos((sqrt(d*x + c)*a + b)/sqrt(d*x + c)) + 2*(d*x + c)*sin((sqrt(d*x + c)*a
 + b)/sqrt(d*x + c)))/d
```

Fricas [A] time = 2.16723, size = 360, normalized size = 3.83

$$\frac{b^2 \operatorname{Ci}\left(\frac{b}{\sqrt{dx+c}}\right)\sin(a) + b^2 \operatorname{Ci}\left(-\frac{b}{\sqrt{dx+c}}\right)\sin(a) + 2\,b^2\cos(a)\operatorname{Si}\left(\frac{b}{\sqrt{dx+c}}\right) + 2\,\sqrt{dx+c}b\cos\left(\frac{adx+ac+\sqrt{dx+c}b}{dx+c}\right) + 2\,(dx+c)\,\mathrm{si}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos_integral(b/sqrt(d*x + c))*sin(a) + b^2*cos_integral(-b/sqrt(d*
x + c))*sin(a) + 2*b^2*cos(a)*sin_integral(b/sqrt(d*x + c)) + 2*sqrt(d*x +
c)*b*cos((a*d*x + a*c + sqrt(d*x + c)*b)/(d*x + c)) + 2*(d*x + c)*sin((a*d*
x + a*c + sqrt(d*x + c)*b)/(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{b}{\sqrt{c + dx}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/2)),x)
```

```
[Out] Integral(sin(a + b/sqrt(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{b}{\sqrt{dx + c}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c)), x)
```

3.200 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$

Optimal. Leaf size=276

$$\frac{\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{f}+\frac{\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{2\sin(a)\text{CosIntegra}}{f}$$

```
[Out] (-2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/f + (CosIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] + b/Sqrt[c + d*x]]*Sin[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f
 + (CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c + d*x]]*Sin[a + (
b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f - (2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])
/f - (Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] - b/Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*
f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]])/f
```

Rubi [A] time = 1.20285, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3431, 3303, 3299, 3302, 3345}

$$\frac{\sin\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{f}+\frac{\sin\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{2\sin(a)\text{CosIntegra}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] (-2*CosIntegral[b/Sqrt[c + d*x]]*Sin[a])/f + (CosIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] + b/Sqrt[c + d*x]]*Sin[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f
 + (CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c + d*x]]*Sin[a + (
b*Sqrt[f])/Sqrt[-(d*e) + c*f]])/f - (2*Cos[a]*SinIntegral[b/Sqrt[c + d*x]])
/f - (Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] - b/Sqrt[c + d*x]])/f + (Cos[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*
f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]])/f
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx = -\frac{2\,\mathrm{Subst}\left(\int\left(\frac{d\sin(a+bx)}{fx}+\frac{d(-de+cf)x\sin(a+bx)}{f\left(f+(de-cf)x^2\right)}\right)dx,x,\frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int\frac{\sin(a+bx)}{x}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}+\frac{(2(de-cf))\,\mathrm{Subst}\left(\int\frac{x\sin(a+bx)}{f+(de-cf)x^2}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}$$

$$= \frac{(2(de-cf))\,\mathrm{Subst}\left(\int\left(-\frac{\sqrt{-de+cf}\sin(a+bx)}{2(de-cf)\left(\sqrt{f}-\sqrt{-de+cf}x\right)}+\frac{\sqrt{-de+cf}\sin(a+bx)}{2(de-cf)\left(\sqrt{f}+\sqrt{-de+cf}x\right)}\right)dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f} - \frac{(2\cos(}{}$$

$$= -\frac{2\mathrm{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f}-\frac{2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}-\frac{\sqrt{-de+cf}\,\mathrm{Subst}\left(\int\frac{\sin(a+bx)}{\sqrt{f}-\sqrt{-de+cf}x}dx,x,\frac{1}{\sqrt{c+dx}}\right)}{f}$$

$$= -\frac{2\mathrm{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f}-\frac{2\cos(a)\mathrm{Si}\left(\frac{b}{\sqrt{c+dx}}\right)}{f}+\frac{\left(\sqrt{-de+cf}\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\mathrm{Subst}\left(\int\frac{\sin\left(\frac{b}{\sqrt{-}}\right.}{\sqrt{f}+\sqrt{}}\right.}{f}$$

$$= -\frac{2\mathrm{Ci}\left(\frac{b}{\sqrt{c+dx}}\right)\sin(a)}{f}+\frac{\mathrm{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)\sin\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)}{f}+\frac{\mathrm{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)\sin\left(\right.}{f}$$

Mathematica [F] time = 15.3157, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x), x]
```

Maple [A] time = 0.029, size = 438, normalized size = 1.6

$$-2\,b^2\left(-1/2\,\frac{1}{b^2f}\left(\mathrm{Si}\left(\frac{b}{\sqrt{dx+c}}+a-\frac{acf-ade+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)\cos\left(\frac{acf-ade+\sqrt{b^2cf^2-b^2def}}{cf-de}\right)+\mathrm{Ci}\left(\frac{b}{\sqrt{dx+c}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x)
```

```
[Out] -2*b^2*(-1/2/b^2/f*(Si(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)
^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))
+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*
sin((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-1/2/b^2/f*(Si(b/(
d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-
a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-
a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*
c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/b^2/f*(Si(b/(d*x+c)^(1/2))*cos(a)+Ci(
b/(d*x+c)^(1/2))*sin(a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{\sqrt{dx+c}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

Fricas [C] time = 2.35873, size = 741, normalized size = 2.68

$$2i\,\mathrm{Ei}\left(\frac{ib}{\sqrt{dx+c}}\right)e^{(ia)} - 2i\,\mathrm{Ei}\left(-\frac{ib}{\sqrt{dx+c}}\right)e^{(-ia)} - i\,\mathrm{Ei}\left(-\frac{2\sqrt{\frac{b^2f}{de-cf}}(dx+c)-2i\sqrt{dx+c}\,b}{2(dx+c)}\right)e^{\left(ia+\sqrt{\frac{b^2f}{de-cf}}\right)} - i\,\mathrm{Ei}\left(\frac{2\sqrt{\frac{b^2f}{de-cf}}(dx+c)+2i\sqrt{dx+c}\,b}{2(dx+c)}\right)e^{\left(ia-\right.}$$

$$2f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*I*Ei(I*b/sqrt(d*x + c))*e^(I*a) - 2*I*Ei(-I*b/sqrt(d*x + c))*e^(-I*a
) - I*Ei(-1/2*(2*sqrt(b^2*f/(d*e - c*f))*(d*x + c) - 2*I*sqrt(d*x + c)*b)/(
d*x + c))*e^(I*a + sqrt(b^2*f/(d*e - c*f))) - I*Ei(1/2*(2*sqrt(b^2*f/(d*e -
 c*f))*(d*x + c) + 2*I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(b^2*f/(d*e
 - c*f))) + I*Ei(-1/2*(2*sqrt(b^2*f/(d*e - c*f))*(d*x + c) + 2*I*sqrt(d*x +
 c)*b)/(d*x + c))*e^(-I*a + sqrt(b^2*f/(d*e - c*f))) + I*Ei(1/2*(2*sqrt(b^2
*f/(d*e - c*f))*(d*x + c) - 2*I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a - sqrt(
b^2*f/(d*e - c*f))))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e), x)
```

3.201 $\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2} dx$

Optimal. Leaf size=350

$$-\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}}+\frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}}-\frac{bd\sin\left(a+\right.}{2}$$

```
[Out] -(b*d*Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] - b/Sqrt[c + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) + (b*d*C
os[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e)
+ c*f] + b/Sqrt[c + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) + ((c + d*x)*Si
n[a + b/Sqrt[c + d*x]])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*Sqrt[f])/
Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c +
 d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) - (b*d*Sin[a - (b*Sqrt[f])/Sqrt[-(
d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]])
/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2))
```

Rubi [A] time = 0.929078, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3431, 3341, 3334, 3303, 3299, 3302}

$$-\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}}+\frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt{f}}{\sqrt{cf-de}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(cf-de)^{3/2}}-\frac{bd\sin\left(a+\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] -(b*d*Cos[a + (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[
-(d*e) + c*f] - b/Sqrt[c + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) + (b*d*C
os[a - (b*Sqrt[f])/Sqrt[-(d*e) + c*f]]*CosIntegral[(b*Sqrt[f])/Sqrt[-(d*e)
+ c*f] + b/Sqrt[c + d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) + ((c + d*x)*Si
n[a + b/Sqrt[c + d*x]])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*Sqrt[f])/
Sqrt[-(d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] - b/Sqrt[c +
 d*x]])/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2)) - (b*d*Sin[a - (b*Sqrt[f])/Sqrt[-(
d*e) + c*f]]*SinIntegral[(b*Sqrt[f])/Sqrt[-(d*e) + c*f] + b/Sqrt[c + d*x]])
/(2*Sqrt[f]*(-(d*e) + c*f)^(3/2))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
```

```
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{x\sin(a+bx)}{\left(\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^2\right)^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^2}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\,\text{Subst}\left(\int \left(\frac{d\cos(a+bx)}{2\sqrt{f}\left(\sqrt{f}-\sqrt{-de+cf}x\right)}+\frac{d\cos(a+bx)}{2\sqrt{f}\left(\sqrt{f}+\sqrt{-de+cf}x\right)}\right)dx, x, \frac{1}{\sqrt{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{f}-\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+b}{\sqrt{f}+\sqrt{-de}}\right.}{2\sqrt{f}(de-}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+bx\right)}{\sqrt{f}+\sqrt{-de+cf}x}\,dx, x, \frac{1}{\sqrt{c+dx}}\right)}{2\sqrt{f}(de-cf)} - \frac{(b}{}$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}-\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} + \frac{bd\cos\left(a-\frac{b\sqrt{f}}{\sqrt{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt{f}}{\sqrt{-de+cf}}+\frac{b}{\sqrt{c+dx}}\right)}{2\sqrt{f}(-de+cf)^{3/2}} + \cdots$$

Mathematica [F] time = 180.033, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/Sqrt[c + d*x]]/(e + f*x)^2,x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.052, size = 2724, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x)
```

```
[Out] -2*d*b^2*(sin(a+b/(d*x+c)^(1/2))*(-1/2*a/b^2/f*(a+b/(d*x+c)^(1/2))+1/2*(a^2
*c*f-a^2*d*e-b^2*f)/b^2/f/(c*f-d*e))/(c*f*(a+b/(d*x+c)^(1/2))^2-d*e*(a+b/(d
*x+c)^(1/2))^2-2*(a+b/(d*x+c)^(1/2))*a*c*f+2*(a+b/(d*x+c)^(1/2))*a*d*e+a^2*
c*f-a^2*d*e-b^2*f)-1/4*a/b^2/f/((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(
c*f-d*e)*c*f-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*d*e-a*c*f+
a*d*e)*(Si(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f
-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c
)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f-a
*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-1/4*a/b^2/f/(-(-a*c*f+a*d*e+(
b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*c*f+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/(c*f-d*e)*d*e-a*c*f+a*d*e)*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b
^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*
f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*
e)))+1/4*((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*a*c*f-(a*c*f-
a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*a*d*e-a^2*c*f+a^2*d*e+b^2*f)/b
^2/f/(c*f-d*e)/((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*c*f-(a*
c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*d*e-a*c*f+a*d*e)*(-Si(b/(d
*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c
*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*
f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2
-b^2*d*e*f)^(1/2))/(c*f-d*e)))+1/4*(-(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1
/2))/(c*f-d*e)*a*c*f+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*a
*d*e-a^2*c*f+a^2*d*e+b^2*f)/b^2/f/(c*f-d*e)/(-(-a*c*f+a*d*e+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/(c*f-d*e)*c*f+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f
-d*e)*d*e-a*c*f+a*d*e)*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d
*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f
-d*e))+Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f
-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-a*(sin(a+
b/(d*x+c)^(1/2))*(-1/2/b^2/f*(a+b/(d*x+c)^(1/2))+1/2*a/b^2/f)/(c*f*(a+b/(d*
x+c)^(1/2))^2-d*e*(a+b/(d*x+c)^(1/2))^2-2*(a+b/(d*x+c)^(1/2))*a*c*f+2*(a+b/
(d*x+c)^(1/2))*a*d*e+a^2*c*f-a^2*d*e-b^2*f)-1/4/b^2/f/((a*c*f-a*d*e+(b^2*c*
f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*c*f-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2
))/(c*f-d*e)*d*e-a*c*f+a*d*e)*(Si(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2
-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))
/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/
(c*f-d*e))*sin((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)))-1/4/b^
2/f/(-(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*c*f+(-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e)*d*e-a*c*f+a*d*e)*(Si(b/(d*x+c)^(1/
2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*cos((-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))-Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*
e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*
d*e*f)^(1/2))/(c*f-d*e)))+1/4/b^2/f/(c*f-d*e)*(-Si(b/(d*x+c)^(1/2)+a-(a*c*f
-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((a*c*f-a*d*e+(b^2*c*f^2-
b^2*d*e*f)^(1/2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a-(a*c*f-a*d*e+(b^2*c*f^2-b
```

```
^2*d*e*f)^(1/2))/(c*f-d*e))*cos((a*c*f-a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(
c*f-d*e)))+1/4/b^2/f/(c*f-d*e)*(Si(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f
^2-b^2*d*e*f)^(1/2))/(c*f-d*e))*sin((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))+Ci(b/(d*x+c)^(1/2)+a+(-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/
2))/(c*f-d*e))*cos((-a*c*f+a*d*e+(b^2*c*f^2-b^2*d*e*f)^(1/2))/(c*f-d*e))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt{dx+c}}\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

Fricas [C] time = 2.43122, size = 1003, normalized size = 2.87

$$-\frac{\left(idfx+ide\right)\sqrt{\frac{b^2f}{de-cf}}\operatorname{Ei}\left(-\frac{2\sqrt{\frac{b^2f}{de-cf}}(dx+c)-2i\sqrt{dx+c}\,b}{2\,(dx+c)}\right)e^{\left(ia+\sqrt{\frac{b^2f}{de-cf}}\right)}+\left(-idfx-ide\right)\sqrt{\frac{b^2f}{de-cf}}\operatorname{Ei}\left(\frac{2\sqrt{\frac{b^2f}{de-cf}}(dx+c)+2i\sqrt{dx+c}\,b}{2\,(dx+c)}\right)e^{\left(ia\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((I*d*f*x + I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(-1/2*(2*sqrt(b^2*f/(d*e
- c*f))*(d*x + c) - 2*I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a + sqrt(b^2*f/(d*
e - c*f))) + (-I*d*f*x - I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(1/2*(2*sqrt(b^2*
f/(d*e - c*f))*(d*x + c) + 2*I*sqrt(d*x + c)*b)/(d*x + c))*e^(I*a - sqrt(b^
2*f/(d*e - c*f))) + (-I*d*f*x - I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(-1/2*(2*s
qrt(b^2*f/(d*e - c*f))*(d*x + c) + 2*I*sqrt(d*x + c)*b)/(d*x + c))*e^(-I*a
+ sqrt(b^2*f/(d*e - c*f))) + (I*d*f*x + I*d*e)*sqrt(b^2*f/(d*e - c*f))*Ei(1
/2*(2*sqrt(b^2*f/(d*e - c*f))*(d*x + c) - 2*I*sqrt(d*x + c)*b)/(d*x + c))*e
^(-I*a - sqrt(b^2*f/(d*e - c*f))) - 4*(d*f*x + c*f)*sin((a*d*x + a*c + sqrt
(d*x + c)*b)/(d*x + c)))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/2))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{\sqrt{dx+c}}\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/sqrt(d*x + c))/(f*x + e)^2, x)
```

3.202 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx$

Optimal. Leaf size=390

$$-\frac{2ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{2ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\right.}{3d^3}$$

```
[Out] (b*f^2*(c + d*x)^(3/2)*Cos[a + b/(c + d*x)^(3/2)])/(3*d^3) - (((2*I)/3)*E^(
I*a)*f*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
 ((-I)*b)/(c + d*x)^(3/2)])/d^3 + (((2*I)/3)*f*(d*e - c*f)*((I*b)/(c + d*x)
^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(I*a))
 - ((I/3)*E^(I*a)*(d*e - c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*
Gamma[-2/3, ((-I)*b)/(c + d*x)^(3/2)])/d^3 + ((I/3)*(d*e - c*f)^2*((I*b)/(c
 + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(
I*a)) + (b^2*f^2*CosIntegral[b/(c + d*x)^(3/2)]*Sin[a])/(3*d^3) + (f^2*(c +
 d*x)^3*Sin[a + b/(c + d*x)^(3/2)])/(3*d^3) + (b^2*f^2*Cos[a]*SinIntegral[b
/(c + d*x)^(3/2)])/(3*d^3)
```

Rubi [A] time = 0.428222, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3433, 3423, 2218, 3379, 3297, 3303, 3299, 3302}

$$-\frac{2ie^{ia}f(c+dx)^2\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3}+\frac{2ie^{-ia}f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\right.}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```


```
[Out] (b*f^2*(c + d*x)^(3/2)*Cos[a + b/(c + d*x)^(3/2)])/(3*d^3) - (((2*I)/3)*E^(
I*a)*f*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
 ((-I)*b)/(c + d*x)^(3/2)])/d^3 + (((2*I)/3)*f*(d*e - c*f)*((I*b)/(c + d*x)
^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(I*a))
 - ((I/3)*E^(I*a)*(d*e - c*f)^2*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*
Gamma[-2/3, ((-I)*b)/(c + d*x)^(3/2)])/d^3 + ((I/3)*(d*e - c*f)^2*((I*b)/(c
 + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^3*E^(
I*a)) + (b^2*f^2*CosIntegral[b/(c + d*x)^(3/2)]*Sin[a])/(3*d^3) + (f^2*(c +
 d*x)^3*Sin[a + b/(c + d*x)^(3/2)])/(3*d^3) + (b^2*f^2*Cos[a]*SinIntegral[b
/(c + d*x)^(3/2)])/(3*d^3)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right) dx = \frac{2\,\text{Subst}\left(\int \left((de-cf)^2 x \sin\left(a+\frac{b}{x^3}\right) - 2f(-de+cf)x^3 \sin\left(a+\frac{b}{x^3}\right) + f^2x^5 \sin\right.\right.}{d^3}$$

$$= \frac{\left(2f^2\right)\text{Subst}\left(\int x^5 \sin\left(a+\frac{b}{x^3}\right) dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(4f(de-cf))\,\text{Subst}\left(\int x^3 \text{si}\right.}{d^3}$$

$$= -\frac{\left(2f^2\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3} dx, x, \frac{1}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{(2if(de-cf))\,\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}} x^3 \right.}{d^3}$$

$$= -\frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3}, -\frac{ib}{(c+dx)^{3/2}}\right)}{3d^3} + \frac{2ie^{-ia}f(de-cf)\left(\right.}{}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\right.}{3d^3}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\right.}{3d^3}$$

$$= \frac{bf^2(c+dx)^{3/2}\cos\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{3d^3} - \frac{2ie^{ia}f(de-cf)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\right.}{3d^3}$$

Mathematica [A] time = 2.25228, size = 463, normalized size = 1.19

$$\frac{i\left((\cos(a)-i\sin(a))\left(4f(c+dx)^2\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(de-cf)\text{Gamma}\left(-\frac{4}{3}, \frac{ib}{(c+dx)^{3/2}}\right) + 2(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)^2\text{Gam}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((I/6)*((Cos[a] - I*Sin[a])*(4*f*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(4/3)*
(c + d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)] + 2*(d*e - c*f)^2*((I*b)/(c
+ d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)] - I*b*f^2*
(I*b*ExpIntegralEi[((-I)*b)/(c + d*x)^(3/2)] + (c + d*x)^(3/2)*(Cos[b/(c +
d*x)^(3/2)] - I*Sin[b/(c + d*x)^(3/2)])) + f^2*(c + d*x)^3*(Cos[b/(c + d*x)
^(3/2)] - I*Sin[b/(c + d*x)^(3/2)])) - (Cos[a] + I*Sin[a])*(b^2*f^2*ExpInte
gralEi[(I*b)/(c + d*x)^(3/2)] + 4*f*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^
(4/3)*(c + d*x)^2*Gamma[-4/3, ((-I)*b)/(c + d*x)^(3/2)] + 2*(d*e - c*f)^2*(
((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*b)/(c + d*x)^(3
/2)] + I*b*f^2*(c + d*x)^(3/2)*(Cos[b/(c + d*x)^(3/2)] + I*Sin[b/(c + d*x)^
(3/2)]) + f^2*(c + d*x)^3*(Cos[b/(c + d*x)^(3/2)] + I*Sin[b/(c + d*x)^(3/2)
])))))/d^3
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (fx+e)^2 \sin\left(a+b(dx+c)^{-\frac{3}{2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
```

```
[Out] int((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x)
```

Maxima [B] time = 2.92936, size = 2916, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(4*(d*x + c)^(3/2)*(abs(b)/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3
/2)*a + b)/(d*x + c)^(3/2)) + (((gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/
3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b
/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arc
tan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x
+ c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (I*gamma(1/3, I*b/(d*x + c)^
(3/2)) - I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b
)))*cos(a) + ((-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x
+ c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)
^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0,
b)) - (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*
sin(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(
1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*sin(a))*b)*e^
2/(sqrt(d*x + c)*(abs(b)/(d*x + c)^(3/2))^(1/3)) - 6*(4*(d*x + c)^(3/2)*(ab
s(b)/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) +
(((gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(
1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3,
 -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I
*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*
arctan2(0, b)) + (I*gamma(1/3, I*b/(d*x + c)^(3/2)) - I*gamma(1/3, -I*b/(d*
x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3, I
*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*
arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d
*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b)) - (gamma(1/3, I*b/(d*x + c
)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)
) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*si
n(-1/6*pi + 1/3*arctan2(0, b)))*sin(a))*b)*c*e*f/(sqrt(d*x + c)*d*(abs(b)/(
d*x + c)^(3/2))^(1/3)) + 3*(4*(d*x + c)^(3/2)*(abs(b)/(d*x + c)^(3/2))^(1/3
)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + (((gamma(1/3, I*b/(d*x + c
)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)
) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*co
s(-1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*ga
mma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (I*gamma(
1/3, I*b/(d*x + c)^(3/2)) - I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi
 + 1/3*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*ga
mma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma
(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*p
i + 1/3*arctan2(0, b)) - (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b
/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x +
 c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0,
 b)))*sin(a))*b)*c^2*f^2/(sqrt(d*x + c)*d^2*(abs(b)/(d*x + c)^(3/2))^(1/3))
 + 2*(2*(d*x + c)^3*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 2*(d*x +
 c)^(3/2)*b*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + ((-I*Ei(I*b/(d*x
 + c)^(3/2)) + I*Ei(-I*b/(d*x + c)^(3/2)))*cos(a) + (Ei(I*b/(d*x + c)^(3/2)
) + Ei(-I*b/(d*x + c)^(3/2)))*sin(a))*b^2)*f^2/d^2 + 3*(4*(d*x + c)^3*(abs(
b)/(d*x + c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12
*(d*x + c)^(3/2)*b*(abs(b)/(d*x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a +
```

```
b)/(d*x + c)^(3/2)) + (((-3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) + 3*I*gamma(2
/3, -I*b/(d*x + c)^(3/2)))*cos(1/3*pi + 2/3*arctan2(0, b)) + (-3*I*gamma(2/
3, I*b/(d*x + c)^(3/2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(-1/3*pi
 + 2/3*arctan2(0, b)) - 3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*
b/(d*x + c)^(3/2)))*sin(1/3*pi + 2/3*arctan2(0, b)) + 3*(gamma(2/3, I*b/(d*
x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(-1/3*pi + 2/3*arctan2
(0, b)))*cos(a) - (3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*
x + c)^(3/2)))*cos(1/3*pi + 2/3*arctan2(0, b)) + 3*(gamma(2/3, I*b/(d*x + c
)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(-1/3*pi + 2/3*arctan2(0, b
)) - (3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) - 3*I*gamma(2/3, -I*b/(d*x + c)^(
3/2)))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-3*I*gamma(2/3, I*b/(d*x + c)^(3/
2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(-1/3*pi + 2/3*arctan2(0, b)
))*sin(a))*b^2)*e*f/((d*x + c)*d*(abs(b)/(d*x + c)^(3/2))^(2/3)) - 3*(4*(d*
x + c)^3*(abs(b)/(d*x + c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x +
c)^(3/2)) + 12*(d*x + c)^(3/2)*b*(abs(b)/(d*x + c)^(3/2))^(2/3)*cos(((d*x +
 c)^(3/2)*a + b)/(d*x + c)^(3/2)) + (((-3*I*gamma(2/3, I*b/(d*x + c)^(3/2))
 + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(1/3*pi + 2/3*arctan2(0, b)) +
(-3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)
))*cos(-1/3*pi + 2/3*arctan2(0, b)) - 3*(gamma(2/3, I*b/(d*x + c)^(3/2)) +
gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(1/3*pi + 2/3*arctan2(0, b)) + 3*(gamm
a(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(-1/3*pi
 + 2/3*arctan2(0, b)))*cos(a) - (3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma
(2/3, -I*b/(d*x + c)^(3/2)))*cos(1/3*pi + 2/3*arctan2(0, b)) + 3*(gamma(2/3
, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(-1/3*pi + 2/
3*arctan2(0, b)) - (3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) - 3*I*gamma(2/3, -I
*b/(d*x + c)^(3/2)))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-3*I*gamma(2/3, I*b
/(d*x + c)^(3/2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(-1/3*pi + 2/3
*arctan2(0, b)))*sin(a))*b^2)*c*f^2/((d*x + c)*d^2*(abs(b)/(d*x + c)^(3/2))
^(2/3)))/d
```

Fricas [A] time = 2.79872, size = 1211, normalized size = 3.11

$$-ib^2f^2\mathrm{Ei}\left(\frac{i\sqrt{dx+c}b}{d^2x^2+2cdx+c^2}\right)e^{(ia)}+ib^2f^2\mathrm{Ei}\left(-\frac{i\sqrt{dx+c}b}{d^2x^2+2cdx+c^2}\right)e^{(-ia)}+\left(-3id^2e^2+6icdef-3ic^2f^2\right)(ib)^{\frac{2}{3}}e^{(-ia)}\Gamma\left(\frac{1}{3},\frac{i\sqrt{dx+c}b}{d^2x^2+2cd\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*b^2*f^2*Ei(I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))*e^(I*a) + I
*b^2*f^2*Ei(-I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))*e^(-I*a) + (-3*I*
d^2*e^2 + 6*I*c*d*e*f - 3*I*c^2*f^2)*(I*b)^(2/3)*e^(-I*a)*gamma(1/3, I*sqrt
(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + (3*I*d^2*e^2 - 6*I*c*d*e*f + 3*I*c
^2*f^2)*(-I*b)^(2/3)*e^(I*a)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d
*x + c^2)) - 9*(b*d*e*f - b*c*f^2)*(I*b)^(1/3)*e^(-I*a)*gamma(2/3, I*sqrt(d
*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - 9*(b*d*e*f - b*c*f^2)*(-I*b)^(1/3)*e
^(I*a)*gamma(2/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(b*d*f^
2*x + 9*b*d*e*f - 8*b*c*f^2)*sqrt(d*x + c)*cos((a*d^2*x^2 + 2*a*c*d*x + a*c
^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(d^3*f^2*x^3 + 3*d^3*e
*f*x^2 + 3*d^3*e^2*x + 3*c*d^2*e^2 - 3*c^2*d*e*f + c^3*f^2)*sin((a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^{2}\sin\left(a+\frac{b}{\left(dx+c\right)^{\frac{3}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^(3/2)), x)
```

3.203 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)\,dx$

Optimal. Leaf size=251

$$-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}+\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)}\right.}{3d^2}$$

```
[Out] ((-I/3)*E^(I*a)*f*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
((-I)*b)/(c + d*x)^(3/2)])/d^2 + ((I/3)*f*((I*b)/(c + d*x)^(3/2))^(4/3)*(c
+ d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a)) - ((I/3)*E^(I*a)
*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*b
)/(c + d*x)^(3/2)])/d^2 + ((I/3)*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(2/3)*
(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a))
```

Rubi [A] time = 0.225731, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {3433, 3423, 2218}

$$-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2}+\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(de-cf)\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)}\right.}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```


```
[Out] ((-I/3)*E^(I*a)*f*(((-I)*b)/(c + d*x)^(3/2))^(4/3)*(c + d*x)^2*Gamma[-4/3,
((-I)*b)/(c + d*x)^(3/2)])/d^2 + ((I/3)*f*((I*b)/(c + d*x)^(3/2))^(4/3)*(c
+ d*x)^2*Gamma[-4/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a)) - ((I/3)*E^(I*a)
*(d*e - c*f)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I)*b
)/(c + d*x)^(3/2)])/d^2 + ((I/3)*(d*e - c*f)*((I*b)/(c + d*x)^(3/2))^(2/3)*
(c + d*x)*Gamma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d^2*E^(I*a))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{2\,\text{Subst}\left(\int \left((de-cf)x\sin\left(a+\frac{b}{x^3}\right)+fx^3\sin\left(a+\frac{b}{x^3}\right)\right)dx,x,\sqrt{c+dx}\right)}{d^2}$$

$$= \frac{(2f)\,\text{Subst}\left(\int x^3\sin\left(a+\frac{b}{x^3}\right)dx,x,\sqrt{c+dx}\right)}{d^2} + \frac{(2(de-cf))\,\text{Subst}\left(\int x\sin\left(a+\frac{b}{x}\right.\right.}{d^2}$$

$$= \frac{(if)\,\text{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x^3\,dx,x,\sqrt{c+dx}\right)}{d^2} - \frac{(if)\,\text{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x^3\,dx,x,\sqrt{c+dx}\right)}{d^2} +$$

$$= -\frac{ie^{ia}f\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d^2} + \frac{ie^{-ia}f\left(\frac{ib}{(c+dx)^{3/2}}\right)^{4/3}(c+dx)^2\Gamma\left(-\frac{4}{3}\right.}{3d^2}$$

Mathematica [B] time = 2.65297, size = 835, normalized size = 3.33

$$\frac{9if\cos(a)\left(\frac{2\text{Gamma}\left(\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}-\frac{2\text{Gamma}\left(\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}\right)b^2}{8d^2} - \frac{9f\left(\frac{2\text{Gamma}\left(\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}+\frac{2\text{Gamma}\left(\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)}\right)\sin(a)b^2}{8d^2} + \frac{3e\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (3*b*e*Cos[a]*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d
*x)^(3/2))^(1/3)*Sqrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*
((I*b)/(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x])))/(4*d) - (3*b*c*f*Cos[a]*((2*
Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*S
qrt[c + d*x]) + (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(
3/2))^(1/3)*Sqrt[c + d*x])))/(4*d^2) + (((9*I)/8)*b^2*f*Cos[a]*((2*Gamma[2/
3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)
) - (2*Gamma[2/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3)*
(c + d*x))))/d^2 + (e*(c + d*x)*Cos[b/(c + d*x)^(3/2)]*Sin[a])/d + (((3*I)/
4)*b*e*((2*Gamma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/
2))^(1/3)*Sqrt[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/
(c + d*x)^(3/2))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d - (((3*I)/4)*b*c*f*((2*Gam
ma[1/3, ((-I)*b)/(c + d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Sqrt
[c + d*x]) - (2*Gamma[1/3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2
))^(1/3)*Sqrt[c + d*x]))*Sin[a])/d^2 - (9*b^2*f*((2*Gamma[2/3, ((-I)*b)/(c
+ d*x)^(3/2)])/(3*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)) + (2*Gamma[2/
3, (I*b)/(c + d*x)^(3/2)])/(3*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)))*Sin
[a])/(8*d^2) + (f*Sqrt[c + d*x]*Cos[b/(c + d*x)^(3/2)]*(3*b*Cos[a] - 2*c*Sq
rt[c + d*x]*Sin[a] + (c + d*x)^(3/2)*Sin[a]))/(2*d^2) + (e*(c + d*x)*Cos[a]
*Sin[b/(c + d*x)^(3/2)])/d + (f*Sqrt[c + d*x]*(-2*c*Sqrt[c + d*x]*Cos[a] +
(c + d*x)^(3/2)*Cos[a] - 3*b*Sin[a])*Sin[b/(c + d*x)^(3/2)])/(2*d^2)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (fx+e)\sin\left(a+b(dx+c)^{-\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)
```

[Out] int((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x)

Maxima [B] time = 2.07836, size = 1623, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")

[Out] 1/8*(2*(4*(d*x + c)^(3/2)*(abs(b)/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + (((gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (I*gamma(1/3, I*b/(d*x + c)^(3/2)) - I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b)) - (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*sin(a))*b)*e/(sqrt(d*x + c)*(abs(b)/(d*x + c)^(3/2))^(1/3)) - 2*(4*(d*x + c)^(3/2)*(abs(b)/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + (((gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (I*gamma(1/3, I*b/(d*x + c)^(3/2)) - I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*cos(a) + ((-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b)) - (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*sin(a))*b)*c*f/(sqrt(d*x + c)*d*(abs(b)/(d*x + c)^(3/2))^(1/3)) + (4*(d*x + c)^3*(abs(b)/(d*x + c)^(3/2))^(2/3)*sin(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + 12*(d*x + c)^(3/2)*b*(abs(b)/(d*x + c)^(3/2))^(2/3)*cos(((d*x + c)^(3/2)*a + b)/(d*x + c)^(3/2)) + (((-3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(1/3*pi + 2/3*arctan2(0, b)) + (-3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(-1/3*pi + 2/3*arctan2(0, b)) - 3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(1/3*pi + 2/3*arctan2(0, b)) + 3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(-1/3*pi + 2/3*arctan2(0, b)))*cos(a) - (3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(1/3*pi + 2/3*arctan2(0, b)) + 3*(gamma(2/3, I*b/(d*x + c)^(3/2)) + gamma(2/3, -I*b/(d*x + c)^(3/2)))*cos(-1/3*pi + 2/3*arctan2(0, b)) - (3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) - 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(1/3*pi + 2/3*arctan2(0, b)) - (-3*I*gamma(2/3, I*b/(d*x + c)^(3/2)) + 3*I*gamma(2/3, -I*b/(d*x + c)^(3/2)))*sin(-1/3*pi + 2/3*arctan2(0, b)))*sin(a))*b^2)*f/((d*x + c)*d*(abs(b)/(d*x + c)^(3/2))^(2/3)))/d

---

Fricas [A] time = 2.04125, size = 825, normalized size = 3.29

$$-\frac{3\,(i\,b)^{\frac{1}{3}}\,bfe^{(-i\,a)}\Gamma\left(\frac{2}{3},\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)+3\,(-i\,b)^{\frac{1}{3}}\,bfe^{(i\,a)}\Gamma\left(\frac{2}{3},-\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)-\left(-2i\,de+2i\,cf\right)(i\,b)^{\frac{2}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{3},\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(I*b)^(1/3)*b*f*e^(-I*a)*gamma(2/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*
c*d*x + c^2)) + 3*(-I*b)^(1/3)*b*f*e^(I*a)*gamma(2/3, -I*sqrt(d*x + c)*b/(d
^2*x^2 + 2*c*d*x + c^2)) - (-2*I*d*e + 2*I*c*f)*(I*b)^(2/3)*e^(-I*a)*gamma(
1/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2)) - (2*I*d*e - 2*I*c*f)*(-I
*b)^(2/3)*e^(I*a)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*x + c^2))
- 6*sqrt(d*x + c)*b*f*cos((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)
/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(d^2*f*x^2 + 2*d^2*e*x + 2*c*d*e - c^2*f)*s
in((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2*c*d*x + c
^2)))/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(3/2)), x)
```

3.204 $\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx$

Optimal. Leaf size=115

$$\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

```
[Out] ((-I/3)*E^(I*a)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I
)*b)/(c + d*x)^(3/2)])/d + ((I/3)*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*G
amma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d*E^(I*a))
```

Rubi [A] time = 0.0822319, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3363, 3423, 2218}

$$\frac{ie^{-ia}(c+dx)\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}-\frac{ie^{ia}(c+dx)\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}\text{Gamma}\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] ((-I/3)*E^(I*a)*(((-I)*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*Gamma[-2/3, ((-I
)*b)/(c + d*x)^(3/2)])/d + ((I/3)*((I*b)/(c + d*x)^(3/2))^(2/3)*(c + d*x)*G
amma[-2/3, (I*b)/(c + d*x)^(3/2)])/(d*E^(I*a))
```

Rule 3363

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)dx = \frac{2\,\mathrm{Subst}\left(\int x\sin\left(a+\frac{b}{x^3}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{i\,\mathrm{Subst}\left(\int e^{-ia-\frac{ib}{x^3}}x\,dx, x, \sqrt{c+dx}\right)}{d} - \frac{i\,\mathrm{Subst}\left(\int e^{ia+\frac{ib}{x^3}}x\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{ie^{ia}\left(-\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{ib}{(c+dx)^{3/2}}\right)}{3d} + \frac{ie^{-ia}\left(\frac{ib}{(c+dx)^{3/2}}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},\frac{ib}{(c+dx)^{3/2}}\right)}{3d}$$

Mathematica [A] time = 0.439614, size = 166, normalized size = 1.44

$$\frac{b(\cos(a)-i\sin(a))\sqrt[3]{-\frac{ib}{(c+dx)^{3/2}}}\mathrm{Gamma}\left(\frac{1}{3},\frac{ib}{(c+dx)^{3/2}}\right)+b(\cos(a)+i\sin(a))\sqrt[3]{\frac{ib}{(c+dx)^{3/2}}}\mathrm{Gamma}\left(\frac{1}{3},-\frac{ib}{(c+dx)^{3/2}}\right)+2(c+dx)^{3/}}{2d\sqrt{c+dx}\sqrt[3]{\frac{b^2}{(c+dx)^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)],x]
```

```
[Out] (b*(((-I)*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, (I*b)/(c + d*x)^(3/2)]*(Cos[
a] - I*Sin[a]) + b*((I*b)/(c + d*x)^(3/2))^(1/3)*Gamma[1/3, ((-I)*b)/(c + d
*x)^(3/2)]*(Cos[a] + I*Sin[a]) + 2*(b^2/(c + d*x)^3)^(1/3)*(c + d*x)^(3/2)*
Sin[a + b/(c + d*x)^(3/2)])/(2*d*(b^2/(c + d*x)^3)^(1/3)*Sqrt[c + d*x])
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(dx+c)^{-\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(3/2)),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(3/2)),x)
```

Maxima [B] time = 1.38339, size = 516, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)^(3/2)*(abs(b)/(d*x + c)^(3/2))^(1/3)*sin(((d*x + c)^(3/2)*
a + b)/(d*x + c)^(3/2)) + (((gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -
I*b/(d*x + c)^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*
x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2
(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)
^(3/2)))*sin(1/6*pi + 1/3*arctan2(0, b)) + (I*gamma(1/3, I*b/(d*x + c)^(3/2
```

```
)) - I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*
cos(a) + ((-I*gamma(1/3, I*b/(d*x + c)^(3/2)) + I*gamma(1/3, -I*b/(d*x + c)
^(3/2)))*cos(1/6*pi + 1/3*arctan2(0, b)) + (-I*gamma(1/3, I*b/(d*x + c)^(3/
2)) + I*gamma(1/3, -I*b/(d*x + c)^(3/2)))*cos(-1/6*pi + 1/3*arctan2(0, b))
- (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3, -I*b/(d*x + c)^(3/2)))*sin(
1/6*pi + 1/3*arctan2(0, b)) + (gamma(1/3, I*b/(d*x + c)^(3/2)) + gamma(1/3,
 -I*b/(d*x + c)^(3/2)))*sin(-1/6*pi + 1/3*arctan2(0, b)))*sin(a))*b)/(sqrt(
d*x + c)*d*(abs(b)/(d*x + c)^(3/2))^(1/3))
```

Fricas [A] time = 1.94411, size = 359, normalized size = 3.12

$$\frac{-i\,(i\,b)^{\frac{2}{3}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{3},\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)+i\,(-i\,b)^{\frac{2}{3}}\,e^{(i\,a)}\Gamma\left(\frac{1}{3},-\frac{i\,\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)+2\,(dx+c)\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*(I*b)^(2/3)*e^(-I*a)*gamma(1/3, I*sqrt(d*x + c)*b/(d^2*x^2 + 2*c*d*
x + c^2)) + I*(-I*b)^(2/3)*e^(I*a)*gamma(1/3, -I*sqrt(d*x + c)*b/(d^2*x^2 +
 2*c*d*x + c^2)) + 2*(d*x + c)*sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*
x + c)*b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{(c+dx)^{\frac{3}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(3/2)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+\frac{b}{(dx+c)^{\frac{3}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2)), x)
```

3.205 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi [A] time = 0.0135741, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]

[Out] Defer[Int][Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

Mathematica [A] time = 13.1115, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x),x]

[Out] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x), x]
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(a+b\,(dx+c)^{-\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2
*c*d*x + c^2))/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e), x)
```

3.206 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0150614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]

[Out] Defer[Int][Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$$

Mathematica [A] time = 15.474, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{3/2}}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2,x]

[Out] Integrate[Sin[a + b/(c + d*x)^(3/2)]/(e + f*x)^2, x]
```

Maple [A] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^2}\sin\left(a+b\left(dx+c\right)^{-\frac{3}{2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sin\left(\frac{ad^2x^2+2\,acdx+ac^2+\sqrt{dx+c}b}{d^2x^2+2\,cdx+c^2}\right)}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + sqrt(d*x + c)*b)/(d^2*x^2 + 2
*c*d*x + c^2))/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(3/2))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{3}{2}}}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(3/2))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(3/2))/(f*x + e)^2, x)
```

3.207 $\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=633

$$\frac{30f(c+dx)^{4/3}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{360f(c+dx)^{2/3}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}+\frac{6\sqrt[3]{c+dx}(de-cf)^2\sin\left(\right.}{b^2d^3}$$

```
[Out] (-120960*f^2*Cos[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*(d*e - c*f)^2*Cos[a
 + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a
 + b*(c + d*x)^(1/3)])/(b^5*d^3) + (60480*f^2*(c + d*x)^(2/3)*Cos[a + b*(c
+ d*x)^(1/3)])/(b^7*d^3) - (3*(d*e - c*f)^2*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d^3) + (120*f*(d*e - c*f)*(c + d*x)*Cos[a + b*(c + d*x)^(1/
3)])/(b^3*d^3) - (5040*f^2*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5
*d^3) - (6*f*(d*e - c*f)*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3
) + (168*f^2*(c + d*x)^2*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (3*f^2*(c
+ d*x)^(8/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*f*(d*e - c*f)*Sin[a
 + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*f^2*(c + d*x)^(1/3)*Sin[a + b*(c
 + d*x)^(1/3)])/(b^8*d^3) + (6*(d*e - c*f)^2*(c + d*x)^(1/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^2*d^3) - (360*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^4*d^3) + (20160*f^2*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/
(b^6*d^3) + (30*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) - (1008*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d^3)
+ (24*f^2*(c + d*x)^(7/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^3)
```

Rubi [A] time = 0.647044, antiderivative size = 633, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3431, 3296, 2638, 2637}

$$\frac{30f(c+dx)^{4/3}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{360f(c+dx)^{2/3}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}+\frac{6\sqrt[3]{c+dx}(de-cf)^2\sin\left(\right.}{b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```


```
[Out] (-120960*f^2*Cos[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*(d*e - c*f)^2*Cos[a
 + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a
 + b*(c + d*x)^(1/3)])/(b^5*d^3) + (60480*f^2*(c + d*x)^(2/3)*Cos[a + b*(c
+ d*x)^(1/3)])/(b^7*d^3) - (3*(d*e - c*f)^2*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d^3) + (120*f*(d*e - c*f)*(c + d*x)*Cos[a + b*(c + d*x)^(1/
3)])/(b^3*d^3) - (5040*f^2*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5
*d^3) - (6*f*(d*e - c*f)*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3
) + (168*f^2*(c + d*x)^2*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (3*f^2*(c
+ d*x)^(8/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*f*(d*e - c*f)*Sin[a
 + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*f^2*(c + d*x)^(1/3)*Sin[a + b*(c
 + d*x)^(1/3)])/(b^8*d^3) + (6*(d*e - c*f)^2*(c + d*x)^(1/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^2*d^3) - (360*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c +
 d*x)^(1/3)])/(b^4*d^3) + (20160*f^2*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/
(b^6*d^3) + (30*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) - (1008*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d^3)
+ (24*f^2*(c + d*x)^(7/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^3)
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
```

```
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{3\,\text{Subst}\left(\int \left(\frac{(de-cf)^2x^2\sin(a+bx)}{d^2}+\frac{2f(de-cf)x^5\sin(a+bx)}{d^2}+\frac{f^2x^8\sin(a+bx)}{d^2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3f^2\right)\text{Subst}\left(\int x^8\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}+\frac{(6f(de-cf))\,\text{Subst}\left(\int x^5\sin(a+\right.}{d^3}$$

$$= -\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}-\frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+\right.}{bd^3}$$

$$= -\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}-\frac{6f(de-cf)(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+\right.}{bd^3}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}+\frac{120f}{}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{3(de-cf)^2(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}+\frac{120f}{}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}-\frac{3(de}{}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}-\frac{3(de}{}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}+\frac{604}{}$$

$$= \frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720f(de-cf)\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}+\frac{604}{}$$

$$= -\frac{120960f^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}+\frac{6(de-cf)^2\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{720f(de-cf)}{}$$

Mathematica [A] time = 2.50915, size = 256, normalized size = 0.4

$$\frac{6b\sin\left(a+b\sqrt[3]{c+dx}\right)\left(b^6d\sqrt[3]{c+dx}(e+fx)(3cf+d(e+4fx))-12b^4f(c+dx)^{2/3}(9cf+5de+14dfx)+120b^2f(27cf+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(40320*f^2 - 20160*b^2*f^2*(c + d*x)^(2/3) + b^8*d^2*(c + d*x)^(2/3)*(e
 + f*x)^2 + 240*b^4*f*(c + d*x)^(1/3)*(6*c*f + d*(e + 7*f*x)) - 2*b^6*(9*c^
2*f^2 + 18*c*d*f*(e + 2*f*x) + d^2*(e^2 + 20*e*f*x + 28*f^2*x^2)))*Cos[a +
b*(c + d*x)^(1/3)] + 6*b*(-20160*f^2*(c + d*x)^(1/3) - 12*b^4*f*(c + d*x)^(
2/3)*(5*d*e + 9*c*f + 14*d*f*x) + b^6*d*(c + d*x)^(1/3)*(e + f*x)*(3*c*f +
d*(e + 4*f*x)) + 120*b^2*f*(27*c*f + d*(e + 28*f*x)))*Sin[a + b*(c + d*x)^(
1/3)])/(b^9*d^3)
```

Maple [B] time = 0.013, size = 2704, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^3/b^3*(c^2*f^2*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b
*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d^2*e^2*(-(a+
b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(
d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+1/b^6*f^2*(-(a+b*(d*x+c)^(1/3))^8*cos
(a+b*(d*x+c)^(1/3))+8*(a+b*(d*x+c)^(1/3))^7*sin(a+b*(d*x+c)^(1/3))+56*(a+b*
(d*x+c)^(1/3))^6*cos(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(
d*x+c)^(1/3))-1680*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+6720*(a+b*(
d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+20160*(a+b*(d*x+c)^(1/3))^2*cos(a+b*
(d*x+c)^(1/3))-40320*cos(a+b*(d*x+c)^(1/3))-40320*(a+b*(d*x+c)^(1/3))*sin(a
+b*(d*x+c)^(1/3)))-a^2*c^2*f^2*cos(a+b*(d*x+c)^(1/3))-2*a*c^2*f^2*(sin(a+b*
(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2*a*d^2*e^2*(sin
(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-56/b^6*a^3*
f^2*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*
sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(
a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*
(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+70/b^6*a^4*f^2*(-(a+b*(d*x+c)^(
1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3
))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a+b*(d*x+c)^(1/3)
)-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*f^2*(-(a+b*(d*x
+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)
^(1/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3
)))-1/b^6*a^8*f^2*cos(a+b*(d*x+c)^(1/3))-a^2*d^2*e^2*cos(a+b*(d*x+c)^(1/3))
+28/b^6*a^6*f^2*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d
*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-8/b^6*a^7*f^2*(s
in(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2/b^3*c*f
^2*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*s
in(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a
+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(
a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-8/b^6*a*f^2*(-(a+b*(d*x+c)^(1/3)
)^7*cos(a+b*(d*x+c)^(1/3))+7*(a+b*(d*x+c)^(1/3))^6*sin(a+b*(d*x+c)^(1/3))+4
2*(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))-210*(a+b*(d*x+c)^(1/3))^4*si
n(a+b*(d*x+c)^(1/3))-840*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+2520*
(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-5040*sin(a+b*(d*x+c)^(1/3))+50
40*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+28/b^6*a^2*f^2*(-(a+b*(d*x+c
)^(1/3))^6*cos(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(
1/3))+30*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3
))^3*sin(a+b*(d*x+c)^(1/3))-360*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3)
)+720*cos(a+b*(d*x+c)^(1/3))+720*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3))
)+2*a^2*c*d*e*f*cos(a+b*(d*x+c)^(1/3))+10/b^3*a*c*f^2*(-(a+b*(d*x+c)^(1/3))
^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+12
*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(a+b*(d*x+c)^(1/3))-24*
```

```
(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-20/b^3*a^2*c*f^2*(-(a+b*(d*x+c)
^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1
/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))
+20/b^3*a^3*c*f^2*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*
(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-10/b^3*a^4*c*f
^2*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2*c*
d*e*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3
))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+2/b^3*d*e*f*(-(a+b*(d*x+c)
^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1
/3))+20*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))
^2*sin(a+b*(d*x+c)^(1/3))+120*sin(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3)
)*cos(a+b*(d*x+c)^(1/3)))-2/b^3*a^5*c*f^2*cos(a+b*(d*x+c)^(1/3))-20/b^3*a^3
*d*e*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/
3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*a^4*d*e*f*(sin(a+b
*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+20/b^3*a^2*d*e*
f*(-(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*si
n(a+b*(d*x+c)^(1/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos(a+b
*(d*x+c)^(1/3)))+4*a*c*d*e*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*co
s(a+b*(d*x+c)^(1/3)))+2/b^3*a^5*d*e*f*cos(a+b*(d*x+c)^(1/3))-10/b^3*a*d*e*f
*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*sin
(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*cos(
a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3))))
```

Maxima [B] time = 1.42847, size = 2904, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*e^2*cos((d*x + c)^(1/3)*b + a) - 2*a^2*c*e*f*cos((d*x + c)^(1/3)*b
+ a)/d + a^2*c^2*f^2*cos((d*x + c)^(1/3)*b + a)/d^2 - 2*(((d*x + c)^(1/3)*b
 + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a*e^2 + 4*((
(d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b +
 a))*a*c*e*f/d - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - si
n((d*x + c)^(1/3)*b + a))*a*c^2*f^2/d^2 - 2*a^5*e*f*cos((d*x + c)^(1/3)*b +
 a)/(b^3*d) + 2*a^5*c*f^2*cos((d*x + c)^(1/3)*b + a)/(b^3*d^2) + ((((d*x +
c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b +
a)*sin((d*x + c)^(1/3)*b + a))*e^2 + 10*(((d*x + c)^(1/3)*b + a)*cos((d*x +
 c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*e*f/(b^3*d) - 2*((((d*x
+ c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b
+ a)*sin((d*x + c)^(1/3)*b + a))*c*e*f/d - 10*(((d*x + c)^(1/3)*b + a)*cos(
(d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*c*f^2/(b^3*d^2) +
((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^
(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c^2*f^2/d^2 + a^8*f^2*cos((d*x + c
)^(1/3)*b + a)/(b^6*d^2) - 20*((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c
)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*a^3*
e*f/(b^3*d) - 8*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((
d*x + c)^(1/3)*b + a))*a^7*f^2/(b^6*d^2) + 20*((((d*x + c)^(1/3)*b + a)^2 -
 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1
/3)*b + a))*a^3*c*f^2/(b^3*d^2) + 20*((((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
 c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b + a)^
2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*e*f/(b^3*d) + 28*((((d*x + c)^(1/3)*
b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d
*x + c)^(1/3)*b + a))*a^6*f^2/(b^6*d^2) - 20*((((d*x + c)^(1/3)*b + a)^3 -
6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)
```

```
*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*c*f^2/(b^3*d^2) - 10*((((d*x
 + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/
3)*b + a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((
d*x + c)^(1/3)*b + a))*a*e*f/(b^3*d) - 56*((((d*x + c)^(1/3)*b + a)^3 - 6*(
d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b
+ a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^5*f^2/(b^6*d^2) + 10*((((d*x + c)
^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b
+ a) - 4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x +
 c)^(1/3)*b + a))*a*c*f^2/(b^3*d^2) + 2*((((d*x + c)^(1/3)*b + a)^5 - 20*((
d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3
)*b + a) - 5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24
)*sin((d*x + c)^(1/3)*b + a))*e*f/(b^3*d) + 70*((((d*x + c)^(1/3)*b + a)^4
- 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) - 4*(((d*x
+ c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a)
)*a^4*f^2/(b^6*d^2) - 2*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b
 + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5*(((
d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^
(1/3)*b + a))*c*f^2/(b^3*d^2) - 56*((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x +
 c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b +
 a) - 5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin
((d*x + c)^(1/3)*b + a))*a^3*f^2/(b^6*d^2) + 28*((((d*x + c)^(1/3)*b + a)^6
 - 30*((d*x + c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*cos(
(d*x + c)^(1/3)*b + a) - 6*(((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)
*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*a^2*
f^2/(b^6*d^2) - 8*((((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)*b + a)^
5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)*cos((d
*x + c)^(1/3)*b + a) - 7*(((d*x + c)^(1/3)*b + a)^6 - 30*((d*x + c)^(1/3)*b
 + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*sin((d*x + c)^(1/3)*b + a))*
a*f^2/(b^6*d^2) + ((((d*x + c)^(1/3)*b + a)^8 - 56*((d*x + c)^(1/3)*b + a)^
6 + 1680*((d*x + c)^(1/3)*b + a)^4 - 20160*((d*x + c)^(1/3)*b + a)^2 + 4032
0)*cos((d*x + c)^(1/3)*b + a) - 8*(((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c
)^(1/3)*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b -
 5040*a)*sin((d*x + c)^(1/3)*b + a))*f^2/(b^6*d^2))/(b^3*d)
```

Fricas [A] time = 1.76319, size = 776, normalized size = 1.23

$$\frac{3\left(\left(56\,b^6d^2f^2x^2+2\,b^6d^2e^2+36\,b^6cdef+18\left(b^6c^2-2240\right)f^2+8\left(5\,b^6d^2ef+9\,b^6cdf^2\right)x-\left(b^8d^2f^2x^2+2\,b^8d^2efx+b\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((56*b^6*d^2*f^2*x^2 + 2*b^6*d^2*e^2 + 36*b^6*c*d*e*f + 18*(b^6*c^2 - 224
0)*f^2 + 8*(5*b^6*d^2*e*f + 9*b^6*c*d*f^2)*x - (b^8*d^2*f^2*x^2 + 2*b^8*d^2
*e*f*x + b^8*d^2*e^2 - 20160*b^2*f^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*f^2*x
+ b^4*d*e*f + 6*b^4*c*f^2)*(d*x + c)^(1/3))*cos((d*x + c)^(1/3)*b + a) + 2*
(3360*b^3*d*f^2*x + 120*b^3*d*e*f + 3240*b^3*c*f^2 - 12*(14*b^5*d*f^2*x + 5
*b^5*d*e*f + 9*b^5*c*f^2)*(d*x + c)^(2/3) + (4*b^7*d^2*f^2*x^2 + b^7*d^2*e^
2 + 3*b^7*c*d*e*f - 20160*b*f^2 + (5*b^7*d^2*e*f + 3*b^7*c*d*f^2)*x)*(d*x +
 c)^(1/3))*sin((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^2\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [B] time = 1.62132, size = 2103, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(f^2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b
^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1
/3)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x +
c)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*
b^3*c + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*
x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c
)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/
3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a^8 - 2*b^6*c^2 + 40*((d*
x + c)^(1/3)*b + a)^3*b^3*c - 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c + 120*(
(d*x + c)^(1/3)*b + a)*a^2*b^3*c - 40*a^3*b^3*c - 56*((d*x + c)^(1/3)*b + a
)^6 + 336*((d*x + c)^(1/3)*b + a)^5*a - 840*((d*x + c)^(1/3)*b + a)^4*a^2 +
 1120*((d*x + c)^(1/3)*b + a)^3*a^3 - 840*((d*x + c)^(1/3)*b + a)^2*a^4 + 3
36*((d*x + c)^(1/3)*b + a)*a^5 - 56*a^6 - 240*((d*x + c)^(1/3)*b + a)*b^3*c
 + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b +
 a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a
)*a^3 + 1680*a^4 - 20160*((d*x + c)^(1/3)*b + a)^2 + 40320*((d*x + c)^(1/3)
*b + a)*a - 20160*a^2 + 40320)*cos((d*x + c)^(1/3)*b + a)/(b^8*d^2) - 2*(((
d*x + c)^(1/3)*b + a)*b^6*c^2 - a*b^6*c^2 - 5*((d*x + c)^(1/3)*b + a)^4*b^3
*c + 20*((d*x + c)^(1/3)*b + a)^3*a*b^3*c - 30*((d*x + c)^(1/3)*b + a)^2*a^
2*b^3*c + 20*((d*x + c)^(1/3)*b + a)*a^3*b^3*c - 5*a^4*b^3*c + 4*((d*x + c)
^(1/3)*b + a)^7 - 28*((d*x + c)^(1/3)*b + a)^6*a + 84*((d*x + c)^(1/3)*b +
a)^5*a^2 - 140*((d*x + c)^(1/3)*b + a)^4*a^3 + 140*((d*x + c)^(1/3)*b + a)^
3*a^4 - 84*((d*x + c)^(1/3)*b + a)^2*a^5 + 28*((d*x + c)^(1/3)*b + a)*a^6 -
 4*a^7 + 60*((d*x + c)^(1/3)*b + a)^2*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a
*b^3*c + 60*a^2*b^3*c - 168*((d*x + c)^(1/3)*b + a)^5 + 840*((d*x + c)^(1/3
)*b + a)^4*a - 1680*((d*x + c)^(1/3)*b + a)^3*a^2 + 1680*((d*x + c)^(1/3)*b
 + a)^2*a^3 - 840*((d*x + c)^(1/3)*b + a)*a^4 + 168*a^5 - 120*b^3*c + 3360*
((d*x + c)^(1/3)*b + a)^3 - 10080*((d*x + c)^(1/3)*b + a)^2*a + 10080*((d*x
 + c)^(1/3)*b + a)*a^2 - 3360*a^3 - 20160*(d*x + c)^(1/3)*b)*sin((d*x + c)^
(1/3)*b + a)/(b^8*d^2)) - (2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b -
 (((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d
*x + c)^(1/3)*b + a)/b^2)*e^2 - 2*f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*(
(d*x + c)^(1/3)*b + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*
((d*x + c)^(1/3)*b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x +
 c)^(1/3)*b + a)^2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20
*((d*x + c)^(1/3)*b + a)^3 - 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)
^(1/3)*b + a)*a^2 - 20*a^3 - 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b +
 a)/b^5 - (2*((d*x + c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)
*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a
^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2
 - 120*((d*x + c)^(1/3)*b + a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/
b^5)*e/d)/(b*d)
```

3.208 $\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=288

$$\frac{6\sqrt[3]{c+dx}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180f(c}{}$$

```
[Out] (6*(d*e - c*f)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^2) - (360*f*(c + d*x)^(1/
3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*(d*e - c*f)*(c + d*x)^(2/3)*C
os[a + b*(c + d*x)^(1/3)])/(b*d^2) + (60*f*(c + d*x)*Cos[a + b*(c + d*x)^(1
/3)])/(b^3*d^2) - (3*f*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^2)
+ (360*f*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2) + (6*(d*e - c*f)*(c + d*x)^(
1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^2) - (180*f*(c + d*x)^(2/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^4*d^2) + (15*f*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^
(1/3)])/(b^2*d^2)
```

Rubi [A] time = 0.269347, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3431, 3296, 2638, 2637}

$$\frac{6\sqrt[3]{c+dx}(de-cf)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{15f(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180f(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```


```
[Out] (6*(d*e - c*f)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d^2) - (360*f*(c + d*x)^(1/
3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*(d*e - c*f)*(c + d*x)^(2/3)*C
os[a + b*(c + d*x)^(1/3)])/(b*d^2) + (60*f*(c + d*x)*Cos[a + b*(c + d*x)^(1
/3)])/(b^3*d^2) - (3*f*(c + d*x)^(5/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d^2)
+ (360*f*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2) + (6*(d*e - c*f)*(c + d*x)^(
1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d^2) - (180*f*(c + d*x)^(2/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^4*d^2) + (15*f*(c + d*x)^(4/3)*Sin[a + b*(c + d*x)^
(1/3)])/(b^2*d^2)
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e+fx)\sin\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{3\,\text{Subst}\left(\int\left(\frac{(de-cf)x^2\sin(a+bx)}{d}+\frac{fx^5\sin(a+bx)}{d}\right)dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{(3f)\,\text{Subst}\left(\int x^5\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^2} + \frac{(3(de-cf))\,\text{Subst}\left(\int x^2\sin(a+bx)\right.}{d^2}$$

$$= -\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{(15}{}$$

$$= -\frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{3f(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6(d}{}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60f(c+}{}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3(de-cf)(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60f(c+}{}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3(de-cf)(c+}{}$$

$$= \frac{6(de-cf)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{360f\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3(de-cf)(c+}{}$$

Mathematica [A] time = 0.604134, size = 147, normalized size = 0.51

$$\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)\left(2b^4de\sqrt[3]{c+dx}+f\left(b^4\sqrt[3]{c+dx}(3c+5dx)-60b^2(c+dx)^{2/3}+120\right)\right)-3b\cos\left(a+b\sqrt[3]{c+dx}\right)\left(b^4d(c\right.}{b^6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*b*(120*f*(c + d*x)^(1/3) + b^4*d*(c + d*x)^(2/3)*(e + f*x) - 2*b^2*(9*c
*f + d*(e + 10*f*x)))*Cos[a + b*(c + d*x)^(1/3)] + 3*(2*b^4*d*e*(c + d*x)^(
1/3) + f*(120 - 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))
)*Sin[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Maple [B] time = 0.009, size = 801, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^2/b^3*(-c*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d
*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+d*e*(-(a+b*(d*x+
c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^
(1/3))*sin(a+b*(d*x+c)^(1/3)))+2*a*c*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)
^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2*a*d*e*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c
)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+a^2*c*f*cos(a+b*(d*x+c)^(1/3))-a^2*d*e*cos
```

```
(a+b*(d*x+c)^(1/3))+1/b^3*f*(-(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+
5*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cos
(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+120*sin
(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-5/b^3*a
*f*(-(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*s
in(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-24*co
s(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*
a^2*f*(-(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^
2*sin(a+b*(d*x+c)^(1/3))-6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*cos
(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*f*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(
1/3))+2*cos(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3))
)+5/b^3*a^4*f*(sin(a+b*(d*x+c)^(1/3))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(
1/3)))+1/b^3*a^5*f*cos(a+b*(d*x+c)^(1/3)))
```

Maxima [B] time = 1.09276, size = 919, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*e*cos((d*x + c)^(1/3)*b + a) - a^2*c*f*cos((d*x + c)^(1/3)*b + a)/d
 - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1
/3)*b + a))*a*e + 2*(((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) - s
in((d*x + c)^(1/3)*b + a))*a*c*f/d - a^5*f*cos((d*x + c)^(1/3)*b + a)/(b^3*
d) + ((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x
+ c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*e + 5*(((d*x + c)^(1/3)*b + a
)*cos((d*x + c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a^4*f/(b^3*d) -
((((d*x + c)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^
(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a))*c*f/d - 10*((((d*x + c)^(1/3)*b +
a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x +
 c)^(1/3)*b + a))*a^3*f/(b^3*d) + 10*((((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
 c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) - 3*(((d*x + c)^(1/3)*b + a)^
2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^2*f/(b^3*d) - 5*((((d*x + c)^(1/3)*b +
 a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) - 4*(
((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*
b + a))*a*f/(b^3*d) + ((((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b +
 a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) - 5*(((d*
x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1
/3)*b + a))*f/(b^3*d))/(b^3*d)
```

Fricas [A] time = 1.67047, size = 362, normalized size = 1.26

$$\frac{3\left(\left(20\,b^3dfx + 2\,b^3de + 18\,b^3cf - 120\,(dx+c)^{\frac{1}{3}}bf - \left(b^5dfx + b^5de\right)(dx+c)^{\frac{2}{3}}\right)\cos\left((dx+c)^{\frac{1}{3}}b + a\right) - \left(60\,(dx+c)^{\frac{2}{3}}b^2\right.\right.}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((20*b^3*d*f*x + 2*b^3*d*e + 18*b^3*c*f - 120*(d*x + c)^(1/3)*b*f - (b^5*
d*f*x + b^5*d*e)*(d*x + c)^(2/3))*cos((d*x + c)^(1/3)*b + a) - (60*(d*x + c
)^(2/3)*b^2*f - (5*b^4*d*f*x + 2*b^4*d*e + 3*b^4*c*f)*(d*x + c)^(1/3) - 120
```

```
*f)*sin((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [A] time = 1.39707, size = 613, normalized size = 2.13

$$3\left(\left(\frac{2\,(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}-\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)e+\frac{f\left(\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2b^3c-2\left((dx+c)^{\frac{1}{3}}b+a\right)ab^3c+a^2b^3c-\left((dx+c)^{\frac{1}{3}}b\cdots\right.\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*((2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*b +
a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b^
2)*e + f*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^
3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a
 - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*
((d*x + c)^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3
- 60*((d*x + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 - 20*a^3
- 120*(d*x + c)^(1/3)*b)*cos((d*x + c)^(1/3)*b + a)/b^5 - (2*((d*x + c)^(1/
3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(
1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b
+ a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x + c)^(1/3)*b +
a)*a + 60*a^2 - 120)*sin((d*x + c)^(1/3)*b + a)/b^5)/d)/(b*d)
```

3.209 $\int \sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=85

$$\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[Out] (6*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d) + (6*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rubi [A] time = 0.057381, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3361, 3296, 2638}

$$\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^(2/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b*d) + (6*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin\left(a+b\sqrt[3]{c+dx}\right)dx &= \frac{3\,\text{Subst}\left(\int x^2\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d}\\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\,\text{Subst}\left(\int x\cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{bd}\\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{6\,\text{Subst}\left(\int \sin(a+b\right.}{b^2d}\\
&= \frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}\end{aligned}$$

Mathematica [A] time = 0.107848, size = 65, normalized size = 0.76

$$\frac{\left(6-3b^2(c+dx)^{2/3}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+6b\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] ((6 - 3*b^2*(c + d*x)^(2/3))*Cos[a + b*(c + d*x)^(1/3)] + 6*b*(c + d*x)^(1/
3)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A] time = 0.009, size = 134, normalized size = 1.6

$$3\,\frac{-\left(a+b\sqrt[3]{dx+c}\right)^2\cos\left(a+b\sqrt[3]{dx+c}\right)+2\,\cos\left(a+b\sqrt[3]{dx+c}\right)+2\,\left(a+b\sqrt[3]{dx+c}\right)\sin\left(a+b\sqrt[3]{dx+c}\right)-2\,a\left(\sin\left(a+b\right.\right.}{db^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d/b^3*(-(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1
/3))+2*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3))-2*a*(sin(a+b*(d*x+c)^(1/3
))-(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-a^2*cos(a+b*(d*x+c)^(1/3)))
```

Maxima [A] time = 0.965139, size = 162, normalized size = 1.91

$$-\frac{3\left(a^2\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)a+\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\right.\right.}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*cos((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*cos((d*x +
c)^(1/3)*b + a) - sin((d*x + c)^(1/3)*b + a))*a + (((d*x + c)^(1/3)*b + a)^
2 - 2)*cos((d*x + c)^(1/3)*b + a) - 2*((d*x + c)^(1/3)*b + a)*sin((d*x + c)
^(1/3)*b + a))/(b^3*d)
```

Fricas [A] time = 1.70394, size = 155, normalized size = 1.82

$$\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2-2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*b*sin((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2 -
2)*cos((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] time = 1.51942, size = 95, normalized size = 1.12

$$\begin{cases} x\sin(a) & \text{for } b = 0 \wedge d = 0 \\ x\sin\left(a + b\sqrt[3]{c}\right) & \text{for } d = 0 \\ x\sin(a) & \text{for } b = 0 \\ -\frac{3(c+dx)^{\frac{2}{3}}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{6\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & Eq(d, 0)), (x*sin(a + b*c**(1/3)), Eq(d, 0)
), (x*sin(a), Eq(b, 0)), (-3*(c + d*x)**(2/3)*cos(a + b*(c + d*x)**(1/3))/(
b*d) + 6*(c + d*x)**(1/3)*sin(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cos(a +
b*(c + d*x)**(1/3))/(b**3*d), True))
```

Giac [A] time = 1.22201, size = 111, normalized size = 1.31

$$\frac{3\left(\frac{2(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b} - \frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 - 2\left((dx+c)^{\frac{1}{3}}b+a\right)a + a^2 - 2\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a)/b - (((d*x + c)^(1/3)*b + a
)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*cos((d*x + c)^(1/3)*b + a)/b^2
)/(b*d)
```

3.210 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$

Optimal. Leaf size=396

$$\frac{\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}+\frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}+\cdots$$

```
[Out] (CosIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - (b*
(d*e - c*f)^(1/3))/f^(1/3)])/f + (CosIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/
3))/f^(1/3) - b*(c + d*x)^(1/3)]*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f
^(1/3)])/f + (CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c +
 d*x)^(1/3)]*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)])/f - (Cos[a
+ ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(1/3)*b*(d*e
- c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/f + (Cos[a - (b*(d*e - c*f)^(1/
3))/f^(1/3)]*SinIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]
)/f + (Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^
(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/f
```

Rubi [A] time = 1.38987, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3431, 3303, 3299, 3302}

$$\frac{\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)}{f}+\frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{f}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```


```
[Out] (CosIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - (b*
(d*e - c*f)^(1/3))/f^(1/3)])/f + (CosIntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/
3))/f^(1/3) - b*(c + d*x)^(1/3)]*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f
^(1/3)])/f + (CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c +
 d*x)^(1/3)]*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)])/f - (Cos[a
+ ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^(1/3)*b*(d*e
- c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/f + (Cos[a - (b*(d*e - c*f)^(1/
3))/f^(1/3)]*SinIntegral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)]
)/f + (Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinIntegral[((-1)^
(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/f
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx = \frac{3\,\text{Subst}\left(\int\left(\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left(\sqrt[3]{de-cf}+\sqrt[3]{f}x\right)}+\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{-1}\sqrt[3]{de-cf}+\sqrt[3]{f}x\right)}+\frac{(de-cf)\sin(a+bx)}{3f^{2/3}\left(e-\frac{cf}{d}\right)\left((-1)^{2/3}\sqrt[3]{de-cf}\right.}\right.\right.}{d}$$

$$= \frac{\text{Subst}\left(\int\frac{\sin(a+bx)}{\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx,x,\sqrt[3]{c+dx}\right)}{f^{2/3}}+\frac{\text{Subst}\left(\int\frac{\sin(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx,x,\sqrt[3]{c+dx}\right)}{f^{2/3}}+\text{Sul}$$

$$= \frac{\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}\left(\int\frac{\sin\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{\sqrt[3]{de-cf}+\sqrt[3]{f}x}\,dx,x,\sqrt[3]{c+dx}\right)}{f^{2/3}}-\frac{\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Subst}}{}$$

$$= \frac{\text{Ci}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\sqrt[3]{c+dx}\right)\sin\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)}{f}+\frac{\text{Ci}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{}}{\sqrt[3]{}}\right.}{f}$$

Mathematica [C] time = 1.75552, size = 118, normalized size = 0.3

$$\frac{i\left(\text{RootSum}\left[\#1^3f-cf+de\&,e^{-i\#1b-ia}\text{Ei}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]-\text{RootSum}\left[\#1^3f-cf+de\&,e^{i\#1b+ia}\text{Ei}\left(ib\left(\sqrt[3]{c}\right.\right.\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] ((I/2)*(RootSum[d*e - c*f + f*#1^3 & , E^((-I)*a - I*b*#1)*ExpIntegralEi[(-
I)*b*((c + d*x)^(1/3) - #1)] & ] - RootSum[d*e - c*f + f*#1^3 & , E^(I*a +
I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #1)] & ]))/f
```

Maple [C] time = 0.019, size = 327, normalized size = 0.8

$$3\,\frac{1}{b^3}\left(1/3\,\frac{b^3}{f}\sum_{\_R1=\text{RootOf}\left(-cfb^3+deb^3+f\_Z^3-3af\_Z^2+3a^2f\_Z-a^3f\right)}\frac{\_R1^2\left(-\text{Si}\left(-b\sqrt[3]{dx+c}+\_R1-a\right)\cos(\_R1)+\text{Ci}\left(b\sqrt[3]{dx+}\right.\right.}{\_R1^2-2\_R1\,a+a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x)
```

```
[Out] 3/b^3*(1/3*b^3/f*sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)
*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_
Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))-2/3*b^3*a/f*sum(_R1/(_R1^2-2*_R1*a+a^2)
*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),
_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))+1/3*a^2*b^
3/f*sum(1/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d
*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+
3*_Z*a^2*f-a^3*f)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

Fricas [C] time = 1.93127, size = 1119, normalized size = 2.83

$$i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,b^3de-i\,b^3cf}{f}\right)^{\frac{1}{3}}\right)e^{\left(\frac{1}{2}\left(i\sqrt{3}+1\right)\left(\frac{i\,b^3de-i\,b^3cf}{f}\right)^{\frac{1}{3}}-i\,a\right)}+i\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{i\,b^3de-i\,b^3c}{f}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*c
*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I*
a) + I*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*c*
f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I*
a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3*c
*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3) + I
*a) - I*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3*c
*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3) +
I*a) - I*Ei(I*(d*x + c)^(1/3)*b + ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3))*e^(I*
a - ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)) + I*Ei(-I*(d*x + c)^(1/3)*b + ((I*b
^3*d*e - I*b^3*c*f)/f)^(1/3))*e^(-I*a - ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3)))
/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e), x)
```

3.211 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=555

$$-\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\right.}{3f^{4/3}(de-cf)^{2/3}}$$

```
[Out] -((-1)^(1/3)*b*d*Cos[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*CosInteg
ral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(3*f^(4/
3)*(d*e - c*f)^(2/3)) + (b*d*Cos[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*CosInte
gral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(d*e -
c*f)^(2/3)) + ((-1)^(2/3)*b*d*Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1
/3)]*CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/
3)])/(3*f^(4/3)*(d*e - c*f)^(2/3)) - Sin[a + b*(c + d*x)^(1/3)]/(f*(e + f*x
)) - ((-1)^(1/3)*b*d*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinI
ntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(3*f
^(4/3)*(d*e - c*f)^(2/3)) - (b*d*Sin[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*Sin
Integral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(d*
e - c*f)^(2/3)) - ((-1)^(2/3)*b*d*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/
f^(1/3)]*SinIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)
^(1/3)])/(3*f^(4/3)*(d*e - c*f)^(2/3))
```

Rubi [A] time = 2.12414, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3431, 3341, 3334, 3303, 3299, 3302}

$$-\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}}+\frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+b\right.}{3f^{4/3}(de-cf)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```


```
[Out] -((-1)^(1/3)*b*d*Cos[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*CosInteg
ral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(3*f^(4/
3)*(d*e - c*f)^(2/3)) + (b*d*Cos[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*CosInte
gral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(d*e -
c*f)^(2/3)) + ((-1)^(2/3)*b*d*Cos[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1
/3)]*CosIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/
3)])/(3*f^(4/3)*(d*e - c*f)^(2/3)) - Sin[a + b*(c + d*x)^(1/3)]/(f*(e + f*x
)) - ((-1)^(1/3)*b*d*Sin[a + ((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3)]*SinI
ntegral[((-1)^(1/3)*b*(d*e - c*f)^(1/3))/f^(1/3) - b*(c + d*x)^(1/3)])/(3*f
^(4/3)*(d*e - c*f)^(2/3)) - (b*d*Sin[a - (b*(d*e - c*f)^(1/3))/f^(1/3)]*Sin
Integral[(b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)^(1/3)])/(3*f^(4/3)*(d*
e - c*f)^(2/3)) - ((-1)^(2/3)*b*d*Sin[a - ((-1)^(2/3)*b*(d*e - c*f)^(1/3))/
f^(1/3)]*SinIntegral[((-1)^(2/3)*b*(d*e - c*f)^(1/3))/f^(1/3) + b*(c + d*x)
^(1/3)])/(3*f^(4/3)*(d*e - c*f)^(2/3))
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
```

```
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e+fx)^2}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(e-\frac{cf}{d}+\frac{fx^3}{d}\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} + \frac{b\,\text{Subst}\left(\int \frac{\cos(a+bx)}{e-\frac{cf}{d}+\frac{fx^3}{d}}\,dx, x, \sqrt[3]{c+dx}\right)}{f}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} + \frac{b\,\text{Subst}\left(\int \left(-\frac{\sqrt[3]{de-cf}\cos(a+bx)}{3\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{de-cf}-\sqrt[3]{f}x\right)} - \frac{\sqrt[3]{de-cf}\cos(a+bx)}{3\left(e-\frac{cf}{d}\right)\left(-\sqrt[3]{de-cf}+\sqrt[3]{-1}\sqrt[3]{f}x\right)} - \frac{}{3\left(e-\frac{c}{d}\right.}\right.\right.}{f}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{-\sqrt[3]{de-cf}-\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a}{-\sqrt[3]{de-cf}+}\right.}{3f(de}$$

$$= -\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{f(e+fx)} - \frac{\left(bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}+bx\right)}{-\sqrt[3]{de-cf}-\sqrt[3]{f}x}\,dx, x, \sqrt[3]{c+dx}\right)}{3f(de-cf)^{2/3}} - \frac{(bc}{}$$

$$= -\frac{\sqrt[3]{-1}bd\cos\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{de-cf}}{\sqrt[3]{f}}-b\sqrt[3]{c+dx}\right)}{3f^{4/3}(de-cf)^{2/3}} + \frac{bd\cos\left(a-\frac{b\sqrt[3]{de-cf}}{\sqrt[3]{f}}\right)\text{Ci}\left(\frac{b\sqrt[3]{de-}}{\sqrt[3]{f}}\right.}{3f^{4/3}(de-cf)^{2/}}$$

Mathematica [C] time = 1.11035, size = 180, normalized size = 0.32

$$\frac{bd\text{RootSum}\left[\#1^3f-cf+de\&, \frac{e^{-i\#1b-ia}\text{Ei}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + bd\text{RootSum}\left[\#1^3f-cf+de\&, \frac{e^{i\#1b+ia}\text{Ei}\left(ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + \cdots}{6f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] (((3*I)*(-1 + E^((2*I)*(a + b*(c + d*x)^(1/3))))*f)/(E^(I*(a + b*(c + d*x)^
(1/3)))*(e + f*x)) + b*d*RootSum[d*e - c*f + f*#1^3 & , (E^((-I)*a - I*b*#1
)*ExpIntegralEi[(-I)*b*((c + d*x)^(1/3) - #1)])/#1^2 & ] + b*d*RootSum[d*e
- c*f + f*#1^3 & , (E^(I*a + I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3) - #
1)])/#1^2 & ])/(6*f^2)
```

Maple [C] time = 0.067, size = 1175, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x)
```

```
[Out] 3*d/b^3*(sin(a+b*(d*x+c)^(1/3))*(-2/3*a*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))^2
+a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3))-1/3*b^3*(b^3*c*f-b^3*d*e+a^3*f)/(c*f
-d*e)/f)/(-c*f*b^3+d*e*b^3+(a+b*(d*x+c)^(1/3))^3*f-3*(a+b*(d*x+c)^(1/3))^2*
a*f+3*(a+b*(d*x+c)^(1/3))*a^2*f-a^3*f)-2/9*a*b^3/f*sum(_R1/(c*f-d*e)/(_R1^2
```

```
-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+
a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f
))+1/9*b^3/f^2*sum((b^3*c*f-b^3*d*e+2*_RR1^2*a*f-3*_RR1*a^2*f+a^3*f)/(c*f-d
*e)/(_RR1^2-2*_RR1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_RR1-a)*sin(_RR1)+Ci(b*(d*x+
c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+
3*_Z*a^2*f-a^3*f))+sin(a+b*(d*x+c)^(1/3))*(2/3*a*b^3/(c*f-d*e)*(a+b*(d*x+c)
^(1/3))^2-2/3*a^2*b^3/(c*f-d*e)*(a+b*(d*x+c)^(1/3)))/(-c*f*b^3+d*e*b^3+(a+b
*(d*x+c)^(1/3))^3*f-3*(a+b*(d*x+c)^(1/3))^2*a*f+3*(a+b*(d*x+c)^(1/3))*a^2*f
-a^3*f)+2/9*a*b^3/f*sum((_R1+a)/(c*f-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+
c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^
3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))-2/9*a*b^3/f*sum(_RR1/(_R
R1-a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/3)+_RR1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_
RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f
-a^3*f))+b^6*a^2*(sin(a+b*(d*x+c)^(1/3))*(-1/3/b^3/(c*f-d*e)*(a+b*(d*x+c)^(
1/3))+1/3*a/b^3/(c*f-d*e))/(-c*f*b^3+d*e*b^3+(a+b*(d*x+c)^(1/3))^3*f-3*(a+b
*(d*x+c)^(1/3))^2*a*f+3*(a+b*(d*x+c)^(1/3))*a^2*f-a^3*f)-2/9/b^3/f*sum(1/(c
*f-d*e)/(_R1^2-2*_R1*a+a^2)*(-Si(-b*(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b*(d*x
+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf(-b^3*c*f+b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*
_Z*a^2*f-a^3*f))+1/9/b^3/f*sum(1/(_RR1-a)/(c*f-d*e)*(Si(-b*(d*x+c)^(1/3)+_R
R1-a)*sin(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf(-b^3*c*f+
b^3*d*e+_Z^3*f-3*_Z^2*a*f+3*_Z*a^2*f-a^3*f))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.12379, size = 1789, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/12*((I*d*f*x + I*d*e - sqrt(3)*(d*f*x + d*e))*((I*b^3*d*e - I*b^3*c*f)/f
)^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*
c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I
*a) + (I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*((I*b^3*d*e - I*b^3*c*f)/f)
^(1/3)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((I*b^3*d*e - I*b^3*c*
f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((I*b^3*d*e - I*b^3*c*f)/f)^(1/3) - I*
a) + (-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e))*((-I*b^3*d*e + I*b^3*c*f)/f
)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(-I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3*
c*f)/f)^(1/3))*e^(1/2*(I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3) +
I*a) + (-I*d*f*x - I*d*e - sqrt(3)*(d*f*x + d*e))*((-I*b^3*d*e + I*b^3*c*f)
/f)^(1/3)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*sqrt(3) - 1)*((-I*b^3*d*e + I*b^3
*c*f)/f)^(1/3))*e^(1/2*(-I*sqrt(3) + 1)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)
+ I*a) + (2*I*d*f*x + 2*I*d*e)*((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3)*Ei(I*(d*x
 + c)^(1/3)*b + ((-I*b^3*d*e + I*b^3*c*f)/f)^(1/3))*e^(I*a - ((-I*b^3*d*e +
 I*b^3*c*f)/f)^(1/3)) + (-2*I*d*f*x - 2*I*d*e)*((I*b^3*d*e - I*b^3*c*f)/f)^
```

```
(1/3)*Ei(-I*(d*x + c)^(1/3)*b + ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3))*e^(-I*a
- ((I*b^3*d*e - I*b^3*c*f)/f)^(1/3)) + 12*(d*e - c*f)*sin((d*x + c)^(1/3)*b
 + a))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{\left(e + fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx + c)^{\frac{1}{3}}b + a\right)}{\left(fx + e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(f*x + e)^2, x)
```

3.212 $\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right) dx$

Optimal. Leaf size=513

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}+\frac{6f(c+dx)^{2/3}(de-cf}{b}$$

```
[Out] (6*f*(d*e - c*f)*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^3) - (3*(d*e - c*f)^2*(
c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^3) + (105*f^2*(c + d*x)*C
os[a + b*(c + d*x)^(2/3)])/(8*b^3*d^3) - (3*f*(d*e - c*f)*(c + d*x)^(4/3)*C
os[a + b*(c + d*x)^(2/3)])/(b*d^3) - (3*f^2*(c + d*x)^(7/3)*Cos[a + b*(c +
d*x)^(2/3)])/(2*b*d^3) + (3*(d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^3) + (315*f^2*Sqrt[Pi/2]*Cos[a]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(16*b^(9/2)*d^3) + (315*f^2*
Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(16*b^(9/2)
*d^3) - (3*(d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(
1/3)]*Sin[a])/(2*b^(3/2)*d^3) - (315*f^2*(c + d*x)^(1/3)*Sin[a + b*(c + d*x
)^(2/3)])/(16*b^4*d^3) + (6*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c + d*
x)^(2/3)])/(b^2*d^3) + (21*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(2/3)])/
(4*b^2*d^3)
```

Rubi [A] time = 0.535216, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3433, 3385, 3354, 3352, 3351, 3379, 3296, 2638, 3386, 3353}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)^2\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)^2S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^3}+\frac{6f(c+dx)^{2/3}(de-cf}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```


```
[Out] (6*f*(d*e - c*f)*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^3) - (3*(d*e - c*f)^2*(
c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^3) + (105*f^2*(c + d*x)*C
os[a + b*(c + d*x)^(2/3)])/(8*b^3*d^3) - (3*f*(d*e - c*f)*(c + d*x)^(4/3)*C
os[a + b*(c + d*x)^(2/3)])/(b*d^3) - (3*f^2*(c + d*x)^(7/3)*Cos[a + b*(c +
d*x)^(2/3)])/(2*b*d^3) + (3*(d*e - c*f)^2*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^3) + (315*f^2*Sqrt[Pi/2]*Cos[a]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(16*b^(9/2)*d^3) + (315*f^2*
Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(16*b^(9/2)
*d^3) - (3*(d*e - c*f)^2*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(
1/3)]*Sin[a])/(2*b^(3/2)*d^3) - (315*f^2*(c + d*x)^(1/3)*Sin[a + b*(c + d*x
)^(2/3)])/(16*b^4*d^3) + (6*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a + b*(c + d*
x)^(2/3)])/(b^2*d^3) + (21*f^2*(c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(2/3)])/
(4*b^2*d^3)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+b(c+dx)^{2/3}\right) dx = \frac{3\,\text{Subst}\left(\int \left((de-cf)^2x^2 \sin\left(a+bx^2\right) - 2f(-de+cf)x^5 \sin\left(a+bx^2\right) + f^2x^8 \right.\right.}{d^3}$$

$$= \frac{\left(3f^2\right)\text{Subst}\left(\int x^8 \sin\left(a+bx^2\right) dx, x, \sqrt[3]{c+dx}\right)}{d^3} + \frac{(6f(de-cf))\,\text{Subst}\left(\int x^5 \right.}{}$$

$$= -\frac{3(de-cf)^2 \sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f^2(c+dx)^{7/3}\cos\left(a+b(c+dx\right.}{2bd^3}$$

$$= -\frac{3(de-cf)^2 \sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} - \frac{3f(de-cf)(c+dx)^{4/3}\cos\left(a+\right.}{bd^3}$$

$$= -\frac{3(de-cf)^2 \sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3} + \frac{105f^2(c+dx)\cos\left(a+b(c+dx\right.}{8b^3d^3}$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2 \sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2 \sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$

$$= \frac{6f(de-cf)\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^3} - \frac{3(de-cf)^2 \sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^3}$$

Mathematica [C] time = 2.31289, size = 432, normalized size = 0.84

$$-\frac{3i\left(\left(\cos\left(a+b(c+dx)^{2/3}\right) - i\sin\left(a+b(c+dx)^{2/3}\right)\right)\left(2\sqrt{b}\left(-8ib^3d^2\sqrt[3]{c+dx}(e+fx)^2 + 4b^2f(c+dx)^{2/3}(cf-8de-7\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (((-3*I)/64)*((Cos[a] + I*Sin[a])*((1 + I)*((-105*I)*f^2 + 8*b^3*(d*e - c*f
)^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]] + 2*Sqrt[b]
*(-105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 + 4*
b^2*f*(c + d*x)^(2/3)*(8*d*e - c*f + 7*d*f*x) + (2*I)*b*f*(16*d*e + 19*c*f
+ 35*d*f*x))*(Cos[b*(c + d*x)^(2/3)] + I*Sin[b*(c + d*x)^(2/3)])) + (2*Sqrt
[b]*(105*f^2*(c + d*x)^(1/3) - (8*I)*b^3*d^2*(c + d*x)^(1/3)*(e + f*x)^2 +
4*b^2*f*(c + d*x)^(2/3)*(-8*d*e + c*f - 7*d*f*x) + (2*I)*b*f*(16*d*e + 19*c
*f + 35*d*f*x)) + (1 + I)*((105*I)*f^2 + 8*b^3*(d*e - c*f)^2)*Sqrt[Pi/2]*Er
f[((1 + I)*Sqrt[b]*(c + d*x)^(1/3))/Sqrt[2]]*(Cos[b*(c + d*x)^(2/3)] + I*Si
n[b*(c + d*x)^(2/3)]))*(Cos[a + b*(c + d*x)^(2/3)] - I*Sin[a + b*(c + d*x)^
(2/3)])))/(b^(9/2)*d^3)
```

Maple [A] time = 0.01, size = 395, normalized size = 0.8

$$3\frac{1}{d^3}\left(-1/2\frac{f^2(dx+c)^{7/3}\cos\left(a+b(dx+c)^{2/3}\right)}{b} + 7/2\frac{f^2}{b}\left(1/2\frac{(dx+c)^{5/3}\sin\left(a+b(dx+c)^{2/3}\right)}{b} - 5/2\frac{1}{b}\left(-1/2\frac{(dx+c)\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] 3/d^3*(-1/2*f^2/b*(d*x+c)^(7/3)*cos(a+b*(d*x+c)^(2/3))+7/2*f^2/b*(1/2/b*(d*
x+c)^(5/3)*sin(a+b*(d*x+c)^(2/3))-5/2/b*(-1/2/b*(d*x+c)*cos(a+b*(d*x+c)^(2/
3))+3/2/b*(1/2/b*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(2/3))-1/4/b^(3/2)*2^(1/2)*P
i^(1/2)*(cos(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*Fre
snelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))))-1/2*(-2*c*f^2+2*d*e*f)/b*
(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))+2*(-2*c*f^2+2*d*e*f)/b*(1/2/b*(d*x+c)^
(2/3)*sin(a+b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))-1/2*(c^2*f^2-2
*c*d*e*f+d^2*e^2)/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4*(c^2*f^2-2*c*d
*e*f+d^2*e^2)/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/
2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2)
)))
```

Maxima [C] time = 2.38944, size = 1769, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/128*(8*(8*(d*x + c)^(1/3)*abs(b)*cos((d*x + c)^(2/3)*b + a) - sqrt(pi)*(
((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) - I*si
n(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a)
- (I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arctan2(0, b)) +
 sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a)
)*erf((d*x + c)^(1/3)*sqrt(I*b)) + ((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(
-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arctan2(0, b)) - I*sin(-1
/4*pi + 1/2*arctan2(0, b)))*cos(a) - (-I*cos(1/4*pi + 1/2*arctan2(0, b)) -
I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(
-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*sqrt
(abs(b)))*e^2/(b*abs(b)) - 16*(8*(d*x + c)^(1/3)*abs(b)*cos((d*x + c)^(2/3)
*b + a) - sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*a
rctan2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arc
tan2(0, b)))*cos(a) - (I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi +
1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*ar
ctan2(0, b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(I*b)) + ((cos(1/4*pi + 1/2*a
rctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arcta
n2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (-I*cos(1/4*pi + 1
/2*arctan2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*a
rctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x + c)^(1/
3)*sqrt(-I*b)))*sqrt(abs(b)))*c*e*f/(b*d*abs(b)) + 8*(8*(d*x + c)^(1/3)*abs
(b)*cos((d*x + c)^(2/3)*b + a) - sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, b)
) + cos(-1/4*pi + 1/2*arctan2(0, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) +
I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (I*cos(1/4*pi + 1/2*arctan2(0,
 b)) + I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b))
 - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(I*b))
 + ((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I
*sin(1/4*pi + 1/2*arctan2(0, b)) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(
a) - (-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b
)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*si
n(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*sqrt(abs(b)))*c^2*f^2/(b*d^2*abs(b))
 - 128*(2*(d*x + c)^(2/3)*b*sin((d*x + c)^(2/3)*b + a) - ((d*x + c)^(4/3)*b
^2 - 2)*cos((d*x + c)^(2/3)*b + a))*e*f/(b^3*d) + 128*(2*(d*x + c)^(2/3)*b*
sin((d*x + c)^(2/3)*b + a) - ((d*x + c)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*
b + a))*c*f^2/(b^3*d^2) + (sqrt(pi)*(((-105*I*cos(1/4*pi + 1/2*arctan2(0, b
)) - 105*I*cos(-1/4*pi + 1/2*arctan2(0, b)) - 105*sin(1/4*pi + 1/2*arctan2(
0, b)) + 105*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (105*cos(1/4*pi + 1
```

```
/2*arctan2(0, b)) + 105*cos(-1/4*pi + 1/2*arctan2(0, b)) - 105*I*sin(1/4*pi
 + 1/2*arctan2(0, b)) + 105*I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf
((d*x + c)^(1/3)*sqrt(I*b)) + ((105*I*cos(1/4*pi + 1/2*arctan2(0, b)) + 105
*I*cos(-1/4*pi + 1/2*arctan2(0, b)) - 105*sin(1/4*pi + 1/2*arctan2(0, b)) +
 105*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (105*cos(1/4*pi + 1/2*arcta
n2(0, b)) + 105*cos(-1/4*pi + 1/2*arctan2(0, b)) + 105*I*sin(1/4*pi + 1/2*a
rctan2(0, b)) - 105*I*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x +
c)^(1/3)*sqrt(-I*b)))*sqrt(abs(b)) + 16*(4*(d*x + c)^(7/3)*b^3*abs(b) - 35*
(d*x + c)*b*abs(b))*cos((d*x + c)^(2/3)*b + a) - 56*(4*(d*x + c)^(5/3)*b^2*
abs(b) - 15*(d*x + c)^(1/3)*abs(b))*sin((d*x + c)^(2/3)*b + a))*f^2/(b^4*d^
2*abs(b)))/d
```

Fricas [A] time = 2.07091, size = 782, normalized size = 1.52

$$3\left(\sqrt{2}\left(105\,\pi f^2\sin\left(a\right)+8\,\pi\left(b^3d^2e^2-2\,b^3cdef+b^3c^2f^2\right)\cos\left(a\right)\right)\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\left(105\,\pi f^2\cos\left(a\right)-8\,\pi\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/32*(sqrt(2)*(105*pi*f^2*sin(a) + 8*pi*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*
c^2*f^2)*cos(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi))
 + sqrt(2)*(105*pi*f^2*cos(a) - 8*pi*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2
*f^2)*sin(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi)) +
4*(35*b^2*d*f^2*x + 16*b^2*d*e*f + 19*b^2*c*f^2 - 4*(b^4*d^2*f^2*x^2 + 2*b^
4*d^2*e*f*x + b^4*d^2*e^2)*(d*x + c)^(1/3))*cos((d*x + c)^(2/3)*b + a) - 2*
(105*(d*x + c)^(1/3)*b*f^2 - 4*(7*b^3*d*f^2*x + 8*b^3*d*e*f - b^3*c*f^2)*(d
*x + c)^(2/3))*sin((d*x + c)^(2/3)*b + a))/(b^5*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)^2\sin\left(a+b\left(c+dx\right)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] time = 1.33173, size = 1049, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/64*(f^2*((I*sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 - 105)*erf(-1/2*sqrt(2)*(d*x
+ c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^4*(-I*b/abs(b) + 1)*s
qrt(abs(b))) - 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c)^(4/3)*b^3*c +
```

```
8*I*(d*x + c)^(1/3)*b^3*c^2 - 28*(d*x + c)^(5/3)*b^2 + 32*(d*x + c)^(2/3)*b
^2*c - (70*I*d*x + 70*I*c)*b + 32*I*b*c + 105*(d*x + c)^(1/3))*e^(I*(d*x +
c)^(2/3)*b + I*a)/b^4)/d^2 + (I*sqrt(2)*sqrt(pi)*(-8*I*b^3*c^2 + 105)*erf(-
1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^4*(I
*b/abs(b) + 1)*sqrt(abs(b))) - 2*I*(8*I*(d*x + c)^(7/3)*b^3 - 16*I*(d*x + c
)^(4/3)*b^3*c + 8*I*(d*x + c)^(1/3)*b^3*c^2 + 28*(d*x + c)^(5/3)*b^2 - 32*(
d*x + c)^(2/3)*b^2*c - (70*I*d*x + 70*I*c)*b + 32*I*b*c - 105*(d*x + c)^(1/
3))*e^(-I*(d*x + c)^(2/3)*b - I*a)/b^4)/d^2) + 8*(sqrt(2)*sqrt(pi)*erf(-1/2
*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/a
bs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3
)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b)))
 + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3)*b + I*a)/b + 2*(d*x + c)^(1/3)*e^
(-I*(d*x + c)^(2/3)*b - I*a)/b)*e^2 - 16*(sqrt(2)*sqrt(pi)*c*erf(-1/2*sqrt(
2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b)
+ 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I
*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2
*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c - 2*(d*x + c)^(2/3)*b -
 2*I)*e^(I*(d*x + c)^(2/3)*b + I*a)/b^3 + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d
*x + c)^(1/3)*b^2*c + 2*(d*x + c)^(2/3)*b - 2*I)*e^(-I*(d*x + c)^(2/3)*b -
I*a)/b^3)*f*e/d)/d
```

3.213 $\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=243

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}+\frac{3f(c+dx)^{2/3}\sin\left(a+b\right.}{b^2d^2}$$

```
[Out] (3*f*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^2) - (3*(d*e - c*f)*(c + d*x)^(1/3)
*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^2) - (3*f*(c + d*x)^(4/3)*Cos[a + b*(c
+ d*x)^(2/3)])/(2*b*d^2) + (3*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^2) - (3*(d*e - c*f)*Sqrt[Pi/2]*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d^2) + (3*f
*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)])/(b^2*d^2)
```

Rubi [A] time = 0.264173, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3433, 3385, 3354, 3352, 3351, 3379, 3296, 2638}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d^2}+\frac{3f(c+dx)^{2/3}\sin\left(a+b\right.}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (3*f*Cos[a + b*(c + d*x)^(2/3)])/(b^3*d^2) - (3*(d*e - c*f)*(c + d*x)^(1/3)
*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d^2) - (3*f*(c + d*x)^(4/3)*Cos[a + b*(c
+ d*x)^(2/3)])/(2*b*d^2) + (3*(d*e - c*f)*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b
]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d^2) - (3*(d*e - c*f)*Sqrt[Pi/2]*
FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d^2) + (3*f
*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)])/(b^2*d^2)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\sin\left(a+b(c+dx)^{2/3}\right)dx &= \frac{3\,\text{Subst}\left(\int\left((de-cf)x^2\sin\left(a+bx^2\right)+fx^5\sin\left(a+bx^2\right)\right)dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\ &= \frac{(3f)\,\text{Subst}\left(\int x^5\sin\left(a+bx^2\right)dx,x,\sqrt[3]{c+dx}\right)}{d^2}+\frac{(3(de-cf))\,\text{Subst}\left(\int x^2\sin\left(a+\right.\right.}{d^2}\\ &= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}+\frac{(3f)\,\text{Subst}\left(\int x^2\sin(a+bx)\,dx,x,(c\right.}{2d^2}\\ &= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}+\\ &= -\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}+\\ &= \frac{3f\cos\left(a+b(c+dx)^{2/3}\right)}{b^3d^2}-\frac{3(de-cf)\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd^2}-\frac{3f(c+dx)^{4}}{}\end{aligned}$$

Mathematica [A] time = 0.824316, size = 213, normalized size = 0.88

$$\frac{3\left(\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)-\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)-2b^2de\sqrt[3]{c+dx}\cos\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (3*(4*f*Cos[a + b*(c + d*x)^(2/3)] - 2*b^2*d*e*(c + d*x)^(1/3)*Cos[a + b*(c
 + d*x)^(2/3)] - 2*b^2*d*f*x*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] + b
```

```
^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)] - b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*
x)^(1/3)]*Sin[a] + 4*b*f*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)]))/(4*b^
3*d^2)
```

Maple [A] time = 0.009, size = 175, normalized size = 0.7

$$3\,\frac{1}{d^2}\left(-1/2\,\frac{f\,(dx+c)^{4/3}\cos\left(a+b\,(dx+c)^{2/3}\right)}{b}+2\,\frac{f}{b}\left(1/2\,\frac{(dx+c)^{2/3}\sin\left(a+b\,(dx+c)^{2/3}\right)}{b}+1/2\,\frac{\cos\left(a+b\,(dx+c)^{2/3}\right)}{b^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] 3/d^2*(-1/2*f/b*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(2/3))+2*f/b*(1/2/b*(d*x+c)^(
2/3)*sin(a+b*(d*x+c)^(2/3))+1/2/b^2*cos(a+b*(d*x+c)^(2/3)))-1/2*(-c*f+d*e)/
b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4*(-c*f+d*e)/b^(3/2)*2^(1/2)*Pi^(1
/2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*Fresnel
S((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] time = 1.88412, size = 832, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*((8*(d*x + c)^(1/3)*abs(b)*cos((d*x + c)^(2/3)*b + a) - sqrt(pi)*(((c
os(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) - I*sin(1
/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (
I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arctan2(0, b)) + si
n(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*e
rf((d*x + c)^(1/3)*sqrt(I*b)) + ((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/
4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arctan2(0, b)) - I*sin(-1/4*
pi + 1/2*arctan2(0, b)))*cos(a) - (-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*c
os(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/
4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*sqrt(ab
s(b)))*e/(b*abs(b)) - (8*(d*x + c)^(1/3)*abs(b)*cos((d*x + c)^(2/3)*b + a)
- sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0
, b)) - I*sin(1/4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan2(0,
b)))*cos(a) - (I*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arct
an2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0,
 b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(I*b)) + ((cos(1/4*pi + 1/2*arctan2(0
, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arctan2(0, b)
) - I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (-I*cos(1/4*pi + 1/2*arcta
n2(0, b)) - I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0
, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(
-I*b)))*sqrt(abs(b)))*c*f/(b*d*abs(b)) - 8*(2*(d*x + c)^(2/3)*b*sin((d*x +
c)^(2/3)*b + a) - ((d*x + c)^(4/3)*b^2 - 2)*cos((d*x + c)^(2/3)*b + a))*f/(
b^3*d))/d
```

Fricas [A] time = 1.87867, size = 450, normalized size = 1.85

$$\frac{3\left(\sqrt{2}\pi\left(bde-bcf\right)\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\pi\left(bde-bcf\right)\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)+4\left(dx+c\right)^{\frac{2}{3}}bf\sin\right.}{4\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/4*(sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x
+ c)^(1/3)*sqrt(b/pi)) - sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*fresnel_sin(
sqrt(2)*(d*x + c)^(1/3)*sqrt(b/pi))*sin(a) + 4*(d*x + c)^(2/3)*b*f*sin((d*x
 + c)^(2/3)*b + a) - 2*((b^2*d*f*x + b^2*d*e)*(d*x + c)^(1/3) - 2*f)*cos((d
*x + c)^(2/3)*b + a))/(b^3*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)\sin\left(a+b\left(c+dx\right)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] time = 1.2604, size = 549, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/8*((sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*
sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)
*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(
b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3)*b
 + I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)*e - (sqrt(2
)*sqrt(pi)*c*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*sqrt(abs(b)
))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*c*erf(-1/2
*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/ab
s(b) + 1)*sqrt(abs(b))) + 2*I*(I*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^
2*c - 2*(d*x + c)^(2/3)*b - 2*I)*e^(I*(d*x + c)^(2/3)*b + I*a)/b^3 + 2*I*(I
*(d*x + c)^(4/3)*b^2 - I*(d*x + c)^(1/3)*b^2*c + 2*(d*x + c)^(2/3)*b - 2*I)
*e^(-I*(d*x + c)^(2/3)*b - I*a)/b^3)*f/d)/d
```

3.214 $\int \sin\left(a+b(c+dx)^{2/3}\right) dx$

Optimal. Leaf size=130

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) + (3*Sqrt[Pi/2]*Cos
[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d) - (3*Sqrt[P
i/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d)
```

Rubi [A] time = 0.0739272, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3363, 3385, 3354, 3352, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d}-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) + (3*Sqrt[Pi/2]*Cos
[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)])/(2*b^(3/2)*d) - (3*Sqrt[P
i/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(2*b^(3/2)*d)
```

Rule 3363

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a+b(c+dx)^{2/3}\right)dx = \frac{3\,\text{Subst}\left(\int x^2\sin\left(a+bx^2\right)dx, x, \sqrt[3]{c+dx}\right)}{d}$$
$$= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\,\text{Subst}\left(\int \cos\left(a+bx^2\right)dx, x, \sqrt[3]{c+dx}\right)}{2bd}$$
$$= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{(3\cos(a))\,\text{Subst}\left(\int \cos\left(bx^2\right)dx, x, \sqrt[3]{c+dx}\right)}{2bd} - \frac{(3\sin$$
$$= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{3\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{2b^{3/2}d} - \frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+}\right.}{2b^{3/2}d}$$

Mathematica [A] time = 0.147802, size = 114, normalized size = 0.88

$$-\frac{3\left(-\sqrt{2\pi}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)+\sqrt{2\pi}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+2\sqrt{b}\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)\right)}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(2*Sqrt[b]*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)] - Sqrt[2*Pi]*Cos[
a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)] + Sqrt[2*Pi]*FresnelS[Sqrt[
b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(4*b^(3/2)*d)
```

Maple [A] time = 0.006, size = 86, normalized size = 0.7

$$3\frac{1}{d}\left(-1/2\frac{\sqrt[3]{dx+c}\cos\left(a+b\left(dx+c\right)^{2/3}\right)}{b}+1/4\frac{\sqrt{2}\sqrt{\pi}}{b^{3/2}}\left(\cos\left(a\right)\text{FresnelC}\left(\frac{\sqrt[3]{dx+c}\sqrt{b}\sqrt{2}}{\sqrt{\pi}}\right)-\sin\left(a\right)\text{FresnelS}\left(\frac{\sqrt[3]{dx+c}\sqrt{}}{\sqrt{\pi}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] 3/d*(-1/2/b*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(2/3))+1/4/b^(3/2)*2^(1/2)*Pi^(1/
2)*(cos(a)*FresnelC((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*FresnelS
((d*x+c)^(1/3)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Maxima [C] time = 1.62291, size = 373, normalized size = 2.87

$$-\frac{3\left(8\left(dx+c\right)^{\frac{1}{3}}|b|\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)-\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,b\right)\right)+\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,b\right)\right)-i\sin\left(\frac{1}{4}\pi+\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] -3/16*(8*(d*x + c)^(1/3)*abs(b)*cos((d*x + c)^(2/3)*b + a) - sqrt(pi)*(((co
s(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4*pi + 1/2*arctan2(0, b)) - I*sin(1/
```

```
4*pi + 1/2*arctan2(0, b)) + I*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (I
*cos(1/4*pi + 1/2*arctan2(0, b)) + I*cos(-1/4*pi + 1/2*arctan2(0, b)) + sin
(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*er
f((d*x + c)^(1/3)*sqrt(I*b)) + ((cos(1/4*pi + 1/2*arctan2(0, b)) + cos(-1/4
*pi + 1/2*arctan2(0, b)) + I*sin(1/4*pi + 1/2*arctan2(0, b)) - I*sin(-1/4*p
i + 1/2*arctan2(0, b)))*cos(a) - (-I*cos(1/4*pi + 1/2*arctan2(0, b)) - I*co
s(-1/4*pi + 1/2*arctan2(0, b)) + sin(1/4*pi + 1/2*arctan2(0, b)) - sin(-1/4
*pi + 1/2*arctan2(0, b)))*sin(a))*erf((d*x + c)^(1/3)*sqrt(-I*b)))*sqrt(abs
(b)))/(b*d*abs(b))
```

Fricas [A] time = 1.79082, size = 297, normalized size = 2.28

$$\frac{3\left(\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}(dx+c)^{\frac{1}{3}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)-2\left(dx+c\right)^{\frac{1}{3}}b\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)\right)}{4\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 3/4*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*(d*x + c)^(1/3)*sqrt(
b/pi)) - sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*(d*x + c)^(1/3)*sqrt(b/p
i))*sin(a) - 2*(d*x + c)^(1/3)*b*cos((d*x + c)^(2/3)*b + a))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(a+b\left(c+dx\right)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] time = 1.19314, size = 230, normalized size = 1.77

$$-\frac{3\left(\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{2}(dx+c)^{\frac{1}{3}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{2\,(dx+c)^{\frac{1}{3}}e^{\left(i(dx+c)^{\frac{2}{3}}b+ia\right)}}{b}+\frac{2\,(dx+c)^{\frac{1}{3}}e^{\left(-i(dx+c)^{\frac{2}{3}}b-i\cdots\right.}}{b}\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/8*(sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(-I*b/abs(b) + 1)*s
qrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + sqrt(2)*sqrt(pi)*
erf(-1/2*sqrt(2)*(d*x + c)^(1/3)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b
*(I*b/abs(b) + 1)*sqrt(abs(b))) + 2*(d*x + c)^(1/3)*e^(I*(d*x + c)^(2/3)*b
+ I*a)/b + 2*(d*x + c)^(1/3)*e^(-I*(d*x + c)^(2/3)*b - I*a)/b)/d
```

3.215 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi [A] time = 0.0133951, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

Mathematica [A] time = 21.8758, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x), x]
```

Maple [A] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(a+b\,(dx+c)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\left(c+dx\right)^{\frac{2}{3}}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e), x)
```

3.216 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi [A] time = 0.01335, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2} dx = \int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2} dx$$

Mathematica [A] time = 20.7271, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(e+fx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^2} \sin\left(a+b\left(dx+c\right)^{\frac{2}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{f^2x^2+2efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e+fx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(f*x+e)**2,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e + f*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(f*x + e)^2, x)
```

3.217 $\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal. Leaf size=855

result too large to display

```
[Out] (b^5*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(120*d^3) -
(b^7*f^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(120960*d^3) + (b*(d*e
 - c*f)^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^3) - (b^3*f*(d*e
 - c*f)*(c + d*x)*Cos[a + b/(c + d*x)^(1/3)])/(60*d^3) + (b^5*f^2*(c + d*x)
^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(20160*d^3) + (b*f*(d*e - c*f)*(c + d*x)
^(5/3)*Cos[a + b/(c + d*x)^(1/3)])/(5*d^3) - (b^3*f^2*(c + d*x)^2*Cos[a + b
/(c + d*x)^(1/3)])/(1008*d^3) + (b*f^2*(c + d*x)^(8/3)*Cos[a + b/(c + d*x)^
(1/3)])/(24*d^3) - (b^9*f^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(120960*
d^3) + (b^3*(d*e - c*f)^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(2*d^3) +
(b^6*f*(d*e - c*f)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(120*d^3) + (b^8*
f^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(120960*d^3) - (b^2*(d*e -
c*f)^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^3) + (b^4*f*(d*e -
c*f)*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(120*d^3) - (b^6*f^2*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/(60480*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin
[a + b/(c + d*x)^(1/3)])/d^3 - (b^2*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b
/(c + d*x)^(1/3)])/(20*d^3) + (b^4*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^
(1/3)])/(5040*d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])
/d^3 - (b^2*f^2*(c + d*x)^(7/3)*Sin[a + b/(c + d*x)^(1/3)])/(168*d^3) + (f^
2*(c + d*x)^3*Sin[a + b/(c + d*x)^(1/3)])/(3*d^3) + (b^6*f*(d*e - c*f)*Cos[
a]*SinIntegral[b/(c + d*x)^(1/3)])/(120*d^3) + (b^9*f^2*Sin[a]*SinIntegral[
b/(c + d*x)^(1/3)])/(120960*d^3) - (b^3*(d*e - c*f)^2*Sin[a]*SinIntegral[b/
(c + d*x)^(1/3)])/(2*d^3)
```

Rubi [A] time = 1.05102, antiderivative size = 855, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3431, 3297, 3303, 3299, 3302}

$$-\frac{f^2\cos(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)b^9}{120960d^3}+\frac{f^2\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)b^9}{120960d^3}+\frac{f^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)b^8}{120960d^3}-\frac{f^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{\cdots}}\right.}{120960d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b^5*f*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(120*d^3) -
(b^7*f^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(120960*d^3) + (b*(d*e
 - c*f)^2*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^3) - (b^3*f*(d*e
 - c*f)*(c + d*x)*Cos[a + b/(c + d*x)^(1/3)])/(60*d^3) + (b^5*f^2*(c + d*x)
^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(20160*d^3) + (b*f*(d*e - c*f)*(c + d*x)
^(5/3)*Cos[a + b/(c + d*x)^(1/3)])/(5*d^3) - (b^3*f^2*(c + d*x)^2*Cos[a + b
/(c + d*x)^(1/3)])/(1008*d^3) + (b*f^2*(c + d*x)^(8/3)*Cos[a + b/(c + d*x)^
(1/3)])/(24*d^3) - (b^9*f^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(120960*
d^3) + (b^3*(d*e - c*f)^2*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(2*d^3) +
(b^6*f*(d*e - c*f)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(120*d^3) + (b^8*
f^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(120960*d^3) - (b^2*(d*e -
c*f)^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^3) + (b^4*f*(d*e -
c*f)*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(120*d^3) - (b^6*f^2*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/(60480*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin
[a + b/(c + d*x)^(1/3)])/d^3 - (b^2*f*(d*e - c*f)*(c + d*x)^(4/3)*Sin[a + b
```

```
/(c + d*x)^(1/3)])/(20*d^3) + (b^4*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^
(1/3)])/(5040*d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])
/d^3 - (b^2*f^2*(c + d*x)^(7/3)*Sin[a + b/(c + d*x)^(1/3)])/(168*d^3) + (f^
2*(c + d*x)^3*Sin[a + b/(c + d*x)^(1/3)])/(3*d^3) + (b^6*f*(d*e - c*f)*Cos[
a]*SinIntegral[b/(c + d*x)^(1/3)])/(120*d^3) + (b^9*f^2*Sin[a]*SinIntegral[
b/(c + d*x)^(1/3)])/(120960*d^3) - (b^3*(d*e - c*f)^2*Sin[a]*SinIntegral[b/
(c + d*x)^(1/3)])/(2*d^3)
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) dx = -\frac{3\,\text{Subst}\left(\int \left(\frac{f^2\sin(a+bx)}{d^2x^{10}}+\frac{2f(de-cf)\sin(a+bx)}{d^2x^7}+\frac{(de-cf)^2\sin(a+bx)}{d^2x^4}\right)dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{(3f^2)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{10}}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d^3} - \frac{(6f(de-cf))\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^7}dx, x, \frac{1}{\sqrt[3]{c+\cdots}}\right)}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^3} + \frac{f^2(c+\cdots}{}$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} + \cdots$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} + \frac{bf(de-cf)(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{5d^3} + \cdots$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^3f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} + \cdots$$

$$= \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \frac{b^3f(de-cf)(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{60d^3} + \cdots$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \cdots$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} + \frac{b(de-cf)^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^3} - \cdots$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} + \frac{b(de-c\cdots}{}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} + \frac{b(de-c\cdots}{}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} + \frac{b(de-c\cdots}{}$$

$$= \frac{b^5f(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^3} - \frac{b^7f^2(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120960d^3} + \frac{b(de-c\cdots}{}$$

Mathematica [C] time = 4.58093, size = 929, normalized size = 1.09

$$-\frac{i\left((\cos(a)+i\sin(a))\left(-if^2\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)b^9-1008cf^2\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)b^6+1008def\,\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)b^6+60480id^2e^2\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)b^3+\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] ((-I/241920)*((Cos[a] + I*Sin[a])*((60480*I)*b^3*d^2*e^2*ExpIntegralEi[(I*b
)/(c + d*x)^(1/3)] + 1008*b^6*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] -
(120960*I)*b^3*c*d*e*f*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - I*b^9*f^2*Exp
IntegralEi[(I*b)/(c + d*x)^(1/3)] - 1008*b^6*c*f^2*ExpIntegralEi[(I*b)/(c +
 d*x)^(1/3)] + (60480*I)*b^3*c^2*f^2*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] +
 (c + d*x)^(1/3)*(b^8*f^2 - I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*x)
^(2/3) + (24*I)*b^3*f*(c + d*x)^(2/3)*(-84*d*e + 79*c*f - 5*d*f*x) + (6*I)*
b^5*f*(168*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*e - 41*c
*f + d*f*x) + 40320*(c + d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e
```

```
^2 + 3*e*f*x + f^2*x^2)) + (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d*f
*(48*e + 7*f*x) + d^2*(60*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2*f
^2 - 2*c*d*f*(399*e + 16*f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2)))*(Cos
[b/(c + d*x)^(1/3)] + I*Sin[b/(c + d*x)^(1/3)])) - ((c + d*x)^(1/3)*(b^8*f^
2 + I*b^7*f^2*(c + d*x)^(1/3) - 2*b^6*f^2*(c + d*x)^(2/3) - (6*I)*b^5*f*(16
8*d*e - 167*c*f + d*f*x) + 24*b^4*f*(c + d*x)^(1/3)*(42*d*e - 41*c*f + d*f*
x) + (24*I)*b^3*f*(c + d*x)^(2/3)*(84*d*e - 79*c*f + 5*d*f*x) + 40320*(c +
d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))
- (1008*I)*b*(c + d*x)^(1/3)*(41*c^2*f^2 - 2*c*d*f*(48*e + 7*f*x) + d^2*(60
*e^2 + 24*e*f*x + 5*f^2*x^2)) - 144*b^2*(383*c^2*f^2 - 2*c*d*f*(399*e + 16*
f*x) + d^2*(420*e^2 + 42*e*f*x + 5*f^2*x^2))) + I*b^3*(-60480*d^2*e^2 + 100
8*((-I)*b^3 + 120*c)*d*e*f + (b^6 + (1008*I)*b^3*c - 60480*c^2)*f^2)*ExpInt
egralEi[((-I)*b)/(c + d*x)^(1/3)]*(Cos[b/(c + d*x)^(1/3)] + I*Sin[b/(c + d*
x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)] - I*Sin[a + b/(c + d*x)^(1/3)])))/d
^3
```

Maple [A] time = 0.078, size = 936, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x)
```

```
[Out] -3/d^3*b^3*(d^2*e^2*(-1/3*sin(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/6*cos(a+b/(d
*x+c)^(1/3))*(d*x+c)^(2/3)/b^2+1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/b+1
/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+c^2*f^2*(-1/3
*sin(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/6*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(2/3
)/b^2+1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/b+1/6*Si(b/(d*x+c)^(1/3))*si
n(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+b^6*f^2*(-1/9*sin(a+b/(d*x+c)^(1/3))*(
d*x+c)^3/b^9-1/72*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(8/3)/b^8+1/504*sin(a+b/(d
*x+c)^(1/3))*(d*x+c)^(7/3)/b^7+1/3024*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^2/b^6-
1/15120*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(5/3)/b^5-1/60480*cos(a+b/(d*x+c)^(1
/3))*(d*x+c)^(4/3)/b^4+1/181440*sin(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3+1/362880
*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(2/3)/b^2-1/362880*sin(a+b/(d*x+c)^(1/3))*(
d*x+c)^(1/3)/b-1/362880*Si(b/(d*x+c)^(1/3))*sin(a)+1/362880*Ci(b/(d*x+c)^(1
/3))*cos(a))-2*c*d*e*f*(-1/3*sin(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/6*cos(a+b
/(d*x+c)^(1/3))*(d*x+c)^(2/3)/b^2+1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/
b+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+2*b^3*d*e*
f*(-1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^2/b^6-1/30*cos(a+b/(d*x+c)^(1/3))*(d
*x+c)^(5/3)/b^5+1/120*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(4/3)/b^4+1/360*cos(a+
b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/720*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(2/3)/b^2
-1/720*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/b-1/720*Si(b/(d*x+c)^(1/3))*cos
(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a))-2*b^3*c*f^2*(-1/6*sin(a+b/(d*x+c)^(1/
3))*(d*x+c)^2/b^6-1/30*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(5/3)/b^5+1/120*sin(a
+b/(d*x+c)^(1/3))*(d*x+c)^(4/3)/b^4+1/360*cos(a+b/(d*x+c)^(1/3))*(d*x+c)/b^
3-1/720*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(2/3)/b^2-1/720*cos(a+b/(d*x+c)^(1/3
))*(d*x+c)^(1/3)/b-1/720*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3
))*sin(a)))
```

Maxima [C] time = 2.10109, size = 1354, normalized size = 1.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/241920*(60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(
a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 +
 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x
 + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))
)*e^2 - 120960*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a
) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 +
2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x
+ c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))
*c*e*f/d + 60480*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos
(a) + (I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3
+ 2*(d*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*
x + c)^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)
))*c^2*f^2/d^2 + 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(
1/3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a)
)*b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*co
s(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*
x + c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(
1/3)))*e*f/d - 1008*(((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/
3)))*cos(a) + (Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*
b^6 + 2*((d*x + c)^(1/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*cos(
((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*x
+ c)^(4/3)*b^2 + 120*(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/
3)))*c*f^2/d^2 - (((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos
(a) - (-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^9
 + 2*((d*x + c)^(2/3)*b^7 - 6*(d*x + c)^(4/3)*b^5 + 120*(d*x + c)^2*b^3 - 5
040*(d*x + c)^(8/3)*b)*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d
*x + c)^(1/3)*b^8 - 2*(d*x + c)*b^6 + 24*(d*x + c)^(5/3)*b^4 - 720*(d*x + c
)^(7/3)*b^2 + 40320*(d*x + c)^3)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3
)))*f^2/d^2)/d
```

Fricas [A] time = 2.17636, size = 1670, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -1/241920*(2*(120*b^3*d^2*f^2*x^2 + 2016*b^3*c*d*e*f - 1896*b^3*c^2*f^2 + 4
8*(42*b^3*d^2*e*f - 37*b^3*c*d*f^2)*x - (5040*b*d^2*f^2*x^2 + 60480*b*d^2*e
^2 - 96768*b*c*d*e*f - (b^7 - 41328*b*c^2)*f^2 + 2016*(12*b*d^2*e*f - 7*b*c
*d*f^2)*x)*(d*x + c)^(2/3) - 6*(b^5*d*f^2*x + 168*b^5*d*e*f - 167*b^5*c*f^2
)*(d*x + c)^(1/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - ((604
80*b^3*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*cos(a) + 1
008*(b^6*d*e*f - b^6*c*f^2)*sin(a))*cos_integral(b/(d*x + c)^(1/3)) - ((604
80*b^3*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*cos(a) + 1
008*(b^6*d*e*f - b^6*c*f^2)*sin(a))*cos_integral(-b/(d*x + c)^(1/3)) - 2*(4
0320*d^3*f^2*x^3 + 120960*d^3*e*f*x^2 + 120960*c*d^2*e^2 - 120960*c^2*d*e*f
 - 2*(b^6*c - 20160*c^3)*f^2 - 2*(b^6*d*f^2 - 60480*d^3*e^2)*x + 24*(b^4*d*
f^2*x + 42*b^4*d*e*f - 41*b^4*c*f^2)*(d*x + c)^(2/3) - (720*b^2*d^2*f^2*x^2
 + 60480*b^2*d^2*e^2 - 114912*b^2*c*d*e*f - (b^8 - 55152*b^2*c^2)*f^2 + 288
*(21*b^2*d^2*e*f - 16*b^2*c*d*f^2)*x)*(d*x + c)^(1/3))*sin((a*d*x + a*c + (
d*x + c)^(2/3)*b)/(d*x + c)) - 2*(1008*(b^6*d*e*f - b^6*c*f^2)*cos(a) - (60
480*b^3*d^2*e^2 - 120960*b^3*c*d*e*f - (b^9 - 60480*b^3*c^2)*f^2)*sin(a))*s
```

```
in_integral(b/(d*x + c)^(1/3)))/d^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e+fx)^2 \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^(1/3)), x)
```

3.218 $\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\,dx$

Optimal. Leaf size=419

$$\frac{b^3\cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}+\frac{b^6f\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}-\frac{b^3\sin(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}-\frac{b^2\sqrt[3]{c+dx}(}{}$$

```
[Out] (b^5*f*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(240*d^2) + (b*(d*e - c*
f)*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^2) - (b^3*f*(c + d*x)*C
os[a + b/(c + d*x)^(1/3)])/(120*d^2) + (b*f*(c + d*x)^(5/3)*Cos[a + b/(c +
d*x)^(1/3)])/(10*d^2) + (b^3*(d*e - c*f)*Cos[a]*CosIntegral[b/(c + d*x)^(1/
3)])/(2*d^2) + (b^6*f*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(240*d^2) - (b
^2*(d*e - c*f)*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2) + (b^4*f
*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(240*d^2) + ((d*e - c*f)*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/d^2 - (b^2*f*(c + d*x)^(4/3)*Sin[a + b/(c
+ d*x)^(1/3)])/(40*d^2) + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2
) + (b^6*f*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(240*d^2) - (b^3*(d*e - c
*f)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2*d^2)
```

Rubi [A] time = 0.504016, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3431, 3297, 3303, 3299, 3302}

$$\frac{b^3\cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}+\frac{b^6f\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2}-\frac{b^3\sin(a)(de-cf)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2}-\frac{b^2\sqrt[3]{c+dx}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```


```
[Out] (b^5*f*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(240*d^2) + (b*(d*e - c*
f)*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d^2) - (b^3*f*(c + d*x)*C
os[a + b/(c + d*x)^(1/3)])/(120*d^2) + (b*f*(c + d*x)^(5/3)*Cos[a + b/(c +
d*x)^(1/3)])/(10*d^2) + (b^3*(d*e - c*f)*Cos[a]*CosIntegral[b/(c + d*x)^(1/
3)])/(2*d^2) + (b^6*f*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(240*d^2) - (b
^2*(d*e - c*f)*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2) + (b^4*f
*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(240*d^2) + ((d*e - c*f)*(c +
d*x)*Sin[a + b/(c + d*x)^(1/3)])/d^2 - (b^2*f*(c + d*x)^(4/3)*Sin[a + b/(c
+ d*x)^(1/3)])/(40*d^2) + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^(1/3)])/(2*d^2
) + (b^6*f*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(240*d^2) - (b^3*(d*e - c
*f)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2*d^2)
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{3\,\text{Subst}\left(\int\left(\frac{f\sin(a+bx)}{dx^7}+\frac{(de-cf)\sin(a+bx)}{dx^4}\right)dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{(3f)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^7}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^2} - \frac{(3(de-cf))\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^4}\,dx,x,\frac{1}{\sqrt[3]{c+\cdots}}\right)}{d^2}\cdots$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{(bf)\,\text{Subst}\left(\int\cdots\right)}{\cdots}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} + \frac{(de-cf)\cdots}{\cdots}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} + \frac{bf(c+dx)^{5/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{10d^2} - \frac{b^2(de-c\cdots}{\cdots}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)\cdots}{\cdots}$$

$$= \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+dx)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{120d^2} + \frac{bf(c+dx)\cdots}{\cdots}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+d\cdots}{\cdots}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+d\cdots}{\cdots}$$

$$= \frac{b^5f\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{240d^2} + \frac{b(de-cf)(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d^2} - \frac{b^3f(c+d\cdots}{\cdots}$$

Mathematica [A] time = 0.855827, size = 540, normalized size = 1.29

$$\frac{b^3f\left(b^3\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)-120c\cos(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+120c\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\cdots}{240d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (e*(c + d*x)^(1/3)*Cos[b/(c + d*x)^(1/3)]*(b*(c + d*x)^(1/3)*Cos[a] - b^2*S
in[a] + 2*(c + d*x)^(2/3)*Sin[a]))/(2*d) + (f*(c + d*x)^(1/3)*Cos[b/(c + d*
x)^(1/3)]*(b^5*Cos[a] - 120*b*c*(c + d*x)^(1/3)*Cos[a] - 2*b^3*(c + d*x)^(2
/3)*Cos[a] + 24*b*(c + d*x)^(4/3)*Cos[a] + 120*b^2*c*Sin[a] + b^4*(c + d*x)
^(1/3)*Sin[a] - 240*c*(c + d*x)^(2/3)*Sin[a] - 6*b^2*(c + d*x)*Sin[a] + 120
*(c + d*x)^(5/3)*Sin[a]))/(240*d^2) + (e*(c + d*x)^(1/3)*(-(b^2*Cos[a]) + 2
*(c + d*x)^(2/3)*Cos[a] - b*(c + d*x)^(1/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)])
/(2*d) + (f*(c + d*x)^(1/3)*(120*b^2*c*Cos[a] + b^4*(c + d*x)^(1/3)*Cos[a]
- 240*c*(c + d*x)^(2/3)*Cos[a] - 6*b^2*(c + d*x)*Cos[a] + 120*(c + d*x)^(5/
3)*Cos[a] - b^5*Sin[a] + 120*b*c*(c + d*x)^(1/3)*Sin[a] + 2*b^3*(c + d*x)^(
2/3)*Sin[a] - 24*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(1/3)])/(240*d^2
) + (b^3*e*(Cos[a]*CosIntegral[b/(c + d*x)^(1/3)] - Sin[a]*SinIntegral[b/(c
 + d*x)^(1/3)]))/(2*d) + (b^3*f*(-120*c*Cos[a]*CosIntegral[b/(c + d*x)^(1/3
)] + b^3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a] + b^3*Cos[a]*SinIntegral[b/(
c + d*x)^(1/3)] + 120*c*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)]))/(240*d^2)
```

Maple [A] time = 0.03, size = 391, normalized size = 0.9

$$-3\,{\frac{{b}^{3}}{{d}^{2}}}\left(-cf\left(-1/3\,{\frac{dx+c}{{b}^{3}}}\sin\left(a+{\frac{b}{\sqrt[3]{dx+c}}}\right)-1/6\,{\frac{(dx+c)^{2/3}}{{b}^{2}}}\cos\left(a+{\frac{b}{\sqrt[3]{dx+c}}}\right)+1/6\,{\frac{\sqrt[3]{dx+c}}{b}}\sin\left(a+{\frac{b}{\sqrt[3]{dx+c}}}\right)+1/6\,S\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x)
```

```
[Out] -3/d^2*b^3*(-c*f*(-1/3*sin(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/6*cos(a+b/(d*x+
c)^(1/3))*(d*x+c)^(2/3)/b^2+1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/b+1/6*
Si(b/(d*x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))+d*e*(-1/3*sin(a+
b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/6*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(2/3)/b^2+1
/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/b+1/6*Si(b/(d*x+c)^(1/3))*sin(a)-1/
6*Ci(b/(d*x+c)^(1/3))*cos(a))+b^3*f*(-1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^2/
b^6-1/30*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(5/3)/b^5+1/120*sin(a+b/(d*x+c)^(1/
3))*(d*x+c)^(4/3)/b^4+1/360*cos(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/720*sin(a+
b/(d*x+c)^(1/3))*(d*x+c)^(2/3)/b^2-1/720*cos(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/
3)/b-1/720*Si(b/(d*x+c)^(1/3))*cos(a)-1/720*Ci(b/(d*x+c)^(1/3))*sin(a)))
```

Maxima [C] time = 1.57241, size = 618, normalized size = 1.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/480*(120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) +
(I*Ei(I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d
*x + c)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)
^(1/3)*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*e -
 120*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*Ei(
I*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*x + c
)^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)
*b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*c*f/d + (
```

```
((-I*Ei(I*b/(d*x + c)^(1/3)) + I*Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (Ei(I*b
/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^6 + 2*((d*x + c)^(1
/3)*b^5 - 2*(d*x + c)*b^3 + 24*(d*x + c)^(5/3)*b)*cos(((d*x + c)^(1/3)*a +
b)/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4 - 6*(d*x + c)^(4/3)*b^2 + 120*
(d*x + c)^2)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))*f/d)/d
```

Fricas [A] time = 2.03869, size = 814, normalized size = 1.94

$$2\left((dx+c)^{\frac{1}{3}}b^5f-2\,b^3dfx-2\,b^3cf+24\left(bdfx+5\,bde-4\,bcf\right)(dx+c)^{\frac{2}{3}}\right)\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)+\left(b^6f\sin\left(a\right)+120\left(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/480*(2*((d*x + c)^(1/3)*b^5*f - 2*b^3*d*f*x - 2*b^3*c*f + 24*(b*d*f*x + 5
*b*d*e - 4*b*c*f)*(d*x + c)^(2/3))*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d
*x + c)) + (b^6*f*sin(a) + 120*(b^3*d*e - b^3*c*f)*cos(a))*cos_integral(b/(
d*x + c)^(1/3)) + (b^6*f*sin(a) + 120*(b^3*d*e - b^3*c*f)*cos(a))*cos_integ
ral(-b/(d*x + c)^(1/3)) + 2*((d*x + c)^(2/3)*b^4*f + 120*d^2*f*x^2 + 240*d^
2*e*x + 240*c*d*e - 120*c^2*f - 6*(b^2*d*f*x + 20*b^2*d*e - 19*b^2*c*f)*(d*
x + c)^(1/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) + 2*(b^6*f*c
os(a) - 120*(b^3*d*e - b^3*c*f)*sin(a))*sin_integral(b/(d*x + c)^(1/3)))/d^
2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(1/3)), x)
```

3.219 $\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$

Optimal. Leaf size=136

$$\frac{b^3\cos(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^3\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d}+\frac{b(c+d}{}$$

```
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d) + (b^3*Cos[a]*CosInteg
ral[b/(c + d*x)^(1/3)])/(2*d) - (b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1
/3)])/(2*d) + ((c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/d - (b^3*Sin[a]*SinInt
egral[b/(c + d*x)^(1/3)])/(2*d)
```

Rubi [A] time = 0.161785, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3361, 3297, 3303, 3299, 3302}

$$\frac{b^3\cos(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^3\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}-\frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d}+\frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d}+\frac{b(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d) + (b^3*Cos[a]*CosInteg
ral[b/(c + d*x)^(1/3)])/(2*d) - (b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1
/3)])/(2*d) + ((c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/d - (b^3*Sin[a]*SinInt
egral[b/(c + d*x)^(1/3)])/(2*d)
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$
$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$
$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{b^2\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+}}\right.}{2d}$$
$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{b}{}$$
$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d} + \frac{(}{}$$
$$= \frac{b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{b^3\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} - \frac{b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d} + \frac{(c+}{}$$

Mathematica [A] time = 0.10153, size = 133, normalized size = 0.98

$$\frac{b^3\cos(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) - b^3\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right) - b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) + 2c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) + 2dx\sin\left(a+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] (b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^3*Cos[a]*CosIntegral[b/(c
 + d*x)^(1/3)] + 2*c*Sin[a + b/(c + d*x)^(1/3)] + 2*d*x*Sin[a + b/(c + d*x)
^(1/3)] - b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] - b^3*Sin[a]*SinIn
tegral[b/(c + d*x)^(1/3)])/(2*d)
```

Maple [A] time = 0.016, size = 108, normalized size = 0.8

$$-3\,\frac{b^3}{d}\left(-1/3\,\frac{dx+c}{b^3}\sin\left(a+\frac{b}{\sqrt[3]{dx+c}}\right) - 1/6\,\frac{(dx+c)^{2/3}}{b^2}\cos\left(a+\frac{b}{\sqrt[3]{dx+c}}\right) + 1/6\,\frac{\sqrt[3]{dx+c}}{b}\sin\left(a+\frac{b}{\sqrt[3]{dx+c}}\right) + 1/6\,\text{Si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3)),x)
```

```
[Out] -3/d*b^3*(-1/3*sin(a+b/(d*x+c)^(1/3))*(d*x+c)/b^3-1/6*cos(a+b/(d*x+c)^(1/3)
)*(d*x+c)^(2/3)/b^2+1/6*sin(a+b/(d*x+c)^(1/3))*(d*x+c)^(1/3)/b+1/6*Si(b/(d*
x+c)^(1/3))*sin(a)-1/6*Ci(b/(d*x+c)^(1/3))*cos(a))
```

Maxima [C] time = 1.2256, size = 186, normalized size = 1.37

$$\frac{\left(\left(\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)+\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\cos(a)+\left(i\,\mathrm{Ei}\left(\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)-i\,\mathrm{Ei}\left(-\frac{ib}{(dx+c)^{\frac{1}{3}}}\right)\right)\sin(a)\right)b^3+2\,(dx+c)^{\frac{2}{3}}b\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)-2\,(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/4*(((Ei(I*b/(d*x + c)^(1/3)) + Ei(-I*b/(d*x + c)^(1/3)))*cos(a) + (I*Ei(I
*b/(d*x + c)^(1/3)) - I*Ei(-I*b/(d*x + c)^(1/3)))*sin(a))*b^3 + 2*(d*x + c)
^(2/3)*b*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)) - 2*((d*x + c)^(1/3)*
b^2 - 2*d*x - 2*c)*sin(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3)))/d
```

Fricas [A] time = 1.82137, size = 412, normalized size = 3.03

$$\frac{b^3\cos(a)\,\mathrm{Ci}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+b^3\cos(a)\,\mathrm{Ci}\left(-\frac{b}{(dx+c)^{\frac{1}{3}}}\right)-2\,b^3\sin(a)\,\mathrm{Si}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)+2\,(dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-2\left((dx+\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*cos(a)*cos_integral(b/(d*x + c)^(1/3)) + b^3*cos(a)*cos_integral(-
b/(d*x + c)^(1/3)) - 2*b^3*sin(a)*sin_integral(b/(d*x + c)^(1/3)) + 2*(d*x
+ c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 2*((d*x + c
)^(1/3)*b^2 - 2*d*x - 2*c)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))
)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3)), x)
```

3.220 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx$

Optimal. Leaf size=434

$$\frac{\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right.}{}$$

```
[Out] (-3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/f + (CosIntegral[(b*f^(1/3))/(d*
e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - (b*f^(1/3))/(d*e - c*f)^(1/3)])
/f + (CosIntegral[((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1
/3)]*Sin[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f + (CosIntegral[((
-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - ((-1)^(
2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f - (3*Cos[a]*SinIntegral[b/(c + d*x)^(
1/3)])/f - (Cos[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(
(-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1/3)])/f + (Cos[a -
(b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(d*e - c*f)^(1/3) +
b/(c + d*x)^(1/3)])/f + (Cos[a - ((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*
SinIntegral[((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)])/
f
```

Rubi [A] time = 1.92008, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3431, 3303, 3299, 3302, 3345}

$$\frac{\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\sin\left(a+\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{f}+\frac{\sin\left(a-\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```


```
[Out] (-3*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/f + (CosIntegral[(b*f^(1/3))/(d*
e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - (b*f^(1/3))/(d*e - c*f)^(1/3)])
/f + (CosIntegral[((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1
/3)]*Sin[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f + (CosIntegral[((
-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)]*Sin[a - ((-1)^(
2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)])/f - (3*Cos[a]*SinIntegral[b/(c + d*x)^(
1/3)])/f - (Cos[a + ((-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(
(-1)^(1/3)*b*f^(1/3))/(d*e - c*f)^(1/3) - b/(c + d*x)^(1/3)])/f + (Cos[a -
(b*f^(1/3))/(d*e - c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(d*e - c*f)^(1/3) +
b/(c + d*x)^(1/3)])/f + (Cos[a - ((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3)]*
SinIntegral[((-1)^(2/3)*b*f^(1/3))/(d*e - c*f)^(1/3) + b/(c + d*x)^(1/3)])/
f
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{e+fx}\,dx = -\frac{3\,\text{Subst}\left(\int \left(\frac{d\sin(a+bx)}{fx}+\frac{d(-de+cf)x^2\sin(a+bx)}{f\left(f+(de-cf)x^3\right)}\right)dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f} + \frac{(3(de-cf))\,\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{f+(de-cf)x^3}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f}$$

$$= \frac{(3(de-cf))\,\text{Subst}\left(\int \left(\frac{\sin(a+bx)}{3(de-cf)^{2/3}\left(\sqrt[3]{f}+\sqrt[3]{de-cf}x\right)}+\frac{\sin(a+bx)}{3(de-cf)^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{f}+\sqrt[3]{de-cf}x\right)}+\frac{\sin(a+b}{3(de-cf)^{2/3}\left((-1)^{2/3}\right.}\cdots\right.\right.}{f}$$

$$= -\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\sqrt[3]{de-cf}\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{f}+\sqrt[3]{de-cf}x}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{f} + \cdots$$

$$= -\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} - \frac{3\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{f} + \frac{\left(\sqrt[3]{de-cf}\cos\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-c}}\cdots\right)}{\sqrt[3]{f}+\sqrt[3]{d}\cdots}\right.}{f}$$

$$= -\frac{3\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{f} + \frac{\text{Ci}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}+\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a-\frac{b\sqrt[3]{f}}{\sqrt[3]{de-cf}}\right)}{f} + \frac{\text{Ci}\left(\frac{\sqrt[3]{-1}b\sqrt[3]{f}}{\sqrt[3]{de-cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)\sin\left(a\cdots\right.}{f}$$

Mathematica [C] time = 2.74273, size = 170, normalized size = 0.39

$$\frac{i\left((\cos(a)-i\sin(a))\left(\text{RootSum}\left[\#1^3f-cf+de\&, e^{-\frac{ib}{\#1}}\text{Ei}\left(-ib\left(\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{\#1}\right)\right)\&\right]-3\text{Ei}\left(-\frac{ib}{\sqrt[3]{c+dx}}\right)\right)+(\cos(a)+i\sin(a\cdots\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x),x]
```

```
[Out] ((I/2)*((-3*ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] + RootSum[d*e - c*f + f
*#1^3 & , ExpIntegralEi[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1) &
 ])*(Cos[a] - I*Sin[a]) + (3*ExpIntegralEi[(I*b)/(c + d*x)^(1/3)] - RootSum
[d*e - c*f + f*#1^3 & , E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(-1/3) -
#1^(-1))] & ])*(Cos[a] + I*Sin[a])))/f
```

Maple [C] time = 0.027, size = 156, normalized size = 0.4

$$-3\,b^3\left(-1/3\,\frac{1}{b^3 f}\sum_{\_R1=\mathrm{RootOf}\left(\left(cf-de\right)\_Z^3+\left(-3\,acf+3\,ade\right)\_Z^2+\left(3\,a^2cf-3\,a^2de\right)\_Z-a^3cf+a^3de-b^3f\right)}-\mathrm{Si}\left(-\frac{b}{\sqrt[3]{dx+c}}+\_R1-a\right)\cos\left(\_R1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x)
```

```
[Out] -3*b^3*(-1/3/b^3/f*sum(-Si(-b/(d*x+c)^(1/3)+_R1-a)*cos(_R1)+Ci(b/(d*x+c)^(1
/3)-_R1+a)*sin(_R1),_R1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^
2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f))+1/b^3/f*(Si(b/(d*x+c)^(1/3))*co
s(a)+Ci(b/(d*x+c)^(1/3))*sin(a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

Fricas [C] time = 2.34048, size = 1359, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(I*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x -
sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*
(I*sqrt(3) + 1) - I*a) - I*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e -
 c*f))^(1/3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*
f/(d*e - c*f))^(1/3)*(I*sqrt(3) + 1) + I*a) + I*Ei(1/2*(-2*I*(d*x + c)^(2/3
)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x +
 c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) - I*a) - I*Ei(1/2*
(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x
```

```
 + I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) +
1) + I*a) + 3*I*Ei(I*b/(d*x + c)^(1/3))*e^(I*a) - 3*I*Ei(-I*b/(d*x + c)^(1/
3))*e^(-I*a) - I*Ei((I*(d*x + c)^(2/3)*b + (-I*b^3*f/(d*e - c*f))^(1/3)*(d*
x + c))/(d*x + c))*e^(I*a - (-I*b^3*f/(d*e - c*f))^(1/3)) + I*Ei((-I*(d*x +
 c)^(2/3)*b + (I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(-I*a - (
I*b^3*f/(d*e - c*f))^(1/3)))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{\sqrt[3]{c+dx}}\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e), x)
```

3.221 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=566

$$-\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}+$$

```
[Out] -(b*d*Cos[a + (b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[(b*f^(1/3))/(-(
d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) -
((-1)^(2/3)*b*d*Cos[a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosInt
egral[((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*
f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((-1)^(1/3)*b*d*Cos[a - ((-1)^(1/3)*b*f^(1/
3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)
^(1/3) + b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((c + d*x)*
Sin[a + b/(c + d*x)^(1/3)])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*f^(1/
3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/
(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) - ((-1)^(2/3)*b*d*Sin[a
+ ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[((-1)^(2/3)*b*f^
(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)
^(4/3)) - ((-1)^(1/3)*b*d*Sin[a - ((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/
3)]*SinIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) + b/(c + d*x)^(
1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3))
```

Rubi [A] time = 2.62981, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3431, 3341, 3334, 3303, 3299, 3302}

$$-\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}-\frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{cf-de}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(cf-de)^{4/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```


```
[Out] -(b*d*Cos[a + (b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[(b*f^(1/3))/(-(
d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) -
((-1)^(2/3)*b*d*Cos[a + ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*CosInt
egral[((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*
f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((-1)^(1/3)*b*d*Cos[a - ((-1)^(1/3)*b*f^(1/
3))/(-(d*e) + c*f)^(1/3)]*CosIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)
^(1/3) + b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) + ((c + d*x)*
Sin[a + b/(c + d*x)^(1/3)])/((d*e - c*f)*(e + f*x)) - (b*d*Sin[a + (b*f^(1/
3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[(b*f^(1/3))/(-(d*e) + c*f)^(1/3) - b/
(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3)) - ((-1)^(2/3)*b*d*Sin[a
+ ((-1)^(2/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3)]*SinIntegral[((-1)^(2/3)*b*f^
(1/3))/(-(d*e) + c*f)^(1/3) - b/(c + d*x)^(1/3)])/(3*f^(2/3)*(-(d*e) + c*f)
^(4/3)) - ((-1)^(1/3)*b*d*Sin[a - ((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/
3)]*SinIntegral[((-1)^(1/3)*b*f^(1/3))/(-(d*e) + c*f)^(1/3) + b/(c + d*x)^(
1/3)])/(3*f^(2/3)*(-(d*e) + c*f)^(4/3))
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e+fx)^2}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^3\right)^2}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\frac{f}{d}+\left(e-\frac{cf}{d}\right)x^3}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{b\,\text{Subst}\left(\int \left(\frac{d\cos(a+bx)}{3f^{2/3}\left(\sqrt[3]{f}-\sqrt[3]{-de+cf}x\right)} + \frac{d\cos(a+bx)}{3f^{2/3}\left(\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{-de+cf}x\right)} + \frac{d\cos}{3f^{2/3}\left(\sqrt[3]{f}-(-\right.}\right.\right.}{de-cf}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt[3]{f}-\sqrt[3]{-de+cf}x}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cos(a+b}{\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{-}}\right.}{3f^{2/3}(de-}$$

$$= \frac{(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(de-cf)(e+fx)} - \frac{\left(bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-bx\right)}{\sqrt[3]{f}-\sqrt[3]{-de+cf}x}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(de-cf)} - \frac{\left(bd\right.}{}$$

$$= -\frac{bd\cos\left(a+\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Ci}\left(\frac{b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{b}{\sqrt[3]{c+dx}}\right)}{3f^{2/3}(-de+cf)^{4/3}} - \frac{(-1)^{2/3}bd\cos\left(a+\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}\right)\text{Ci}\left(\frac{(-1)^{2/3}b\sqrt[3]{f}}{\sqrt[3]{-de+cf}}-\frac{}{\sqrt[3]{c}}\right.}{3f^{2/3}(-de+cf)^{4/3}}$$

Mathematica [C] time = 1.2292, size = 313, normalized size = 0.55

$$\frac{(\cos(a)+i\sin(a))\left(bd(e+fx)\text{RootSum}\left[\#1^3f-cf+de\&, \frac{\text{Ei}\left(\frac{ib}{\sqrt[3]{c+dx}}\right)-e^{\frac{ib}{\#1}}\text{Ei}\left(ib\left(\frac{1}{\sqrt[3]{c+dx}}-\frac{1}{\#1}\right)\right)}{\#1}\&\right]+(c+dx)\left(-3f\sin\left(\frac{b}{\sqrt[3]{c+dx}}\right)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(e + f*x)^2,x]
```

```
[Out] ((Cos[a] + I*Sin[a])*(b*d*(e + f*x)*RootSum[d*e - c*f + f*#1^3 & , (ExpInte
gralEi[(I*b)/(c + d*x)^(1/3)] - E^((I*b)/#1)*ExpIntegralEi[I*b*((c + d*x)^(
-1/3) - #1^(-1))])/#1 & ] + (c + d*x)*((3*I)*f*Cos[b/(c + d*x)^(1/3)] - 3*f
*Sin[b/(c + d*x)^(1/3)])) + I*(-3*c*f - 3*d*f*x + b*d*(e + f*x)*RootSum[d*e
 - c*f + f*#1^3 & , (ExpIntegralEi[((-I)*b)/(c + d*x)^(1/3)] - ExpIntegralE
i[(-I)*b*((c + d*x)^(-1/3) - #1^(-1))]/E^((I*b)/#1))/#1 & ]*((-I)*Cos[b/(c
+ d*x)^(1/3)] + Sin[b/(c + d*x)^(1/3)]))*(Cos[a + b/(c + d*x)^(1/3)] - I*Si
n[a + b/(c + d*x)^(1/3)]))/(6*f*(-(d*e) + c*f)*(e + f*x))
```

Maple [C] time = 0.083, size = 1556, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x)
```

```
[Out] -3*d*b^3*(sin(a+b/(d*x+c)^(1/3))*(-2/3*a/b^3/f*(a+b/(d*x+c)^(1/3))^2+a^2/b^
3/f*(a+b/(d*x+c)^(1/3))-1/3*(a^3*c*f-a^3*d*e+b^3*f)/b^3/f/(c*f-d*e))/(c*f*(
```

```
a+b/(d*x+c)^(1/3))^3-d*e*(a+b/(d*x+c)^(1/3))^3-3*(a+b/(d*x+c)^(1/3))^2*a*c*
f+3*(a+b/(d*x+c)^(1/3))^2*a*d*e+3*(a+b/(d*x+c)^(1/3))*a^2*c*f-3*(a+b/(d*x+c
)^(1/3))*a^2*d*e-a^3*c*f+a^3*d*e-b^3*f)-2/9*a/b^3/f*sum(_R1/(_R1^2*c*f-_R1^
2*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c)^(1/3)+_R1-a)
*cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*f-d*e)*_Z^3+(-3
*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f))+1/9/b
^3/f*sum((2*_RR1^2*a*c*f-2*_RR1^2*a*d*e-3*_RR1*a^2*c*f+3*_RR1*a^2*d*e+a^3*c
*f-a^3*d*e+b^3*f)/(c*f-d*e)/(_RR1^2*c*f-_RR1^2*d*e-2*_RR1*a*c*f+2*_RR1*a*d*
e+a^2*c*f-a^2*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*sin(_RR1)+Ci(b/(d*x+c)^(1/3
)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-3*a*c*f+3*a*d*e)*_Z^2+(3*
a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f))+sin(a+b/(d*x+c)^(1/3))*(2/3*a
/b^3/f*(a+b/(d*x+c)^(1/3))^2-2/3*a^2/b^3/f*(a+b/(d*x+c)^(1/3)))/(c*f*(a+b/(
d*x+c)^(1/3))^3-d*e*(a+b/(d*x+c)^(1/3))^3-3*(a+b/(d*x+c)^(1/3))^2*a*c*f+3*(
a+b/(d*x+c)^(1/3))^2*a*d*e+3*(a+b/(d*x+c)^(1/3))*a^2*c*f-3*(a+b/(d*x+c)^(1/
3))*a^2*d*e-a^3*c*f+a^3*d*e-b^3*f)+2/9*a/b^3/f*sum((_R1+a)/(_R1^2*c*f-_R1^2
*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c)^(1/3)+_R1-a)*
cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*f-d*e)*_Z^3+(-3*
a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f))-2/9*a/
b^3/f*sum(_RR1/(_RR1*c*f-_RR1*d*e-a*c*f+a*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)
*sin(_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3
+(-3*a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f))+a
^2*(sin(a+b/(d*x+c)^(1/3))*(-1/3/b^3/f*(a+b/(d*x+c)^(1/3))+1/3*a/b^3/f)/(c*
f*(a+b/(d*x+c)^(1/3))^3-d*e*(a+b/(d*x+c)^(1/3))^3-3*(a+b/(d*x+c)^(1/3))^2*a
*c*f+3*(a+b/(d*x+c)^(1/3))^2*a*d*e+3*(a+b/(d*x+c)^(1/3))*a^2*c*f-3*(a+b/(d*
x+c)^(1/3))*a^2*d*e-a^3*c*f+a^3*d*e-b^3*f)-2/9/b^3/f*sum(1/(_R1^2*c*f-_R1^2
*d*e-2*_R1*a*c*f+2*_R1*a*d*e+a^2*c*f-a^2*d*e)*(-Si(-b/(d*x+c)^(1/3)+_R1-a)*
cos(_R1)+Ci(b/(d*x+c)^(1/3)-_R1+a)*sin(_R1)),_R1=RootOf((c*f-d*e)*_Z^3+(-3*
a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f))+1/9/b^
3/f*sum(1/(_RR1*c*f-_RR1*d*e-a*c*f+a*d*e)*(Si(-b/(d*x+c)^(1/3)+_RR1-a)*sin(
_RR1)+Ci(b/(d*x+c)^(1/3)-_RR1+a)*cos(_RR1)),_RR1=RootOf((c*f-d*e)*_Z^3+(-3*
a*c*f+3*a*d*e)*_Z^2+(3*a^2*c*f-3*a^2*d*e)*_Z-a^3*c*f+a^3*d*e-b^3*f)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

Fricas [C] time = 2.7331, size = 1908, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/12*((I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x - I*d*e + sqrt(3)*(d*f*x + d*e
))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt
```

```
(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(I*b^3*f/(d*e - c*f))^(1/3)*(I*s
qrt(3) + 1) - I*a) + (-I*b^3*f/(d*e - c*f))^(1/3)*(I*d*f*x + I*d*e - sqrt(3
)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b - (-I*b^3*f/(d*e - c*f))^(1/
3)*(d*x - sqrt(3)*(-I*d*x - I*c) + c))/(d*x + c))*e^(1/2*(-I*b^3*f/(d*e - c
*f))^(1/3)*(I*sqrt(3) + 1) + I*a) + (I*b^3*f/(d*e - c*f))^(1/3)*(-I*d*f*x -
 I*d*e - sqrt(3)*(d*f*x + d*e))*Ei(1/2*(-2*I*(d*x + c)^(2/3)*b - (I*b^3*f/(
d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c))*e^(1/2*(I*b
^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) - I*a) + (-I*b^3*f/(d*e - c*f))^(1
/3)*(I*d*f*x + I*d*e + sqrt(3)*(d*f*x + d*e))*Ei(1/2*(2*I*(d*x + c)^(2/3)*b
 - (-I*b^3*f/(d*e - c*f))^(1/3)*(d*x - sqrt(3)*(I*d*x + I*c) + c))/(d*x + c
))*e^(1/2*(-I*b^3*f/(d*e - c*f))^(1/3)*(-I*sqrt(3) + 1) + I*a) + (-I*b^3*f/
(d*e - c*f))^(1/3)*(-2*I*d*f*x - 2*I*d*e)*Ei((I*(d*x + c)^(2/3)*b + (-I*b^3
*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(I*a - (-I*b^3*f/(d*e - c*f))
^(1/3)) + (I*b^3*f/(d*e - c*f))^(1/3)*(2*I*d*f*x + 2*I*d*e)*Ei((-I*(d*x + c
)^(2/3)*b + (I*b^3*f/(d*e - c*f))^(1/3)*(d*x + c))/(d*x + c))*e^(-I*a - (I*
b^3*f/(d*e - c*f))^(1/3)) - 12*(d*f*x + c*f)*sin((a*d*x + a*c + (d*x + c)^(
2/3)*b)/(d*x + c)))/(d*e^2*f - c*e*f^2 + (d*e*f^2 - c*f^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(f*x + e)^2, x)
```

3.222 $\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal. Leaf size=630

$$\frac{b^3 f \cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}+\frac{2\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)^2\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d^3}+\frac{2\sqrt{2\pi}b^{3/2}\cos(a)(de-c}{d^3}$$

```
[Out] (2*b*(d*e - c*f)^2*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^3 - (8*b^3
*f^2*(c + d*x)*Cos[a + b/(c + d*x)^(2/3)])/(315*d^3) + (b*f*(d*e - c*f)*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*d^3) + (2*b*f^2*(c + d*x)^(7/3)
*Cos[a + b/(c + d*x)^(2/3)])/(21*d^3) + (b^3*f*(d*e - c*f)*Cos[a]*CosIntegr
al[b/(c + d*x)^(2/3)])/(2*d^3) - (16*b^(9/2)*f^2*Sqrt[2*Pi]*Cos[a]*FresnelC
[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/(315*d^3) + (2*b^(3/2)*(d*e - c*f)^
2*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^3 + (
2*b^(3/2)*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^
(1/3)]*Sin[a])/d^3 + (16*b^(9/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)]*Sin[a])/(315*d^3) + (16*b^4*f^2*(c + d*x)^(1/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(315*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(2*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)
^(2/3)])/d^3 - (4*b^2*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^(2/3)])/(105*
d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(2/3)])/d^3 + (f^2*(c
 + d*x)^3*Sin[a + b/(c + d*x)^(2/3)])/(3*d^3) - (b^3*f*(d*e - c*f)*Sin[a]*S
inIntegral[b/(c + d*x)^(2/3)])/(2*d^3)
```

Rubi [A] time = 0.747672, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {3433, 3409, 3387, 3388, 3353, 3352, 3351, 3379, 3297, 3303, 3299, 3302, 3354}

$$\frac{b^3 f \cos(a)(de-cf)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}+\frac{2\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)^2\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d^3}+\frac{2\sqrt{2\pi}b^{3/2}\cos(a)(de-c}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```


```
[Out] (2*b*(d*e - c*f)^2*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^3 - (8*b^3
*f^2*(c + d*x)*Cos[a + b/(c + d*x)^(2/3)])/(315*d^3) + (b*f*(d*e - c*f)*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*d^3) + (2*b*f^2*(c + d*x)^(7/3)
*Cos[a + b/(c + d*x)^(2/3)])/(21*d^3) + (b^3*f*(d*e - c*f)*Cos[a]*CosIntegr
al[b/(c + d*x)^(2/3)])/(2*d^3) - (16*b^(9/2)*f^2*Sqrt[2*Pi]*Cos[a]*FresnelC
[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/(315*d^3) + (2*b^(3/2)*(d*e - c*f)^
2*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^3 + (
2*b^(3/2)*(d*e - c*f)^2*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^
(1/3)]*Sin[a])/d^3 + (16*b^(9/2)*f^2*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)]*Sin[a])/(315*d^3) + (16*b^4*f^2*(c + d*x)^(1/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(315*d^3) - (b^2*f*(d*e - c*f)*(c + d*x)^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(2*d^3) + ((d*e - c*f)^2*(c + d*x)*Sin[a + b/(c + d*x)
^(2/3)])/d^3 - (4*b^2*f^2*(c + d*x)^(5/3)*Sin[a + b/(c + d*x)^(2/3)])/(105*
d^3) + (f*(d*e - c*f)*(c + d*x)^2*Sin[a + b/(c + d*x)^(2/3)])/d^3 + (f^2*(c
 + d*x)^3*Sin[a + b/(c + d*x)^(2/3)])/(3*d^3) - (b^3*f*(d*e - c*f)*Sin[a]*S
inIntegral[b/(c + d*x)^(2/3)])/(2*d^3)
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int (e+fx)^2 \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx = \frac{3\,\text{Subst}\left(\int \left((de-cf)^2x^2\sin\left(a+\frac{b}{x^2}\right)-2f(-de+cf)x^5\sin\left(a+\frac{b}{x^2}\right)+f^2x^8\sin\left(a\right.\right.\right.}{d^3}$$

$$= \frac{\left(3f^2\right)\text{Subst}\left(\int x^8\sin\left(a+\frac{b}{x^2}\right)dx,x,\sqrt[3]{c+dx}\right)}{d^3}+\frac{(6f(de-cf))\,\text{Subst}\left(\int x^5\sin\left(\right.\right.}{d^3}$$

$$= -\frac{\left(3f^2\right)\text{Subst}\left(\int \frac{\sin\left(a+bx^2\right)}{x^{10}}dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d^3}-\frac{(3f(de-cf))\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}dx,x,\right.}{d^3}$$

$$= \frac{(de-cf)^2(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}+\frac{f(de-cf)(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}+\frac{f^2}{}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}+\frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}+\frac{bf(de-cf)(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^3}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}+\frac{bf(d}{}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}+\frac{bf(d}{}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}+\frac{bf(d}{}$$

$$= \frac{2b(de-cf)^2\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^3}-\frac{8b^3f^2(c+dx)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{315d^3}+\frac{bf(d}{}$$

Mathematica [C] time = 2.93087, size = 613, normalized size = 0.97

$$\frac{ie^{-ia}\left(4\sqrt[4]{-1}\sqrt{\pi}e^{2ia}b^{3/2}\left(f^2\left(8b^3+315ic^2\right)-630icdef+315id^2e^2\right)\text{Erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}}{\sqrt[3]{c+dx}}\right)-\sqrt[3]{c+dx}e^{i\left(2a+\frac{b}{(c+dx)^{2/3}}\right)}\left(3b^2f\sqrt[3]{c+dx}(97c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((I/1260)*(((c + d*x)^(1/3)*(32*b^4*f^2 + (16*I)*b^3*f^2*(c + d*x)^(2/3) +
3*b^2*f*(c + d*x)^(1/3)*(-105*d*e + 97*c*f - 8*d*f*x) - (15*I)*b*(84*d^2*e^
2 + 21*d*e*f*(-7*c + d*x) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c +
 d*x)^(2/3)*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))
))/E^((I*b)/(c + d*x)^(2/3)) - E^(I*(2*a + b/(c + d*x)^(2/3)))*(c + d*x)^(1
/3)*(32*b^4*f^2 - (16*I)*b^3*f^2*(c + d*x)^(2/3) + 3*b^2*f*(c + d*x)^(1/3)*
(-105*d*e + 97*c*f - 8*d*f*x) + (15*I)*b*(84*d^2*e^2 + 21*d*e*f*(-7*c + d*x
) + f^2*(67*c^2 - 13*c*d*x + 4*d^2*x^2)) + 210*(c + d*x)^(2/3)*(c^2*f^2 - c
*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))) + 4*(-1)^(1/4)*b^(3/2)
*E^((2*I)*a)*((315*I)*d^2*e^2 - (630*I)*c*d*e*f + (8*b^3 + (315*I)*c^2)*f^2
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b])/(c + d*x)^(1/3)] - 4*(-1)^(1/4)*b^(3/2
)*(315*d^2*e^2 - 630*c*d*e*f + ((8*I)*b^3 + 315*c^2)*f^2)*Sqrt[Pi]*Erfi[((-
1)^(3/4)*Sqrt[b])/(c + d*x)^(1/3)] + (315*I)*b^3*f*(-(d*e) + c*f)*ExpIntegr
```

```
alEi[((-I)*b)/(c + d*x)^(2/3)] + (315*I)*b^3*E^((2*I)*a)*f*(-(d*e) + c*f)*E
xpIntegralEi[(I*b)/(c + d*x)^(2/3)]))/(d^3*E^(I*a))
```

Maple [A] time = 0.017, size = 452, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] 3/d^3*(1/3*(c^2*f^2-2*c*d*e*f+d^2*e^2)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*(
c^2*f^2-2*c*d*e*f+d^2*e^2)*b*(-(d*x+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)
*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+
sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))))+1/6*(-2*c*f^2+2*d
*e*f)*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))-1/3*(-2*c*f^2+2*d*e*f)*b*(-1/4*(d*x+
c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d*x+c)^(2/3)*sin(a+b/(d*x+c)^(
2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*sin(a)*Si(b/(d*x+c)^(2/3)))))+1
/9*f^2*(d*x+c)^3*sin(a+b/(d*x+c)^(2/3))-2/9*f^2*b*(-1/7*(d*x+c)^(7/3)*cos(a
+b/(d*x+c)^(2/3))-2/7*b*(-1/5*(d*x+c)^(5/3)*sin(a+b/(d*x+c)^(2/3))+2/5*b*(-
1/3*(d*x+c)*cos(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*x+c)^(1/3)*sin(a+b/(d*x+c)^(2
/3))+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*
x+c)^(1/3))-sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)))))))
```

Maxima [C] time = 2.91142, size = 3386, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/1260*(630*(4*(d*x + c)^(2/3)*b*sqrt(abs(b)/(d*x + c)^(2/3))*cos(((d*x + c
)^(2/3)*a + b)/(d*x + c)^(2/3)) + (((I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/
3))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi +
1/2*arctan2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*s
qrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0,
 b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(
-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(e
rf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3
))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + ((sqrt(pi)*(erf(sqrt(I
*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))
*cos(1/4*pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))
) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*
arctan2(0, b)) + (-I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + I*sqrt
(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b))
 + (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(
-I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^2
+ 2*(d*x + c)^(4/3)*sqrt(abs(b)/(d*x + c)^(2/3))*sin(((d*x + c)^(2/3)*a + b
)/(d*x + c)^(2/3)))*e^2/((d*x + c)^(1/3)*sqrt(abs(b)/(d*x + c)^(2/3))) - 12
60*(4*(d*x + c)^(2/3)*b*sqrt(abs(b)/(d*x + c)^(2/3))*cos(((d*x + c)^(2/3)*a
 + b)/(d*x + c)^(2/3)) + (((I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1)
 - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*arcta
n2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(
```

erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (s
qrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x
 + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(erf(sqrt(I
*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))
*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + ((sqrt(pi)*(erf(sqrt(I*b/(d*x +
 c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*
pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) +
sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0
, b)) + (-I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + I*sqrt(pi)*(erf
(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) + (I*sqr
t(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x
 + c)^(2/3))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^2 + 2*(d*x
+ c)^(4/3)*sqrt(abs(b)/(d*x + c)^(2/3))*sin(((d*x + c)^(2/3)*a + b)/(d*x +
c)^(2/3)))*c*e*f/((d*x + c)^(1/3)*d*sqrt(abs(b)/(d*x + c)^(2/3))) + 630*(4*
(d*x + c)^(2/3)*b*sqrt(abs(b)/(d*x + c)^(2/3))*cos(((d*x + c)^(2/3)*a + b)/
(d*x + c)^(2/3)) + (((I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*s
qrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*arctan2(0,
b)) + (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sq
rt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (sqrt(pi
)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^
(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(erf(sqrt(I*b/(d*
x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(-
1/4*pi + 1/2*arctan2(0, b)))*cos(a) + ((sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2
/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1
/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(p
i)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b))
 + (-I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + I*sqrt(pi)*(erf(sqrt(
-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) + (I*sqrt(pi)*
(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^
(2/3))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^2 + 2*(d*x + c)^(
4/3)*sqrt(abs(b)/(d*x + c)^(2/3))*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/
3)))*c^2*f^2/((d*x + c)^(1/3)*d^2*sqrt(abs(b)/(d*x + c)^(2/3))) + 315*(((Ei
(I*b/(d*x + c)^(2/3)) + Ei(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x +
 c)^(2/3)) - I*Ei(-I*b/(d*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*
cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(
d*x + c)^2)*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*e*f/d - 315*(((Ei
(I*b/(d*x + c)^(2/3)) + Ei(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x +
 c)^(2/3)) - I*Ei(-I*b/(d*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*
cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(
d*x + c)^2)*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*c*f^2/d^2 - 4*(((
4*(sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/
(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*arctan2(0, b)) + 4*(sqrt(pi)*(erf(
sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3)))
 - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) - (4*I*sqrt(pi)*(erf(sqrt(I*b/(d*x
+ c)^(2/3))) - 1) - 4*I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin
(1/4*pi + 1/2*arctan2(0, b)) - (-4*I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3)
)) - 1) + 4*I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*pi +
 1/2*arctan2(0, b)))*cos(a) - ((4*I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))
) - 1) - 4*I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1
/2*arctan2(0, b)) + (4*I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - 4*
I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2
(0, b)) + 4*(sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(
sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - 4*(sqrt
(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x +
c)^(2/3))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^5 + 2*(4*(d*x
+ c)^(4/3)*b^3*sqrt(abs(b)/(d*x + c)^(2/3)) - 15*(d*x + c)^(8/3)*b*sqrt(abs
(b)/(d*x + c)^(2/3)))*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)) - (16*(d
*x + c)^(2/3)*b^4*sqrt(abs(b)/(d*x + c)^(2/3)) - 12*(d*x + c)^2*b^2*sqrt(ab
s(b)/(d*x + c)^(2/3)) + 105*(d*x + c)^(10/3)*sqrt(abs(b)/(d*x + c)^(2/3)))*

```
sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*f^2/((d*x + c)^(1/3)*d^2*sqrt
(abs(b)/(d*x + c)^(2/3))))/d
```

Fricas [A] time = 2.43705, size = 1303, normalized size = 2.07

$$315\left(b^3def - b^3cf^2\right)\cos(a)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right) + 315\left(b^3def - b^3cf^2\right)\cos(a)\operatorname{Ci}\left(-\frac{b}{(dx+c)^{\frac{2}{3}}}\right) - 8\sqrt{2}\left(8\pi b^4f^2\cos(a) - 315\pi(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 1/1260*(315*(b^3*d*e*f - b^3*c*f^2)*cos(a)*cos_integral(b/(d*x + c)^(2/3))
+ 315*(b^3*d*e*f - b^3*c*f^2)*cos(a)*cos_integral(-b/(d*x + c)^(2/3)) - 8*s
qrt(2)*(8*pi*b^4*f^2*cos(a) - 315*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*
sin(a))*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) + 8*sqrt
(2)*(8*pi*b^4*f^2*sin(a) + 315*pi*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cos
(a))*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3)) - 630*(b^3*
d*e*f - b^3*c*f^2)*sin(a)*sin_integral(b/(d*x + c)^(2/3)) - 2*(16*b^3*d*f^2
*x + 16*b^3*c*f^2 - 15*(4*b*d^2*f^2*x^2 + 84*b*d^2*e^2 - 147*b*c*d*e*f + 67
*b*c^2*f^2 + (21*b*d^2*e*f - 13*b*c*d*f^2)*x)*(d*x + c)^(1/3))*cos((a*d*x +
 a*c + (d*x + c)^(1/3)*b)/(d*x + c)) + 2*(210*d^3*f^2*x^3 + 630*d^3*e*f*x^2
 + 32*(d*x + c)^(1/3)*b^4*f^2 + 630*d^3*e^2*x + 630*c*d^2*e^2 - 630*c^2*d*e
*f + 210*c^3*f^2 - 3*(8*b^2*d*f^2*x + 105*b^2*d*e*f - 97*b^2*c*f^2)*(d*x +
c)^(2/3))*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^2 \sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)**2*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)^2 \sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sin(a + b/(d*x + c)^(2/3)), x)
```

3.223 $\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$

Optimal. Leaf size=318

$$\frac{b^3 f\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{2\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{2\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2}-$$

```
[Out] (2*b*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^2 + (b*f*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d^2) + (b^3*f*Cos[a]*CosIntegra
l[b/(c + d*x)^(2/3)])/(4*d^2) + (2*b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fr
esnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^2 + (2*b^(3/2)*(d*e - c*f)*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d^2 - (b^
2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])/(4*d^2) + ((d*e - c*f)*(c +
 d*x)*Sin[a + b/(c + d*x)^(2/3)])/d^2 + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^
(2/3)])/(2*d^2) - (b^3*f*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Rubi [A] time = 0.384618, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3433, 3409, 3387, 3388, 3353, 3352, 3351, 3379, 3297, 3303, 3299, 3302}

$$\frac{b^3 f\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d^2}+\frac{2\sqrt{2\pi}b^{3/2}\sin(a)(de-cf)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d^2}+\frac{2\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```



```
[Out] (2*b*(d*e - c*f)*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d^2 + (b*f*(c
+ d*x)^(4/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d^2) + (b^3*f*Cos[a]*CosIntegra
l[b/(c + d*x)^(2/3)])/(4*d^2) + (2*b^(3/2)*(d*e - c*f)*Sqrt[2*Pi]*Cos[a]*Fr
esnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d^2 + (2*b^(3/2)*(d*e - c*f)*
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d^2 - (b^
2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])/(4*d^2) + ((d*e - c*f)*(c +
 d*x)*Sin[a + b/(c + d*x)^(2/3)])/d^2 + (f*(c + d*x)^2*Sin[a + b/(c + d*x)^
(2/3)])/(2*d^2) - (b^3*f*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
```

```
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{3\,\text{Subst}\left(\int \left((de-cf)x^2\sin\left(a+\frac{b}{x^2}\right)+fx^5\sin\left(a+\frac{b}{x^2}\right)\right)dx, x, \sqrt[3]{c+dx}\right)}{d^2}$$

$$= \frac{(3f)\,\text{Subst}\left(\int x^5\sin\left(a+\frac{b}{x^2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d^2} + \frac{(3(de-cf))\,\text{Subst}\left(\int x^2\sin\left(a+\right.\right.}{d^2}$$

$$= -\frac{(3f)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^4}dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d^2} - \frac{(3(de-cf))\,\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^4}dx, x, \right.}{d^2}$$

$$= \frac{(de-cf)(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{f(c+dx)^2\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d^2} - \frac{(bf)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{(de-c}{}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} - \frac{b^2f(c+}{}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{2b^{3/2}(de}{}$$

$$= \frac{2b(de-cf)\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d^2} + \frac{bf(c+dx)^{4/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d^2} + \frac{b^3f\cos}{}$$

Mathematica [A] time = 1.17299, size = 378, normalized size = 1.19

$$\frac{b^3f\cos(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)+8\sqrt{2\pi}b^{3/2}\cos(a)(de-cf)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+8\sqrt{2\pi}b^{3/2}de\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)-8\sqrt{2\pi}b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (8*b*d*e*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] - 7*b*c*f*(c + d*x)^(1/
3)*Cos[a + b/(c + d*x)^(2/3)] + b*d*f*x*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)
^(2/3)] + b^3*f*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)] + 8*b^(3/2)*(d*e - c*
f)*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 8*b^(
3/2)*d*e*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] -
 8*b^(3/2)*c*f*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Si
n[a] + 4*c*d*e*Sin[a + b/(c + d*x)^(2/3)] - 2*c^2*f*Sin[a + b/(c + d*x)^(2/
3)] + 4*d^2*e*x*Sin[a + b/(c + d*x)^(2/3)] + 2*d^2*f*x^2*Sin[a + b/(c + d*x
)^(2/3)] - b^2*f*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] - b^3*f*Sin[a]*
SinIntegral[b/(c + d*x)^(2/3)])/(4*d^2)
```

Maple [A] time = 0.017, size = 225, normalized size = 0.7

$$3\frac{1}{d^2}\left(-1/3\,(cf-de)(dx+c)\sin\left(a+\frac{b}{(dx+c)^{2/3}}\right)+2/3\,(cf-de)\,b\left(-\sqrt[3]{dx+c}\cos\left(a+\frac{b}{(dx+c)^{2/3}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos(a)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] 3/d^2*(-1/3*(c*f-d*e)*(d*x+c)*sin(a+b/(d*x+c)^(2/3))+2/3*(c*f-d*e)*b*(-(d*x
+c)^(1/3)*cos(a+b/(d*x+c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(
b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^
(1/2)/(d*x+c)^(1/3))))+1/6*f*(d*x+c)^2*sin(a+b/(d*x+c)^(2/3))-1/3*f*b*(-1/4
*(d*x+c)^(4/3)*cos(a+b/(d*x+c)^(2/3))-1/2*b*(-1/2*(d*x+c)^(2/3)*sin(a+b/(d*
x+c)^(2/3))+b*(1/2*cos(a)*Ci(b/(d*x+c)^(2/3))-1/2*sin(a)*Si(b/(d*x+c)^(2/3)
)))))
```

Maxima [C] time = 1.9979, size = 1628, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(4*(d*x + c)^(2/3)*b*sqrt(abs(b)/(d*x + c)^(2/3))*cos(((d*x + c)^(2/
3)*a + b)/(d*x + c)^(2/3)) + (((I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3)))
- 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*a
rctan2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(p
i)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b))
+ (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/
(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(erf(sq
rt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) -
 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + ((sqrt(pi)*(erf(sqrt(I*b/(d
*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(
1/4*pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1
) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arcta
n2(0, b)) + (-I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + I*sqrt(pi)*
(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) + (I
*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/
(d*x + c)^(2/3))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^2 + 2*(
d*x + c)^(4/3)*sqrt(abs(b)/(d*x + c)^(2/3))*sin(((d*x + c)^(2/3)*a + b)/(d*
x + c)^(2/3)))*e/((d*x + c)^(1/3)*sqrt(abs(b)/(d*x + c)^(2/3))) - 4*(4*(d*x
 + c)^(2/3)*b*sqrt(abs(b)/(d*x + c)^(2/3))*cos(((d*x + c)^(2/3)*a + b)/(d*x
 + c)^(2/3)) + (((I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(
pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*arctan2(0, b))
+ (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(-
I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(e
rf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3
))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(erf(sqrt(I*b/(d*x +
c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(-1/4*
pi + 1/2*arctan2(0, b)))*cos(a) + ((sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))
) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*a
rctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(
erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (-
I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + I*sqrt(pi)*(erf(sqrt(-I*b
/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) + (I*sqrt(pi)*(erf
(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3
))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^2 + 2*(d*x + c)^(4/3)
*sqrt(abs(b)/(d*x + c)^(2/3))*sin(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))
*c*f/((d*x + c)^(1/3)*d*sqrt(abs(b)/(d*x + c)^(2/3))) + (((Ei(I*b/(d*x + c)
^(2/3)) + Ei(-I*b/(d*x + c)^(2/3)))*cos(a) + (I*Ei(I*b/(d*x + c)^(2/3)) - I
*Ei(-I*b/(d*x + c)^(2/3)))*sin(a))*b^3 + 2*(d*x + c)^(4/3)*b*cos(((d*x + c)
^(2/3)*a + b)/(d*x + c)^(2/3)) - 2*((d*x + c)^(2/3)*b^2 - 2*(d*x + c)^2)*si
```

```
n(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3)))*f/d)/d
```

Fricas [A] time = 2.072, size = 771, normalized size = 2.42

$$\frac{b^3 f\cos(a)\operatorname{Ci}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+b^3 f\cos(a)\operatorname{Ci}\left(-\frac{b}{(dx+c)^{\frac{2}{3}}}\right)-2\,b^3 f\sin(a)\operatorname{Si}\left(\frac{b}{(dx+c)^{\frac{2}{3}}}\right)+16\,\sqrt{2}\pi\left(bde-bcf\right)\sqrt{\frac{b}{\pi}}\cos(a)\operatorname{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*f*cos(a)*cos_integral(b/(d*x + c)^(2/3)) + b^3*f*cos(a)*cos_integr
al(-b/(d*x + c)^(2/3)) - 2*b^3*f*sin(a)*sin_integral(b/(d*x + c)^(2/3)) + 1
6*sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/p
i)/(d*x + c)^(1/3)) + 16*sqrt(2)*pi*(b*d*e - b*c*f)*sqrt(b/pi)*fresnel_cos(
sqrt(2)*sqrt(b/pi)/(d*x + c)^(1/3))*sin(a) + 2*(b*d*f*x + 8*b*d*e - 7*b*c*f
)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)) + 2*(2*d
^2*f*x^2 + 4*d^2*e*x - (d*x + c)^(2/3)*b^2*f + 4*c*d*e - 2*c^2*f)*sin((a*d*
x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e + f*x)*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(fx+e\right)\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sin(a + b/(d*x + c)^(2/3)), x)
```

3.224 $\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2\pi}b^{3/2}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{2\sqrt{2\pi}b^{3/2}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{}{(}\right.}{d}$$

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d + (2*b^(3/2)*Sqrt[2*Pi]*
Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d + (2*b^(3/2)*Sqrt[
2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d + ((c + d*x)
*Sin[a + b/(c + d*x)^(2/3)])/d
```

Rubi [A] time = 0.11186, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3363, 3409, 3387, 3388, 3353, 3352, 3351}

$$\frac{2\sqrt{2\pi}b^{3/2}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{2\sqrt{2\pi}b^{3/2}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{}{(}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/d + (2*b^(3/2)*Sqrt[2*Pi]*
Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)])/d + (2*b^(3/2)*Sqrt[
2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/d + ((c + d*x)
*Sin[a + b/(c + d*x)^(2/3)])/d
```

Rule 3363

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Module[{k = Denominator[n]}, Dist[k/f, Subst[Int[x^(k - 1)*(a + b
*Sin[c + d*x^(k*n)])^p, x], x, (e + f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f}, x] && IGtQ[p, 0] && FractionQ[n]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
```

```
LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx &= \frac{3\,\mathrm{Subst}\left(\int x^2\sin\left(a+\frac{b}{x^2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= -\frac{3\,\mathrm{Subst}\left(\int \frac{\sin\left(a+bx^2\right)}{x^4}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= \frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{\cos\left(a+bx^2\right)}{x^2}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{\left(4b^2\right)\mathrm{Subst}\left(\int \sin\left(a+bx^2\right)\right.}{d}\\
&= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{\left(4b^2\cos(a)\right)\mathrm{Subst}\left(\int \sin\left(bx\right.\right.}{d}\\
&= \frac{2b\sqrt[3]{c+dx}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d}+\frac{2b^{3/2}\sqrt{2\pi}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d}+\frac{2b^{3/2}\sqrt{2\pi}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d}+
\end{aligned}
$$

Mathematica [A] time = 0.151286, size = 146, normalized size = 1.04

$$
\frac{2\sqrt{2\pi}b^{3/2}\sin(a)\mathrm{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)+2\sqrt{2\pi}b^{3/2}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+c\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+dx\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+2b\sqrt[3]{c+d}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(2/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Cos[
a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)] + 2*b^(3/2)*Sqrt[2*Pi]*Fr
esnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)
^(2/3)] + d*x*Sin[a + b/(c + d*x)^(2/3)])/d
```

Maple [A] time = 0.013, size = 105, normalized size = 0.7

$$3\frac{1}{d}\left(1/3\ (dx+c)\sin\left(a+\frac{b}{(dx+c)^{2/3}}\right)-2/3\,b\left(-\sqrt[3]{dx+c}\cos\left(a+\frac{b}{(dx+c)^{2/3}}\right)-\sqrt{b}\sqrt{2}\sqrt{\pi}\left(\cos\left(a\right)\mathrm{FresnelS}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{\pi}\sqrt[3]{dx}}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] 3/d*(1/3*(d*x+c)*sin(a+b/(d*x+c)^(2/3))-2/3*b*(-(d*x+c)^(1/3)*cos(a+b/(d*x+
c)^(2/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2
)/(d*x+c)^(1/3))+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/(d*x+c)^(1/3)))))
```

Maxima [C] time = 1.44145, size = 716, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)^(2/3)*b*sqrt(abs(b)/(d*x + c)^(2/3))*cos(((d*x + c)^(2/3)*
a + b)/(d*x + c)^(2/3)) + (((I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1
) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4*pi + 1/2*arct
an2(0, b)) + (I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*
(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(0, b)) + (
sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*
x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(pi)*(erf(sqrt(
I*b/(d*x + c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1)
)*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) + ((sqrt(pi)*(erf(sqrt(I*b/(d*x
+ c)^(2/3))) - 1) + sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(1/4
*pi + 1/2*arctan2(0, b)) + (sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) +
 sqrt(pi)*(erf(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*cos(-1/4*pi + 1/2*arctan2(
0, b)) + (-I*sqrt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) + I*sqrt(pi)*(er
f(sqrt(-I*b/(d*x + c)^(2/3))) - 1))*sin(1/4*pi + 1/2*arctan2(0, b)) + (I*sq
rt(pi)*(erf(sqrt(I*b/(d*x + c)^(2/3))) - 1) - I*sqrt(pi)*(erf(sqrt(-I*b/(d*
x + c)^(2/3))) - 1))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*b^2 + 2*(d*x
 + c)^(4/3)*sqrt(abs(b)/(d*x + c)^(2/3))*sin(((d*x + c)^(2/3)*a + b)/(d*x +
 c)^(2/3)))/((d*x + c)^(1/3)*d*sqrt(abs(b)/(d*x + c)^(2/3)))
```

Fricas [A] time = 2.0195, size = 406, normalized size = 2.88

$$\frac{2\sqrt{2}\pi b\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)+2\sqrt{2}\pi b\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{(dx+c)^{\frac{1}{3}}}\right)\sin\left(a\right)+2\left(dx+c\right)^{\frac{1}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)+\left(dx+c\right)\sin\left(\frac{adx}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] (2*sqrt(2)*pi*b*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/(d*x + c)^
(1/3)) + 2*sqrt(2)*pi*b*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/(d*x + c)
^(1/3))*sin(a) + 2*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/3)*b)/
```

```
(d*x + c)) + (d*x + c)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3)), x)
```

3.225 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi [A] time = 0.013414, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
```

```
[Out] Defer[Int][Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx=\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Mathematica [A] time = 29.6677, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x),x]
```

```
[Out] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x), x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{fx+e}\sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e), x)
```

3.226 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi [A] time = 0.0138288, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]

[Out] Defer[Int][Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx = \int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(e+fx)^2}\,dx$$

Mathematica [F] time = 180.033, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(e + f*x)^2,x]

[Out] $Aborted
```

Maple [A] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{(fx+e)^2}\sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{f^2x^2+2efx+e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(f^2*x^2 + 2*e*f*
x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(f*x + e)^2, x)
```

3.227 $\int (ce+dex)^{4/3} \sin\left(a+b\sqrt[3]{c+dx}\right) dx$

Optimal. Leaf size=289

$$\frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d}+\frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+\right.}{b^6d}$$

```
[Out] (2160*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^7*d*(c + d*x)^(1
/3)) - (1080*e*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3
)])/(b^5*d) + (90*e*(c + d*x)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)
])/(b^3*d) - (3*e*(c + d*x)^(5/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(
1/3)])/(b*d) + (2160*e*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^6
*d) - (360*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)]
)/(b^4*d) + (18*e*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(
1/3)])/(b^2*d)
```

Rubi [A] time = 0.268518, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2638}

$$\frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{360e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d}+\frac{2160e\sqrt[3]{e(c+dx)}\sin\left(a+\right.}{b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```


```
[Out] (2160*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^7*d*(c + d*x)^(1
/3)) - (1080*e*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3
)])/(b^5*d) + (90*e*(c + d*x)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)
])/(b^3*d) - (3*e*(c + d*x)^(5/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(
1/3)])/(b*d) + (2160*e*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^6
*d) - (360*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)]
)/(b^4*d) + (18*e*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(
1/3)])/(b^2*d)
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (ce+dex)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{3\,\text{Subst}\left(\int x^2\left(ex^3\right)^{4/3}\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^6\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$
$$= -\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{\left(18e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^5\right.}{bd\sqrt[3]{c}}$$
$$= -\frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{18e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\sin}{b^2d}$$
$$= \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+\right.}{bd}$$
$$= \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3e(c+dx)^{5/3}\sqrt[3]{e(c+dx)}\cos\left(a+\right.}{bd}$$
$$= -\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(\right.}{b^3d}$$
$$= -\frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d} + \frac{90e(c+dx)\sqrt[3]{e(c+dx)}\cos\left(\right.}{b^3d}$$
$$= \frac{2160e\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^7d\sqrt[3]{c+dx}} - \frac{1080e\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{\right.}}{b^5d}$$

Mathematica [A] time = 0.547398, size = 226, normalized size = 0.78

$$\frac{3(e(c+dx))^{4/3}\left(\sin\left(b\sqrt[3]{c+dx}\right)\left(\sin(a)\left(b^6(c+dx)^2-30b^4(c+dx)^{4/3}+360b^2(c+dx)^{2/3}-720\right)+6b\cos(a)\left(b^4(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(e*(c + d*x))^(4/3)*(-(Cos[b*(c + d*x)^(1/3)]*((-720 + 360*b^2*(c + d*x)
^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Cos[a] - 6*b*(120*(c + d
*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Sin[a])) + (6*b*(120*(c
 + d*x)^(1/3) - 20*b^2*(c + d*x) + b^4*(c + d*x)^(5/3))*Cos[a] + (-720 + 36
0*b^2*(c + d*x)^(2/3) - 30*b^4*(c + d*x)^(4/3) + b^6*(c + d*x)^2)*Sin[a])*S
in[b*(c + d*x)^(1/3)]))/(b^7*d*(c + d*x)^(4/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{4}{3}}\sin\left(a+b\sqrt[3]{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 7.63023, size = 568, normalized size = 1.97

$$\frac{3\left(\left(30\,b^4d^2ex^2+60\,b^4cdex+30\,b^4c^2e-\left(b^6d^2ex^2+2\,b^6cdex+\left(b^6c^2-720\right)e\right)(dx+c)^{\frac{2}{3}}-360\left(b^2dex+b^2ce\right)(dx+c)^{\frac{1}{3}}\right)(de \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((30*b^4*d^2*e*x^2 + 60*b^4*c*d*e*x + 30*b^4*c^2*e - (b^6*d^2*e*x^2 + 2*b
^6*c*d*e*x + (b^6*c^2 - 720)*e)*(d*x + c)^(2/3) - 360*(b^2*d*e*x + b^2*c*e)
*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*cos((d*x + c)^(1/3)*b + a) + 6*(120*b
*d*e*x + 120*b*c*e - 20*(b^3*d*e*x + b^3*c*e)*(d*x + c)^(2/3) + (b^5*d^2*e*
x^2 + 2*b^5*c*d*e*x + b^5*c^2*e)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*sin((
d*x + c)^(1/3)*b + a))/(b^7*d^2*x + b^7*c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25369, size = 594, normalized size = 2.06

$$-\frac{3\left(c\left(\frac{\left((dxe+ce)b^3e^3-6\,(dxe+ce)^{\frac{1}{3}}be^{\frac{11}{3}}\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{8}{3}\right)}}{b^4}-\frac{3\left((dxe+ce)^{\frac{2}{3}}b^2e^{\frac{10}{3}}-2\,e^4\right)e^{\left(-\frac{8}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^4}\right)-\left(\left(\left((dxe+ce \cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(c*((((d*x*e + c*e)*b^3*e^3 - 6*(d*x*e + c*e)^(1/3)*b*e^(11/3))*cos(((d*x
*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-8/3)/b^4 - 3*((d*x*e + c*e)^(2
/3)*b^2*e^(10/3) - 2*e^4)*e^(-8/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e
)*e^(-1))/b^4) - ((((d*x*e + c*e)*b^3*c*e^3 - 6*(d*x*e + c*e)^(1/3)*b*c*e^(
11/3))*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-8/3)/b^4 - 3*(
(d*x*e + c*e)^(2/3)*b^2*c*e^(10/3) - 2*c*e^4)*e^(-8/3)*sin(((d*x*e + c*e)^(
1/3)*b*e^(2/3) + a*e)*e^(-1))/b^4)*e - ((d*x*e + c*e)^2*b^6*e^5 - 30*(d*x*e
 + c*e)^(4/3)*b^4*e^(17/3) + 360*(d*x*e + c*e)^(2/3)*b^2*e^(19/3) - 720*e^7
)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-14/3)/b^7 + 6*((d*x
*e + c*e)^(5/3)*b^5*e^(16/3) - 20*(d*x*e + c*e)*b^3*e^6 + 120*(d*x*e + c*e)
^(1/3)*b*e^(20/3))*e^(-14/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-
1))/b^7)*e^(-1))/d
```

3.228 $\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=202

$$\frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}-\frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}$$

```
[Out] (36*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (72*(e*(c + d
*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d*(c + d*x)^(2/3)) - (3*(c + d*
x)^(2/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (72*(e*(c
+ d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (12*(c
+ d*x)^(1/3)*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rubi [A] time = 0.177942, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2638}

$$\frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{72(e(c+dx))^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}-\frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (36*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (72*(e*(c + d
*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^5*d*(c + d*x)^(2/3)) - (3*(c + d*
x)^(2/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (72*(e*(c
+ d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (12*(c
+ d*x)^(1/3)*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (ce+dex)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{3\,\text{Subst}\left(\int x^2\left(ex^3\right)^{2/3}\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$

$$= -\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{\left(12(e(c+dx))^{2/3}\right)\text{Subst}\left(\int\right.}{bd(c}$$

$$= -\frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{12\sqrt[3]{c+dx}(e(c+dx))^{2/3}\sin}{b^2d}$$

$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c}\right.}{bd}$$

$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{3(c+dx)^{2/3}(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c}\right.}{bd}$$

$$= \frac{36(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} - \frac{72(e(c+dx))^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^5d(c+dx)^{2/3}} -$$

Mathematica [A] time = 0.283815, size = 111, normalized size = 0.55

$$-\frac{3(e(c+dx))^{2/3}\left(\left(b^4(c+dx)^{4/3}-12b^2(c+dx)^{2/3}+24\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-4b\left(b^2(c+dx)-6\sqrt[3]{c+dx}\right)\sin\left(a+b\sqrt[3]{c}\right.}{b^5d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(2/3)*((24 - 12*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(4/3)
)*Cos[a + b*(c + d*x)^(1/3)] - 4*b*(-6*(c + d*x)^(1/3) + b^2*(c + d*x))*Sin
[a + b*(c + d*x)^(1/3)]))/(b^5*d*(c + d*x)^(2/3))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{2}{3}}\sin\left(a+b\sqrt[3]{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 7.76001, size = 356, normalized size = 1.76

$$\frac{3\left(\left(12\,b^2dx+12\,b^2c-\left(b^4dx+b^4c\right)(dx+c)^{\frac{2}{3}}-24\,(dx+c)^{\frac{1}{3}}\right)(dex+ce)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-4\,(dex+ce)^{\frac{2}{3}}\left(6\,(dx+c)^{\frac{2}{3}}b\right.\right.}{b^5d^2x+b^5cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((12*b^2*d*x + 12*b^2*c - (b^4*d*x + b^4*c)*(d*x + c)^(2/3) - 24*(d*x + c
)^(1/3))*(d*e*x + c*e)^(2/3)*cos((d*x + c)^(1/3)*b + a) - 4*(d*e*x + c*e)^(
2/3)*(6*(d*x + c)^(2/3)*b - (b^3*d*x + b^3*c)*(d*x + c)^(1/3))*sin((d*x + c
)^(1/3)*b + a))/(b^5*d^2*x + b^5*c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\,(c+dx))^{\frac{2}{3}}\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [A] time = 1.17336, size = 198, normalized size = 0.98

$$-\frac{3\left(\frac{\left((dxe+ce)^{\frac{4}{3}}b^4e^{\frac{11}{3}}-12\,(dxe+ce)^{\frac{2}{3}}b^2e^{\frac{13}{3}}+24\,e^5\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{10}{3}\right)}}{b^5}-\frac{4\left((dxe+ce)b^3e^4-6\,(dxe+ce)^{\frac{1}{3}}be^{\frac{14}{3}}\right)e^{\left(-\frac{10}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(\right.\right.}}{b^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(((d*x*e + c*e)^(4/3)*b^4*e^(11/3) - 12*(d*x*e + c*e)^(2/3)*b^2*e^(13/3)
 + 24*e^5)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-10/3)/b^5
- 4*((d*x*e + c*e)*b^3*e^4 - 6*(d*x*e + c*e)^(1/3)*b*e^(14/3))*e^(-10/3)*si
n(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^5)*e^(-1)/d
```

3.229 $\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=160

$$\frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-3(c$$

```
[Out] (18*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (18*(e*(c + d
*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (9*(c + d*
x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rubi [A] time = 0.137502, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2637}

$$\frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{18\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^4d\sqrt[3]{c+dx}}+\frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-3(c$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (18*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^3*d) - (3*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d) - (18*(e*(c + d
*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^4*d*(c + d*x)^(1/3)) + (9*(c + d*
x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d)
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt[3]{ce+dex}\sin\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{3\,\text{Subst}\left(\int x^2\sqrt[3]{ex^3}\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^3\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$
$$= -\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{\left(9\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^2\cos(a+\right.}{bd\sqrt[3]{c+dx}}$$
$$= -\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{9\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+b\sqrt[3]{c-\right.}{b^2d}$$
$$= \frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}+$$
$$= \frac{18\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd}-$$

Mathematica [A] time = 0.210573, size = 97, normalized size = 0.61

$$-\frac{3\sqrt[3]{e(c+dx)}\left(\left(b^3(c+dx)-6b\sqrt[3]{c+dx}\right)\cos\left(a+b\sqrt[3]{c+dx}\right)-3\left(b^2(c+dx)^{2/3}-2\right)\sin\left(a+b\sqrt[3]{c+dx}\right)\right)}{b^4d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(1/3)*((-6*b*(c + d*x)^(1/3) + b^3*(c + d*x))*Cos[a + b*(
c + d*x)^(1/3)] - 3*(-2 + b^2*(c + d*x)^(2/3))*Sin[a + b*(c + d*x)^(1/3)]))
/(b^4*d*(c + d*x)^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sqrt[3]{dex+ce}\sin\left(a+b\sqrt[3]{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 7.27118, size = 317, normalized size = 1.98

$$\frac{3\left(\left(6\,bdx+6\,bc-\left(b^3dx+b^3c\right)(dx+c)^{\frac{2}{3}}\right)(dex+ce)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+3\,(dex+ce)^{\frac{1}{3}}\left(\left(b^2dx+b^2c\right)(dx+c)^{\frac{1}{3}}-2\,(\right.\right.}{b^4d^2x+b^4cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((6*b*d*x + 6*b*c - (b^3*d*x + b^3*c)*(d*x + c)^(2/3))*(d*e*x + c*e)^(1/3
)*cos((d*x + c)^(1/3)*b + a) + 3*(d*e*x + c*e)^(1/3)*((b^2*d*x + b^2*c)*(d*
x + c)^(1/3) - 2*(d*x + c)^(2/3))*sin((d*x + c)^(1/3)*b + a))/(b^4*d^2*x +
b^4*c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{e\,(c+dx)}\sin\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(1/3)), x)
```

Giac [A] time = 1.19032, size = 173, normalized size = 1.08

$$-\frac{3\left(\frac{\left((dxe+ce)b^3e^3-6\,(dxe+ce)^{\frac{1}{3}}be^{\frac{11}{3}}\right)\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{8}{3}\right)}}{b^4}-\frac{3\left((dxe+ce)^{\frac{2}{3}}b^2e^{\frac{10}{3}}-2\,e^4\right)e^{\left(-\frac{8}{3}\right)}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^4}\right)e^{(-1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*(((d*x*e + c*e)*b^3*e^3 - 6*(d*x*e + c*e)^(1/3)*b*e^(11/3))*cos(((d*x*e
+ c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-8/3)/b^4 - 3*((d*x*e + c*e)^(2/3)
*b^2*e^(10/3) - 2*e^4)*e^(-8/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e
^(-1))/b^4)*e^(-1)/d
```

3.230 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=85

$$\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}$$

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(1/3)) +
 (3*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.0698091, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2637}

$$\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(1/3)) +
 (3*(c + d*x)^(1/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\sqrt[3]{ex^3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{e(c+dx)}}$$
$$= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}} + \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}}$$
$$= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd\sqrt[3]{e(c+dx)}} + \frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

Mathematica [A] time = 0.0755826, size = 70, normalized size = 0.82

$$\frac{3\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right) - 3b(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(c + d*x)^(1/3)*Sin[a
+ b*(c + d*x)^(1/3)])/(b^2*d*(e*(c + d*x))^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \sin\left(a+b\sqrt[3]{dx+c}\right)\frac{1}{\sqrt[3]{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 7.24124, size = 219, normalized size = 2.58

$$-\frac{3\left((dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}b\cos\left((dx+c)^{\frac{1}{3}}b+a\right) - (dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^2d^2ex+b^2cde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] -3*((d*e*x + c*e)^(2/3)*(d*x + c)^(2/3)*b*cos((d*x + c)^(1/3)*b + a) - (d*e
*x + c*e)^(2/3)*(d*x + c)^(1/3)*sin((d*x + c)^(1/3)*b + a))/(b^2*d^2*e*x +
b^2*c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + b\sqrt[3]{c + dx}\right)}{\sqrt[3]{e\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Giac [A] time = 1.19676, size = 112, normalized size = 1.32

$$-\frac{3\left(\frac{(dxe+ce)^{\frac{1}{3}}\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\frac{1}{3}}}{b}-\frac{e^{\frac{2}{3}}\sin\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)}{b^2}\right)e^{(-1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] -3*((d*x*e + c*e)^(1/3)*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e
^(1/3)/b - e^(2/3)*sin(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))/b^2)*e
^(-1)/d
```

3.231 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=42

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Rubi [A] time = 0.0515908, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3431, 15, 2638}

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{2/3}}\,dx &= \frac{3\,\mathrm{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(ex^3\right)^{2/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}\\
&= -\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0678411, size = 42, normalized size = 1.

$$-\frac{3(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{bd(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sin\left(a+b\sqrt[3]{dx+c}\right)(dex+ce)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [A] time = 1.0494, size = 31, normalized size = 0.74

$$-\frac{3\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{bde^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] -3*cos((d*x + c)^(1/3)*b + a)/(b*d*e^(2/3))
```

Fricas [A] time = 1.66474, size = 120, normalized size = 2.86

$$-\frac{3\,(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{bd^2ex+bcde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

```
[Out] -3*(d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*cos((d*x + c)^(1/3)*b + a)/(b*d^2*e*
x + b*c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e\,(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Giac [A] time = 1.17757, size = 47, normalized size = 1.12

$$-\frac{3\cos\left(\left((dxe+ce)^{\frac{1}{3}}be^{\frac{2}{3}}+ae\right)e^{(-1)}\right)e^{\left(-\frac{2}{3}\right)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] -3*cos(((d*x*e + c*e)^(1/3)*b*e^(2/3) + a*e)*e^(-1))*e^(-2/3)/(b*d)
```

3.232 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=120

$$\frac{3b\cos(a)\sqrt[3]{c+dx}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3b\sin(a)\sqrt[3]{c+dx}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

```
[Out] (3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*
b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))
^(1/3))
```

Rubi [A] time = 0.146167, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3431, 15, 3297, 3303, 3299, 3302}

$$\frac{3b\cos(a)\sqrt[3]{c+dx}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3b\sin(a)\sqrt[3]{c+dx}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*
b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)])/(d*e*(e*(c + d*x))
^(1/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{4/3}}\,dx &= \frac{3\,\mathrm{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(ex^3\right)^{4/3}}\,dx,x,\sqrt[3]{c+dx}\right)}{d} \\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} \\
&= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}+\frac{\left(3b\sqrt[3]{c+dx}\right)\mathrm{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} \\
&= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}+\frac{\left(3b\sqrt[3]{c+dx}\cos(a)\right)\mathrm{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{\left(3b\sqrt[3]{c+\cdots}\right.}{} \\
&= \frac{3b\sqrt[3]{c+dx}\cos(a)\mathrm{Ci}\left(b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3b\sqrt[3]{c+dx}\sin(a)\mathrm{Si}\left(b\sqrt[3]{c+dx}\cdots\right.}{de\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.14493, size = 85, normalized size = 0.71

$$-\frac{3\left(-b\cos(a)\sqrt[3]{c+dx}\,\mathrm{CosIntegral}\left(b\sqrt[3]{c+dx}\right)+b\sin(a)\sqrt[3]{c+dx}\,\mathrm{Si}\left(b\sqrt[3]{c+dx}\right)+\sin\left(a+b\sqrt[3]{c+dx}\right)\right)}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b*(c + d*x)^(1/3)]) + Sin[a + b
*(c + d*x)^(1/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b*(c + d*x)^(1/3)]
))/(d*e*(e*(c + d*x))^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \sin\left(a+b\sqrt[3]{dx+c}\right)(dex+ce)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dex+ce)^{\frac{2}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(e\,(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(1/3))/(e*(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(4/3), x)
```

3.233 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=175

$$-\frac{3b^2\sin(a)(c+dx)^{2/3}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b^2\cos(a)(c+dx)^{2/3}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b\sqrt[3]{c}}{}$$

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(2/3
)) - (3*b^2*(c + d*x)^(2/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(2*d*e*(
e*(c + d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(
2/3)) - (3*b^2*(c + d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/(2*d*
e*(e*(c + d*x))^(2/3))
```

Rubi [A] time = 0.181199, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3431, 15, 3297, 3303, 3299, 3302}

$$-\frac{3b^2\sin(a)(c+dx)^{2/3}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b^2\cos(a)(c+dx)^{2/3}\text{Si}\left(b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b\sqrt[3]{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```


```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(2/3
)) - (3*b^2*(c + d*x)^(2/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(2*d*e*(
e*(c + d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(2*d*e*(e*(c + d*x))^(
2/3)) - (3*b^2*(c + d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/(2*d*
e*(e*(c + d*x))^(2/3))
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{5/3}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{\left(ex^3\right)^{5/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \sqrt[3]{c+dx}\right)}{de(e(c+dx))^{2/3}}$$

$$= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{\left(3b^2(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx}{x}\right.}{2de(e(c+dx))^{2/3}}$$

$$= -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{\left(3b^2(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int\right.}{2de(e(c+dx))}$$

$$= -\frac{3b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b^2(c+dx)^{2/3}\text{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right.}{2de(e(c+dx))^{2/3}}$$

Mathematica [A] time = 0.181336, size = 115, normalized size = 0.66

$$-\frac{3\left(b^2\sin(a)(c+dx)^{2/3}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)+b^2\cos(a)(c+dx)^{2/3}\text{Si}\left(b\sqrt[3]{c+dx}\right)+\sin\left(a+b\sqrt[3]{c+dx}\right)+b\sqrt[3]{c+dx}\,\text{co}\right.}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*(b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x)^(2/3)*Cos
Integral[b*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(1/3)] + b^2*(c +
d*x)^(2/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)]))/(2*d*e*(e*(c + d*x))^(2/
3))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \sin\left(a+b\sqrt[3]{dx+c}\right)(dex+ce)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)

[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")

[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dex+ce)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")

[Out] integral((d*e*x + c*e)^(1/3)*sin((d*x + c)^(1/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")

[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(5/3), x)
```

3.234 $\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal. Leaf size=267

$$\frac{b^4\sin(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^4\cos(a)\sqrt[3]{c+dx}\,\text{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{b^3\cos\left(a+b\sqrt[3]{\ldots}\right.}{8de^2\sqrt[3]{e(c+\ldots}}$$

```
[Out] (b^3*Cos[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(e*(c + d*x))^(1/3)) - (b*Cos[a +
 b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) + (b^4*(
c + d*x)^(1/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(8*d*e^2*(e*(c + d*x)
)^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)*(e*(c + d*x))^
(1/3)) + (b^2*Sin[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(c + d*x)^(1/3)*(e*(c +
d*x))^(1/3)) + (b^4*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/
(8*d*e^2*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.262629, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3431, 15, 3297, 3303, 3299, 3302}

$$\frac{b^4\sin(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^4\cos(a)\sqrt[3]{c+dx}\,\text{Si}\left(b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}}+\frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}+\frac{b^3\cos\left(a+b\sqrt[3]{\ldots}\right.}{8de^2\sqrt[3]{e(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (b^3*Cos[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(e*(c + d*x))^(1/3)) - (b*Cos[a +
 b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) + (b^4*(
c + d*x)^(1/3)*CosIntegral[b*(c + d*x)^(1/3)]*Sin[a])/(8*d*e^2*(e*(c + d*x)
)^(1/3)) - (3*Sin[a + b*(c + d*x)^(1/3)])/(4*d*e^2*(c + d*x)*(e*(c + d*x))^
(1/3)) + (b^2*Sin[a + b*(c + d*x)^(1/3)])/(8*d*e^2*(c + d*x)^(1/3)*(e*(c +
d*x))^(1/3)) + (b^4*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b*(c + d*x)^(1/3)])/
(8*d*e^2*(e*(c + d*x))^(1/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+b\sqrt[3]{c+dx}\right)}{(ce+dex)^{7/3}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^2\sin(a+bx)}{(ex^3)^{7/3}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}\,dx, x, \sqrt[3]{c+dx}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^4}\,dx, x, \sqrt[3]{c+dx}\right)}{4de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} - \frac{\left(b^2\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,d\ldots\right.}{4de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{b^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{\left(b^3\sqrt[3]{\ldots}\right.}{\ldots}$$

$$= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{b^2\sin\left(\ldots\right.}{8de^2\sqrt[3]{c\ldots}}$$

$$= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{3\sin\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)\sqrt[3]{e(c+dx)}} + \frac{b^2\sin\left(\ldots\right.}{8de^2\sqrt[3]{c\ldots}}$$

$$= \frac{b^3\cos\left(a+b\sqrt[3]{c+dx}\right)}{8de^2\sqrt[3]{e(c+dx)}} - \frac{b\cos\left(a+b\sqrt[3]{c+dx}\right)}{4de^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{b^4\sqrt[3]{c+dx}\,\text{Ci}\left(b\sqrt[3]{c+dx}\right)\sin(a)}{8de^2\sqrt[3]{e(c+dx)}} - \ldots$$

Mathematica [A] time = 0.355332, size = 184, normalized size = 0.69

$$\frac{b^4\sin(a)(c+dx)^{4/3}\text{CosIntegral}\left(b\sqrt[3]{c+dx}\right) + b^4\cos(a)(c+dx)^{4/3}\text{Si}\left(b\sqrt[3]{c+dx}\right) + b^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right) + \ldots}{8de(e(c+dx))\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (b^3*c*Cos[a + b*(c + d*x)^(1/3)] + b^3*d*x*Cos[a + b*(c + d*x)^(1/3)] - 2*
b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*CosInteg
```

```
ral[b*(c + d*x)^(1/3)]*Sin[a] - 6*Sin[a + b*(c + d*x)^(1/3)] + b^2*(c + d*x
)^(2/3)*Sin[a + b*(c + d*x)^(1/3)] + b^4*(c + d*x)^(4/3)*Cos[a]*SinIntegral
[b*(c + d*x)^(1/3)])/(8*d*e*(e*(c + d*x))^(4/3))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \sin\left(a+b\sqrt[3]{dx+c}\right)(dex+ce)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dex+ce)^{\frac{2}{3}}\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(1/3)*b + a)/(d^3*e^3*x^3 + 3*c*
d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(1/3)*b + a)/(d*e*x + c*e)^(7/3), x)
```

3.235 $\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=267

$$-\frac{45\sqrt{\pi}e\cos(a)\sqrt[3]{e(c+dx)}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}+\frac{45\sqrt{\pi}e\sin(a)\sqrt[3]{e(c+dx)}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}+\frac{15e(c+dx)^{2/3}\sqrt[3]{e}}{}$$

```
[Out] (45*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(8*b^3*d) - (3*e*(c +
 d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) - (45*e
*Sqrt[Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(8*Sqrt[2]*b^(7/2)*d*(c + d*x)^(1/3)) + (45*e*Sqrt[Pi]*(e*(c + d*x)
)^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2]*b^(
7/2)*d*(c + d*x)^(1/3)) + (15*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a +
 b*(c + d*x)^(2/3)])/(4*b^2*d)
```

Rubi [A] time = 0.273075, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3435, 3417, 3415, 3385, 3386, 3354, 3352, 3351}

$$-\frac{45\sqrt{\pi}e\cos(a)\sqrt[3]{e(c+dx)}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}+\frac{45\sqrt{\pi}e\sin(a)\sqrt[3]{e(c+dx)}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{8\sqrt{2}b^{7/2}d\sqrt[3]{c+dx}}+\frac{15e(c+dx)^{2/3}\sqrt[3]{e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```


```
[Out] (45*e*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(8*b^3*d) - (3*e*(c +
 d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d) - (45*e
*Sqrt[Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(8*Sqrt[2]*b^(7/2)*d*(c + d*x)^(1/3)) + (45*e*Sqrt[Pi]*(e*(c + d*x)
)^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2]*b^(
7/2)*d*(c + d*x)^(1/3)) + (15*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a +
 b*(c + d*x)^(2/3)])/(4*b^2*d)
```


Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (ce+dex)^{4/3}\sin\left(a+b(c+dx)^{2/3}\right)dx = \frac{\text{Subst}\left(\int (ex)^{4/3}\sin\left(a+bx^{2/3}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\left(e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{4/3}\sin\left(a+bx^{2/3}\right)dx, x, c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{6}\sin\left(a+bx^{2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{\left(15e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\right.}{2}$$

$$= -\frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd} + \frac{15e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}{4b^2}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(\right.}{2bd}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(\right.}{2bd}$$

$$= \frac{45e\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{8b^3d} - \frac{3e(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+b(\right.}{2bd}$$

Mathematica [A] time = 0.738247, size = 175, normalized size = 0.66

$$-\frac{3(e(c+dx))^{4/3}\left(2\sqrt{b}\left(\sqrt[3]{c+dx}\left(4b^2(c+dx)^{4/3}-15\right)\cos\left(a+b(c+dx)^{2/3}\right)-10b(c+dx)\sin\left(a+b(c+dx)^{2/3}\right)\right)+15\sqrt{}\right.}{16b^{7/2}d(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(4/3)*(15*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(
c + d*x)^(1/3)] - 15*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)
]*Sin[a] + 2*Sqrt[b]*((c + d*x)^(1/3)*(-15 + 4*b^2*(c + d*x)^(4/3))*Cos[a +
 b*(c + d*x)^(2/3)] - 10*b*(c + d*x)*Sin[a + b*(c + d*x)^(2/3)])))/(16*b^(7
/2)*d*(c + d*x)^(4/3))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{4}{3}} \sin\left(a + b\,(dx + c)^{\frac{2}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dex + ce)^{\frac{4}{3}} \sin\left((dx + c)^{\frac{2}{3}} b + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(4/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.33883, size = 639, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/32*((-8*I*(-I*(d*x*e + c*e)^(2/3)*b*c*e^(-2/3) + c)*e^(I*(d*x*e + c*e)^(
2/3)*b*e^(-2/3) + I*a + 7/3)/b^2 - 8*I*(-I*(d*x*e + c*e)^(2/3)*b*c*e^(-2/3)
 - c)*e^(-I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a + 7/3)/b^2 - 2*I*(4*I*(d*x
*e + c*e)^(5/3)*b^2*e^(-4/3) - 10*(d*x*e + c*e)*b*e^(-2/3) - 15*I*(d*x*e +
c*e)^(1/3))*e^(I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) + I*a + 2)/b^3 - 2*I*(4*I*(
d*x*e + c*e)^(5/3)*b^2*e^(-4/3) + 10*(d*x*e + c*e)*b*e^(-2/3) - 15*I*(d*x*e
 + c*e)^(1/3))*e^(-I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a + 2)/b^3 - 15*sqr
t(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a + 2)/(sqrt(-I*b*
e^(-2/3))*b^3) - 15*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))*e
^(-I*a + 2)/(sqrt(I*b*e^(-2/3))*b^3))*e^(-1) + 8*((d*x*e + c*e)^(2/3)*b*cos
((d*x*e + c*e)^(2/3)*b*e^(-2/3) + a)*e^(-2/3) + (d*x*e + c*e)^(2/3)*b*cos(-
(d*x*e + c*e)^(2/3)*b*e^(-2/3) - a)*e^(-2/3) - sin((d*x*e + c*e)^(2/3)*b*e^
(-2/3) + a) + sin(-(d*x*e + c*e)^(2/3)*b*e^(-2/3) - a))*c*e^(4/3)/b^2)/d
```

3.236 $\int (ce + dex)^{2/3} \sin\left(a + b(c + dx)^{2/3}\right) dx$

Optimal. Leaf size=227

$$-\frac{9\sqrt{\pi}\sin(a)(e(c+dx))^{2/3}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}-\frac{9\sqrt{\pi}\cos(a)(e(c+dx))^{2/3}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}+\frac{9(e(c+dx))^{2/3}\sin}{4b^2d}$$

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 - (9*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)^(1/3)])/(4*Sqrt[2]*b^(5/2)*d*(c + d*x)^(2/3)) - (9*Sqrt[Pi]*(e*(c + d*
x))^(2/3)*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(4*Sqrt[2]*b
^(5/2)*d*(c + d*x)^(2/3)) + (9*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(2/3
)])/(4*b^2*d*(c + d*x)^(1/3))
```

Rubi [A] time = 0.197811, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3435, 3417, 3415, 3385, 3386, 3353, 3352, 3351}

$$-\frac{9\sqrt{\pi}\sin(a)(e(c+dx))^{2/3}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}-\frac{9\sqrt{\pi}\cos(a)(e(c+dx))^{2/3}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{4\sqrt{2}b^{5/2}d(c+dx)^{2/3}}+\frac{9(e(c+dx))^{2/3}\sin}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 - (9*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c +
d*x)^(1/3)])/(4*Sqrt[2]*b^(5/2)*d*(c + d*x)^(2/3)) - (9*Sqrt[Pi]*(e*(c + d*
x))^(2/3)*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(4*Sqrt[2]*b
^(5/2)*d*(c + d*x)^(2/3)) + (9*(e*(c + d*x))^(2/3)*Sin[a + b*(c + d*x)^(2/3
)])/(4*b^2*d*(c + d*x)^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int (ce+dex)^{2/3}\sin\left(a+b(c+dx)^{2/3}\right)dx &= \frac{\text{Subst}\left(\int (ex)^{2/3}\sin\left(a+bx^{2/3}\right)dx,x,c+dx\right)}{d}\\
&= \frac{(e(c+dx))^{2/3}\,\text{Subst}\left(\int x^{2/3}\sin\left(a+bx^{2/3}\right)dx,x,c+dx\right)}{d(c+dx)^{2/3}}\\
&= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+bx^2\right)dx,x,\sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}\\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{\left(9(e(c+dx))^{2/3}\right)\text{Subst}\left(\int\right.}{2bd}\\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{9(e(c+dx))^{2/3}\sin\left(a+b(\right.}{4b^2d\sqrt[3]{c+dx}}\\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{9(e(c+dx))^{2/3}\sin\left(a+b(\right.}{4b^2d\sqrt[3]{c+dx}}\\
&= -\frac{3\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}-\frac{9\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)S}{4\sqrt{2}b^{5/2}d(c+}
\end{aligned}
$$

Mathematica [A] time = 0.45467, size = 160, normalized size = 0.7

$$
-\frac{3(e(c+dx))^{2/3}\left(3\sqrt{2\pi}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)+3\sqrt{2\pi}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+2\sqrt{b}\left(2b(c+dx)\cos\left(a\right.\right.\right.}{8b^{5/2}d(c+dx)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(2/3)*(3*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)] + 3*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*
Sin[a] + 2*Sqrt[b]*(2*b*(c + d*x)*Cos[a + b*(c + d*x)^(2/3)] - 3*(c + d*x)^
(1/3)*Sin[a + b*(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(c + d*x)^(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{2}{3}} \sin \left(a + b (dx + c)^{\frac{2}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dex + ce)^{\frac{2}{3}} \sin \left((dx + c)^{\frac{2}{3}} b + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(2/3)*b + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e (c + dx))^{\frac{2}{3}} \sin \left(a + b (c + dx)^{\frac{2}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(2/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [C] time = 1.2228, size = 284, normalized size = 1.25

$$-\frac{3\left(-\frac{2i\left(2i\,(dxe+ce)be^{\left(-\frac{2}{3}\right)}-3\,(dxe+ce)^{\frac{1}{3}}\right)e^{\left(i\,(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+i\,a+\frac{4}{3}\right)}}{b^2}-\frac{2i\left(2i\,(dxe+ce)be^{\left(-\frac{2}{3}\right)}+3\,(dxe+ce)^{\frac{1}{3}}\right)e^{\left(-i\,(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-i\,a+\frac{4}{3}\right)}}{b^2}+\frac{3i\sqrt{\pi}\,\mathrm{erf}\left(-(dxe+c\right.}{\sqrt{-}}\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/16*(-2*I*(2*I*(d*x*e + c*e)*b*e^(-2/3) - 3*(d*x*e + c*e)^(1/3))*e^(I*(d*
x*e + c*e)^(2/3)*b*e^(-2/3) + I*a + 4/3)/b^2 - 2*I*(2*I*(d*x*e + c*e)*b*e^(
-2/3) + 3*(d*x*e + c*e)^(1/3))*e^(-I*(d*x*e + c*e)^(2/3)*b*e^(-2/3) - I*a +
 4/3)/b^2 + 3*I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I
*a + 4/3)/(sqrt(-I*b*e^(-2/3))*b^2) - 3*I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)
*sqrt(I*b*e^(-2/3)))*e^(-I*a + 4/3)/(sqrt(I*b*e^(-2/3))*b^2))*e^(-1)/d
```

3.237 $\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx$

Optimal. Leaf size=89

$$\frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}-\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 + (3*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(2/3)])/(2*b^2*d*(c + d*x)^(1
/3))
```

Rubi [A] time = 0.0861527, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3435, 3381, 3379, 3296, 2637}

$$\frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx)^{2/3}\right)}{2b^2d\sqrt[3]{c+dx}}-\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d)
 + (3*(e*(c + d*x))^(1/3)*Sin[a + b*(c + d*x)^(2/3)])/(2*b^2*d*(c + d*x)^(1
/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{ce+dex}\sin\left(a+b(c+dx)^{2/3}\right)\,dx &= \frac{\text{Subst}\left(\int \sqrt[3]{ex}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d}\\ &= \frac{\sqrt[3]{e(c+dx)}\,\text{Subst}\left(\int \sqrt[3]{x}\sin\left(a+bx^{2/3}\right)\,dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}\\ &= \frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,(c+dx)^{2/3}\right)}{2d\sqrt[3]{c+dx}}\\ &= -\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \cos(a\right.}{2bd\sqrt[3]{c+}}\\ &= -\frac{3\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd}+\frac{3\sqrt[3]{e(c+dx)}\sin\left(a+b(c+dx\right.}{2b^2d\sqrt[3]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0604944, size = 72, normalized size = 0.81

$$-\frac{3\sqrt[3]{e(c+dx)}\left(b(c+dx)^{2/3}\cos\left(a+b(c+dx)^{2/3}\right)-\sin\left(a+b(c+dx)^{2/3}\right)\right)}{2b^2d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b*(c + d*x)^(2/3)],x]
```

```
[Out] (-3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(2/3)] - Sin
[a + b*(c + d*x)^(2/3)]))/(2*b^2*d*(c + d*x)^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \sqrt[3]{dex+ce}\sin\left(a+b\left(dx+c\right)^{\frac{2}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 7.91939, size = 232, normalized size = 2.61

$$-\frac{3\left((bdx+bc)(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)-(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)\right)}{2\left(b^2d^2x+b^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] -3/2*((b*d*x + b*c)*(d*e*x + c*e)^(1/3)*(d*x + c)^(1/3)*cos((d*x + c)^(2/3)
*b + a) - (d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*sin((d*x + c)^(2/3)*b + a))/(
b^2*d^2*x + b^2*c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[3]{e\,(c+dx)}\sin\left(a+b\,(c+dx)^{\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b*(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b*(c + d*x)**(2/3)), x)
```

Giac [A] time = 1.20059, size = 171, normalized size = 1.92

$$-\frac{3\left((dxe+ce)^{\frac{2}{3}}b\cos\left((dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+a\right)e^{\left(-\frac{2}{3}\right)}+(dxe+ce)^{\frac{2}{3}}b\cos\left(-(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-a\right)e^{\left(-\frac{2}{3}\right)}-\sin\left((dxe+ce)^{\frac{2}{3}}be\right.\right.}{4\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b*(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] -3/4*((d*x*e + c*e)^(2/3)*b*cos((d*x*e + c*e)^(2/3)*b*e^(-2/3) + a)*e^(-2/3
) + (d*x*e + c*e)^(2/3)*b*cos(-(d*x*e + c*e)^(2/3)*b*e^(-2/3) - a)*e^(-2/3)
 - sin((d*x*e + c*e)^(2/3)*b*e^(-2/3) + a) + sin(-(d*x*e + c*e)^(2/3)*b*e^(
-2/3) - a))*e^(1/3)/(b^2*d)
```

3.238 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}} dx$

Optimal. Leaf size=44

$$-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.0628817, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3435, 3381, 3379, 2638}

$$-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{\sqrt[3]{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{\sqrt[3]{ex}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{\sqrt[3]{x}}\,dx, x, c+dx\right)}{d\sqrt[3]{e(c+dx)}}$$
$$= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, (c+dx)^{2/3}\right)}{2d\sqrt[3]{e(c+dx)}}$$
$$= -\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

Mathematica [A] time = 0.074525, size = 44, normalized size = 1.

$$-\frac{3\sqrt[3]{c+dx}\cos\left(a+b(c+dx)^{2/3}\right)}{2bd\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sin\left(a+b\left(dx+c\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [A] time = 1.04933, size = 31, normalized size = 0.7

$$-\frac{3\cos\left(\left(dx+c\right)^{\frac{2}{3}}b+a\right)}{2\,bde^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/2*cos((d*x + c)^(2/3)*b + a)/(b*d*e^(1/3))
```

Fricas [A] time = 1.71172, size = 123, normalized size = 2.8

$$-\frac{3\,(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{2}{3}}b+a\right)}{2\left(bd^{2}ex+bcde\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(d*e*x + c*e)^(2/3)*(d*x + c)^(1/3)*cos((d*x + c)^(2/3)*b + a)/(b*d^2*
e*x + b*c*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+b\,(c+dx)^{\frac{2}{3}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Giac [A] time = 1.22065, size = 70, normalized size = 1.59

$$-\frac{3\left(\cos\left((dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}+a\right)+\cos\left(-(dxe+ce)^{\frac{2}{3}}be^{\left(-\frac{2}{3}\right)}-a\right)\right)e^{\left(-\frac{1}{3}\right)}}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] -3/4*(cos((d*x*e + c*e)^(2/3)*b*e^(-2/3) + a) + cos(-(d*x*e + c*e)^(2/3)*b*
e^(-2/3) - a))*e^(-1/3)/(b*d)
```

3.239 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=133

$$\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(Sqrt[b]*d*(e*(c + d*x))^(2/3)) + (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(Sqrt[b]*d*(e*(c + d*x))^
(2/3))
```

Rubi [A] time = 0.124505, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3435, 3417, 3383, 3353, 3352, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}}+\frac{3\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```


```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^
(1/3)])/(Sqrt[b]*d*(e*(c + d*x))^(2/3)) + (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(Sqrt[b]*d*(e*(c + d*x))^
(2/3))
```


Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3383

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{2/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{(ex)^{2/3}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)^{2/3}\,\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{x^{2/3}}\,dx, x, c+dx\right)}{d(e(c+dx))^{2/3}}$$
$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$
$$= \frac{\left(3(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}} + \frac{\left(3(c+dx)^{2/3}\sin(a)\right)\text{Subst}\,(}{d(e(c+}$$
$$= \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{\sqrt{b}d(e(c+dx))^{2/3}} + \frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

Mathematica [A] time = 0.147337, size = 96, normalized size = 0.72

$$\frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\left(\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)+\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\right)}{\sqrt{b}d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*Sqrt[Pi/2]*(c + d*x)^(2/3)*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)
^(1/3)] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[b]*d*
(e*(c + d*x))^(2/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \sin\left(a+b\left(dx+c\right)^{\frac{2}{3}}\right)\left(dex+ce\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^(2/3)*b + a)/(d*e*x + c*e)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^{\frac{2}{3}}\right)}{(e(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Giac [C] time = 1.13899, size = 113, normalized size = 0.85

$$-\frac{3\left(-\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(ia)}}{\sqrt{-ibe^{\left(-\frac{2}{3}\right)}}}+\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-(dxe+ce)^{\frac{1}{3}}\sqrt{ibe^{\left(-\frac{2}{3}\right)}}\right)e^{(-ia)}}{\sqrt{ibe^{\left(-\frac{2}{3}\right)}}}\right)e^{(-1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] -3/4*(-I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(-I*b*e^(-2/3)))*e^(I*a)/sqr
t(-I*b*e^(-2/3)) + I*sqrt(pi)*erf(-(d*x*e + c*e)^(1/3)*sqrt(I*b*e^(-2/3)))*
e^(-I*a)/sqrt(I*b*e^(-2/3)))*e^(-1)/d
```

3.240 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}} dx$

Optimal. Leaf size=168

$$\frac{3\sqrt{2\pi}\sqrt{b}\cos(a)\sqrt[3]{c+dx}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{2\pi}\sqrt{b}\sin(a)\sqrt[3]{c+dx}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b(c+d\right.}{de\sqrt[3]{e(c+dx}}$$

```
[Out] (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)
^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(d*e*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.16265, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3435, 3417, 3415, 3387, 3354, 3352, 3351}

$$\frac{3\sqrt{2\pi}\sqrt{b}\cos(a)\sqrt[3]{c+dx}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{2\pi}\sqrt{b}\sin(a)\sqrt[3]{c+dx}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}-\frac{3\sin\left(a+b(c+d\right.}{de\sqrt[3]{e(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*(c
 + d*x)^(1/3)])/(d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)
^(1/3)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a])/(d*e*(e*(c + d*
x))^(1/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(d*e*(e*(c + d*x))^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
```

```
LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{4/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{(ex)^{4/3}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{x^{4/3}}\,dx, x, c+dx\right)}{de\sqrt[3]{e(c+dx)}}\\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin\left(a+bx^{2}\right)}{x^{2}}\,dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}\\
&= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(6b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos\left(a+bx^2\right)dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}}\\
&= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{de\sqrt[3]{e(c+dx)}} + \frac{\left(6b\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)dx, x, \sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{(6b\sqrt[3]{}}{}\\
&= \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{b}\sqrt{2\pi}\sqrt[3]{c+dx}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)\sin(a)}{de\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.24195, size = 133, normalized size = 0.79

$$-\frac{3\left(\sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)\sqrt[3]{c+dx}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[3]{c+dx}\right)+\sqrt{2\pi}\sqrt{b}\sin(a)\sqrt[3]{c+dx}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[3]{c+dx}\right)+\sin\left(a+b(c+\right.\right.}{de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi
]*(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(1/3)*FresnelS[Sqrt[b]*S
qrt[2/Pi]*(c + d*x)^(1/3)]*Sin[a] + Sin[a + b*(c + d*x)^(2/3)]))/(d*e*(e*(c
 + d*x))^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sin\left(a + b (dx + c)^{\frac{2}{3}}\right) (dex + ce)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dex + ce)^{\frac{2}{3}} \sin\left((dx + c)^{\frac{2}{3}} b + a\right)}{d^2 e^2 x^2 + 2 cde^2 x + c^2 e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx + c)^{\frac{2}{3}} b + a\right)}{(dex + ce)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(d*e*x + c*e)^(4/3), x)
```

3.241 $\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=126

$$\frac{3b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(2*d*e*(e*(c + d*x))^(2/3)) -
 (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
 d*x))^(2/3))
```

Rubi [A] time = 0.153494, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3435, 3381, 3379, 3297, 3303, 3299, 3302}

$$\frac{3b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}-\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```


```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
d*x))^(2/3)) - (3*Sin[a + b*(c + d*x)^(2/3)])/(2*d*e*(e*(c + d*x))^(2/3)) -
 (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)])/(2*d*e*(e*(c +
 d*x))^(2/3))
```


Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+b(c+dx)^{2/3}\right)}{(ce+dex)^{5/3}}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{(ex)^{5/3}}\,dx, x, c+dx\right)}{d}\\
&= \frac{(c+dx)^{2/3}\,\mathrm{Subst}\left(\int \frac{\sin\left(a+bx^{2/3}\right)}{x^{5/3}}\,dx, x, c+dx\right)}{de(e(c+dx))^{2/3}}\\
&= \frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}\\
&= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}}\\
&= -\frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2/3}\cos(a)\right)\mathrm{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, (c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{(3b}{}\\
&= \frac{3b(c+dx)^{2/3}\cos(a)\mathrm{Ci}\left(b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3\sin\left(a+b(c+dx)^{2/3}\right)}{2de(e(c+dx))^{2/3}} - \frac{3b(c+dx)^{2/3}\sin(a)\mathrm{Si}\left(b(\right.}{2de(e(c+dx))^{2/}}
\end{aligned}$$

Mathematica [A] time = 0.15564, size = 87, normalized size = 0.69

$$-\frac{3\left(-b\cos(a)(c+dx)^{2/3}\mathrm{CosIntegral}\left(b(c+dx)^{2/3}\right)+b\sin(a)(c+dx)^{2/3}\mathrm{Si}\left(b(c+dx)^{2/3}\right)+\sin\left(a+b(c+dx)^{2/3}\right)\right)}{2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (-3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b*(c + d*x)^(2/3)]) + Sin[a + b
*(c + d*x)^(2/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b*(c + d*x)^(2/3)]
))/(2*d*e*(e*(c + d*x))^(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \sin\left(a+b(dx+c)^{\frac{2}{3}}\right)(dex+ce)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dex+ce)^{\frac{1}{3}}\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((d*x + c)^(2/3)*b + a)/(d^2*e^2*x^2 + 2*c*
d*e^2*x + c^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^{\frac{2}{3}}b+a\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^(2/3)*b + a)/(d*e*x + c*e)^(5/3), x)
```

3.242 $\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$

Optimal. Leaf size=247

$$-\frac{b^4\sin(a)\sqrt[3]{e(c+dx)}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}-\frac{b^4\cos(a)\sqrt[3]{e(c+dx)}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}-\frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}$$

```
[Out] -(b^3*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(8*d) + (b*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4*(e*(c +
d*x))^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(8*d*(c + d*x)^(1/3)) -
(b^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(8*d)
+ (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4
*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(8*d*(c + d*x)^
(1/3))
```

Rubi [A] time = 0.242208, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3431, 15, 3297, 3303, 3299, 3302}

$$-\frac{b^4\sin(a)\sqrt[3]{e(c+dx)}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}-\frac{b^4\cos(a)\sqrt[3]{e(c+dx)}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{8d\sqrt[3]{c+dx}}-\frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```


```
[Out] -(b^3*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(8*d) + (b*(c + d*x)^
(2/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4*(e*(c +
d*x))^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(8*d*(c + d*x)^(1/3)) -
(b^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(8*d)
+ (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(4*d) - (b^4
*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(8*d*(c + d*x)^
(1/3))
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx = -\frac{3\,\text{Subst}\left(\int \frac{\sqrt[3]{\frac{e}{x^3}}\sin(a+bx)}{x^4}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^5}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{\left(3b\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^4}dx, x, \ldots\right)}{4d\sqrt[3]{c+dx}}$$

$$= \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} + \ldots$$

$$= \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \ldots$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c \ldots}}{\ldots}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^2\sqrt[3]{c \ldots}}{\ldots}$$

$$= -\frac{b^3\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{8d} + \frac{b(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{4d} - \frac{b^4\sqrt[3]{e(\ldots}}{\ldots}$$

Mathematica [A] time = 0.340315, size = 208, normalized size = 0.84

$$-\frac{\sqrt[3]{e(c+dx)}\left(b^4\sin(a)\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+b^4\cos(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+b^2(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+b^3\sqrt[3]{c+dx}\cos\left(a+\ldots\right.\right.}{8d\sqrt[3]{c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)],x]
```

```
[Out] -((e*(c + d*x))^(1/3)*(-2*b*c*Cos[a + b/(c + d*x)^(1/3)] - 2*b*d*x*Cos[a +
b/(c + d*x)^(1/3)] + b^3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)] + b^4*C
osIntegral[b/(c + d*x)^(1/3)]*Sin[a] - 6*c*(c + d*x)^(1/3)*Sin[a + b/(c + d
*x)^(1/3)] - 6*d*x*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)] + b^2*(c + d*
x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)] + b^4*Cos[a]*SinIntegral[b/(c + d*x)^(1
/3)]))/(8*d*(c + d*x)^(1/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \sqrt[3]{dex+ce}\,\sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dex+ce)^{\frac{1}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{e(c+dx)}\,\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(1/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{1}{3}} \sin\left(a + \frac{b}{(dx + c)^{\frac{1}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(1/3)*sin(a + b/(d*x + c)^(1/3)), x)
```

3.243 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=168

$$\frac{3b^2\sin(a)\sqrt[3]{c+dx}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b^2\cos(a)\sqrt[3]{c+dx}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b(c+dx)^{2/3}}{2d\sqrt[3]{e}}$$

```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x))^(1/3))
+ (3*b^2*(c + d*x)^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(2*d*(e*(c
+ d*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x)
)^(1/3)) + (3*b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2
*d*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.165201, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3431, 15, 3297, 3303, 3299, 3302}

$$\frac{3b^2\sin(a)\sqrt[3]{c+dx}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b^2\cos(a)\sqrt[3]{c+dx}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b(c+dx)^{2/3}}{2d\sqrt[3]{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```


```
[Out] (3*b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x))^(1/3))
+ (3*b^2*(c + d*x)^(1/3)*CosIntegral[b/(c + d*x)^(1/3)]*Sin[a])/(2*d*(e*(c
+ d*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(2*d*(e*(c + d*x)
)^(1/3)) + (3*b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(1/3)])/(2
*d*(e*(c + d*x))^(1/3))
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{ce+dex}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{\frac{e}{x^3}}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx \ldots\right.}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b^2\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \frac{\sin(\ldots}{x}\ldots\right.}{2d\sqrt[3]{e(c+dx)}}$$

$$= \frac{3b(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b^2\sqrt[3]{c+dx}\,\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)\sin(a)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{e(c+dx)}} + \ldots$$

Mathematica [A] time = 0.183383, size = 131, normalized size = 0.78

$$\frac{3\left(b^2\sin(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right) + b^2\cos(a)\sqrt[3]{c+dx}\,\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right) + c\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) + dx\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right) + b(c+\ldots\right.}{2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*CosI
ntegral[b/(c + d*x)^(1/3)]*Sin[a] + c*Sin[a + b/(c + d*x)^(1/3)] + d*x*Sin[
a + b/(c + d*x)^(1/3)] + b^2*(c + d*x)^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)
^(1/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)\frac{1}{\sqrt[3]{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)}{(dex+ce)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*e*x + c*e)^(1/
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{\sqrt[3]{e\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex + ce)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(1/3), x)
```

3.244 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=116

$$-\frac{3b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c + d*x
))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)
) + (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c +
d*x))^(2/3))
```

Rubi [A] time = 0.129129, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3431, 15, 3297, 3303, 3299, 3302}

$$-\frac{3b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3b\sin(a)(c+dx)^{2/3}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (-3*b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c + d*x
))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)
) + (3*b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*x)^(1/3)])/(d*(e*(c +
d*x))^(2/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{2/3}}\,dx &= -\frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{2/3}x^4}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d}\\
&= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}\\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}\\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(3b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,\frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{\left(3b(c+dx)^{2,\ldots}\right.}{}\\
&= -\frac{3b(c+dx)^{2/3}\cos(a)\text{Ci}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3b(c+dx)^{2/3}\sin(a)\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.181369, size = 88, normalized size = 0.76

$$\frac{3\left(-b\cos(a)(c+dx)^{2/3}\text{CosIntegral}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+b\sin(a)(c+dx)^{2/3}\text{Si}\left(\frac{b}{\sqrt[3]{c+dx}}\right)+(c+dx)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*(-(b*(c + d*x)^(2/3)*Cos[a]*CosIntegral[b/(c + d*x)^(1/3)]) + (c + d*x)*
Sin[a + b/(c + d*x)^(1/3)] + b*(c + d*x)^(2/3)*Sin[a]*SinIntegral[b/(c + d*
x)^(1/3)]))/(d*(e*(c + d*x))^(2/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)(dex+ce)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)}{(dex+ce)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c))/(d*e*x + c*e)^(2/
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(e(c+dx))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(2/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(1/3))/(e*(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")

[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(2/3), x)
```

3.245 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=45

$$\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.0474572, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3431, 15, 2638}

$$\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(1/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{4/3}}\,dx = -\frac{3\,\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{4/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$
$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\mathrm{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}}$$
$$= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde\sqrt[3]{e(c+dx)}}$$

Mathematica [A] time = 0.0664153, size = 42, normalized size = 0.93

$$\frac{3(c+dx)^{4/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bd(e(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*(e*(c + d*x))^(4/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)(dex+ce)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [A] time = 1.03975, size = 42, normalized size = 0.93

$$\frac{3\cos\left(\frac{(dx+c)^{\frac{1}{3}}a+b}{(dx+c)^{\frac{1}{3}}}\right)}{bde^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] 3*cos(((d*x + c)^(1/3)*a + b)/(d*x + c)^(1/3))/(b*d*e^(4/3))
```

Fricas [A] time = 2.07689, size = 154, normalized size = 3.42

$$\frac{3\,(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)}{bd^2e^2x+bcde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] 3*(d*e*x + c*e)^(2/3)*(d*x + c)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)
/(d*x + c))/(b*d^2*e^2*x + b*c*d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex + ce)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(4/3), x)
```

3.246 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=91

$$\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}-\frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(2/3))
- (3*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e*(e*(c + d*x))^(2/
3))
```

Rubi [A] time = 0.0758568, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2637}

$$\frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}-\frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(1/3)*Cos[a + b/(c + d*x)^(1/3)])/(b*d*e*(e*(c + d*x))^(2/3))
- (3*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e*(e*(c + d*x))^(2/
3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{5/3}}\,dx = -\frac{3\,\mathrm{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{5/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de(e(c+dx))^{2/3}}$$

$$= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{\left(3(c+dx)^{2/3}\right)\mathrm{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}}$$

$$= \frac{3\sqrt[3]{c+dx}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde(e(c+dx))^{2/3}} - \frac{3(c+dx)^{2/3}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de(e(c+dx))^{2/3}}$$

Mathematica [A] time = 0.0877367, size = 72, normalized size = 0.79

$$\frac{3(c+dx)^{5/3}\left(\frac{\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b\sqrt[3]{c+dx}} - \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2}\right)}{d(e(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(5/3)*(Cos[a + b/(c + d*x)^(1/3)]/(b*(c + d*x)^(1/3)) - Sin[a
+ b/(c + d*x)^(1/3)]/b^2))/(d*(e*(c + d*x))^(5/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)(dex+ce)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 7.57984, size = 282, normalized size = 3.1

$$\frac{3\left((dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-(dex+ce)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)\right)}{b^2d^2e^2x+b^2cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] 3*((d*e*x + c*e)^(1/3)*(d*x + c)^(1/3)*b*cos((a*d*x + a*c + (d*x + c)^(2/3)
*b)/(d*x + c)) - (d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*sin((a*d*x + a*c + (d*
x + c)^(2/3)*b)/(d*x + c)))/(b^2*d^2*e^2*x + b^2*c*d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(5/3), x)
```

3.247 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal. Leaf size=172

$$\frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}}-\frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}-\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}$$

```
[Out] (-18*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(1/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) - (9*S
in[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3))
+ (18*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c + d*x))^
(1/3))
```

Rubi [A] time = 0.147866, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2637}

$$\frac{18\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2\sqrt[3]{e(c+dx)}}-\frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}-\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}}+\frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-18*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(1/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)) - (9*S
in[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3))
+ (18*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c + d*x))^
(1/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{7/3}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{7/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2\sqrt[3]{e(c+dx)}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{\left(9\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{bde^2\sqrt[3]{e(c+dx)}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} + \frac{\left(18\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx\right.}{b^2de^2\sqrt[3]{e(c+dx)}}$$

$$= -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} + \frac{\left(18\sqrt[3]{c+dx}\right)\text{Su}}{}$$

$$= -\frac{18\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} - \frac{9\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} + \frac{18\sqrt[3]{c+dx}\sin\left(\right.}{b^4de^2\sqrt[3]{e(c}}$$

Mathematica [A] time = 0.166364, size = 107, normalized size = 0.62

$$-\frac{3\left(3\sqrt[3]{c+dx}\left(b^2\sqrt[3]{c+dx}-2c-2dx\right)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+\left(6b(c+dx)-b^3\sqrt[3]{c+dx}\right)\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)\right)}{b^4de(e(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-3*((-(b^3*(c + d*x)^(1/3)) + 6*b*(c + d*x))*Cos[a + b/(c + d*x)^(1/3)] +
3*(c + d*x)^(1/3)*(-2*c - 2*d*x + b^2*(c + d*x)^(1/3))*Sin[a + b/(c + d*x)^
(1/3)]))/(b^4*d*e*(e*(c + d*x))^(4/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)(dex+ce)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 8.88699, size = 382, normalized size = 2.22

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b^3-6\,bdx-6\,bc\right)(dex+ce)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right)-3\,(dex+ce)^{\frac{2}{3}}\left((dx+c)^{\frac{2}{3}}b^2-2\,(dx+c)^{\frac{4}{3}}\right)\sin\left(\frac{adx+ac+}{dx}\right.\right.}{b^4d^3e^3x^2+2\,b^4cd^2e^3x+b^4c^2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(1/3)*b^3 - 6*b*d*x - 6*b*c)*(d*e*x + c*e)^(2/3)*cos((a*d*x +
 a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 3*(d*e*x + c*e)^(2/3)*((d*x + c)^(2/
3)*b^2 - 2*(d*x + c)^(4/3))*sin((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)
))/(b^4*d^3*e^3*x^2 + 2*b^4*c*d^2*e^3*x + b^4*c^2*d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(7/3), x)
```

3.248 $\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal. Leaf size=217

$$\frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}-\frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}}+\frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}}-\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}}+\frac{3}{bde^2(c}$$

```
[Out] (-36*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(2/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(2/3)) + (72*
(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(b^5*d*e^2*(e*(c + d*x))^(2/3))
 - (12*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))
^(2/3)) + (72*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c
+ d*x))^(2/3))
```

Rubi [A] time = 0.188688, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3431, 15, 3296, 2638}

$$\frac{72\sqrt[3]{c+dx}\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^4de^2(e(c+dx))^{2/3}}-\frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}}+\frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}}-\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}}+\frac{3}{bde^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] (-36*Cos[a + b/(c + d*x)^(1/3)])/(b^3*d*e^2*(e*(c + d*x))^(2/3)) + (3*Cos[a
 + b/(c + d*x)^(1/3)])/(b*d*e^2*(c + d*x)^(2/3)*(e*(c + d*x))^(2/3)) + (72*
(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(1/3)])/(b^5*d*e^2*(e*(c + d*x))^(2/3))
 - (12*Sin[a + b/(c + d*x)^(1/3)])/(b^2*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))
^(2/3)) + (72*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(1/3)])/(b^4*d*e^2*(e*(c
+ d*x))^(2/3))
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{(ce+dex)^{8/3}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\left(\frac{e}{x^3}\right)^{8/3}x^4}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{\left(12(c+dx)^{2/3}\right)\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{bde^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{\left(36(c+dx)^{2/3}\right)\text{Subst}\left(\int x^2\sin\ldots\right.}{b^2de^2(e(c+dx\ldots}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{\left(72(c+\ldots\right.}{\ldots}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} - \frac{12\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^2de^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{72\sqrt[3]{c+\ldots}}{b^4d\ldots}$$

$$= -\frac{36\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^3de^2(e(c+dx))^{2/3}} + \frac{3\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{bde^2(c+dx)^{2/3}(e(c+dx))^{2/3}} + \frac{72(c+dx)^{2/3}\cos\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)}{b^5de^2(e(c+dx))^{2/3}} - \frac{1\ldots}{b^2de^2\ldots}$$

Mathematica [A] time = 0.275825, size = 112, normalized size = 0.52

$$\frac{(c+dx)^{4/3}\left(12b\left(b^2\left(-\sqrt[3]{c+dx}\right)+6c+6dx\right)\sin\left(a+\frac{b}{\sqrt[3]{c+dx}}\right)+3\left(-12b^2(c+dx)^{2/3}+b^4+24(c+dx)^{4/3}\right)\cos\left(a+\frac{b}{\sqrt[3]{c+d\ldots}}\right.\ldots}{b^5d(e(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(1/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] ((c + d*x)^(4/3)*(3*(b^4 - 12*b^2*(c + d*x)^(2/3) + 24*(c + d*x)^(4/3))*Cos
[a + b/(c + d*x)^(1/3)] + 12*b*(6*c + 6*d*x - b^2*(c + d*x)^(1/3))*Sin[a +
b/(c + d*x)^(1/3)]))/(b^5*d*(e*(c + d*x))^(8/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \sin\left(a+b\frac{1}{\sqrt[3]{dx+c}}\right)(dex+ce)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 8.78944, size = 437, normalized size = 2.01

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b^4 - 12\,b^2dx - 12\,b^2c + 24\,(dx+c)^{\frac{5}{3}}\right)(dex+ce)^{\frac{1}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{2}{3}}b}{dx+c}\right) - 4\left((dx+c)^{\frac{2}{3}}b^3 - 6\,(bdx+bc)(dx+c\right.\right.}{b^5d^3e^3x^2 + 2\,b^5cd^2e^3x + b^5c^2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(1/3)*b^4 - 12*b^2*d*x - 12*b^2*c + 24*(d*x + c)^(5/3))*(d*e*
x + c*e)^(1/3)*cos((a*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)) - 4*((d*x +
 c)^(2/3)*b^3 - 6*(b*d*x + b*c)*(d*x + c)^(1/3))*(d*e*x + c*e)^(1/3)*sin((a
*d*x + a*c + (d*x + c)^(2/3)*b)/(d*x + c)))/(b^5*d^3*e^3*x^2 + 2*b^5*c*d^2*
e^3*x + b^5*c^2*d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(1/3))/(d*e*x+c*e)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(1/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(1/3))/(d*e*x + c*e)^(8/3), x)
```

3.249 $\int (ce+dex)^{4/3} \sin\left(a+\frac{b}{(c+dx)^{2/3}}\right) dx$

Optimal. Leaf size=299

$$-\frac{8\sqrt{2\pi}b^{7/2}e\sin(a)\sqrt[3]{e(c+dx)}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}-\frac{8\sqrt{2\pi}b^{7/2}e\cos(a)\sqrt[3]{e(c+dx)}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}-\frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}{35d}$$

```
[Out] (-8*b^3*e*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) + (6*b*e*(
c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) - (8*
b^(7/2)*e*Sqrt[2*Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)])/(35*d*(c + d*x)^(1/3)) - (8*b^(7/2)*e*Sqrt[2*Pi]*(e*(c
 + d*x))^(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(35*d
*(c + d*x)^(1/3)) - (4*b^2*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b/
(c + d*x)^(2/3)])/(35*d) + (3*e*(c + d*x)^2*(e*(c + d*x))^(1/3)*Sin[a + b/(
c + d*x)^(2/3)])/(7*d)
```

Rubi [A] time = 0.287398, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3435, 3417, 3415, 3409, 3387, 3388, 3353, 3352, 3351}

$$-\frac{8\sqrt{2\pi}b^{7/2}e\sin(a)\sqrt[3]{e(c+dx)}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}-\frac{8\sqrt{2\pi}b^{7/2}e\cos(a)\sqrt[3]{e(c+dx)}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{35d\sqrt[3]{c+dx}}-\frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (-8*b^3*e*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) + (6*b*e*(
c + d*x)^(4/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(35*d) - (8*
b^(7/2)*e*Sqrt[2*Pi]*(e*(c + d*x))^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)])/(35*d*(c + d*x)^(1/3)) - (8*b^(7/2)*e*Sqrt[2*Pi]*(e*(c
 + d*x))^(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(35*d
*(c + d*x)^(1/3)) - (4*b^2*e*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3)*Sin[a + b/
(c + d*x)^(2/3)])/(35*d) + (3*e*(c + d*x)^2*(e*(c + d*x))^(1/3)*Sin[a + b/(
c + d*x)^(2/3)])/(7*d)
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (ce+dex)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{\text{Subst}\left(\int (ex)^{4/3}\sin\left(a+\frac{b}{x^{2/3}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\left(e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^{4/3}\sin\left(a+\frac{b}{x^{2/3}}\right)dx, x, c+dx\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int x^6\sin\left(a+\frac{b}{x^2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d\sqrt[3]{c+dx}}$$

$$= -\frac{\left(3e\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^8}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c+dx}}$$

$$= \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{7d} - \frac{\left(6be\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+}{x^6}\right.}{7d\sqrt[3]{c+dx}}$$

$$= \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{3e(c+dx)^2\sqrt[3]{e(c+dx)}\sin\left(a\right.}{7d}$$

$$= \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} - \frac{4b^2e(c+dx)^{2/3}\sqrt[3]{e(c+dx)}\sin}{35d}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{}{(c+}\right.}{35d}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{}{(c+}\right.}{35d}$$

$$= -\frac{8b^3e\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{35d} + \frac{6be(c+dx)^{4/3}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{}{(c+}\right.}{35d}$$

Mathematica [A] time = 0.906377, size = 237, normalized size = 0.79

$$\frac{(e(c+dx))^{4/3}\left(-\frac{8\sqrt{2\pi}b^{7/2}\left(\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)+\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\right)}{(c+dx)^{4/3}} + \frac{\cos\left(\frac{b}{(c+dx)^{2/3}}\right)\left(-4b^2\sin(a)(c+dx)^{2/3}-8b^3\cos(a)+6b\cos(a)(c+dx)^{4/3}+15\sin\right.}{c+dx}\right.}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(4/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((e*(c + d*x))^(4/3)*((Cos[b/(c + d*x)^(2/3)]*(-8*b^3*Cos[a] + 6*b*(c + d*x
)^(4/3)*Cos[a] - 4*b^2*(c + d*x)^(2/3)*Sin[a] + 15*(c + d*x)^2*Sin[a]))/(c
+ d*x) - (8*b^(7/2)*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(c + d
*x)^(4/3) + ((-4*b^2*(c + d*x)^(2/3)*Cos[a] + 15*(c + d*x)^2*Cos[a] + 8*b^3
*Sin[a] - 6*b*(c + d*x)^(4/3)*Sin[a])*Sin[b/(c + d*x)^(2/3)])/(c + d*x)))/(
35*d)
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dex+ce)^{\frac{4}{3}} \sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(4/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(4/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{4}{3}} \sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(4/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(4/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

3.250 $\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$

Optimal. Leaf size=262

$$\frac{4\sqrt{2}\sqrt{\pi}b^{5/2}\cos(a)(e(c+dx))^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4\sqrt{2}\sqrt{\pi}b^{5/2}\sin(a)(e(c+dx))^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4b^2(e(c+dx))^{2/3}\sin\left(a\right.}{5d\sqrt[3]{c+dx}}$$

```
[Out] (2*b*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(5*d)
+ (4*Sqrt[2]*b^(5/2)*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/(c + d*x)^(1/3)])/(5*d*(c + d*x)^(2/3)) - (4*Sqrt[2]*b^(5/2)*Sq
rt[Pi]*(e*(c + d*x))^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*S
in[a])/(5*d*(c + d*x)^(2/3)) - (4*b^2*(e*(c + d*x))^(2/3)*Sin[a + b/(c + d*
x)^(2/3)])/(5*d*(c + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(5*d)
```

Rubi [A] time = 0.227576, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3435, 3417, 3415, 3409, 3387, 3388, 3354, 3352, 3351}

$$\frac{4\sqrt{2}\sqrt{\pi}b^{5/2}\cos(a)(e(c+dx))^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4\sqrt{2}\sqrt{\pi}b^{5/2}\sin(a)(e(c+dx))^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{5d(c+dx)^{2/3}}-\frac{4b^2(e(c+dx))^{2/3}\sin\left(a\right.}{5d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (2*b*(c + d*x)^(1/3)*(e*(c + d*x))^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(5*d)
+ (4*Sqrt[2]*b^(5/2)*Sqrt[Pi]*(e*(c + d*x))^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*
Sqrt[2/Pi])/(c + d*x)^(1/3)])/(5*d*(c + d*x)^(2/3)) - (4*Sqrt[2]*b^(5/2)*Sq
rt[Pi]*(e*(c + d*x))^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*S
in[a])/(5*d*(c + d*x)^(2/3)) - (4*b^2*(e*(c + d*x))^(2/3)*Sin[a + b/(c + d*
x)^(2/3)])/(5*d*(c + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(2/3)*Sin[a +
 b/(c + d*x)^(2/3)])/(5*d)
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& IntegerQ[p] && FractionQ[n]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (ce+dex)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{\text{Subst}\left(\int (ex)^{2/3}\sin\left(a+\frac{b}{x^{2/3}}\right)dx, x, c+dx\right)}{d}$$
$$= \frac{(e(c+dx))^{2/3}\,\text{Subst}\left(\int x^{2/3}\sin\left(a+\frac{b}{x^{2/3}}\right)dx, x, c+dx\right)}{d(c+dx)^{2/3}}$$
$$= \frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+\frac{b}{x^2}\right)dx, x, \sqrt[3]{c+dx}\right)}{d(c+dx)^{2/3}}$$
$$= -\frac{\left(3(e(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sin(a+bx^2)}{x^6}dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(c+dx)^{2/3}}$$
$$= \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{\left(6b(e(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\cos(a+bx}{x^4}\right.}{5d(c+dx)^{2/3}}$$
$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{3(c+dx)(e(c+dx))^{2/3}\sin\left(a+\frac{}{(c\right.}}{5d}$$
$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}}$$
$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} - \frac{4b^2(e(c+dx))^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d\sqrt[3]{c+dx}}$$
$$= \frac{2b\sqrt[3]{c+dx}(e(c+dx))^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{5d} + \frac{4\sqrt{2}b^{5/2}\sqrt{\pi}(e(c+dx))^{2/3}\cos(a)\text{C}}{5d(c+dx)^{2/3}}$$

Mathematica [A] time = 0.363622, size = 228, normalized size = 0.87

$$\frac{(e(c+dx))^{2/3}\left(4\sqrt{2\pi}b^{5/2}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)-4\sqrt{2\pi}b^{5/2}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)-4b^2\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+3c(c+d\right.}{5d(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] ((e*(c + d*x))^(2/3)*(2*b*c*Cos[a + b/(c + d*x)^(2/3)] + 2*b*d*x*Cos[a + b/
(c + d*x)^(2/3)] + 4*b^(5/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]
)/(c + d*x)^(1/3)] - 4*b^(5/2)*Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
+ d*x)^(1/3)]*Sin[a] - 4*b^2*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)] + 3
*c*(c + d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)] + 3*d*x*(c + d*x)^(2/3)*Sin[a
 + b/(c + d*x)^(2/3)]))/(5*d*(c + d*x)^(2/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{2}{3}}\sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dex+ce)^{\frac{2}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(2/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex+ce)^{\frac{2}{3}}\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(2/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(2/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

3.251 $\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx$

Optimal. Leaf size=168

$$\frac{3b^2\sin(a)\sqrt[3]{e(c+dx)}\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}+\frac{3b^2\cos(a)\sqrt[3]{e(c+dx)}\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}$$

```
[Out] (3*b*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d)
+ (3*b^2*(e*(c + d*x))^(1/3)*CosIntegral[b/(c + d*x)^(2/3)]*Sin[a])/(4*d*(c
 + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*d) + (3*b^2*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]
)/(4*d*(c + d*x)^(1/3))
```

Rubi [A] time = 0.181942, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3435, 3381, 3379, 3297, 3303, 3299, 3302}

$$\frac{3b^2\sin(a)\sqrt[3]{e(c+dx)}\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}+\frac{3b^2\cos(a)\sqrt[3]{e(c+dx)}\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{4d\sqrt[3]{c+dx}}+\frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (3*b*(c + d*x)^(1/3)*(e*(c + d*x))^(1/3)*Cos[a + b/(c + d*x)^(2/3)])/(4*d)
+ (3*b^2*(e*(c + d*x))^(1/3)*CosIntegral[b/(c + d*x)^(2/3)]*Sin[a])/(4*d*(c
 + d*x)^(1/3)) + (3*(c + d*x)*(e*(c + d*x))^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*d) + (3*b^2*(e*(c + d*x))^(1/3)*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]
)/(4*d*(c + d*x)^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \sqrt[3]{ce+dex}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)dx = \frac{\text{Subst}\left(\int \sqrt[3]{ex}\sin\left(a+\frac{b}{x^{2/3}}\right)dx,x,c+dx\right)}{d}$$
$$= \frac{\sqrt[3]{e(c+dx)}\,\text{Subst}\left(\int \sqrt[3]{x}\sin\left(a+\frac{b}{x^{2/3}}\right)dx,x,c+dx\right)}{d\sqrt[3]{c+dx}}$$
$$= -\frac{\left(3\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^3}dx,x,\frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{c+dx}}$$
$$= \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} - \frac{\left(3b\sqrt[3]{e(c+dx)}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}d\right.}{4d\sqrt[3]{c+dx}}$$
$$= \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx}\right.}{4d}$$
$$= \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3(c+dx)\sqrt[3]{e(c+dx)}\sin\left(a+\frac{b}{(c+dx}\right.}{4d}$$
$$= \frac{3b\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4d} + \frac{3b^2\sqrt[3]{e(c+dx)}\text{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)\sin(a)}{4d\sqrt[3]{c+dx}}$$

Mathematica [A] time = 0.303161, size = 113, normalized size = 0.67

$$\frac{3\sqrt[3]{e(c+dx)}\left(b^2\sin(a)\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)+b^2\cos(a)\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)^{4/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)+b(c+dx)^{2/3}\text{co}\right.}{4d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)],x]
```

```
[Out] (3*(e*(c + d*x))^(1/3)*(b*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)] + b^2*
CosIntegral[b/(c + d*x)^(2/3)]*Sin[a] + (c + d*x)^(4/3)*Sin[a + b/(c + d*x)
^(2/3)] + b^2*Cos[a]*SinIntegral[b/(c + d*x)^(2/3)]))/(4*d*(c + d*x)^(1/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sqrt[3]{dex + ce}\,\sin\left(a + b\,(dx + c)^{-\frac{2}{3}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

```
[Out] int((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dex + ce)^{\frac{1}{3}}\,\sin\left(\frac{adx + ac + (dx + c)^{\frac{1}{3}}b}{dx + c}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{e\,(c + dx)}\,\sin\left(a + \frac{b}{(c + dx)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(1/3)*sin(a+b/(d*x+c)**(2/3)),x)
```

```
[Out] Integral((e*(c + d*x))**(1/3)*sin(a + b/(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dex + ce)^{\frac{1}{3}} \sin\left(a + \frac{b}{(dx + c)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(1/3)*sin(a+b/(d*x+c)^(2/3)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(1/3)*sin(a + b/(d*x + c)^(2/3)), x)
```

3.252 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx$

Optimal. Leaf size=122

$$-\frac{3b\cos(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b\sin(a)\sqrt[3]{c+dx}\,\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)])/(2*d*(e*(c + d
*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(2*d*(e*(c + d*x))^(
1/3)) + (3*b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(2*d*(e
*(c + d*x))^(1/3))
```

Rubi [A] time = 0.145402, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3435, 3381, 3379, 3297, 3303, 3299, 3302}

$$-\frac{3b\cos(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3b\sin(a)\sqrt[3]{c+dx}\,\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (-3*b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)])/(2*d*(e*(c + d
*x))^(1/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(2*d*(e*(c + d*x))^(
1/3)) + (3*b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*x)^(2/3)])/(2*d*(e
*(c + d*x))^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{\sqrt[3]{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{\sqrt[3]{ex}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{\sqrt[3]{x}}\,dx, x, c+dx\right)}{d\sqrt[3]{e(c+dx)}}\\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}}\\
&= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} - \frac{\left(3b\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{\left(3b\sqrt[3]{c \cdots}\right.}{\cdots}\\
&= -\frac{3b\sqrt[3]{c+dx}\cos(a)\text{Ci}\left(\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2d\sqrt[3]{e(c+dx)}} + \frac{3b\sqrt[3]{c+dx}\sin(a)\text{Si}\left(\frac{b}{(c+dx \cdots}\right.}{2d\sqrt[3]{e(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.212919, size = 90, normalized size = 0.74

$$
\frac{3\left(-b\cos(a)\sqrt[3]{c+dx}\,\text{CosIntegral}\left(\frac{b}{(c+dx)^{2/3}}\right)+b\sin(a)\sqrt[3]{c+dx}\,\text{Si}\left(\frac{b}{(c+dx)^{2/3}}\right)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2d\sqrt[3]{e(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(1/3),x]
```

```
[Out] (3*(-(b*(c + d*x)^(1/3)*Cos[a]*CosIntegral[b/(c + d*x)^(2/3)]) + (c + d*x)*
Sin[a + b/(c + d*x)^(2/3)] + b*(c + d*x)^(1/3)*Sin[a]*SinIntegral[b/(c + d*
```

```
x)^(2/3)]))/(2*d*(e*(c + d*x))^(1/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \sin\left(a + b(dx + c)^{-\frac{2}{3}}\right) \frac{1}{\sqrt[3]{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{(dex + ce)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*e*x + c*e)^(1/
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{\sqrt[3]{e(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(1/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex + ce)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(1/3), x)
```

3.253 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx$

Optimal. Leaf size=164

$$-\frac{3\sqrt{2\pi}\sqrt{b}\cos(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3\sqrt{2\pi}\sqrt{b}\sin(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

```
[Out] (-3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])
/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)) + (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x
)^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(d*(e*(c + d
*x))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(d*(e*(c + d*x))^(2/
3))
```

Rubi [A] time = 0.157629, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3435, 3417, 3415, 3359, 3387, 3354, 3352, 3351}

$$-\frac{3\sqrt{2\pi}\sqrt{b}\cos(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3\sqrt{2\pi}\sqrt{b}\sin(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}+\frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```


```
[Out] (-3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])
/(c + d*x)^(1/3)])/(d*(e*(c + d*x))^(2/3)) + (3*Sqrt[b]*Sqrt[2*Pi]*(c + d*x
)^(2/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(d*(e*(c + d
*x))^(2/3)) + (3*(c + d*x)*Sin[a + b/(c + d*x)^(2/3)])/(d*(e*(c + d*x))^(2/
3))
```


Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3359

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Sin[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{2/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{2/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{2/3}}\,dx, x, c+dx\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a+\frac{b}{x^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d(e(c+dx))^{2/3}}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(6b(c+dx)^{2/3}\right)\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}}$$

$$= \frac{3(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{d(e(c+dx))^{2/3}} - \frac{\left(6b(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{(6\ldots}{\ldots}$$

$$= -\frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{d(e(c+dx))^{2/3}} + \frac{3\sqrt{b}\sqrt{2\pi}(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{d(e(c+dx))^{2/3}} + \frac{3(c+dx)\,\text{s}\ldots}{d(e(\ldots}$$

Mathematica [A] time = 0.327287, size = 136, normalized size = 0.83

$$\frac{3\left(\sqrt{2\pi}\left(-\sqrt{b}\right)\cos(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)+\sqrt{2\pi}\sqrt{b}\sin(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+(c+dx)\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{d(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(2/3),x]
```

```
[Out] (3*(-(Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)]) + Sqrt[b]*Sqrt[2*Pi]*(c + d*x)^(2/3)*FresnelS[(Sqrt[b]
*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] + (c + d*x)*Sin[a + b/(c + d*x)^(2/3)]
))/(d*(e*(c + d*x))^(2/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)(dex+ce)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{(dex+ce)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c))/(d*e*x + c*e)^(2/
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{\frac{2}{3}}}\right)}{(e(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(2/3),x)
```

```
[Out] Integral(sin(a + b/(c + d*x)**(2/3))/(e*(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(2/3), x)
```

3.254 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx$

Optimal. Leaf size=141

$$-\frac{3\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}$$

```
[Out] (-3*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(Sqrt[2]*Sqrt[b]*d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[Pi]*(c + d*x)^
(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(Sqrt[2]*Sqrt[
b]*d*e*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.137753, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3435, 3417, 3383, 3353, 3352, 3351}

$$-\frac{3\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}-\frac{3\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(Sqrt[2]*Sqrt[b]*d*e*(e*(c + d*x))^(1/3)) - (3*Sqrt[Pi]*(c + d*x)^
(1/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(Sqrt[2]*Sqrt[
b]*d*e*(e*(c + d*x))^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3383

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)], x_Symbol] :> Dist[2/n, Subst[I
nt[Sin[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{4/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{4/3}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{4/3}}\,dx, x, c+dx\right)}{de\sqrt[3]{e(c+dx)}} \\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}} \\
&= -\frac{\left(3\sqrt[3]{c+dx}\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de\sqrt[3]{e(c+dx)}} - \frac{\left(3\sqrt[3]{c+dx}\sin(a)\right)\text{Subst}\left(\int \cos\left(b\ldots\right.\right.}{de\sqrt[3]{e(c+dx)}} \\
&= -\frac{3\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}} - \frac{3\sqrt{\pi}\sqrt[3]{c+dx}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\sin(a)}{\sqrt{2}\sqrt{b}de\sqrt[3]{e(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.170769, size = 96, normalized size = 0.68

$$
-\frac{3\sqrt{\frac{\pi}{2}}(c+dx)^{4/3}\left(\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)+\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)\right)}{\sqrt{b}d(e(c+dx))^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(4/3),x]
```

```
[Out] (-3*Sqrt[Pi/2]*(c + d*x)^(4/3)*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d
*x)^(1/3)] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a]))/(Sqrt[
b]*d*(e*(c + d*x))^(4/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$
\int \sin\left(a+b(dx+c)^{-\frac{2}{3}}\right)(dex+ce)^{-\frac{4}{3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dex+ce)^{\frac{2}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
))/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(4/3), x)
```

3.255 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx$

Optimal. Leaf size=47

$$\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

```
[Out] (3*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e*(e*(c + d*x))^(2/3)
)
```

Rubi [A] time = 0.066342, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3435, 3381, 3379, 2638}

$$\frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(2/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e*(e*(c + d*x))^(2/3)
)
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{5/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{5/3}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)^{2/3}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{5/3}}\,dx, x, c+dx\right)}{de(e(c+dx))^{2/3}}$$
$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2de(e(c+dx))^{2/3}}$$
$$= \frac{3(c+dx)^{2/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde(e(c+dx))^{2/3}}$$

Mathematica [A] time = 0.0819698, size = 44, normalized size = 0.94

$$\frac{3(c+dx)^{5/3}\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bd(e(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(5/3),x]
```

```
[Out] (3*(c + d*x)^(5/3)*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*(e*(c + d*x))^(5/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)(dex+ce)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x)
```

Maxima [A] time = 1.03258, size = 42, normalized size = 0.89

$$\frac{3\cos\left(\frac{(dx+c)^{\frac{2}{3}}a+b}{(dx+c)^{\frac{2}{3}}}\right)}{2\,bde^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="maxima")
```

```
[Out] 3/2*cos(((d*x + c)^(2/3)*a + b)/(d*x + c)^(2/3))/(b*d*e^(5/3))
```

Fricas [A] time = 1.46317, size = 157, normalized size = 3.34

$$\frac{3\left(dex+ce\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{2\left(bd^{2}e^{2}x+bcde^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="fricas")
```

```
[Out] 3/2*(d*e*x + c*e)^(1/3)*(d*x + c)^(2/3)*cos((a*d*x + a*c + (d*x + c)^(1/3)*
b)/(d*x + c))/(b*d^2*e^2*x + b*c*d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{\left(dex+ce\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(5/3), x)
```

3.256 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}-\frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/
3)) - (3*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)])/(2*b^2*d*e^2*(e*(c + d
*x))^(1/3))
```

Rubi [A] time = 0.0955332, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3435, 3381, 3379, 3296, 2637}

$$\frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}}-\frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(1/
3)) - (3*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)])/(2*b^2*d*e^2*(e*(c + d
*x))^(1/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3381

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{7/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{7/3}}\,dx, x, c+dx\right)}{d}\\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{7/3}}\,dx, x, c+dx\right)}{de^2\sqrt[3]{e(c+dx)}}\\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2de^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{e(c+dx)}}\\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}\sqrt[3]{e(c+dx)}} - \frac{3\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2b^2de^2\sqrt[3]{e(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.108754, size = 72, normalized size = 0.76

$$-\frac{3(c+dx)^{5/3}\left((c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)-b\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)}{2b^2d(e(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(5/3)*(-(b*Cos[a + b/(c + d*x)^(2/3)]) + (c + d*x)^(2/3)*Sin[
a + b/(c + d*x)^(2/3)]))/(2*b^2*d*(e*(c + d*x))^(7/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)(dex+ce)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 6.57708, size = 317, normalized size = 3.34

$$\frac{3\left((dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}b\cos\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)-(dex+ce)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)\right)}{2\left(b^2d^3e^3x^2+2\,b^2cd^2e^3x+b^2c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="fricas")
```

```
[Out] 3/2*((d*e*x + c*e)^(2/3)*(d*x + c)^(2/3)*b*cos((a*d*x + a*c + (d*x + c)^(1/
3)*b)/(d*x + c)) - (d*e*x + c*e)^(2/3)*(d*x + c)^(4/3)*sin((a*d*x + a*c + (
d*x + c)^(1/3)*b)/(d*x + c)))/(b^2*d^3*e^3*x^2 + 2*b^2*c*d^2*e^3*x + b^2*c^
2*d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(7/3), x)
```

3.257 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx$

Optimal. Leaf size=237

$$\frac{9\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}+\frac{9\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}-\frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}}+\frac{3\,\mathrm{co}}{2bde^2\sqrt[3]{}}$$

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(2/
3)) + (9*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
 + d*x)^(1/3)])/(4*b^(5/2)*d*e^2*(e*(c + d*x))^(2/3)) + (9*Sqrt[Pi/2]*(c +
d*x)^(2/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(4*b^(5/2
)*d*e^2*(e*(c + d*x))^(2/3)) - (9*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*b^2*d*e^2*(e*(c + d*x))^(2/3))
```

Rubi [A] time = 0.245137, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3435, 3417, 3415, 3409, 3385, 3386, 3353, 3352, 3351}

$$\frac{9\sqrt{\frac{\pi}{2}}\sin(a)(c+dx)^{2/3}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}+\frac{9\sqrt{\frac{\pi}{2}}\cos(a)(c+dx)^{2/3}S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}}-\frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}}+\frac{3\,\mathrm{co}}{2bde^2\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] (3*Cos[a + b/(c + d*x)^(2/3)])/(2*b*d*e^2*(c + d*x)^(1/3)*(e*(c + d*x))^(2/
3)) + (9*Sqrt[Pi/2]*(c + d*x)^(2/3)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c
 + d*x)^(1/3)])/(4*b^(5/2)*d*e^2*(e*(c + d*x))^(2/3)) + (9*Sqrt[Pi/2]*(c +
d*x)^(2/3)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(4*b^(5/2
)*d*e^2*(e*(c + d*x))^(2/3)) - (9*(c + d*x)^(1/3)*Sin[a + b/(c + d*x)^(2/3)
])/(4*b^2*d*e^2*(e*(c + d*x))^(2/3))
```

Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& IntegerQ[p] && FractionQ[n]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{8/3}}\,dx = \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{8/3}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^{2/3}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{8/3}}\,dx, x, c+dx\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^6}\,dx, x, \sqrt[3]{c+dx}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= -\frac{\left(3(c+dx)^{2/3}\right)\text{Subst}\left(\int x^4\sin\left(a+bx^2\right)dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} - \frac{\left(9(c+dx)^{2/3}\right)\text{Subst}\left(\int x^2\cos\left(a+bx^2\right)dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2bde^2(e(c+dx))^{2/3}}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\right)\text{Subst}\left(\int \sin\left(a\right.\right.}{4b^2de^2(e(c+d}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} - \frac{9\sqrt[3]{c+dx}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^2(e(c+dx))^{2/3}} + \frac{\left(9(c+dx)^{2/3}\cos(a)\right)\text{Subst}\left(\int\right.}{4b^2de^2(e(c+}$$

$$= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^2\sqrt[3]{c+dx}(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{4b^{5/2}de^2(e(c+dx))^{2/3}} + \frac{9\sqrt{\frac{\pi}{2}}(c+dx)^{2/3}C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right.}{4b^{5/2}de^2(e(c+dx))^{2}}$$

Mathematica [A] time = 0.907013, size = 165, normalized size = 0.7

$$\frac{(c+dx)^{5/3}\left(9\sqrt{2\pi}\sin(a)(c+dx)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)+9\sqrt{2\pi}\cos(a)(c+dx)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)+6\sqrt{b}\left(2b\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)-3(\right.\right.}{8b^{5/2}d(e(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(8/3),x]
```

```
[Out] ((c + d*x)^(5/3)*(9*Sqrt[2*Pi]*(c + d*x)*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
])/(c + d*x)^(1/3)] + 9*Sqrt[2*Pi]*(c + d*x)*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/
(c + d*x)^(1/3)]*Sin[a] + 6*Sqrt[b]*(2*b*Cos[a + b/(c + d*x)^(2/3)] - 3*(c
+ d*x)^(2/3)*Sin[a + b/(c + d*x)^(2/3)])))/(8*b^(5/2)*d*(e*(c + d*x))^(8/3)
)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \sin\left(a+b\left(dx+c\right)^{-\frac{2}{3}}\right)\left(dex+ce\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dex+ce)^{\frac{1}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(1/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
))/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(8/3), x)
```

3.258 $\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx$

Optimal. Leaf size=277

$$\frac{45\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{45\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}-\frac{45\cos\left(a\right.}{8b^3de^3\sqrt[3]{}}$$

```
[Out] (-45*Cos[a + b/(c + d*x)^(2/3)])/(8*b^3*d*e^3*(e*(c + d*x))^(1/3)) + (3*Cos
[a + b/(c + d*x)^(2/3)])/(2*b*d*e^3*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)) +
(45*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(8*Sqrt[2]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (45*Sqrt[Pi]*(c +
d*x)^(1/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2
]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (15*Sin[a + b/(c + d*x)^(2/3)])/(4*b
^2*d*e^3*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3))
```

Rubi [A] time = 0.264657, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3435, 3417, 3415, 3409, 3385, 3386, 3354, 3352, 3351}

$$\frac{45\sqrt{\pi}\cos(a)\sqrt[3]{c+dx}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{45\sqrt{\pi}\sin(a)\sqrt[3]{c+dx}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}}-\frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}}-\frac{45\cos\left(a\right.}{8b^3de^3\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```


```
[Out] (-45*Cos[a + b/(c + d*x)^(2/3)])/(8*b^3*d*e^3*(e*(c + d*x))^(1/3)) + (3*Cos
[a + b/(c + d*x)^(2/3)])/(2*b*d*e^3*(c + d*x)^(4/3)*(e*(c + d*x))^(1/3)) +
(45*Sqrt[Pi]*(c + d*x)^(1/3)*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)
^(1/3)])/(8*Sqrt[2]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (45*Sqrt[Pi]*(c +
d*x)^(1/3)*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a])/(8*Sqrt[2
]*b^(7/2)*d*e^3*(e*(c + d*x))^(1/3)) - (15*Sin[a + b/(c + d*x)^(2/3)])/(4*b
^2*d*e^3*(c + d*x)^(2/3)*(e*(c + d*x))^(1/3))
```


Rule 3435

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[((h*x)/f)^m*(a + b
*Sin[c + d*x^n])^p, x], x, e + f*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m
}, x] && IGtQ[p, 0] && EqQ[f*g - e*h, 0]
```

Rule 3417

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Sin[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p
] && FractionQ[n]
```

Rule 3415

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
```

```
b*Sin[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rule 3409

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> -Subst[Int[(a + b*Sin[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{(ce+dex)^{10/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{(ex)^{10/3}}\,dx, x, c+dx\right)}{d} \\
&= \frac{\sqrt[3]{c+dx}\,\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^{2/3}}\right)}{x^{10/3}}\,dx, x, c+dx\right)}{de^3\sqrt[3]{e(c+dx)}} \\
&= \frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int \frac{\sin\left(a+\frac{b}{x^2}\right)}{x^8}\,dx, x, \sqrt[3]{c+dx}\right)}{de^3\sqrt[3]{e(c+dx)}} \\
&= -\frac{\left(3\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^6\sin\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{de^3\sqrt[3]{e(c+dx)}} \\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{\left(15\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^4\cos\left(a+bx^2\right)\,dx, x, \frac{1}{\sqrt[3]{c+dx}}\right)}{2bde^3\sqrt[3]{e(c+dx)}} \\
&= \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{\left(45\sqrt[3]{c+dx}\right)\text{Subst}\left(\int x^2\sin \cdots\right.}{4b^2de^3\sqrt[3]{e(c \cdots}} \\
&= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{(45\cdots}{\cdots} \\
&= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} - \frac{15\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{4b^2de^3(c+dx)^{2/3}\sqrt[3]{e(c+dx)}} + \frac{(45\cdots}{\cdots} \\
&= -\frac{45\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{8b^3de^3\sqrt[3]{e(c+dx)}} + \frac{3\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)}{2bde^3(c+dx)^{4/3}\sqrt[3]{e(c+dx)}} + \frac{45\sqrt{\pi}\sqrt[3]{c+dx}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt[3]{c+dx}}\right)}{8\sqrt{2}b^{7/2}de^3\sqrt[3]{e(c+dx)}} - \cdots
\end{aligned}
$$

Mathematica [A] time = 1.25353, size = 192, normalized size = 0.69

$$
\frac{(e(c+dx))^{2/3}\left(-6\sqrt{b}\left(\left(15(c+dx)^{4/3}-4b^2\right)\cos\left(a+\frac{b}{(c+dx)^{2/3}}\right)+10b(c+dx)^{2/3}\sin\left(a+\frac{b}{(c+dx)^{2/3}}\right)\right)+45\sqrt{2\pi}\cos(a)(c+\cdots\right.}{16b^{7/2}de^4(c+dx)^{7/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b/(c + d*x)^(2/3)]/(c*e + d*e*x)^(10/3),x]
```

```
[Out] ((e*(c + d*x))^(2/3)*(45*Sqrt[2*Pi]*(c + d*x)^(5/3)*Cos[a]*FresnelC[(Sqrt[b
]*Sqrt[2/Pi])/(c + d*x)^(1/3)] - 45*Sqrt[2*Pi]*(c + d*x)^(5/3)*FresnelS[(Sq
rt[b]*Sqrt[2/Pi])/(c + d*x)^(1/3)]*Sin[a] - 6*Sqrt[b]*((-4*b^2 + 15*(c + d*
x)^(4/3))*Cos[a + b/(c + d*x)^(2/3)] + 10*b*(c + d*x)^(2/3)*Sin[a + b/(c +
d*x)^(2/3)])))/(16*b^(7/2)*d*e^4*(c + d*x)^(7/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$
\int \sin\left(a+b\,(dx+c)^{-\frac{2}{3}}\right)(dex+ce)^{-\frac{10}{3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

```
[Out] int(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dex+ce)^{\frac{2}{3}}\sin\left(\frac{adx+ac+(dx+c)^{\frac{1}{3}}b}{dx+c}\right)}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)^(2/3)*sin((a*d*x + a*c + (d*x + c)^(1/3)*b)/(d*x + c
))/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4
*e^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)**(2/3))/(d*e*x+c*e)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+\frac{b}{(dx+c)^{\frac{2}{3}}}\right)}{(dex+ce)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b/(d*x+c)^(2/3))/(d*e*x+c*e)^(10/3),x, algorithm="giac")
```

```
[Out] integrate(sin(a + b/(d*x + c)^(2/3))/(d*e*x + c*e)^(10/3), x)
```

3.259 $\int (ex)^m \sin(a+b(c+dx)^n)\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((ex)^m \sin(a+b(c+dx)^n),x\right)$$

```
[Out] Unintegrable[(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Rubi [A] time = 0.009114, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \sin(a+b(c+dx)^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] Defer[Int][(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Rubi steps

$$\int (ex)^m \sin(a+b(c+dx)^n)\,dx = \int (ex)^m \sin(a+b(c+dx)^n)\,dx$$

Mathematica [A] time = 6.09264, size = 0, normalized size = 0.

$$\int (ex)^m \sin(a+b(c+dx)^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] Integrate[(e*x)^m*Sin[a + b*(c + d*x)^n], x]
```

Maple [A] time = 0.187, size = 0, normalized size = 0.

$$\int (ex)^m \sin\left(a+b\,(dx+c)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int((e*x)^m*sin(a+b*(d*x+c)^n),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sin\left((dx+c)^n b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sin\left((dx+c)^n b + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sin((d*x + c)^n*b + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sin\left(a + b\left(c + dx\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral((e*x)**m*sin(a + b*(c + d*x)**n), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sin\left((dx+c)^n b + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sin((d*x + c)^n*b + a), x)
```

3.260 $\int x^3 \sin(a + b(c + dx)^n)\, dx$

Optimal. Leaf size=503

$$\frac{3ie^{ia}c^2(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^4n}-\frac{ie^{ia}c^3(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n}$$

```
[Out] ((-I/2)*c^3*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^4*n*((-
I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c^3*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x
)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + (((3*I)/2)*c^2*E^(I*a)*(c
+ d*x)^2*Gamma[2/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(2/n))
 - (((3*I)/2)*c^2*(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n*(
I*b*(c + d*x)^n)^(2/n)) - (((3*I)/2)*c*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*
b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(3/n)) + (((3*I)/2)*c*(c + d*x)
^3*Gamma[3/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n)) + (
(I/2)*E^(I*a)*(c + d*x)^4*Gamma[4/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c
 + d*x)^n)^(4/n)) - ((I/2)*(c + d*x)^4*Gamma[4/n, I*b*(c + d*x)^n])/(d^4*E^
(I*a)*n*(I*b*(c + d*x)^n)^(4/n))
```

Rubi [A] time = 0.391409, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3433, 3365, 2208, 3423, 2218}

$$\frac{3ie^{ia}c^2(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^4n}-\frac{ie^{ia}c^3(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[a + b*(c + d*x)^n],x]
```


```
[Out] ((-I/2)*c^3*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^4*n*((-
I)*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c^3*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x
)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + (((3*I)/2)*c^2*E^(I*a)*(c
+ d*x)^2*Gamma[2/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(2/n))
 - (((3*I)/2)*c^2*(c + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n*(
I*b*(c + d*x)^n)^(2/n)) - (((3*I)/2)*c*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*
b*(c + d*x)^n])/(d^4*n*((-I)*b*(c + d*x)^n)^(3/n)) + (((3*I)/2)*c*(c + d*x)
^3*Gamma[3/n, I*b*(c + d*x)^n])/(d^4*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n)) + (
(I/2)*E^(I*a)*(c + d*x)^4*Gamma[4/n, (-I)*b*(c + d*x)^n])/(d^4*n*((-I)*b*(c
 + d*x)^n)^(4/n)) - ((I/2)*(c + d*x)^4*Gamma[4/n, I*b*(c + d*x)^n])/(d^4*E^
(I*a)*n*(I*b*(c + d*x)^n)^(4/n))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \sin\left(a+b(c+dx)^n\right)\,dx &= \frac{\text{Subst}\left(\int\left(-c^3\sin\left(a+bx^n\right)+3c^2x\sin\left(a+bx^n\right)-3cx^2\sin\left(a+bx^n\right)+x^3\sin\left(a+bx^n\right.\right.}{d^4}\\ &= \frac{\text{Subst}\left(\int x^3\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^4}-\frac{(3c)\,\text{Subst}\left(\int x^2\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^4}\\ &= \frac{i\,\text{Subst}\left(\int e^{-ia-ibx^n}x^3\,dx,x,c+dx\right)}{2d^4}-\frac{i\,\text{Subst}\left(\int e^{ia+ibx^n}x^3\,dx,x,c+dx\right)}{2d^4}-\frac{(3ic)\,\text{Subst}}{}\\ &= -\frac{ic^3e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^4n}+\frac{ic^3e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}}{2d^4n}\end{aligned}$$

Mathematica [F] time = 11.5296, size = 0, normalized size = 0.

$$\int x^3 \sin\left(a+b(c+dx)^n\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] Integrate[x^3*Sin[a + b*(c + d*x)^n], x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int x^3 \sin\left(a+b\left(dx+c\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(x^3*sin(a+b*(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sin \left((dx+c)^n b + a \right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^3*sin((d*x + c)^n*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \sin \left((dx+c)^n b + a \right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(x^3*sin((d*x + c)^n*b + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sin \left(a + b (c+dx)^n \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(x**3*sin(a + b*(c + d*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sin \left((dx+c)^n b + a \right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*sin((d*x + c)^n*b + a), x)
```

3.261 $\int x^2 \sin\left(a + b(c + dx)^n\right) dx$

Optimal. Leaf size=369

$$\frac{ie^{ia}c^2(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n}-\frac{ie^{-ia}c^2(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n}$$

```
[Out] ((I/2)*c^2*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^n^(-1)) - ((I/2)*c^2*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)
^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) - (I*c*E^(I*a)*(c + d*x)^2*Ga
mma[2/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I)*b*(c + d*x)^n)^(2/n)) + (I*c*(c
+ d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(2/n
)) + ((I/2)*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^(3/n)) - ((I/2)*(c + d*x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(
d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n))
```

Rubi [A] time = 0.249425, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3433, 3365, 2208, 3423, 2218}

$$\frac{ie^{ia}c^2(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n}-\frac{ie^{-ia}c^2(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2d^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*(c + d*x)^n],x]
```


```
[Out] ((I/2)*c^2*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^n^(-1)) - ((I/2)*c^2*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)
^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) - (I*c*E^(I*a)*(c + d*x)^2*Ga
mma[2/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I)*b*(c + d*x)^n)^(2/n)) + (I*c*(c
+ d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(2/n
)) + ((I/2)*E^(I*a)*(c + d*x)^3*Gamma[3/n, (-I)*b*(c + d*x)^n])/(d^3*n*((-I
)*b*(c + d*x)^n)^(3/n)) - ((I/2)*(c + d*x)^3*Gamma[3/n, I*b*(c + d*x)^n])/(
d^3*E^(I*a)*n*(I*b*(c + d*x)^n)^(3/n))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^2 \sin(a+b(c+dx)^n)\,dx = \frac{\text{Subst}\left(\int \left(c^2\sin(a+bx^n)-2cx\sin(a+bx^n)+x^2\sin(a+bx^n)\right)dx,x,c+dx\right)}{d^3}$$
$$= \frac{\text{Subst}\left(\int x^2\sin(a+bx^n)\,dx,x,c+dx\right)}{d^3} - \frac{(2c)\,\text{Subst}\left(\int x\sin(a+bx^n)\,dx,x,c+dx\right)}{d^3} + \cdots$$
$$= \frac{i\,\text{Subst}\left(\int e^{-ia-ibx^n}x^2\,dx,x,c+dx\right)}{2d^3} - \frac{i\,\text{Subst}\left(\int e^{ia+ibx^n}x^2\,dx,x,c+dx\right)}{2d^3} - \frac{(ic)\,\text{Subst}\left(\int e \cdots\right.}{\cdots}$$
$$= \frac{ic^2e^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^3n} - \frac{ic^2e^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n}\cdots\right.}{2d^3n}$$

Mathematica [F] time = 7.57962, size = 0, normalized size = 0.

$$\int x^2\sin(a+b(c+dx)^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sin[a + b*(c + d*x)^n],x]

[Out] Integrate[x^2*Sin[a + b*(c + d*x)^n], x]
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x^2\sin\left(a+b\,(dx+c)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a+b*(d*x+c)^n),x)

[Out] int(x^2*sin(a+b*(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left((dx+c)^n b+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sin((d*x + c)^n*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\sin\left((dx+c)^n b+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(x^2*sin((d*x + c)^n*b + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left(a+b(c+dx)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(x**2*sin(a + b*(c + d*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sin\left((dx+c)^n b+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin((d*x + c)^n*b + a), x)
```

3.262 $\int x \sin\left(a + b(c + dx)^n\right) dx$

Optimal. Leaf size=243

$$\frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\operatorname{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^2n}-\frac{ie^{ia}c(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}+$$

```
[Out] ((-I/2)*c*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^2*n*((-I)
*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n]
)/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*E^(I*a)*(c + d*x)^2*Gam
ma[2/n, (-I)*b*(c + d*x)^n])/(d^2*n*((-I)*b*(c + d*x)^n)^(2/n)) - ((I/2)*(c
 + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^(2/
n))
```

Rubi [A] time = 0.131454, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3433, 3365, 2208, 3423, 2218}

$$\frac{ie^{ia}(c+dx)^2\left(-ib(c+dx)^n\right)^{-2/n}\operatorname{Gamma}\left(\frac{2}{n},-ib(c+dx)^n\right)}{2d^2n}-\frac{ie^{ia}c(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}+$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*(c + d*x)^n],x]
```


```
[Out] ((-I/2)*c*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d^2*n*((-I)
*b*(c + d*x)^n)^n^(-1)) + ((I/2)*c*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n]
)/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*E^(I*a)*(c + d*x)^2*Gam
ma[2/n, (-I)*b*(c + d*x)^n])/(d^2*n*((-I)*b*(c + d*x)^n)^(2/n)) - ((I/2)*(c
 + d*x)^2*Gamma[2/n, I*b*(c + d*x)^n])/(d^2*E^(I*a)*n*(I*b*(c + d*x)^n)^(2/
n))
```


Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x\sin\left(a+b(c+dx)^n\right)\,dx=\frac{\text{Subst}\left(\int\left(-c\sin\left(a+bx^n\right)+x\sin\left(a+bx^n\right)\right)\,dx,x,c+dx\right)}{d^2}$$
$$=\frac{\text{Subst}\left(\int x\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^2}-\frac{c\,\text{Subst}\left(\int\sin\left(a+bx^n\right)\,dx,x,c+dx\right)}{d^2}$$
$$=\frac{i\,\text{Subst}\left(\int e^{-ia-ibx^n}x\,dx,x,c+dx\right)}{2d^2}-\frac{i\,\text{Subst}\left(\int e^{ia+ibx^n}x\,dx,x,c+dx\right)}{2d^2}-\frac{(ic)\,\text{Subst}\left(\int e\right.}{}$$
$$=-\frac{ice^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2d^2n}+\frac{ice^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\Gamma\left(\right.}{2d^2n}$$

Mathematica [A] time = 0.828501, size = 192, normalized size = 0.79

$$\frac{(c+dx)\left((\sin(a)-i\cos(a))\left(-ib(c+dx)^n\right)^{-2/n}\left(c\left(-ib(c+dx)^n\right)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)-(c+dx)\text{Gamma}\left(\frac{2}{n},-i\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((c + d*x)*((((c*((-I)*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*b*(c + d*x)^
n] - (c + d*x)*Gamma[2/n, (-I)*b*(c + d*x)^n])*((-I)*Cos[a] + Sin[a]))/((-I
)*b*(c + d*x)^n)^(2/n) + ((c*(I*b*(c + d*x)^n)^n^(-1)*Gamma[n^(-1), I*b*(c
+ d*x)^n] - (c + d*x)*Gamma[2/n, I*b*(c + d*x)^n])*(I*Cos[a] + Sin[a]))/(I*
b*(c + d*x)^n)^(2/n)))/(2*d^2*n)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int x\sin\left(a+b\left(dx+c\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(x*sin(a+b*(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(\left(dx+c\right)^{n}b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(x*sin((d*x + c)^n*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x\sin\left((dx+c)^n b+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(x*sin((d*x + c)^n*b + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(a+b\left(c+dx\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(x*sin(a + b*(c + d*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin\left((dx+c)^n b+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(x*sin((d*x + c)^n*b + a), x)
```

3.263 $\int \sin \left(a+b(c+dx)^n\right) \, dx$

Optimal. Leaf size=117

$$\frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

```
[Out] ((I/2)*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d*n*((-I)*b*(c
 + d*x)^n)^n^(-1)) - ((I/2)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n])/(d*E^
(I*a)*n*(I*b*(c + d*x)^n)^n^(-1))
```

Rubi [A] time = 0.0293592, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3365, 2208}

$$\frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((I/2)*E^(I*a)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)^n])/(d*n*((-I)*b*(c
 + d*x)^n)^n^(-1)) - ((I/2)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n])/(d*E^
(I*a)*n*(I*b*(c + d*x)^n)^n^(-1))
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \sin \left(a+b(c+dx)^n\right) \, dx &= \frac{1}{2} i \int e^{-ia-ib(c+dx)^n} \, dx-\frac{1}{2} i \int e^{ia+ib(c+dx)^n} \, dx\\ &= \frac{ie^{ia}(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\,\Gamma\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{ie^{-ia}(c+dx)\left(ib(c+dx)^n\right)^{-1/n}\,\Gamma\left(\frac{1}{n},ib(c+dx)^n\right)}{2dn}\end{aligned}$$

Mathematica [A] time = 0.095625, size = 121, normalized size = 1.03

$$\frac{i(\cos(a)+i\sin(a))(c+dx)\left(-ib(c+dx)^n\right)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ib(c+dx)^n\right)}{2dn}-\frac{i(\cos(a)-i\sin(a))(c+dx)\left(ib(c+dx)^n\right)^{-1/n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*(c + d*x)^n],x]
```

```
[Out] ((-I/2)*(c + d*x)*Gamma[n^(-1), I*b*(c + d*x)^n]*(Cos[a] - I*Sin[a]))/(d*n*
(I*b*(c + d*x)^n)^n^(-1)) + ((I/2)*(c + d*x)*Gamma[n^(-1), (-I)*b*(c + d*x)
^n]*(Cos[a] + I*Sin[a]))/(d*n*((-I)*b*(c + d*x)^n)^n^(-1))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(dx+c)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^n),x)
```

```
[Out] int(sin(a+b*(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left((dx+c)^{n}b+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^n*b + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sin\left((dx+c)^{n}b+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^n*b + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a+b\,(c+dx)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**n),x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left((dx+c)^n b + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^n*b + a), x)
```

3.264 $\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sin(a+b(c+dx)^n)}{x},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^n]/x, x]
```

Rubi [A] time = 0.0092419, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^n]/x,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^n]/x, x]
```

Rubi steps

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx = \int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

Mathematica [A] time = 1.68817, size = 0, normalized size = 0.

$$\int \frac{\sin(a+b(c+dx)^n)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^n]/x,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^n]/x, x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\,(dx+c)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^n)/x,x)
```

```
[Out] int(sin(a+b*(d*x+c)^n)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^n b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^n*b + a)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left((dx+c)^n b+a\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^n*b + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\left(c+dx\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**n)/x,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**n)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^n b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^n*b + a)/x, x)
```

3.265 $\int \frac{\sin(a+b(c+dx)^n)}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sin\left(a+b(c+dx)^n\right)}{x^2},x\right)$$

```
[Out] Unintegrable[Sin[a + b*(c + d*x)^n]/x^2, x]
```

Rubi [A] time = 0.009085, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*(c + d*x)^n]/x^2,x]
```

```
[Out] Defer[Int][Sin[a + b*(c + d*x)^n]/x^2, x]
```

Rubi steps

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx=\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

Mathematica [A] time = 1.60642, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b(c+dx)^n\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*(c + d*x)^n]/x^2,x]
```

```
[Out] Integrate[Sin[a + b*(c + d*x)^n]/x^2, x]
```

Maple [A] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\left(dx+c\right)^{n}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*(d*x+c)^n)/x^2,x)
```

```
[Out] int(sin(a+b*(d*x+c)^n)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^n b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left((dx+c)^n b+a\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sin((d*x + c)^n*b + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+b\left(c+dx\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)**n)/x**2,x)
```

```
[Out] Integral(sin(a + b*(c + d*x)**n)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left((dx+c)^n b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*(d*x+c)^n)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin((d*x + c)^n*b + a)/x^2, x)
```

3.266 $\int x^3 \left(a + b\sin\left(c + d(f + gx)^n\right)\right) dx$

Optimal. Leaf size=519

$$\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f\right.}{2g^4n}$$

```
[Out] (a*x^4)/4 - ((I/2)*b*E^(I*c)*f^3*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^4*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f^3*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^n^(-1)) + (((3*I)/2)*
b*E^(I*c)*f^2*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f
 + g*x)^n)^(2/n)) - (((3*I)/2)*b*f^2*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n
])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(2/n)) - (((3*I)/2)*b*E^(I*c)*f*(f + g*
x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(3/n)) + (
((3*I)/2)*b*f*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*
(f + g*x)^n)^(3/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^4*Gamma[4/n, (-I)*d*(f + g
*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) - ((I/2)*b*(f + g*x)^4*Gamma[4/n
, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```

Rubi [A] time = 0.534127, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {14, 3433, 3365, 2208, 3423, 2218}

$$\frac{3ibe^{ic}f^2(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^4n}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f\right.}{2g^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```


```
[Out] (a*x^4)/4 - ((I/2)*b*E^(I*c)*f^3*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^4*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f^3*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^n^(-1)) + (((3*I)/2)*
b*E^(I*c)*f^2*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f
 + g*x)^n)^(2/n)) - (((3*I)/2)*b*f^2*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n
])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(2/n)) - (((3*I)/2)*b*E^(I*c)*f*(f + g*
x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(3/n)) + (
((3*I)/2)*b*f*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*
(f + g*x)^n)^(3/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^4*Gamma[4/n, (-I)*d*(f + g
*x)^n])/(g^4*n*((-I)*d*(f + g*x)^n)^(4/n)) - ((I/2)*b*(f + g*x)^4*Gamma[4/n
, I*d*(f + g*x)^n])/(E^(I*c)*g^4*n*(I*d*(f + g*x)^n)^(4/n))
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+b\sin\left(c+d(f+gx)^n\right)\right)dx &= \int \left(ax^3+bx^3\sin\left(c+d(f+gx)^n\right)\right)dx \\
&= \frac{ax^4}{4}+b\int x^3\sin\left(c+d(f+gx)^n\right)dx \\
&= \frac{ax^4}{4}+\frac{b\,\text{Subst}\left(\int\left(-f^3\sin\left(c+dx^n\right)+3f^2x\sin\left(c+dx^n\right)-3fx^2\sin\left(c+dx^n\right)\right.\right.}{g^4} \\
&= \frac{ax^4}{4}+\frac{b\,\text{Subst}\left(\int x^3\sin\left(c+dx^n\right)dx,x,f+gx\right)}{g^4}-\frac{(3bf)\,\text{Subst}\left(\int x^2\sin\left(c+\right.\right.}{g^4} \\
&= \frac{ax^4}{4}+\frac{(ib)\,\text{Subst}\left(\int e^{-ic-idx^n}x^3\,dx,x,f+gx\right)}{2g^4}-\frac{(ib)\,\text{Subst}\left(\int e^{ic+idx^n}x^3\,dx,x,\right.}{2g^4} \\
&= \frac{ax^4}{4}-\frac{ibe^{ic}f^3(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^4n}+\frac{ibe^{-ic}f^3(f+g}{}
\end{aligned}$$

Mathematica [A] time = 14.6369, size = 539, normalized size = 1.04

$$\frac{1}{4}\left(ax^4-\frac{2ib(\cos(c)-i\sin(c))(f+gx)\left(d^2(f+gx)^{2n}\right)^{-4/n}\left(f^3(\cos(c)+i\sin(c))^2\left(id(f+gx)^n\right)^{4/n}\left(-id(f+gx)^n\right)^{3/n}\text{Ga}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^4 - ((2*I)*b*(f + g*x)*(-(f^3*((-I)*d*(f + g*x)^n)^(4/n)*(I*d*(f + g*x
)^n)^(3/n)*Gamma[n^(-1), I*d*(f + g*x)^n]) - (f + g*x)*(-3*f^2*((-I)*d*(f +
 g*x)^n)^(4/n)*(I*d*(f + g*x)^n)^(2/n)*Gamma[2/n, I*d*(f + g*x)^n] - (f + g
```

```
*x)*(-3*f*((-I)*d*(f + g*x)^n)^(4/n)*(I*d*(f + g*x)^n)^n^(-1)*Gamma[3/n, I*
d*(f + g*x)^n] - (f + g*x)*(-(((-I)*d*(f + g*x)^n)^(4/n)*Gamma[4/n, I*d*(f
+ g*x)^n]) + (I*d*(f + g*x)^n)^(4/n)*Gamma[4/n, (-I)*d*(f + g*x)^n]*(Cos[c]
 + I*Sin[c])^2) + 3*f*((-I)*d*(f + g*x)^n)^n^(-1)*(I*d*(f + g*x)^n)^(4/n)*G
amma[3/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2) + 3*f^2*((-I)*d*(f + g
*x)^n)^(2/n)*(I*d*(f + g*x)^n)^(4/n)*Gamma[2/n, (-I)*d*(f + g*x)^n]*(Cos[c]
 + I*Sin[c])^2) + f^3*((-I)*d*(f + g*x)^n)^(3/n)*(I*d*(f + g*x)^n)^(4/n)*Ga
mma[n^(-1), (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2)*(Cos[c] - I*Sin[c]))
/(g^4*n*(d^2*(f + g*x)^(2*n))^(4/n)))/4
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int x^3 \left(a + b\sin\left(c + d\left(gx + f\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x^3*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}ax^4 + b\int x^3 \sin\left(\left(gx + f\right)^n d + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + b*integrate(x^3*sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bx^3 \sin\left(\left(gx + f\right)^n d + c\right) + ax^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*sin((g*x + f)^n*d + c) + a*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(a + b\sin\left(c + d\left(f + gx\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Integral(x**3*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(\left(gx+f\right)^{n}d+c\right)+a\right)x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x^3, x)
```

3.267 $\int x^2 \left(a+b\sin\left(c+d(f+gx)^n\right)\right)\,dx$

Optimal. Leaf size=383

$$\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx\right.}{2g^3n}$$

```
[Out] (a*x^3)/3 + ((I/2)*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*f^2*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^(-1)) - (I*b*E^(I*c
)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)
^(2/n)) + (I*b*f*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I
*d*(f + g*x)^n)^(2/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f
+ g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)^3*Gamma[
3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```

Rubi [A] time = 0.344696, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {14, 3433, 3365, 2208, 3423, 2218}

$$\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx\right.}{2g^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```


```
[Out] (a*x^3)/3 + ((I/2)*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n
])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*f^2*(f + g*x)*Gamma[n^(-1
), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^(-1)) - (I*b*E^(I*c
)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)
^(2/n)) + (I*b*f*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I
*d*(f + g*x)^n)^(2/n)) + ((I/2)*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f
+ g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) - ((I/2)*b*(f + g*x)^3*Gamma[
3/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(3/n))
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)dx=\int\left(ax^2+bx^2\sin\left(c+d(f+gx)^n\right)\right)dx$$
$$=\frac{ax^3}{3}+b\int x^2\sin\left(c+d(f+gx)^n\right)dx$$
$$=\frac{ax^3}{3}+\frac{b\,\mathrm{Subst}\left(\int\left(f^2\sin\left(c+dx^n\right)-2fx\sin\left(c+dx^n\right)+x^2\sin\left(c+dx^n\right)\right)dx,\right.}{g^3}$$
$$=\frac{ax^3}{3}+\frac{b\,\mathrm{Subst}\left(\int x^2\sin\left(c+dx^n\right)dx,x,f+gx\right)}{g^3}-\frac{(2bf)\,\mathrm{Subst}\left(\int x\sin\left(c+\right.\right.}{g^3}$$
$$=\frac{ax^3}{3}+\frac{(ib)\,\mathrm{Subst}\left(\int e^{-ic-idx^n}x^2\,dx,x,f+gx\right)}{2g^3}-\frac{(ib)\,\mathrm{Subst}\left(\int e^{ic+idx^n}x^2\,dx,x,\right.}{2g^3}$$
$$=\frac{ax^3}{3}+\frac{ibe^{ic}f^2(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^3n}-\frac{ibe^{-ic}f^2(f+g}{}$$

Mathematica [A] time = 12.5848, size = 403, normalized size = 1.05

$$\frac{ax^3}{3}+\frac{ib(\cos(c)-i\sin(c))(f+gx)\left(d^2(f+gx)^{2n}\right)^{-3/n}\left(f^2(\cos(c)+i\sin(c))^2\left(id(f+gx)^n\right)^{3/n}\left(-id(f+gx)^n\right)^{2/n}\mathrm{Gamm}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^3)/3 + ((I/2)*b*(f + g*x)*(-(f^2*((-I)*d*(f + g*x)^n)^(3/n)*(I*d*(f +
g*x)^n)^(2/n)*Gamma[n^(-1), I*d*(f + g*x)^n]) - (f + g*x)*(-2*f*((-I)*d*(f
+ g*x)^n)^(3/n)*(I*d*(f + g*x)^n)^n^(-1)*Gamma[2/n, I*d*(f + g*x)^n] - (f +
 g*x)*(-(((-I)*d*(f + g*x)^n)^(3/n)*Gamma[3/n, I*d*(f + g*x)^n]) + (I*d*(f
+ g*x)^n)^(3/n)*Gamma[3/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2) + 2*f
*((-I)*d*(f + g*x)^n)^n^(-1)*(I*d*(f + g*x)^n)^(3/n)*Gamma[2/n, (-I)*d*(f +
 g*x)^n]*(Cos[c] + I*Sin[c])^2) + f^2*((-I)*d*(f + g*x)^n)^(2/n)*(I*d*(f +
g*x)^n)^(3/n)*Gamma[n^(-1), (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2)*(Cos
[c] - I*Sin[c]))/(g^3*n*(d^2*(f + g*x)^(2*n))^(3/n))
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int x^2 \left(a + b\sin\left(c + d\left(gx + f\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x^2*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}ax^3 + b\int x^2 \sin\left(\left(gx + f\right)^n d + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(bx^2 \sin\left(\left(gx + f\right)^n d + c\right) + ax^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*sin((g*x + f)^n*d + c) + a*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b\sin\left(c + d\left(f + gx\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Integral(x**2*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(\left(gx + f\right)^n d + c\right) + a\right)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x^2, x)
```

3.268 $\int x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)dx$

Optimal. Leaf size=255

$$\frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx\right.}{2g^2n}$$

```
[Out] (a*x^2)/2 - ((I/2)*b*E^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])
/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*E^(I*c
)*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(
2/n)) - ((I/2)*b*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I
*d*(f + g*x)^n)^(2/n))
```

Rubi [A] time = 0.191521, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {14, 3433, 3365, 2208, 3423, 2218}

$$\frac{ibe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{2g^2n}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx\right.}{2g^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^2)/2 - ((I/2)*b*E^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])
/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*f*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*E^(I*c
)*(f + g*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(
2/n)) - ((I/2)*b*(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I
*d*(f + g*x)^n)^(2/n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
```

```
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)dx=\int\left(ax+bx\sin\left(c+d(f+gx)^n\right)\right)dx$$
$$=\frac{ax^2}{2}+b\int x\sin\left(c+d(f+gx)^n\right)dx$$
$$=\frac{ax^2}{2}+\frac{b\,\mathrm{Subst}\left(\int\left(-f\sin\left(c+dx^n\right)+x\sin\left(c+dx^n\right)\right)dx,x,f+gx\right)}{g^2}$$
$$=\frac{ax^2}{2}+\frac{b\,\mathrm{Subst}\left(\int x\sin\left(c+dx^n\right)dx,x,f+gx\right)}{g^2}-\frac{(bf)\,\mathrm{Subst}\left(\int\sin\left(c+dx^n\right)\right.}{g^2}$$
$$=\frac{ax^2}{2}+\frac{(ib)\,\mathrm{Subst}\left(\int e^{-ic-idx^n}x\,dx,x,f+gx\right)}{2g^2}-\frac{(ib)\,\mathrm{Subst}\left(\int e^{ic+idx^n}x\,dx,x,f+\right.}{2g^2}$$
$$=\frac{ax^2}{2}-\frac{ibe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2g^2n}+\frac{ibe^{-ic}f(f+gx)\left(\right.}{}$$

Mathematica [A] time = 0.492889, size = 215, normalized size = 0.84

$$\frac{b(\sin(c)-i\cos(c))(f+gx)\left(-id(f+gx)^n\right)^{-2/n}\left(f\left(-id(f+gx)^n\right)^{\frac{1}{n}}\mathrm{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)-(f+gx)\mathrm{Gamma}\left(\frac{2}{n},\right.\right.}{2g^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] (a*x^2)/2 + (b*(f + g*x)*(f*((-I)*d*(f + g*x)^n)^n^(-1)*Gamma[n^(-1), (-I)*
d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n])*((-I)*Cos[c] + S
in[c]))/(2*g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (b*(f + g*x)*(f*(I*d*(f + g*
x)^n)^n^(-1)*Gamma[n^(-1), I*d*(f + g*x)^n] - (f + g*x)*Gamma[2/n, I*d*(f +
 g*x)^n])*(I*Cos[c] + Sin[c]))/(2*g^2*n*(I*d*(f + g*x)^n)^(2/n))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int x\left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x*(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,ax^2 + b\int x\sin\left(\left(gx+f\right)^n d + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(bx\sin\left(\left(gx+f\right)^n d + c\right) + ax, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x*sin((g*x + f)^n*d + c) + a*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\sin\left(c + d\left(f + gx\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Integral(x*(a + b*sin(c + d*(f + g*x)**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(\left(gx+f\right)^n d + c\right) + a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)*x, x)
```

3.269 $\int \left(a+b\sin\left(c+d(f+gx)^n\right)\right)\,dx$

Optimal. Leaf size=122

$$\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}$$

```
[Out] a*x + ((I/2)*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g*n*((
-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)
^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rubi [A] time = 0.0579811, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3365, 2208}

$$\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*(f + g*x)^n],x]
```

```
[Out] a*x + ((I/2)*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g*n*((
-I)*d*(f + g*x)^n)^n^(-1)) - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)
^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin\left(c+d(f+gx)^n\right)\right)\,dx &= ax+b\int \sin\left(c+d(f+gx)^n\right)\,dx\\ &= ax+\frac{1}{2}(ib)\int e^{-ic-id(f+gx)^n}\,dx-\frac{1}{2}(ib)\int e^{ic+id(f+gx)^n}\,dx\\ &= ax+\frac{ibe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ibe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}\end{aligned}$$

Mathematica [A] time = 0.245949, size = 126, normalized size = 1.03

$$\frac{ib(\cos(c)+i\sin(c))(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{2gn}-\frac{ib(\cos(c)-i\sin(c))(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+gx)^n\right)}{2gn}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*(f + g*x)^n],x]
```

```
[Out] a*x - ((I/2)*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])
)/(g*n*(I*d*(f + g*x)^n)^n^(-1)) + ((I/2)*b*(f + g*x)*Gamma[n^(-1), (-I)*d*
(f + g*x)^n]*(Cos[c] + I*Sin[c]))/(g*n*((-I)*d*(f + g*x)^n)^n^(-1))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int a+b\sin\left(c+d\left(gx+f\right)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(c+d*(g*x+f)^n),x)
```

```
[Out] int(a+b*sin(c+d*(g*x+f)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ax+b\int\sin\left(\left(gx+f\right)^{n}d+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="maxima")
```

```
[Out] a*x + b*integrate(sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b\sin\left(\left(gx+f\right)^{n}d+c\right)+a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="fricas")
```

```
[Out] integral(b*sin((g*x + f)^n*d + c) + a, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+d\left(f+gx\right)^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)**n),x)
```

```
[Out] Integral(a + b*sin(c + d*(f + g*x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b\sin\left(\left(gx+f\right)^{n}d+c\right)+a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d*(g*x+f)^n),x, algorithm="giac")
```

```
[Out] integrate(b*sin((g*x + f)^n*d + c) + a, x)
```

3.270 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x}\,dx$

Optimal. Leaf size=25

$$b\text{Unintegrable}\left(\frac{\sin\left(c+d(f+gx)^n\right)}{x},x\right)+a\log(x)$$

```
[Out] a*Log[x] + b*Unintegrable[Sin[c + d*(f + g*x)^n]/x, x]
```

Rubi [A] time = 0.018133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
```

```
[Out] a*Log[x] + b*Defer[Int][Sin[c + d*(f + g*x)^n]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx &= \int \left(\frac{a}{x}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x}\right)dx\\ &= a\log(x)+b\int \frac{\sin\left(c+d(f+gx)^n\right)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 2.78931, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x, x]
```

Maple [A] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+d\left(gx+f\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\sin\left(\left(gx+f\right)^{n}d+c\right)}{x}\,dx + a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(sin((g*x + f)^n*d + c)/x, x) + a*log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\sin\left(\left(gx+f\right)^{n}d+c\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+d\left(f+gx\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))/x,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(\left(gx+f\right)^{n}d+c\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)/x, x)
```

3.271 $\int \frac{a+b\sin(c+d(f+gx)^n)}{x^2}\,dx$

Optimal. Leaf size=27

$$b\text{Unintegrable}\left(\frac{\sin\left(c+d(f+gx)^n\right)}{x^2},x\right)-\frac{a}{x}$$

```
[Out] -(a/x) + b*Unintegrable[Sin[c + d*(f + g*x)^n]/x^2, x]
```

Rubi [A] time = 0.0184941, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]

[Out] -(a/x) + b*Defer[Int][Sin[c + d*(f + g*x)^n]/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx &= \int \left(\frac{a}{x^2}+\frac{b\sin\left(c+d(f+gx)^n\right)}{x^2}\right)dx\\ &= -\frac{a}{x}+b\int \frac{\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx\end{aligned}$$

Mathematica [A] time = 2.67523, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+d(f+gx)^n\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2,x]

[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])/x^2, x]
```

Maple [A] time = 0.118, size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+d\left(gx+f\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\sin\left(\left(gx+f\right)^n d+c\right)}{x^2}\,dx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="maxima")
```

```
[Out] b*integrate(sin((g*x + f)^n*d + c)/x^2, x) - a/x
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\sin\left(\left(gx+f\right)^n d+c\right)+a}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+d\left(f+gx\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))/x**2,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(\left(gx+f\right)^n d+c\right)+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)/x^2, x)
```

3.272 $\int x^2 \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2 \, dx$

Optimal. Leaf size=856

$$\frac{iabe^{ic}(f+gx)^3\text{Gamma}\left(\frac{3}{n},-id(f+gx)^n\right)\left(-id(f+gx)^n\right)^{-3/n}}{g^3n}+\frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\text{Gamma}\left(\frac{3}{n},-2id(f+gx)^n\right)\left(-id(f\right.}{g^3n}$$

```
[Out] ((2*a^2 + b^2)*f^2*x)/(2*g^2) - ((2*a^2 + b^2)*f*(f + g*x)^2)/(2*g^3) + ((2
*a^2 + b^2)*(f + g*x)^3)/(6*g^3) + (I*a*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1
), (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*f^2*(f
 + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^
(-1)) + (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f^2*(f + g*x)*Gamma[n^(-1), (-2*I)
*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2
*f^2*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*
(f + g*x)^n)^n^(-1)) - ((2*I)*a*b*E^(I*c)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(
f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) + ((2*I)*a*b*f*(f + g*x)^2*
Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) - (2^(
-1 - 2/n)*b^2*E^((2*I)*c)*f*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n])/(
g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) - (2^(-1 - 2/n)*b^2*f*(f + g*x)^2*Gamma[2
/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) + (I*
a*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f +
 g*x)^n)^(3/n)) - (I*a*b*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*
g^3*n*(I*d*(f + g*x)^n)^(3/n)) + (2^(-2 - 3/n)*b^2*E^((2*I)*c)*(f + g*x)^3*
Gamma[3/n, (-2*I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) + (2^(
-2 - 3/n)*b^2*(f + g*x)^3*Gamma[3/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3
*n*(I*d*(f + g*x)^n)^(3/n))
```

Rubi [A] time = 0.965702, antiderivative size = 856, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3433, 3367, 3366, 2208, 3365, 3425, 6, 3424, 2218, 3423}

$$\frac{iabe^{ic}(f+gx)^3\text{Gamma}\left(\frac{3}{n},-id(f+gx)^n\right)\left(-id(f+gx)^n\right)^{-3/n}}{g^3n}+\frac{2^{-2-\frac{3}{n}}b^2e^{2ic}(f+gx)^3\text{Gamma}\left(\frac{3}{n},-2id(f+gx)^n\right)\left(-id(f\right.}{g^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*f^2*x)/(2*g^2) - ((2*a^2 + b^2)*f*(f + g*x)^2)/(2*g^3) + ((2
*a^2 + b^2)*(f + g*x)^3)/(6*g^3) + (I*a*b*E^(I*c)*f^2*(f + g*x)*Gamma[n^(-1
), (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*f^2*(f
 + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^n^
(-1)) + (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f^2*(f + g*x)*Gamma[n^(-1), (-2*I)
*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2
*f^2*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*
(f + g*x)^n)^n^(-1)) - ((2*I)*a*b*E^(I*c)*f*(f + g*x)^2*Gamma[2/n, (-I)*d*(
f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) + ((2*I)*a*b*f*(f + g*x)^2*
Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) - (2^(
-1 - 2/n)*b^2*E^((2*I)*c)*f*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n])/(
g^3*n*((-I)*d*(f + g*x)^n)^(2/n)) - (2^(-1 - 2/n)*b^2*f*(f + g*x)^2*Gamma[2
/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3*n*(I*d*(f + g*x)^n)^(2/n)) + (I*
a*b*E^(I*c)*(f + g*x)^3*Gamma[3/n, (-I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f +
 g*x)^n)^(3/n)) - (I*a*b*(f + g*x)^3*Gamma[3/n, I*d*(f + g*x)^n])/(E^(I*c)*
g^3*n*(I*d*(f + g*x)^n)^(3/n)) + (2^(-2 - 3/n)*b^2*E^((2*I)*c)*(f + g*x)^3*
Gamma[3/n, (-2*I)*d*(f + g*x)^n])/(g^3*n*((-I)*d*(f + g*x)^n)^(3/n)) + (2^(
```

```
-2 - 3/n)*b^2*(f + g*x)^3*Gamma[3/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^3
*n*(I*d*(f + g*x)^n)^(3/n))
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3367

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int x^2 \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2\,dx = \frac{\text{Subst}\left(\int\left(f^2\left(a+b\sin\left(c+dx^n\right)\right)^2-2fx\left(a+b\sin\left(c+dx^n\right)\right)^2+x^2\left(a+b\sin(c\right.\right.}{g^3}$$

$$= \frac{\text{Subst}\left(\int x^2\left(a+b\sin\left(c+dx^n\right)\right)^2\,dx,x,f+gx\right)}{g^3}-\frac{(2f)\,\text{Subst}\left(\int x\left(a+b\sin(c\right.\right.}{g^3}$$

$$= \frac{\text{Subst}\left(\int\left(a^2x^2+\frac{b^2x^2}{2}-\frac{1}{2}b^2x^2\cos\left(2c+2dx^n\right)+2abx^2\sin\left(c+dx^n\right)\right)dx,x,f+\right.}{g^3}$$

$$= \frac{\left(2a^2+b^2\right)f^2x}{2g^2}+\frac{\text{Subst}\left(\int\left(\left(a^2+\frac{b^2}{2}\right)x^2-\frac{1}{2}b^2x^2\cos\left(2c+2dx^n\right)+2abx^2\sin(c\right.\right.}{g^3}$$

$$= \frac{\left(2a^2+b^2\right)f^2x}{2g^2}-\frac{\left(2a^2+b^2\right)f(f+gx)^2}{2g^3}+\frac{\left(2a^2+b^2\right)(f+gx)^3}{6g^3}+\frac{(2ab)\,\text{Subst}\left(\right.}{}$$

$$= \frac{\left(2a^2+b^2\right)f^2x}{2g^2}-\frac{\left(2a^2+b^2\right)f(f+gx)^2}{2g^3}+\frac{\left(2a^2+b^2\right)(f+gx)^3}{6g^3}+\frac{iabe^{ic}f^2(f+g}{}$$

$$= \frac{\left(2a^2+b^2\right)f^2x}{2g^2}-\frac{\left(2a^2+b^2\right)f(f+gx)^2}{2g^3}+\frac{\left(2a^2+b^2\right)(f+gx)^3}{6g^3}+\frac{iabe^{ic}f^2(f+g}{}$$

Mathematica [A] time = 21.6811, size = 786, normalized size = 0.92

$$\frac{1}{12}\left(-\frac{3b(f+gx)\left(4af^2(\sin(c)-i\cos(c))\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)+4af^2(\sin(c)+i\cos(c))\left(id(f+\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (4*a^2*x^3 + 2*b^2*x^3 - (3*b*(f + g*x)*(((-8*I)*a*f*(f + g*x)*Gamma[2/n, I
*d*(f + g*x)^n]*(Cos[c] - I*Sin[c]))/(I*d*(f + g*x)^n)^(2/n) + ((8*I)*a*f*(
f + g*x)*Gamma[2/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g
*x)^n)^(2/n) + (4*a*f^2*Gamma[n^(-1), (-I)*d*(f + g*x)^n]*((-I)*Cos[c] + Si
n[c]))/((-I)*d*(f + g*x)^n)^n^(-1) + (4*a*(f + g*x)^2*Gamma[3/n, (-I)*d*(f
+ g*x)^n]*((-I)*Cos[c] + Sin[c]))/((-I)*d*(f + g*x)^n)^(3/n) + (4*a*f^2*Gam
ma[n^(-1), I*d*(f + g*x)^n]*(I*Cos[c] + Sin[c]))/(I*d*(f + g*x)^n)^n^(-1) +
 (4*a*(f + g*x)^2*Gamma[3/n, I*d*(f + g*x)^n]*(I*Cos[c] + Sin[c]))/(I*d*(f
+ g*x)^n)^(3/n) - (b*f^2*Gamma[n^(-1), (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin
[c])^2*(Cosh[Log[2]/n] - Sinh[Log[2]/n]))/(I*d*(f + g*x)^n)^n^(-1) - (b*f^2
*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Log[2]/n]
- Sinh[Log[2]/n]))/((-I)*d*(f + g*x)^n)^n^(-1) + (2*b*f*(f + g*x)*Gamma[2/n
, (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/
n]))/(I*d*(f + g*x)^n)^(2/n) + (2*b*f*(f + g*x)*Gamma[2/n, (-2*I)*d*(f + g*
x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/n]))/((-I)*d*(f +
 g*x)^n)^(2/n) - (b*(f + g*x)^2*Gamma[3/n, (2*I)*d*(f + g*x)^n]*(Cos[c] - I
*Sin[c])^2*(Cosh[Log[8]/n] - Sinh[Log[8]/n]))/(I*d*(f + g*x)^n)^(3/n) - (b*
(f + g*x)^2*Gamma[3/n, (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Lo
```

```
g[8]/n] - Sinh[Log[8]/n]))/((-I)*d*(f + g*x)^n)^(3/n)))/(g^3*n))/12
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int x^2 \left(a + b\sin\left(c + d\left(gx + f\right)^n\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a^2x^3 + \frac{1}{6}\,b^2x^3 - \frac{1}{2}\,b^2\int x^2\cos\left(2\left(gx + f\right)^n d + 2\,c\right)\,dx + 2\,ab\int x^2\sin\left(\left(gx + f\right)^n d + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*b^2*x^3 - 1/2*b^2*integrate(x^2*cos(2*(g*x + f)^n*d + 2*c
), x) + 2*a*b*integrate(x^2*sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-b^2x^2\cos\left(\left(gx + f\right)^n d + c\right)^2 + 2\,abx^2\sin\left(\left(gx + f\right)^n d + c\right) + \left(a^2 + b^2\right)x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*x^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*x^2*sin((g*x + f)^n*d +
c) + (a^2 + b^2)*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b\sin\left(c + d\left(f + gx\right)^n\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2*x^2, x)
```

3.273 $\int x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2\,dx$

Optimal. Leaf size=556

$$\frac{iabe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n}-\frac{iabe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f\right.}{g^2n}$$

```
[Out] -((2*a^2 + b^2)*f*x)/(2*g) + ((2*a^2 + b^2)*(f + g*x)^2)/(4*g^2) - (I*a*b*E
^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g
*x)^n)^n^(-1)) + (I*a*b*f*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c
)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) - (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f*(f +
 g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-
1)) - (2^(-2 - n^(-1))*b^2*f*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/
(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + (I*a*b*E^(I*c)*(f + g*x)^2*G
amma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) - (I*a*b*
(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^(
2/n)) + (4^(-1 - n^(-1))*b^2*E^((2*I)*c)*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f
 + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (4^(-1 - n^(-1))*b^2*(f +
g*x)^2*Gamma[2/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n
)^(2/n))
```

Rubi [A] time = 0.456812, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3433, 3367, 3366, 2208, 3365, 3425, 6, 3424, 2218, 3423}

$$\frac{iabe^{ic}(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)}{g^2n}-\frac{iabe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f\right.}{g^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] -((2*a^2 + b^2)*f*x)/(2*g) + ((2*a^2 + b^2)*(f + g*x)^2)/(4*g^2) - (I*a*b*E
^(I*c)*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g
*x)^n)^n^(-1)) + (I*a*b*f*(f + g*x)*Gamma[n^(-1), I*d*(f + g*x)^n])/(E^(I*c
)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) - (2^(-2 - n^(-1))*b^2*E^((2*I)*c)*f*(f +
 g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^n^(-
1)) - (2^(-2 - n^(-1))*b^2*f*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n])/
(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n)^n^(-1)) + (I*a*b*E^(I*c)*(f + g*x)^2*G
amma[2/n, (-I)*d*(f + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) - (I*a*b*
(f + g*x)^2*Gamma[2/n, I*d*(f + g*x)^n])/(E^(I*c)*g^2*n*(I*d*(f + g*x)^n)^(
2/n)) + (4^(-1 - n^(-1))*b^2*E^((2*I)*c)*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f
 + g*x)^n])/(g^2*n*((-I)*d*(f + g*x)^n)^(2/n)) + (4^(-1 - n^(-1))*b^2*(f +
g*x)^2*Gamma[2/n, (2*I)*d*(f + g*x)^n])/(E^((2*I)*c)*g^2*n*(I*d*(f + g*x)^n
)^(2/n))
```

Rule 3433

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 3367

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(-f\left(a+b\sin\left(c+dx^n\right)\right)^2+x\left(a+b\sin\left(c+dx^n\right)\right)^2\right)dx,x,f+gx\right)}{g^2}$$
$$=\frac{\text{Subst}\left(\int x\left(a+b\sin\left(c+dx^n\right)\right)^2\,dx,x,f+gx\right)}{g^2}-\frac{f\,\text{Subst}\left(\int\left(a+b\sin\left(c+d\right.\right.\right.}{g^2}$$
$$=\frac{\text{Subst}\left(\int\left(a^2x+\frac{b^2x}{2}-\frac{1}{2}b^2x\cos\left(2c+2dx^n\right)+2abx\sin\left(c+dx^n\right)\right)dx,x,f+g\right.}{g^2}$$
$$=-\frac{\left(2a^2+b^2\right)fx}{2g}+\frac{\text{Subst}\left(\int\left(\left(a^2+\frac{b^2}{2}\right)x-\frac{1}{2}b^2x\cos\left(2c+2dx^n\right)+2abx\sin\left(c\right.\right.\right.}{g^2}$$
$$=-\frac{\left(2a^2+b^2\right)fx}{2g}+\frac{\left(2a^2+b^2\right)(f+gx)^2}{4g^2}+\frac{(2ab)\,\text{Subst}\left(\int x\sin\left(c+dx^n\right)dx,x,\right.}{g^2}$$
$$=-\frac{\left(2a^2+b^2\right)fx}{2g}+\frac{\left(2a^2+b^2\right)(f+gx)^2}{4g^2}-\frac{iabe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma}{g^2n}$$
$$=-\frac{\left(2a^2+b^2\right)fx}{2g}+\frac{\left(2a^2+b^2\right)(f+gx)^2}{4g^2}-\frac{iabe^{ic}f(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma}{g^2n}$$

Mathematica [A] time = 4.53195, size = 552, normalized size = 0.99

$$\frac{4iab(\cos(c)+i\sin(c))(f+gx)^2\left(-id(f+gx)^n\right)^{-2/n}\text{Gamma}\left(\frac{2}{n},-id(f+gx)^n\right)+4abf(\sin(c)-i\cos(c))(f+gx)\left(-id(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (2*a^2*g^2*n*x^2 + b^2*g^2*n*x^2 - ((4*I)*a*b*(f + g*x)^2*Gamma[2/n, I*d*(f
 + g*x)^n]*(Cos[c] - I*Sin[c]))/(I*d*(f + g*x)^n)^(2/n) + ((4*I)*a*b*(f + g
*x)^2*Gamma[2/n, (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g*x)
^n)^(2/n) + (4*a*b*f*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*x)^n]*((-I)*Cos[
c] + Sin[c]))/((-I)*d*(f + g*x)^n)^n^(-1) + (4*a*b*f*(f + g*x)*Gamma[n^(-1)
, I*d*(f + g*x)^n]*(I*Cos[c] + Sin[c]))/(I*d*(f + g*x)^n)^n^(-1) - (b^2*f*(
f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])^2*(Cosh[Log
[2]/n] - Sinh[Log[2]/n]))/(I*d*(f + g*x)^n)^n^(-1) - (b^2*f*(f + g*x)*Gamma
[n^(-1), (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Log[2]/n] - Sinh
[Log[2]/n]))/((-I)*d*(f + g*x)^n)^n^(-1) + (b^2*(f + g*x)^2*Gamma[2/n, (2*I
)*d*(f + g*x)^n]*(Cos[c] - I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/n]))/(
I*d*(f + g*x)^n)^(2/n) + (b^2*(f + g*x)^2*Gamma[2/n, (-2*I)*d*(f + g*x)^n]*
(Cos[c] + I*Sin[c])^2*(Cosh[Log[4]/n] - Sinh[Log[4]/n]))/((-I)*d*(f + g*x)^
n)^(2/n))/(4*g^2*n)
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int x\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x*(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2x^2 + \frac{1}{4}b^2x^2 - \frac{1}{2}b^2\int x\cos\left(2\left(gx+f\right)^n d + 2c\right)dx + 2ab\int x\sin\left(\left(gx+f\right)^n d + c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/4*b^2*x^2 - 1/2*b^2*integrate(x*cos(2*(g*x + f)^n*d + 2*c),
 x) + 2*a*b*integrate(x*sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-b^2x\cos\left(\left(gx+f\right)^n d + c\right)^2 + 2\,abx\sin\left(\left(gx+f\right)^n d + c\right) + \left(a^2+b^2\right)x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*x*cos((g*x + f)^n*d + c)^2 + 2*a*b*x*sin((g*x + f)^n*d + c) +
 (a^2 + b^2)*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\sin\left(c + d\left(f + gx\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral(x*(a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(\left(gx+f\right)^n d + c\right) + a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2*x, x)
```

3.274 $\int \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2\,dx$

Optimal. Leaf size=261

$$\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+g\right.}{gn}$$

```
[Out] ((2*a^2 + b^2)*x)/2 + (I*a*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*
x)^n])/(g*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*
b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g*n*((-I)*d
*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2*(f + g*x)*Gamma[n^(-1), (2*I)*
d*(f + g*x)^n])/(E^((2*I)*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rubi [A] time = 0.149213, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3367, 3366, 2208, 3365}

$$\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}-\frac{iabe^{-ic}(f+gx)\left(id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},id(f+g\right.}{gn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (I*a*b*E^(I*c)*(f + g*x)*Gamma[n^(-1), (-I)*d*(f + g*
x)^n])/(g*n*((-I)*d*(f + g*x)^n)^n^(-1)) - (I*a*b*(f + g*x)*Gamma[n^(-1), I
*d*(f + g*x)^n])/(E^(I*c)*g*n*(I*d*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*
b^2*E^((2*I)*c)*(f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n])/(g*n*((-I)*d
*(f + g*x)^n)^n^(-1)) + (2^(-2 - n^(-1))*b^2*(f + g*x)*Gamma[n^(-1), (2*I)*
d*(f + g*x)^n])/(E^((2*I)*c)*g*n*(I*d*(f + g*x)^n)^n^(-1))
```

Rule 3367

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
```

```
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\int \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2\,dx=\int\left(a^2+\frac{b^2}{2}-\frac{1}{2}b^2\cos\left(2c+2d(f+gx)^n\right)+2ab\sin\left(c+d(f+gx)^n\right)\right)dx$$
$$=\frac{1}{2}\left(2a^2+b^2\right)x+(2ab)\int\sin\left(c+d(f+gx)^n\right)dx-\frac{1}{2}b^2\int\cos\left(2c+2d(f+gx)^n\right)$$
$$=\frac{1}{2}\left(2a^2+b^2\right)x+(iab)\int e^{-ic-id(f+gx)^n}\,dx-(iab)\int e^{ic+id(f+gx)^n}\,dx-\frac{1}{4}b^2\int e^{-2ic-2id(}$$
$$=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{iabe^{ic}(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-id(f+gx)^n\right)}{gn}-\frac{iabe^{-ic}(f}{}$$

Mathematica [A] time = 1.96856, size = 277, normalized size = 1.06

$$\frac{4iab(\cos(c)+i\sin(c))(f+gx)\left(-id(f+gx)^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-id(f+gx)^n\right)-4iab(\cos(c)-i\sin(c))(f+gx)\left(id(f+gx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] (4*a^2*g*n*x + 2*b^2*g*n*x - ((4*I)*a*b*(f + g*x)*Gamma[n^(-1), I*d*(f + g*
x)^n]*(Cos[c] - I*Sin[c]))/(I*d*(f + g*x)^n)^n^(-1) + ((4*I)*a*b*(f + g*x)*
Gamma[n^(-1), (-I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c]))/((-I)*d*(f + g*x)^n)
^n^(-1) + (b^2*(f + g*x)*Gamma[n^(-1), (2*I)*d*(f + g*x)^n]*(Cos[c] - I*Sin
[c])^2*(Cosh[Log[2]/n] - Sinh[Log[2]/n]))/(I*d*(f + g*x)^n)^n^(-1) + (b^2*(
f + g*x)*Gamma[n^(-1), (-2*I)*d*(f + g*x)^n]*(Cos[c] + I*Sin[c])^2*(Cosh[Lo
g[2]/n] - Sinh[Log[2]/n]))/((-I)*d*(f + g*x)^n)^n^(-1))/(4*g*n)
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2x+\frac{1}{2}b^2x-\frac{1}{2}b^2\int\cos\left(2\left(gx+f\right)^nd+2c\right)dx+2ab\int\sin\left(\left(gx+f\right)^nd+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/2*b^2*x - 1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c), x) + 2*a*
b*integrate(sin((g*x + f)^n*d + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-b^2\cos\left(\left(gx+f\right)^n d+c\right)^2+2\,ab\sin\left(\left(gx+f\right)^n d+c\right)+a^2+b^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-b^2*cos((g*x + f)^n*d + c)^2 + 2*a*b*sin((g*x + f)^n*d + c) + a^2
 + b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+d\left(f+gx\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2, x)
```

3.275 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}{x},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Rubi [A] time = 0.024356, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}{x}\,dx$$

Mathematica [A] time = 3.65128, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x, x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}b^2\int\frac{\cos\left(2\left(gx+f\right)^n d+2c\right)}{x}\,dx+2\,ab\int\frac{\sin\left(\left(gx+f\right)^n d+c\right)}{x}\,dx+a^2\log\left(x\right)+\frac{1}{2}b^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*integrate(cos(2*(g*x + f)^n*d + 2*c)/x, x) + 2*a*b*integrate(sin((
g*x + f)^n*d + c)/x, x) + a^2*log(x) + 1/2*b^2*log(x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\cos\left(\left(gx+f\right)^n d+c\right)^2-2\,ab\sin\left(\left(gx+f\right)^n d+c\right)-a^2-b^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a^
2 - b^2)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sin\left(c+d\left(f+gx\right)^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2/x,x)
```

```
[Out] Integral((a + b*sin(c + d*(f + g*x)**n))**2/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2/x, x)
```

3.276 $\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2},x\right)$$

[Out] Unintegrable[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]

Rubi [A] time = 0.0255086, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]

[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]

Rubi steps

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx = \int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Mathematica [A] time = 3.16726, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(c+d(f+gx)^n))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2,x]

[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^2/x^2, x]

Maple [A] time = 0.293, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)

```
[Out] int((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{x} - \frac{b^2 x \int \frac{\cos\left(2\,(gx+f)^n d+2\,c\right)}{x^2}\,dx - 4\,abx \int \frac{\sin\left((gx+f)^n d+c\right)}{x^2}\,dx + b^2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -a^2/x - 1/2*(b^2*x*integrate(cos(2*(g*x + f)^n*d + 2*c)/x^2, x) - 4*a*b*x*
integrate(sin((g*x + f)^n*d + c)/x^2, x) + b^2)/x
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\cos\left((gx+f)^n d+c\right)^2 - 2\,ab\sin\left((gx+f)^n d+c\right) - a^2 - b^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) - a^
2 - b^2)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left((gx+f)^n d+c\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d*(g*x+f)^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^2/x^2, x)
```

3.277 $\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable[x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi [A] time = 0.0275613, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] Defer[Int][x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Rubi steps

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)} dx = \int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)} dx$$

Mathematica [A] time = 1.37546, size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\sin\left(c+d(f+gx)^n\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]),x]
```

```
[Out] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n]), x]
```

Maple [A] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{x^2}{a+b\sin\left(c+d\left(gx+f\right)^n\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b \sin \left(\left(gx + f \right)^n d + c \right) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{b \sin \left(\left(gx + f \right)^n d + c \right) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b \sin \left(\left(gx + f \right)^n d + c \right) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a), x)
```

3.278 $\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

[Out] Unintegrable[x/(a + b*Sin[c + d*(f + g*x)^n]), x]

Rubi [A] time = 0.0183274, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b*Sin[c + d*(f + g*x)^n]),x]

[Out] Defer[Int][x/(a + b*Sin[c + d*(f + g*x)^n]), x]

Rubi steps

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx = \int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [A] time = 1.29552, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]),x]

[Out] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n]), x]

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\sin\left(c+d\left(gx+f\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*sin(c+d*(g*x+f)^n)),x)

[Out] int(x/(a+b*sin(c+d*(g*x+f)^n)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \sin\left(\left(gx+f\right)^{n} d+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{b \sin\left(\left(gx+f\right)^{n} d+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \sin\left(\left(gx+f\right)^{n} d+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a), x)
```

3.279 $\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{a+b\sin\left(c+d(f+gx)^n\right)},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]
```

Rubi [A] time = 0.0056604, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx=\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Mathematica [A] time = 0.266651, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+d(f+gx)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-1), x]
```

Maple [A] time = 0.066, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(1/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(\left(gx+f\right)^{n}d+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b\sin\left(\left(gx+f\right)^{n}d+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(\left(gx+f\right)^{n}d+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin((g*x + f)^n*d + c) + a), x)
```

3.280 $\int \frac{1}{x(a+b\sin(c+d(f+gx)^n))}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi [A] time = 0.0274813, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Mathematica [A] time = 0.660481, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

```
[Out] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Maple [A] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(1/x/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx\sin\left(\left(gx+f\right)^n d+c\right)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*sin((g*x + f)^n*d + c) + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x), x)
```

3.281 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi [A] time = 0.0274604, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx=\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Mathematica [A] time = 0.813269, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])),x]
```

```
[Out] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])), x]
```

Maple [A] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

```
[Out] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(\left(gx+f\right)^{n}d+c\right)+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{bx^{2}\sin\left(\left(gx+f\right)^{n}d+c\right)+ax^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*sin((g*x + f)^n*d + c) + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(\left(gx+f\right)^{n}d+c\right)+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)*x^2), x)
```

3.282 $\int \frac{x^2}{(a+b\sin(c+d(f+gx)^n))^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^2}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2, x]
```

Rubi [A] time = 0.0274452, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] Defer[Int][x^2/(a + b*Sin[c + d*(f + g*x)^n])^2, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [F] time = 180.165, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(a + b*Sin[c + d*(f + g*x)^n])^2,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.4, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2}{b^2\cos\left(\left(gx+f\right)^n d+c\right)^2-2\,ab\sin\left(\left(gx+f\right)^n d+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-x^2/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c)
- a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

3.283 $\int \frac{x}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

[Out] Unintegrable[x/(a + b*Sin[c + d*(f + g*x)^n])^2, x]

Rubi [A] time = 0.0145726, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]

[Out] Defer[Int][x/(a + b*Sin[c + d*(f + g*x)^n])^2, x]

Rubi steps

$$\int \frac{x}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [F] time = 180.133, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

[In] Integrate[x/(a + b*Sin[c + d*(f + g*x)^n])^2,x]

[Out] $Aborted

Maple [A] time = 2.198, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)

[Out] int(x/(a+b*sin(c+d*(g*x+f)^n))^2,x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{b^2\cos\left(\left(gx+f\right)^n d+c\right)^2-2\,ab\sin\left(\left(gx+f\right)^n d+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-x/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*sin((g*x + f)^n*d + c) + a)^2, x)
```

3.284 $\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Rubi [A] time = 0.005769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]

[Out] Defer[Int][(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [A] time = 11.1895, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2),x]

[Out] Integrate[(a + b*Sin[c + d*(f + g*x)^n])^(-2), x]
```

Maple [A] time = 1.366, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+d\left(gx+f\right)^{n}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(1/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] (2*(a*b*g*x + a*b*f)*cos(2*(g*x + f)^n*d + 2*c)*cos((g*x + f)^n*d + c) + 2*
(a*b*g*x + a*b*f)*cos((g*x + f)^n*d + c) - ((a^2*b^2 - b^4)*(g*x + f)^n*d*g
*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos((
g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n*d
 + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(2*
(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f)^
n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) + (
a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*s
in((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f)^
n*d + 2*c))*integrate(-2*(2*(g*x + f)^n*a^2*d*n*cos((g*x + f)^n*d + c)^2 +
2*(g*x + f)^n*a^2*d*n*sin((g*x + f)^n*d + c)^2 + (g*x + f)^n*a*b*d*n*sin((g
*x + f)^n*d + c) - ((g*x + f)^n*a*b*d*n*sin((g*x + f)^n*d + c) - (a*b*n - a
*b)*cos((g*x + f)^n*d + c))*cos(2*(g*x + f)^n*d + 2*c) + (a*b*n - a*b)*cos(
(g*x + f)^n*d + c) + ((g*x + f)^n*a*b*d*n*cos((g*x + f)^n*d + c) + b^2*n -
b^2 + (a*b*n - a*b)*sin((g*x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a
^2*b^2 - b^4)*(g*x + f)^n*d*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b
^2)*(g*x + f)^n*d*n*cos((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^
n*d*n*cos((g*x + f)^n*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(
g*x + f)^n*d*n*sin(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n
*d*n*sin((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*n*sin((g*x
+ f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n - 2*(2*(a^3*b - a*b^3)*(g*x
 + f)^n*d*n*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*n)*cos(2
*(g*x + f)^n*d + 2*c)), x) + 2*(b^2*g*x + b^2*f + (a*b*g*x + a*b*f)*sin((g*
x + f)^n*d + c))*sin(2*(g*x + f)^n*d + 2*c))/((a^2*b^2 - b^4)*(g*x + f)^n*d
*g*n*cos(2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*cos
((g*x + f)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*cos((g*x + f)^n
*d + c)*sin(2*(g*x + f)^n*d + 2*c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n*sin(
2*(g*x + f)^n*d + 2*c)^2 + 4*(a^4 - a^2*b^2)*(g*x + f)^n*d*g*n*sin((g*x + f
)^n*d + c)^2 + 4*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n*sin((g*x + f)^n*d + c) +
 (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n - 2*(2*(a^3*b - a*b^3)*(g*x + f)^n*d*g*n
*sin((g*x + f)^n*d + c) + (a^2*b^2 - b^4)*(g*x + f)^n*d*g*n)*cos(2*(g*x + f
)^n*d + 2*c))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{b^2\cos\left(\left(gx+f\right)^n d+c\right)^2-2\,ab\sin\left(\left(gx+f\right)^n d+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*sin((g*x + f)^n*d + c) -
a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(\left(gx+f\right)^{n}d+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin((g*x + f)^n*d + c) + a)^(-2), x)
```

3.285 $\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi [A] time = 0.0261045, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2}\,dx$$

Mathematica [F] time = 180.108, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2 x \cos\left(\left(gx+f\right)^n d+c\right)^2-2\,abx \sin\left(\left(gx+f\right)^n d+c\right)-\left(a^2+b^2\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x*cos((g*x + f)^n*d + c)^2 - 2*a*b*x*sin((g*x + f)^n*d + c
) - (a^2 + b^2)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin\left(\left(gx+f\right)^n d+c\right)+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x), x)
```

3.286 $\int \frac{1}{x^2(a+b\sin(c+d(f+gx)^n))^2} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi [A] time = 0.0256077, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2} dx = \int \frac{1}{x^2\left(a+b\sin\left(c+d(f+gx)^n\right)\right)^2} dx$$

Mathematica [F] time = 180.128, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + b*Sin[c + d*(f + g*x)^n])^2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.359, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

```
[Out] int(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2x^2\cos\left(\left(gx+f\right)^nd+c\right)^2-2\,abx^2\sin\left(\left(gx+f\right)^nd+c\right)-\left(a^2+b^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*x^2*cos((g*x + f)^n*d + c)^2 - 2*a*b*x^2*sin((g*x + f)^n*d
 + c) - (a^2 + b^2)*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*sin(c+d*(g*x+f)**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sin\left(\left(gx+f\right)^nd+c\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*sin(c+d*(g*x+f)^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin((g*x + f)^n*d + c) + a)^2*x^2), x)
```

3.287 $\int (ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left((ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Rubi [A] time = 0.0239176, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Rubi steps

$$\int (ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p\,dx=\int (ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p\,dx$$

Mathematica [A] time = 2.09684, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+d(f+gx)^n\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Sin[c + d*(f + g*x)^n])^p, x]
```

Maple [A] time = 0.994, size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\sin\left(c+d\left(gx+f\right)^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

```
[Out] int((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((ex)^m \left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sin(c+d*(g*x+f)**n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(b\sin\left(\left(gx+f\right)^n d+c\right)+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sin(c+d*(g*x+f)^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*sin((g*x + f)^n*d + c) + a)^p, x)
```

3.288 $\int (e+fx)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right) dx$

Optimal. Leaf size=224

$$ae^2x + aefx^2 + \frac{1}{3}af^2x^3 + bd^2ef\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right) + \frac{1}{6}bd^3f^2\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right) - bde^2\cos(c)\text{CosIntegral}$$

```
[Out] a*e^2*x + a*e*f*x^2 + (a*f^2*x^3)/3 + b*d*e*f*x*Cos[c + d/x] + (b*d*f^2*x^2
*Cos[c + d/x])/6 - b*d*e^2*Cos[c]*CosIntegral[d/x] + (b*d^3*f^2*Cos[c]*CosI
ntegral[d/x])/6 + b*d^2*e*f*CosIntegral[d/x]*Sin[c] + b*e^2*x*Sin[c + d/x]
- (b*d^2*f^2*x*Sin[c + d/x])/6 + b*e*f*x^2*Sin[c + d/x] + (b*f^2*x^3*Sin[c
+ d/x])/3 + b*d^2*e*f*Cos[c]*SinIntegral[d/x] + b*d*e^2*Sin[c]*SinIntegral[
d/x] - (b*d^3*f^2*Sin[c]*SinIntegral[d/x])/6
```

Rubi [A] time = 0.457095, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3431, 14, 3297, 3303, 3299, 3302}

$$ae^2x + aefx^2 + \frac{1}{3}af^2x^3 + bd^2ef\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right) + \frac{1}{6}bd^3f^2\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right) - bde^2\cos(c)\text{CosIntegral}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

```
[Out] a*e^2*x + a*e*f*x^2 + (a*f^2*x^3)/3 + b*d*e*f*x*Cos[c + d/x] + (b*d*f^2*x^2
*Cos[c + d/x])/6 - b*d*e^2*Cos[c]*CosIntegral[d/x] + (b*d^3*f^2*Cos[c]*CosI
ntegral[d/x])/6 + b*d^2*e*f*CosIntegral[d/x]*Sin[c] + b*e^2*x*Sin[c + d/x]
- (b*d^2*f^2*x*Sin[c + d/x])/6 + b*e*f*x^2*Sin[c + d/x] + (b*f^2*x^3*Sin[c
+ d/x])/3 + b*d^2*e*f*Cos[c]*SinIntegral[d/x] + b*d*e^2*Sin[c]*SinIntegral[
d/x] - (b*d^3*f^2*Sin[c]*SinIntegral[d/x])/6
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx = -\operatorname{Subst}\left(\int\left(\frac{f^2(a+b\sin(c+dx))}{x^4}+\frac{2ef(a+b\sin(c+dx))}{x^3}+\frac{e^2(a+b\sin(c+dx)}{x^2}\right.\right.$$

$$= -\left(e^2\operatorname{Subst}\left(\int\frac{a+b\sin(c+dx)}{x^2}dx,x,\frac{1}{x}\right)\right)-(2ef)\operatorname{Subst}\left(\int\frac{a+b\sin(c+dx)}{x^3}dx,\right.$$

$$= -\left(e^2\operatorname{Subst}\left(\int\left(\frac{a}{x^2}+\frac{b\sin(c+dx)}{x^2}\right)dx,x,\frac{1}{x}\right)\right)-(2ef)\operatorname{Subst}\left(\int\left(\frac{a}{x^3}+\frac{b\sin(c+}{x^3}\right.\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3-\left(be^2\right)\operatorname{Subst}\left(\int\frac{\sin(c+dx)}{x^2}dx,x,\frac{1}{x}\right)-(2bef)\operatorname{Subst}\left(\int\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+be^2x\sin\left(c+\frac{d}{x}\right)+befx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{3}bf^2x^3\sin\left(c+\frac{d}{x}\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)+be^2x\sin\left(c+\right.$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)-bde^2\cos(c)\mathrm{C}$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)-bde^2\cos(c)\mathrm{C}$$

$$= ae^2x+aefx^2+\frac{1}{3}af^2x^3+bdefx\cos\left(c+\frac{d}{x}\right)+\frac{1}{6}bdf^2x^2\cos\left(c+\frac{d}{x}\right)-bde^2\cos(c)\mathrm{C}$$

Mathematica [A] time = 0.597292, size = 150, normalized size = 0.67

$$\frac{1}{6}\left(x\left(2a\left(3e^2+3efx+f^2x^2\right)+b\sin\left(c+\frac{d}{x}\right)\left(-f^2\left(d^2-2x^2\right)+6e^2+6efx\right)+bdf(6e+fx)\cos\left(c+\frac{d}{x}\right)\right)+bd\mathrm{CosIntegr}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*Sin[c + d/x]),x]
```

```
[Out] (b*d*CosIntegral[d/x]*((-6*e^2 + d^2*f^2)*Cos[c] + 6*d*e*f*Sin[c]) + x*(2*a
*(3*e^2 + 3*e*f*x + f^2*x^2) + b*d*f*(6*e + f*x)*Cos[c + d/x] + b*(6*e^2 +
6*e*f*x - f^2*(d^2 - 2*x^2))*Sin[c + d/x]) - b*d*(-6*d*e*f*Cos[c] + (-6*e^2
 + d^2*f^2)*Sin[c])*SinIntegral[d/x])/6
```

Maple [A] time = 0.037, size = 209, normalized size = 0.9

$$-d\left(-\frac{ae^2x}{d}-\frac{aefx^2}{d}-\frac{af^2x^3}{3d}+be^2\left(-\frac{x}{d}\sin\left(c+\frac{d}{x}\right)-\mathrm{Si}\left(\frac{d}{x}\right)\sin(c)+\mathrm{Ci}\left(\frac{d}{x}\right)\cos(c)\right)+2\,befd\left(-1/2\,\frac{x^2}{d^2}\sin\left(c+\frac{d}{x}\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*sin(c+d/x)),x)
```

```
[Out] -d*(-a*e^2*x/d-a/d*e*f*x^2-1/3*a*f^2/d*x^3+b*e^2*(-sin(c+d/x)*x/d-Si(d/x)*s
in(c)+Ci(d/x)*cos(c))+2*b*e*f*d*(-1/2*sin(c+d/x)*x^2/d^2-1/2*cos(c+d/x)*x/d
-1/2*Si(d/x)*cos(c)-1/2*Ci(d/x)*sin(c))+b*f^2*d^2*(-1/3*sin(c+d/x)*x^3/d^3-
1/6*cos(c+d/x)*x^2/d^2+1/6*sin(c+d/x)*x/d+1/6*Si(d/x)*sin(c)-1/6*Ci(d/x)*co
s(c)))
```

Maxima [C] time = 1.34175, size = 348, normalized size = 1.55

$$\frac{1}{3}\,af^2x^3+aefx^2-\frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right)+\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right)+i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right)be^2+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(
I*d/x) + I*Ei(-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e^2 + 1/2*(((-I*
Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2
*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x + d)/x))*b*e*f + 1/12*(((Ei(I*d/x) +
 Ei(-I*d/x))*cos(c) + (I*Ei(I*d/x) - I*Ei(-I*d/x))*sin(c))*d^3 + 2*d*x^2*co
s((c*x + d)/x) - 2*(d^2*x - 2*x^3)*sin((c*x + d)/x))*b*f^2 + a*e^2*x
```

Fricas [A] time = 1.40155, size = 571, normalized size = 2.55

$$\frac{1}{3}\,af^2x^3+aefx^2+ae^2x+\frac{1}{12}\left(12\,bd^2ef\,\mathrm{Si}\left(\frac{d}{x}\right)+\left(bd^3f^2-6\,bde^2\right)\mathrm{Ci}\left(\frac{d}{x}\right)+\left(bd^3f^2-6\,bde^2\right)\mathrm{Ci}\left(-\frac{d}{x}\right)\right)\cos(c)+\frac{1}{6}\left(bd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + a*e^2*x + 1/12*(12*b*d^2*e*f*sin_integral(d/x)
+ (b*d^3*f^2 - 6*b*d*e^2)*cos_integral(d/x) + (b*d^3*f^2 - 6*b*d*e^2)*cos_i
ntegral(-d/x))*cos(c) + 1/6*(b*d*f^2*x^2 + 6*b*d*e*f*x)*cos((c*x + d)/x) +
1/6*(3*b*d^2*e*f*cos_integral(d/x) + 3*b*d^2*e*f*cos_integral(-d/x) - (b*d^
3*f^2 - 6*b*d*e^2)*sin_integral(d/x))*sin(c) + 1/6*(2*b*f^2*x^3 + 6*b*e*f*x
^2 - (b*d^2*f^2 - 6*b*e^2)*x)*sin((c*x + d)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\left(e+fx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((a + b*sin(c + d/x))*(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \left(b \sin \left(c+\frac{d}{x}\right)+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*sin(c + d/x) + a), x)
```

3.289 $\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal. Leaf size=118

$$aex+\frac{1}{2}afx^2+\frac{1}{2}bd^2f\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-bde\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+bde\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

```
[Out] a*e*x + (a*f*x^2)/2 + (b*d*f*x*Cos[c + d/x])/2 - b*d*e*Cos[c]*CosIntegral[d
/x] + (b*d^2*f*CosIntegral[d/x]*Sin[c])/2 + b*e*x*Sin[c + d/x] + (b*f*x^2*S
in[c + d/x])/2 + (b*d^2*f*Cos[c]*SinIntegral[d/x])/2 + b*d*e*Sin[c]*SinInte
gral[d/x]
```

Rubi [A] time = 0.239125, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3431, 14, 3297, 3303, 3299, 3302}

$$aex+\frac{1}{2}afx^2+\frac{1}{2}bd^2f\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-bde\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+bde\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*d*f*x*Cos[c + d/x])/2 - b*d*e*Cos[c]*CosIntegral[d
/x] + (b*d^2*f*CosIntegral[d/x]*Sin[c])/2 + b*e*x*Sin[c + d/x] + (b*f*x^2*S
in[c + d/x])/2 + (b*d^2*f*Cos[c]*SinIntegral[d/x])/2 + b*d*e*Sin[c]*SinInte
gral[d/x]
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx = -\text{Subst}\left(\int\left(\frac{f(a+b\sin(c+dx))}{x^3}+\frac{e(a+b\sin(c+dx))}{x^2}\right)dx,x,\frac{1}{x}\right)$$
$$= -\left(e\,\text{Subst}\left(\int\frac{a+b\sin(c+dx)}{x^2}dx,x,\frac{1}{x}\right)\right)-f\,\text{Subst}\left(\int\frac{a+b\sin(c+dx)}{x^3}dx,x,\frac{1}{x}\right)$$
$$= -\left(e\,\text{Subst}\left(\int\left(\frac{a}{x^2}+\frac{b\sin(c+dx)}{x^2}\right)dx,x,\frac{1}{x}\right)\right)-f\,\text{Subst}\left(\int\left(\frac{a}{x^3}+\frac{b\sin(c+dx)}{x^3}\right)\right.$$
$$= aex+\frac{1}{2}afx^2-(be)\,\text{Subst}\left(\int\frac{\sin(c+dx)}{x^2}dx,x,\frac{1}{x}\right)-(bf)\,\text{Subst}\left(\int\frac{\sin(c+dx)}{x^3}dx,\right.$$
$$= aex+\frac{1}{2}afx^2+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)-(bde)\,\text{Subst}\left(\int\frac{\cos(c+dx)}{x}\right.$$
$$= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}\left(bd^2f\right.$$
$$= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)-bde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+bex\sin\left(c+\frac{d}{x}\right)+\frac{1}{2}bfx^2\sin$$
$$= aex+\frac{1}{2}afx^2+\frac{1}{2}bdfx\cos\left(c+\frac{d}{x}\right)-bde\cos(c)\text{Ci}\left(\frac{d}{x}\right)+\frac{1}{2}bd^2f\text{Ci}\left(\frac{d}{x}\right)\sin(c)+bex\sin$$

Mathematica [A] time = 0.213352, size = 79, normalized size = 0.67

$$\frac{1}{2}\left(x(2e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)+bd\text{CosIntegral}\left(\frac{d}{x}\right)(df\sin(c)-2e\cos(c))+bd\text{Si}\left(\frac{d}{x}\right)(df\cos(c)+2e\sin(c))+bdfx\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*Sin[c + d/x]),x]
```

```
[Out] (b*d*f*x*Cos[c + d/x] + b*d*CosIntegral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) + x
*(2*e + f*x)*(a + b*Sin[c + d/x]) + b*d*(d*f*Cos[c] + 2*e*Sin[c])*SinIntegr
al[d/x])/2
```

Maple [A] time = 0.022, size = 115, normalized size = 1.

$$-d\left(-\frac{aex}{d}-\frac{afx^2}{2d}+be\left(-\frac{x}{d}\sin\left(c+\frac{d}{x}\right)-\text{Si}\left(\frac{d}{x}\right)\sin(c)+\text{Ci}\left(\frac{d}{x}\right)\cos(c)\right)+bfd\left(-\frac{x^2}{2d^2}\sin\left(c+\frac{d}{x}\right)-\frac{x}{2d}\cos\left(c+\frac{d}{x}\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*sin(c+d/x)),x)
```

```
[Out] -d*(-a*e*x/d-1/2*a/d*f*x^2+b*e*(-sin(c+d/x)*x/d-Si(d/x)*sin(c)+Ci(d/x)*cos(
c))+b*f*d*(-1/2*sin(c+d/x)*x^2/d^2-1/2*cos(c+d/x)*x/d-1/2*Si(d/x)*cos(c)-1/
2*Ci(d/x)*sin(c)))
```

Maxima [C] time = 1.22079, size = 207, normalized size = 1.75

$$\frac{1}{2}afx^2 - \frac{1}{2}\left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right) + \mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c) - \left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right) + i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d - 2x\sin\left(\frac{cx+d}{x}\right)\right)be + \frac{1}{4}\left(\left(\left(-i\,\mathrm{Ei}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 - 1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(
-I*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*b*e + 1/4*(((-I*Ei(I*d/x) + I*Ei
(-I*d/x))*cos(c) + (Ei(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x +
d)/x) + 2*x^2*sin((c*x + d)/x))*b*f + a*e*x
```

Fricas [A] time = 1.32544, size = 387, normalized size = 3.28

$$\frac{1}{2}bdfx\cos\left(\frac{cx+d}{x}\right) + \frac{1}{2}afx^2 + aex + \frac{1}{2}\left(bd^2f\,\mathrm{Si}\left(\frac{d}{x}\right) - bde\,\mathrm{Ci}\left(\frac{d}{x}\right) - bde\,\mathrm{Ci}\left(-\frac{d}{x}\right)\right)\cos(c) + \frac{1}{4}\left(bd^2f\,\mathrm{Ci}\left(\frac{d}{x}\right) + bd^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*d*f*x*cos((c*x + d)/x) + 1/2*a*f*x^2 + a*e*x + 1/2*(b*d^2*f*sin_integ
ral(d/x) - b*d*e*cos_integral(d/x) - b*d*e*cos_integral(-d/x))*cos(c) + 1/4
*(b*d^2*f*cos_integral(d/x) + b*d^2*f*cos_integral(-d/x) + 4*b*d*e*sin_inte
gral(d/x))*sin(c) + 1/2*(b*f*x^2 + 2*b*e*x)*sin((c*x + d)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(c + \frac{d}{x}\right)\right)\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((a + b*sin(c + d/x))*(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)\left(b\sin\left(c + \frac{d}{x}\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x)),x, algorithm="giac")

[Out] integrate((f*x + e)*(b*sin(c + d/x) + a), x)
```

3.290 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$

Optimal. Leaf size=38

$$ax - bd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right) + bd\sin(c)\text{Si}\left(\frac{d}{x}\right) + bx\sin\left(c+\frac{d}{x}\right)$$

```
[Out] a*x - b*d*Cos[c]*CosIntegral[d/x] + b*x*Sin[c + d/x] + b*d*Sin[c]*SinIntegr
al[d/x]
```

Rubi [A] time = 0.0783685, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3361, 3297, 3303, 3299, 3302}

$$ax - bd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right) + bd\sin(c)\text{Si}\left(\frac{d}{x}\right) + bx\sin\left(c+\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d/x],x]
```

```
[Out] a*x - b*d*Cos[c]*CosIntegral[d/x] + b*x*Sin[c + d/x] + b*d*Sin[c]*SinIntegr
al[d/x]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)dx = ax + b\int \sin\left(c+\frac{d}{x}\right)dx$$
$$= ax - b\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= ax + bx\sin\left(c+\frac{d}{x}\right) - (bd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= ax + bx\sin\left(c+\frac{d}{x}\right) - (bd\cos(c))\,\text{Subst}\left(\int \frac{\cos(dx)}{x}\,dx, x, \frac{1}{x}\right) + (bd\sin(c))\,\text{Subst}\left(\int \frac{\sin(dx}{x}\right.$$
$$= ax - bd\cos(c)\text{Ci}\left(\frac{d}{x}\right) + bx\sin\left(c+\frac{d}{x}\right) + bd\sin(c)\text{Si}\left(\frac{d}{x}\right)$$

Mathematica [A] time = 0.0303449, size = 50, normalized size = 1.32

$$ax - bd\left(\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right) - \sin(c)\text{Si}\left(\frac{d}{x}\right)\right) + bx\sin(c)\cos\left(\frac{d}{x}\right) + bx\cos(c)\sin\left(\frac{d}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d/x],x]
```

```
[Out] a*x + b*x*Cos[d/x]*Sin[c] + b*x*Cos[c]*Sin[d/x] - b*d*(Cos[c]*CosIntegral[d
/x] - Sin[c]*SinIntegral[d/x])
```

Maple [A] time = 0.01, size = 43, normalized size = 1.1

$$ax - bd\left(-\frac{x}{d}\sin\left(c+\frac{d}{x}\right) - \text{Si}\left(\frac{d}{x}\right)\sin(c) + \text{Ci}\left(\frac{d}{x}\right)\cos(c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(c+d/x),x)
```

```
[Out] a*x-b*d*(-sin(c+d/x)*x/d-Si(d/x)*sin(c)+Ci(d/x)*cos(c))
```

Maxima [C] time = 1.12897, size = 88, normalized size = 2.32

$$-\frac{1}{2}\left(\left(\left(\text{Ei}\left(\frac{id}{x}\right) + \text{Ei}\left(-\frac{id}{x}\right)\right)\cos(c) - \left(-i\,\text{Ei}\left(\frac{id}{x}\right) + i\,\text{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d - 2x\sin\left(\frac{cx+d}{x}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="maxima")
```

```
[Out] -1/2*(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(
c))*d - 2*x*sin((c*x + d)/x))*b + ax
```

Fricas [A] time = 1.30836, size = 163, normalized size = 4.29

$$bd\sin(c)\operatorname{Si}\left(\frac{d}{x}\right)+bx\sin\left(\frac{cx+d}{x}\right)+ax-\frac{1}{2}\left(bd\operatorname{Ci}\left(\frac{d}{x}\right)+bd\operatorname{Ci}\left(-\frac{d}{x}\right)\right)\cos(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="fricas")
```

```
[Out] b*d*sin(c)*sin_integral(d/x) + b*x*sin((c*x + d)/x) + a*x - 1/2*(b*d*cos_in
tegral(d/x) + b*d*cos_integral(-d/x))*cos(c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x)
```

```
[Out] Integral(a + b*sin(c + d/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b\sin\left(c+\frac{d}{x}\right)+a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(c+d/x),x, algorithm="giac")
```

```
[Out] integrate(b*sin(c + d/x) + a, x)
```

3.291 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$

Optimal. Leaf size=103

$$\frac{a\log\left(\frac{e}{x}+f\right)}{f}+\frac{a\log(x)}{f}+\frac{b\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}\right.\right.}{f}$$

```
[Out] (a*Log[f + e/x])/f + (a*Log[x])/f - (b*CosIntegral[d/x]*Sin[c])/f + (b*CosI
ntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (b*Cos[c - (d*f)/e]*SinInte
gral[d*(f/e + x^(-1))])/f - (b*Cos[c]*SinIntegral[d/x])/f
```

Rubi [A] time = 0.278575, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3431, 14, 3303, 3299, 3302, 3317}

$$\frac{a\log\left(\frac{e}{x}+f\right)}{f}+\frac{a\log(x)}{f}+\frac{b\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{b\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x),x]
```

```
[Out] (a*Log[f + e/x])/f + (a*Log[x])/f - (b*CosIntegral[d/x]*Sin[c])/f + (b*CosI
ntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (b*Cos[c - (d*f)/e]*SinInte
gral[d*(f/e + x^(-1))])/f - (b*Cos[c]*SinIntegral[d/x])/f
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx &= -\operatorname{Subst}\left(\int\left(\frac{a+b\sin(c+dx)}{fx}-\frac{e(a+b\sin(c+dx))}{f(f+ex)}\right)dx,x,\frac{1}{x}\right)\\
&= -\frac{\operatorname{Subst}\left(\int\frac{a+b\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{e\operatorname{Subst}\left(\int\frac{a+b\sin(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{f}\\
&= -\frac{\operatorname{Subst}\left(\int\left(\frac{a}{x}+\frac{b\sin(c+dx)}{x}\right)dx,x,\frac{1}{x}\right)}{f}+\frac{e\operatorname{Subst}\left(\int\left(\frac{a}{f+ex}+\frac{b\sin(c+dx)}{f+ex}\right)dx,x,\frac{1}{x}\right)}{f}\\
&= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\operatorname{Subst}\left(\int\frac{\sin(c+dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{(be)\operatorname{Subst}\left(\int\frac{\sin(c+dx)}{f+ex}\,dx,x,\frac{1}{x}\right)}{f}\\
&= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{(b\cos(c))\operatorname{Subst}\left(\int\frac{\sin(dx)}{x}\,dx,x,\frac{1}{x}\right)}{f}+\frac{\left(be\cos\left(c-\frac{df}{e}\right)\right)\operatorname{Subst}\left(\int\right.}{f}\\
&= \frac{a\log\left(f+\frac{e}{x}\right)}{f}+\frac{a\log(x)}{f}-\frac{b\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{b\operatorname{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(c-\frac{df}{e}\right)}{f}+\frac{b\cos\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\right.}{f}
\end{aligned}$$

Mathematica [A] time = 0.201585, size = 83, normalized size = 0.81

$$\frac{a\log(e+fx)+b\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)-b\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)+b\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)-b}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] - b*CosIntegral[d/x]*Sin[c] + b*CosIntegral[d*(f/e + x^(-1)
)]*Sin[c - (d*f)/e] + b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - b*
Cos[c]*SinIntegral[d/x])/f
```

Maple [A] time = 0.023, size = 142, normalized size = 1.4

$$-\frac{a}{f}\ln\left(\frac{d}{x}\right)+\frac{a}{f}\ln\left(e\left(c+\frac{d}{x}\right)-ce+df\right)+\frac{b}{f}\operatorname{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)-\frac{b}{f}\operatorname{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))/(f*x+e),x)
```

```
[Out] -a/f*ln(d/x)+a/f*ln(e*(c+d/x)-c*e+d*f)+b/f*Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e
+d*f)/e)-b/f*Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)-b*cos(c)*Si(d/x)/f-b*
Ci(d/x)*sin(c)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{2\left(\left(fx+e\right)\cos\left(\frac{cx+d}{x}\right)^{2}+\left(fx+e\right)\sin\left(\frac{cx+d}{x}\right)^{2}\right)}\,dx+\int \frac{\sin\left(\frac{cx+d}{x}\right)}{2\left(fx+e\right)}\,dx\right)+\frac{a\log\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/((f*x + e)*cos((c*x + d)/x)^2 + (f*x + e)
*sin((c*x + d)/x)^2), x) + integrate(1/2*sin((c*x + d)/x)/(f*x + e), x)) +
a*log(f*x + e)/f
```

Fricas [A] time = 1.5374, size = 366, normalized size = 3.55

$$-\frac{2\,b\cos\left(c\right)\operatorname{Si}\left(\frac{d}{x}\right)-2\,b\cos\left(-\frac{ce-df}{e}\right)\operatorname{Si}\left(\frac{dfx+de}{ex}\right)-2\,a\log\left(fx+e\right)+\left(b\operatorname{Ci}\left(\frac{d}{x}\right)+b\operatorname{Ci}\left(-\frac{d}{x}\right)\right)\sin\left(c\right)+\left(b\operatorname{Ci}\left(\frac{dfx+de}{ex}\right)+b\,C}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cos(c)*sin_integral(d/x) - 2*b*cos(-(c*e - d*f)/e)*sin_integral((
d*f*x + d*e)/(e*x)) - 2*a*log(f*x + e) + (b*cos_integral(d/x) + b*cos_integ
ral(-d/x))*sin(c) + (b*cos_integral((d*f*x + d*e)/(e*x)) + b*cos_integral(-
(d*f*x + d*e)/(e*x)))*sin(-(c*e - d*f)/e))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x)
```

```
[Out] Integral((a + b*sin(c + d/x))/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)/(f*x + e), x)
```

3.292 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx$

Optimal. Leaf size=94

$$\frac{a}{e\left(\frac{e}{x}+f\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

```
[Out] a/(e*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^2
+ (b*Sin[c + d/x])/(e*(f + e/x)) + (b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e
 + x^(-1))])/e^2
```

Rubi [A] time = 0.221876, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3431, 3317, 3297, 3303, 3299, 3302}

$$\frac{a}{e\left(\frac{e}{x}+f\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{bd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

```
[Out] a/(e*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^2
+ (b*Sin[c + d/x])/(e*(f + e/x)) + (b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e
 + x^(-1))])/e^2
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^2}\,dx &= -\operatorname{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{a}{(f+ex)^2}+\frac{b\sin(c+dx)}{(f+ex)^2}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} - b\operatorname{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(bd)\operatorname{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{\left(bd\cos\left(c-\frac{df}{e}\right)\right)\operatorname{Subst}\left(\int \frac{\cos\left(\frac{df}{e}+dx\right)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e} + \frac{\left(bd\sin\left(c-\frac{d}{e}\right.\right.}{}\\
&= \frac{a}{e\left(f+\frac{e}{x}\right)} - \frac{bd\cos\left(c-\frac{df}{e}\right)\operatorname{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} + \frac{b\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2}
\end{aligned}
$$

Mathematica [A] time = 0.740666, size = 85, normalized size = 0.9

$$
\frac{\frac{e\left(bfx\sin\left(c+\frac{d}{x}\right)-ae\right)}{f(e+fx)} - bd\cos\left(c-\frac{df}{e}\right)\operatorname{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right) + bd\sin\left(c-\frac{df}{e}\right)\operatorname{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x)^2,x]
```

```
[Out] (-(b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]) + (e*(-(a*e) + b*f*x
*Sin[c + d/x]))/(f*(e + f*x)) + b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x
^(-1))])/e^2
```

Maple [A] time = 0.019, size = 144, normalized size = 1.5

$$
-d\left(-\frac{a}{e}\left(e\left(c+\frac{d}{x}\right)-ce+df\right)^{-1} + b\left(-\frac{1}{e}\sin\left(c+\frac{d}{x}\right)\left(e\left(c+\frac{d}{x}\right)-ce+df\right)^{-1} + \frac{1}{e}\left(\frac{1}{e}\operatorname{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-ce+}{e}\right.\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))/(f*x+e)^2,x)
```

```
[Out] -d*(-a/(e*(c+d/x)-c*e+d*f)/e+b*(-sin(c+d/x)/(e*(c+d/x)-c*e+d*f)/e+(Si(d/x+c
+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)
/e)/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\int \frac{\sin\left(\frac{cx+d}{x}\right)}{2\left(f^2x^2+2\,efx+e^2\right)}\,dx+\int \frac{\sin\left(\frac{cx+d}{x}\right)}{2\left(\left(f^2x^2+2\,efx+e^2\right)\cos\left(\frac{cx+d}{x}\right)^2+\left(f^2x^2+2\,efx+e^2\right)\sin\left(\frac{cx+d}{x}\right)^2\right)}\,dx\right)-\frac{a}{f^2x+ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/(f^2*x^2 + 2*e*f*x + e^2), x) + integrate
(1/2*sin((c*x + d)/x)/((f^2*x^2 + 2*e*f*x + e^2)*cos((c*x + d)/x)^2 + (f^2*
x^2 + 2*e*f*x + e^2)*sin((c*x + d)/x)^2), x)) - a/(f^2*x + e*f)
```

Fricas [A] time = 1.33398, size = 382, normalized size = 4.06

$$\frac{2\,befx\sin\left(\frac{cx+d}{x}\right)-2\,ae^2-2\left(bdf^2x+bdef\right)\sin\left(-\frac{ce-df}{e}\right)\operatorname{Si}\left(\frac{dfx+de}{ex}\right)-\left(\left(bdf^2x+bdef\right)\operatorname{Ci}\left(\frac{dfx+de}{ex}\right)+\left(bdf^2x+bdef\right)\operatorname{Ci}\right.}{2\left(e^2f^2x+e^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*e*f*x*sin((c*x + d)/x) - 2*a*e^2 - 2*(b*d*f^2*x + b*d*e*f)*sin(-(c
*e - d*f)/e)*sin_integral((d*f*x + d*e)/(e*x)) - ((b*d*f^2*x + b*d*e*f)*cos
_integral((d*f*x + d*e)/(e*x)) + (b*d*f^2*x + b*d*e*f)*cos_integral(-(d*f*x
 + d*e)/(e*x)))*cos(-(c*e - d*f)/e))/(e^2*f^2*x + e^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(c+\frac{d}{x}\right)+a}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)/(f*x + e)^2, x)
```

3.293 $\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx$

Optimal. Leaf size=233

$$\frac{a}{e^2\left(\frac{e}{x}+f\right)}-\frac{af}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{bd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{bd}{}$$

```
[Out] -(a*f)/(2*e^2*(f + e/x)^2) + a/(e^2*(f + e/x)) - (b*d*f*Cos[c + d/x])/(2*e^
3*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^3 - (
b*d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/(2*e^4) - (b*f*Sin[
c + d/x])/(2*e^2*(f + e/x)^2) + (b*Sin[c + d/x])/(e^2*(f + e/x)) - (b*d^2*f
*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/(2*e^4) + (b*d*Sin[c - (d*
f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^3
```

Rubi [A] time = 0.487439, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3431, 3317, 3297, 3303, 3299, 3302}

$$\frac{a}{e^2\left(\frac{e}{x}+f\right)}-\frac{af}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{bd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{2e^4}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}-\frac{bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

```
[Out] -(a*f)/(2*e^2*(f + e/x)^2) + a/(e^2*(f + e/x)) - (b*d*f*Cos[c + d/x])/(2*e^
3*(f + e/x)) - (b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])/e^3 - (
b*d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/(2*e^4) - (b*f*Sin[
c + d/x])/(2*e^2*(f + e/x)^2) + (b*Sin[c + d/x])/(e^2*(f + e/x)) - (b*d^2*f
*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/(2*e^4) + (b*d*Sin[c - (d*
f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^3
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{a+b\sin\left(c+\frac{d}{x}\right)}{(e+fx)^3}\,dx = -\text{Subst}\left(\int \left(-\frac{f(a+b\sin(c+dx))}{e(f+ex)^3}+\frac{a+b\sin(c+dx)}{e(f+ex)^2}\right)dx, x, \frac{1}{x}\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)}{e}+\frac{f\,\text{Subst}\left(\int \frac{a+b\sin(c+dx)}{(f+ex)^3}\,dx, x, \frac{1}{x}\right)}{e}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{(f+ex)^2}+\frac{b\sin(c+dx)}{(f+ex)^2}\right)dx, x, \frac{1}{x}\right)}{e}+\frac{f\,\text{Subst}\left(\int \left(\frac{a}{(f+ex)^3}+\frac{b\sin(c+dx)}{(f+ex)^3}\right)dx, x, \frac{1}{x}\right)}{e}$$
$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{b\,\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)}{e}+\frac{(bf)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^3}\,dx, x, \right.}{e}$$
$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{(bd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x,\right.}{e^2}$$
$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bf\sin\left(c+\frac{d}{x}\right)}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{b\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{\left(bd^2f\right)\text{Su}}{}$$
$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-\frac{bf\sin\left(c+\right.}{2e^2\left(f+\frac{e}{x}\right)}$$
$$= -\frac{af}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a}{e^2\left(f+\frac{e}{x}\right)}-\frac{bdf\cos\left(c+\frac{d}{x}\right)}{2e^3\left(f+\frac{e}{x}\right)}-\frac{bd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-\frac{bd^2f\text{Ci}\left(\frac{d(f\right.}{}}{}$$

Mathematica [A] time = 1.8573, size = 151, normalized size = 0.65

$$-\frac{\frac{e\left(ae^3+bdf^2x(e+fx)\cos\left(c+\frac{d}{x}\right)-befx(2e+fx)\sin\left(c+\frac{d}{x}\right)\right)}{f(e+fx)^2}+bd\,\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\left(df\sin\left(c-\frac{df}{e}\right)+2e\cos\left(c-\frac{df}{e}\right)\right)+bd\text{Si}\left(\right.}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])/(e + f*x)^3,x]
```

```
[Out] -(b*d*CosIntegral[d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c - (d*
f)/e]) + (e*(a*e^3 + b*d*f^2*x*(e + f*x)*Cos[c + d/x] - b*e*f*x*(2*e + f*x)
*Sin[c + d/x]))/(f*(e + f*x)^2) + b*d*(d*f*Cos[c - (d*f)/e] - 2*e*Sin[c - (
d*f)/e])*SinIntegral[d*(f/e + x^(-1))])/(2*e^4)
```

Maple [B] time = 0.02, size = 527, normalized size = 2.3

$$-d\left(-\frac{a}{e^2}\left(e\left(c+\frac{d}{x}\right)-ce+df\right)^{-1}-\frac{a\left(ce-df\right)}{2\,e^2}\left(e\left(c+\frac{d}{x}\right)-ce+df\right)^{-2}+\frac{\left(ce-df\right)b}{e}\left(-\frac{1}{2\,e}\sin\left(c+\frac{d}{x}\right)\left(e\left(c+\frac{d}{x}\right)-ce+d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))/(f*x+e)^3,x)
```

```
[Out] -d*(-a/e^2/(e*(c+d/x)-c*e+d*f)-1/2*a*(c*e-d*f)/e^2/(e*(c+d/x)-c*e+d*f)^2+(c
*e-d*f)/e*b*(-1/2*sin(c+d/x)/(e*(c+d/x)-c*e+d*f)^2/e+1/2*(-cos(c+d/x)/(e*(c
+d/x)-c*e+d*f)/e-(Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*e
+d*f)/e)*sin((-c*e+d*f)/e)/e)/e)/e)+b/e*(-sin(c+d/x)/(e*(c+d/x)-c*e+d*f)/e+
(Si(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*
e+d*f)/e)/e)/e)+1/2*c*a/(e*(c+d/x)-c*e+d*f)^2/e-c*b*(-1/2*sin(c+d/x)/(e*(c+
d/x)-c*e+d*f)^2/e+1/2*(-cos(c+d/x)/(e*(c+d/x)-c*e+d*f)/e-(Si(d/x+c+(-c*e+d*
f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)/e)/e)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\int\frac{\sin\left(\frac{cx+d}{x}\right)}{2\left(f^3x^3+3\,ef^2x^2+3\,e^2fx+e^3\right)}\,dx+\int\frac{\sin\left(\frac{cx+d}{x}\right)}{2\left(\left(f^3x^3+3\,ef^2x^2+3\,e^2fx+e^3\right)\cos\left(\frac{cx+d}{x}\right)^2+\left(f^3x^3+3\,ef^2x^2+3\,e^2fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] b*(integrate(1/2*sin((c*x + d)/x)/(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)
, x) + integrate(1/2*sin((c*x + d)/x)/((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x +
 e^3)*cos((c*x + d)/x)^2 + (f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*sin((c
*x + d)/x)^2), x)) - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

Fricas [A] time = 1.59395, size = 959, normalized size = 4.12

$$-\frac{2\,ae^4+2\left(\left(bdef^3x^2+2\,bde^2f^2x+bde^3f\right)\mathrm{Ci}\left(\frac{dfx+de}{ex}\right)+\left(bdef^3x^2+2\,bde^2f^2x+bde^3f\right)\mathrm{Ci}\left(-\frac{dfx+de}{ex}\right)+\left(bd^2f^4x^2+2\,bd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*e^4 + 2*((b*d*e*f^3*x^2 + 2*b*d*e^2*f^2*x + b*d*e^3*f)*cos_integr
al((d*f*x + d*e)/(e*x)) + (b*d*e*f^3*x^2 + 2*b*d*e^2*f^2*x + b*d*e^3*f)*cos
_integral(-(d*f*x + d*e)/(e*x)) + (b*d^2*f^4*x^2 + 2*b*d^2*e*f^3*x + b*d^2*
e^2*f^2)*sin_integral((d*f*x + d*e)/(e*x)))*cos(-(c*e - d*f)/e) + 2*(b*d*e*
f^3*x^2 + b*d*e^2*f^2*x)*cos((c*x + d)/x) - ((b*d^2*f^4*x^2 + 2*b*d^2*e*f^3
*x + b*d^2*e^2*f^2)*cos_integral((d*f*x + d*e)/(e*x)) + (b*d^2*f^4*x^2 + 2*
b*d^2*e*f^3*x + b*d^2*e^2*f^2)*cos_integral(-(d*f*x + d*e)/(e*x)) - 4*(b*d*
e*f^3*x^2 + 2*b*d*e^2*f^2*x + b*d*e^3*f)*sin_integral((d*f*x + d*e)/(e*x)))
*sin(-(c*e - d*f)/e) - 2*(b*e^2*f^2*x^2 + 2*b*e^3*f*x)*sin((c*x + d)/x))/(e
^4*f^3*x^2 + 2*e^5*f^2*x + e^6*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \sin\left(c + \frac{d}{x}\right) + a}{\left(fx + e\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)/(f*x + e)^3, x)
```

3.294 $\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$

Optimal. Leaf size=254

$$a^2ex+\frac{1}{2}a^2fx^2+abd^2f\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-2abde\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+2abde\sin(c)\text{Si}\left(\right.$$

```
[Out] a^2*e*x + (a^2*f*x^2)/2 + a*b*d*f*x*Cos[c + d/x] - 2*a*b*d*e*Cos[c]*CosInte
gral[d/x] - b^2*d^2*f*Cos[2*c]*CosIntegral[(2*d)/x] + a*b*d^2*f*CosIntegral
[d/x]*Sin[c] - b^2*d*e*CosIntegral[(2*d)/x]*Sin[2*c] + 2*a*b*e*x*Sin[c + d/
x] + a*b*f*x^2*Sin[c + d/x] + b^2*d*f*x*Cos[c + d/x]*Sin[c + d/x] + b^2*e*x
*Sin[c + d/x]^2 + (b^2*f*x^2*Sin[c + d/x]^2)/2 + a*b*d^2*f*Cos[c]*SinIntegr
al[d/x] + 2*a*b*d*e*Sin[c]*SinIntegral[d/x] - b^2*d*e*Cos[2*c]*SinIntegral[
(2*d)/x] + b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x]
```

Rubi [A] time = 0.616003, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {3431, 3317, 3297, 3303, 3299, 3302, 3314, 29, 3312, 3313, 12}

$$a^2ex+\frac{1}{2}a^2fx^2+abd^2f\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)-2abde\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+2abde\sin(c)\text{Si}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```


```
[Out] a^2*e*x + (a^2*f*x^2)/2 + a*b*d*f*x*Cos[c + d/x] - 2*a*b*d*e*Cos[c]*CosInte
gral[d/x] - b^2*d^2*f*Cos[2*c]*CosIntegral[(2*d)/x] + a*b*d^2*f*CosIntegral
[d/x]*Sin[c] - b^2*d*e*CosIntegral[(2*d)/x]*Sin[2*c] + 2*a*b*e*x*Sin[c + d/
x] + a*b*f*x^2*Sin[c + d/x] + b^2*d*f*x*Cos[c + d/x]*Sin[c + d/x] + b^2*e*x
*Sin[c + d/x]^2 + (b^2*f*x^2*Sin[c + d/x]^2)/2 + a*b*d^2*f*Cos[c]*SinIntegr
al[d/x] + 2*a*b*d*e*Sin[c]*SinIntegral[d/x] - b^2*d*e*Cos[2*c]*SinIntegral[
(2*d)/x] + b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x]
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int (e+fx)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx = -\operatorname{Subst}\left(\int \left(\frac{f(a+b\sin(c+dx))^2}{x^3}+\frac{e(a+b\sin(c+dx))^2}{x^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\left(e\operatorname{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x^2}dx, x, \frac{1}{x}\right)\right) - f\operatorname{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x^3}dx\right.$$

$$= -\left(e\operatorname{Subst}\left(\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin^2(c+dx)}{x^2}\right)dx, x, \frac{1}{x}\right)\right) - f\operatorname{Subst}\left(\int\left(\right.\right.$$

$$= a^2ex+\frac{1}{2}a^2fx^2-(2abe)\operatorname{Subst}\left(\int \frac{\sin(c+dx)}{x^2}dx, x, \frac{1}{x}\right)-\left(b^2e\right)\operatorname{Subst}\left(\int \frac{\sin^2(c+}{x^2}\right.$$

$$= a^2ex+\frac{1}{2}a^2fx^2+2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin\left(c+\frac{d}{x}\right)+b^2dfx\cos\left(c+\frac{d}{x}\right)\sin\left(c\right.$$

$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)+b^2d^2f\log(x)+2abex\sin\left(c+\frac{d}{x}\right)+abfx^2\sin$$

$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\operatorname{Ci}\left(\frac{d}{x}\right)+2abex\sin\left(c+\frac{d}{x}\right)+ab$$

$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\operatorname{Ci}\left(\frac{d}{x}\right)+abd^2f\operatorname{Ci}\left(\frac{d}{x}\right)\sin(c)-b$$

$$= a^2ex+\frac{1}{2}a^2fx^2+abdfx\cos\left(c+\frac{d}{x}\right)-2abde\cos(c)\operatorname{Ci}\left(\frac{d}{x}\right)-b^2d^2f\cos(2c)\operatorname{Ci}\left(\frac{2d}{x}\right)$$

Mathematica [A] time = 0.560838, size = 252, normalized size = 0.99

$$\frac{1}{4}\left(4a^2ex+2a^2fx^2+4abd\text{CosIntegral}\left(\frac{d}{x}\right)(df\sin(c)-2e\cos(c))+4abd^2f\cos(c)\text{Si}\left(\frac{d}{x}\right)+8abde\sin(c)\text{Si}\left(\frac{d}{x}\right)+8abex\,\text{s}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*Sin[c + d/x])^2,x]
```

```
[Out] (4*a^2*e*x + 2*b^2*e*x + 2*a^2*f*x^2 + b^2*f*x^2 + 4*a*b*d*f*x*Cos[c + d/x]
 - 2*b^2*e*x*Cos[2*(c + d/x)] - b^2*f*x^2*Cos[2*(c + d/x)] + 4*a*b*d*CosInt
egral[d/x]*(-2*e*Cos[c] + d*f*Sin[c]) - 4*b^2*d*CosIntegral[(2*d)/x]*(d*f*C
os[2*c] + e*Sin[2*c]) + 8*a*b*e*x*Sin[c + d/x] + 4*a*b*f*x^2*Sin[c + d/x] +
 2*b^2*d*f*x*Sin[2*(c + d/x)] + 4*a*b*d^2*f*Cos[c]*SinIntegral[d/x] + 8*a*b
*d*e*Sin[c]*SinIntegral[d/x] - 4*b^2*d*e*Cos[2*c]*SinIntegral[(2*d)/x] + 4*
b^2*d^2*f*Sin[2*c]*SinIntegral[(2*d)/x])/4
```

Maple [A] time = 0.039, size = 265, normalized size = 1.

$$-d\left(-\frac{a^2ex}{d}-\frac{a^2fx^2}{2d}+2abe\left(-\frac{x}{d}\sin\left(c+\frac{d}{x}\right)-\operatorname{Si}\left(\frac{d}{x}\right)\sin(c)+\operatorname{Ci}\left(\frac{d}{x}\right)\cos(c)\right)+2abfd\left(-1/2\frac{x^2}{d^2}\sin\left(c+\frac{d}{x}\right)-1/2\frac{x}{d}\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*sin(c+d/x))^2,x)
```

```
[Out] -d*(-a^2*e*x/d-1/2*a^2/d*f*x^2+2*a*b*e*(-sin(c+d/x)*x/d-Si(d/x)*sin(c)+Ci(d
/x)*cos(c))+2*a*b*f*d*(-1/2*sin(c+d/x)*x^2/d^2-1/2*cos(c+d/x)*x/d-1/2*Si(d/
x)*cos(c)-1/2*Ci(d/x)*sin(c))-1/2*b^2*e*x/d-1/4*b^2*e*(-2*cos(2*d/x+2*c)*x/
d-4*Si(2*d/x)*cos(2*c)-4*Ci(2*d/x)*sin(2*c))-1/4*b^2*f/d*x^2-1/4*b^2*f*d*(-
cos(2*d/x+2*c)*x^2/d^2+2*sin(2*d/x+2*c)*x/d+4*Si(2*d/x)*sin(2*c)-4*Ci(2*d/x
```

```
)*cos(2*c)))
```

Maxima [C] time = 1.41808, size = 435, normalized size = 1.71

$$\frac{1}{2}a^2fx^2 - \left(\left(\left(\mathrm{Ei}\left(\frac{id}{x}\right) + \mathrm{Ei}\left(-\frac{id}{x}\right)\right)\cos(c) - \left(-i\,\mathrm{Ei}\left(\frac{id}{x}\right) + i\,\mathrm{Ei}\left(-\frac{id}{x}\right)\right)\sin(c)\right)d - 2\,x\sin\left(\frac{cx+d}{x}\right)\right)abe - \frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*f*x^2 - (((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I
*d/x))*sin(c))*d - 2*x*sin((c*x + d)/x))*a*b*e - 1/2*(((-I*Ei(2*I*d/x) + I*
Ei(-2*I*d/x))*cos(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*cos(2
*(c*x + d)/x) - x)*b^2*e + 1/2*(((-I*Ei(I*d/x) + I*Ei(-I*d/x))*cos(c) + (Ei
(I*d/x) + Ei(-I*d/x))*sin(c))*d^2 + 2*d*x*cos((c*x + d)/x) + 2*x^2*sin((c*x
 + d)/x))*a*b*f - 1/4*((2*(Ei(2*I*d/x) + Ei(-2*I*d/x))*cos(2*c) - (-2*I*Ei(
2*I*d/x) + 2*I*Ei(-2*I*d/x))*sin(2*c))*d^2 + x^2*cos(2*(c*x + d)/x) - 2*d*x
*sin(2*(c*x + d)/x) - x^2)*b^2*f + a^2*e*x
```

Fricas [A] time = 1.63403, size = 828, normalized size = 3.26

$$abdfx\cos\left(\frac{cx+d}{x}\right) + \frac{1}{2}\left(a^2+b^2\right)fx^2 + \left(a^2+b^2\right)ex - \frac{1}{2}\left(b^2fx^2 + 2\,b^2ex\right)\cos\left(\frac{cx+d}{x}\right)^2 - \frac{1}{2}\left(b^2d^2f\,\mathrm{Ci}\left(\frac{2\,d}{x}\right) + b^2d^2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] a*b*d*f*x*cos((c*x + d)/x) + 1/2*(a^2 + b^2)*f*x^2 + (a^2 + b^2)*e*x - 1/2*
(b^2*f*x^2 + 2*b^2*e*x)*cos((c*x + d)/x)^2 - 1/2*(b^2*d^2*f*cos_integral(2*
d/x) + b^2*d^2*f*cos_integral(-2*d/x) + 2*b^2*d*e*sin_integral(2*d/x))*cos(
2*c) + (a*b*d^2*f*sin_integral(d/x) - a*b*d*e*cos_integral(d/x) - a*b*d*e*c
os_integral(-d/x))*cos(c) + 1/2*(2*b^2*d^2*f*sin_integral(2*d/x) - b^2*d*e*
cos_integral(2*d/x) - b^2*d*e*cos_integral(-2*d/x))*sin(2*c) + 1/2*(a*b*d^2
*f*cos_integral(d/x) + a*b*d^2*f*cos_integral(-d/x) + 4*a*b*d*e*sin_integra
l(d/x))*sin(c) + (b^2*d*f*x*cos((c*x + d)/x) + a*b*f*x^2 + 2*a*b*e*x)*sin((
c*x + d)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sin\left(c + \frac{d}{x}\right)\right)^2 \left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2*(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*sin(c + d/x) + a)^2, x)
```

3.295 $\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$

Optimal. Leaf size=94

$$a^2x-2abd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)-b^2d\sin(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2d\cos$$

```
[Out] a^2*x - 2*a*b*d*Cos[c]*CosIntegral[d/x] - b^2*d*CosIntegral[(2*d)/x]*Sin[2*
c] + 2*a*b*x*Sin[c + d/x] + b^2*x*Sin[c + d/x]^2 + 2*a*b*d*Sin[c]*SinIntegr
al[d/x] - b^2*d*Cos[2*c]*SinIntegral[(2*d)/x]
```

Rubi [A] time = 0.22635, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3361, 3317, 3297, 3303, 3299, 3302, 3313, 12}

$$a^2x-2abd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)-b^2d\sin(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)-b^2d\cos$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2,x]
```

```
[Out] a^2*x - 2*a*b*d*Cos[c]*CosIntegral[d/x] - b^2*d*CosIntegral[(2*d)/x]*Sin[2*
c] + 2*a*b*x*Sin[c + d/x] + b^2*x*Sin[c + d/x]^2 + 2*a*b*d*Sin[c]*SinIntegr
al[d/x] - b^2*d*Cos[2*c]*SinIntegral[(2*d)/x]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2 dx &= -\text{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a^2}{x^2}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin^2(c+dx)}{x^2}\right)dx, x, \frac{1}{x}\right)\\
&= a^2x-(2ab)\,\text{Subst}\left(\int \frac{\sin(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)-b^2\,\text{Subst}\left(\int \frac{\sin^2(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= a^2x+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)-(2abd)\,\text{Subst}\left(\int \frac{\cos(c+dx)}{x}\,dx, x, \frac{1}{x}\right)-\left(2b^2d\right.\\
&= a^2x+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)-\left(b^2d\right)\text{Subst}\left(\int \frac{\sin(2c+2dx)}{x}\,dx, x, \frac{1}{x}\right)-(2ab\\
&= a^2x-2abd\cos(c)\text{Ci}\left(\frac{d}{x}\right)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)+2abd\sin(c)\text{Si}\left(\frac{d}{x}\right)-\left(b^2d\right.\\
&= a^2x-2abd\cos(c)\text{Ci}\left(\frac{d}{x}\right)-b^2d\text{Ci}\left(\frac{2d}{x}\right)\sin(2c)+2abx\sin\left(c+\frac{d}{x}\right)+b^2x\sin^2\left(c+\frac{d}{x}\right)+2ab
\end{aligned}$$

Mathematica [A] time = 0.144822, size = 105, normalized size = 1.12

$$\frac{1}{2}\left(2a^2x-4abd\cos(c)\text{CosIntegral}\left(\frac{d}{x}\right)+4abd\sin(c)\text{Si}\left(\frac{d}{x}\right)+4abx\sin\left(c+\frac{d}{x}\right)-2b^2d\sin(2c)\text{CosIntegral}\left(\frac{2d}{x}\right)-2b^2d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2,x]
```

```
[Out] (2*a^2*x + b^2*x - b^2*x*Cos[2*(c + d/x)] - 4*a*b*d*Cos[c]*CosIntegral[d/x]
 - 2*b^2*d*CosIntegral[(2*d)/x]*Sin[2*c] + 4*a*b*x*Sin[c + d/x] + 4*a*b*d*S
in[c]*SinIntegral[d/x] - 2*b^2*d*Cos[2*c]*SinIntegral[(2*d)/x])/2
```

Maple [A] time = 0.024, size = 110, normalized size = 1.2

$$-d\left(-\frac{a^2x}{d}+2\,ab\left(-\frac{x}{d}\sin\left(c+\frac{d}{x}\right)-\mathrm{Si}\left(\frac{d}{x}\right)\sin(c)+\mathrm{Ci}\left(\frac{d}{x}\right)\cos(c)\right)-\frac{b^2x}{2\,d}-\frac{b^2}{4}\left(-2\,\frac{x}{d}\cos\left(2\,\frac{d}{x}+2\,c\right)-4\,\mathrm{Si}\left(2\,\frac{d}{x}\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2,x)
```

```
[Out] -d*(-a^2*x/d+2*a*b*(-sin(c+d/x)*x/d-Si(d/x)*sin(c)+Ci(d/x)*cos(c))-1/2*b^2*
x/d-1/4*b^2*(-2*cos(2*d/x+2*c)*x/d-4*Si(2*d/x)*cos(2*c)-4*Ci(2*d/x)*sin(2*c
)))
```

Maxima [C] time = 1.22124, size = 185, normalized size = 1.97

$$-\left(\left(\left(\mathrm{Ei}\left(\frac{i\,d}{x}\right)+\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\cos(c)-\left(-i\,\mathrm{Ei}\left(\frac{i\,d}{x}\right)+i\,\mathrm{Ei}\left(-\frac{i\,d}{x}\right)\right)\sin(c)\right)d-2\,x\sin\left(\frac{cx+d}{x}\right)\right)ab-\frac{1}{2}\left(\left(\left(-i\,\mathrm{Ei}\left(\frac{2i\,d}{x}\right)+i\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(((Ei(I*d/x) + Ei(-I*d/x))*cos(c) - (-I*Ei(I*d/x) + I*Ei(-I*d/x))*sin(c))*
d - 2*x*sin((c*x + d)/x))*a*b - 1/2*(((-I*Ei(2*I*d/x) + I*Ei(-2*I*d/x))*cos
(2*c) + (Ei(2*I*d/x) + Ei(-2*I*d/x))*sin(2*c))*d + x*cos(2*(c*x + d)/x) - x
)*b^2 + a^2*x
```

Fricas [A] time = 1.55331, size = 373, normalized size = 3.97

$$-b^2x\cos\left(\frac{cx+d}{x}\right)^2-b^2d\cos(2\,c)\,\mathrm{Si}\left(\frac{2\,d}{x}\right)+2\,abd\sin(c)\,\mathrm{Si}\left(\frac{d}{x}\right)+2\,abx\sin\left(\frac{cx+d}{x}\right)+\left(a^2+b^2\right)x-\left(abd\,\mathrm{Ci}\left(\frac{d}{x}\right)+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] -b^2*x*cos((c*x + d)/x)^2 - b^2*d*cos(2*c)*sin_integral(2*d/x) + 2*a*b*d*si
n(c)*sin_integral(d/x) + 2*a*b*x*sin((c*x + d)/x) + (a^2 + b^2)*x - (a*b*d*
cos_integral(d/x) + a*b*d*cos_integral(-d/x))*cos(c) - 1/2*(b^2*d*cos_integ
ral(2*d/x) + b^2*d*cos_integral(-2*d/x))*sin(2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2,x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(c+\frac{d}{x}\right)+a\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)^2, x)
```

3.296 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=255

$$\frac{a^2\log\left(\frac{e}{x}+f\right)}{f}+\frac{a^2\log(x)}{f}+\frac{2ab\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{2ab\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{2ab\cos\left(c-\right.}{}$$

```
[Out] -(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/(2*f) + (b^2*Co
s[2*c]*CosIntegral[(2*d)/x])/(2*f) + (a^2*Log[f + e/x])/f + (b^2*Log[f + e/
x])/(2*f) + (a^2*Log[x])/f + (b^2*Log[x])/(2*f) - (2*a*b*CosIntegral[d/x]*S
in[c])/f + (2*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (2*a*
b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/f + (b^2*Sin[2*c - (2*d*f
)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(2*f) - (2*a*b*Cos[c]*SinIntegral[d/x
])/f - (b^2*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Rubi [A] time = 0.66276, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3431, 3317, 3303, 3299, 3302, 3312}

$$\frac{a^2\log\left(\frac{e}{x}+f\right)}{f}+\frac{a^2\log(x)}{f}+\frac{2ab\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{f}-\frac{2ab\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)}{f}+\frac{2ab\cos\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```


```
[Out] -(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/(2*f) + (b^2*Co
s[2*c]*CosIntegral[(2*d)/x])/(2*f) + (a^2*Log[f + e/x])/f + (b^2*Log[f + e/
x])/(2*f) + (a^2*Log[x])/f + (b^2*Log[x])/(2*f) - (2*a*b*CosIntegral[d/x]*S
in[c])/f + (2*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/f + (2*a*
b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/f + (b^2*Sin[2*c - (2*d*f
)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(2*f) - (2*a*b*Cos[c]*SinIntegral[d/x
])/f - (b^2*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{e+fx}\,dx = -\text{Subst}\left(\int\left(\frac{(a+b\sin(c+dx))^2}{fx}-\frac{e(a+b\sin(c+dx))^2}{f(f+ex)}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{x}dx,x,\frac{1}{x}\right)}{f}+\frac{e\,\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{f+ex}dx,x,\frac{1}{x}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a^2}{x}+\frac{2ab\sin(c+dx)}{x}+\frac{b^2\sin^2(c+dx)}{x}\right)dx,x,\frac{1}{x}\right)}{f}+\frac{e\,\text{Subst}\left(\int\left(\frac{a^2}{f+ex}+\frac{2ab\sin(c+dx)}{f+ex}+\frac{b^2 s}{}\right.\right.}{f}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{a^2\log(x)}{f}-\frac{(2ab)\,\text{Subst}\left(\int\frac{\sin(c+dx)}{x}dx,x,\frac{1}{x}\right)}{f}-\frac{b^2\,\text{Subst}\left(\int\frac{\sin^2(c+dx)}{x}dx,x\right.}{f}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{a^2\log(x)}{f}-\frac{b^2\,\text{Subst}\left(\int\left(\frac{1}{2x}-\frac{\cos(2c+2dx)}{2x}\right)dx,x,\frac{1}{x}\right)}{f}+\frac{\left(b^2e\right)\text{Subst}\left(\int\left(\frac{1}{2(f}\right.\right.}{}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a^2\log(x)}{f}+\frac{b^2\log(x)}{2f}-\frac{2ab\text{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{2ab\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right.}{}$$

$$= \frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a^2\log(x)}{f}+\frac{b^2\log(x)}{2f}-\frac{2ab\text{Ci}\left(\frac{d}{x}\right)\sin(c)}{f}+\frac{2ab\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right.}{}$$

$$= -\frac{b^2\cos\left(2c-\frac{2df}{e}\right)\text{Ci}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)}{2f}+\frac{b^2\cos(2c)\text{Ci}\left(\frac{2d}{x}\right)}{2f}+\frac{a^2\log\left(f+\frac{e}{x}\right)}{f}+\frac{b^2\log\left(f+\frac{e}{x}\right)}{2f}+\frac{a}{}$$

Mathematica [A] time = 0.400668, size = 195, normalized size = 0.76

$$2a^2\log(e+fx)+4ab\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)-4ab\sin(c)\text{CosIntegral}\left(\frac{d}{x}\right)+4ab\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x),x]
```

```
[Out] (-(b^2*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))]) + b^2*Cos[2*c]
*CosIntegral[(2*d)/x] + 2*a^2*Log[e + f*x] + b^2*Log[e + f*x] - 4*a*b*CosIn
tegral[d/x]*Sin[c] + 4*a*b*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e] +
 4*a*b*Cos[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + b^2*Sin[2*c - (2*d*
f)/e]*SinIntegral[2*d*(f/e + x^(-1))] - 4*a*b*Cos[c]*SinIntegral[d/x] - b^2
*Sin[2*c]*SinIntegral[(2*d)/x])/(2*f)
```

Maple [A] time = 0.034, size = 321, normalized size = 1.3

$$-\frac{a^2}{f}\ln\left(\frac{d}{x}\right)+\frac{a^2}{f}\ln\left(e\left(c+\frac{d}{x}\right)-ce+df\right)+2\,\frac{ab}{f}\mathrm{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\cos\left(\frac{-ce+df}{e}\right)-2\,\frac{ab}{f}\mathrm{Ci}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e),x)
```

```
[Out] -a^2/f*ln(d/x)+a^2/f*ln(e*(c+d/x)-c*e+d*f)+2*a*b/f*Si(d/x+c+(-c*e+d*f)/e)*c
os((-c*e+d*f)/e)-2*a*b/f*Ci(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)-2*a*b*Ci(
d/x)*sin(c)/f-2*a*b*cos(c)*Si(d/x)/f-1/2*b^2/f*ln(d/x)+1/2*b^2/f*ln(e*(c+d/
x)-c*e+d*f)-1/2*b^2/f*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)-1/2*
b^2/f*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)+1/2*b^2*Ci(2*d/x)*co
s(2*c)/f-1/2*b^2*Si(2*d/x)*sin(2*c)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.47742, size = 776, normalized size = 3.04

$$-\frac{2\,b^2\sin(2\,c)\,\mathrm{Si}\left(\frac{2\,d}{x}\right)+8\,ab\cos(c)\,\mathrm{Si}\left(\frac{d}{x}\right)+2\,b^2\sin\left(-\frac{2\,(ce-df)}{e}\right)\mathrm{Si}\left(\frac{2\,(dfx+de)}{ex}\right)-8\,ab\cos\left(-\frac{ce-df}{e}\right)\mathrm{Si}\left(\frac{dfx+de}{ex}\right)-\left(b^2\,\mathrm{C}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*sin(2*c)*sin_integral(2*d/x) + 8*a*b*cos(c)*sin_integral(d/x) +
 2*b^2*sin(-2*(c*e - d*f)/e)*sin_integral(2*(d*f*x + d*e)/(e*x)) - 8*a*b*co
s(-(c*e - d*f)/e)*sin_integral((d*f*x + d*e)/(e*x)) - (b^2*cos_integral(2*d
/x) + b^2*cos_integral(-2*d/x))*cos(2*c) + (b^2*cos_integral(2*(d*f*x + d*e
)/(e*x)) + b^2*cos_integral(-2*(d*f*x + d*e)/(e*x)))*cos(-2*(c*e - d*f)/e)
- 2*(2*a^2 + b^2)*log(f*x + e) + 4*(a*b*cos_integral(d/x) + a*b*cos_integra
```

```
l(-d/x))*sin(c) + 4*(a*b*cos_integral((d*f*x + d*e)/(e*x)) + a*b*cos_integr
al(-(d*f*x + d*e)/(e*x)))*sin(-(c*e - d*f)/e))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sin\left(c + \frac{d}{x}\right)\right)^2}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*sin(c + d/x))**2/(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(c + \frac{d}{x}\right) + a\right)^2}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)^2/(f*x + e), x)
```

3.297 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=195

$$\frac{a^2}{e\left(\frac{e}{x}+f\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}-\frac{b^2d\,\text{si}}{}$$

```
[Out] a^2/(e*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]
)/e^2 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^2 +
(2*a*b*Sin[c + d/x])/(e*(f + e/x)) + (b^2*Sin[c + d/x]^2)/(e*(f + e/x)) + (
2*a*b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^2 - (b^2*d*Cos[2*
c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/e^2
```

Rubi [A] time = 0.391029, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3431, 3317, 3297, 3303, 3299, 3302, 3313, 12}

$$\frac{a^2}{e\left(\frac{e}{x}+f\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{2abd\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(\frac{e}{x}+f\right)}-\frac{b^2d\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

```
[Out] a^2/(e*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))]
)/e^2 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^2 +
(2*a*b*Sin[c + d/x])/(e*(f + e/x)) + (b^2*Sin[c + d/x]^2)/(e*(f + e/x)) + (
2*a*b*d*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^2 - (b^2*d*Cos[2*
c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/e^2
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^2}\,dx = -\operatorname{Subst}\left(\int \frac{(a+b\sin(c+dx))^2}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\operatorname{Subst}\left(\int \left(\frac{a^2}{(f+ex)^2}+\frac{2ab\sin(c+dx)}{(f+ex)^2}+\frac{b^2\sin^2(c+dx)}{(f+ex)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - (2ab)\operatorname{Subst}\left(\int \frac{\sin(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right) - b^2\operatorname{Subst}\left(\int \frac{\sin^2(c+dx)}{(f+ex)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{(2abd)\operatorname{Subst}\left(\int \frac{\cos(c+dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e} - \frac{\left(2b^2d\right)\cdots}{\cdots}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} - \frac{\left(b^2d\right)\operatorname{Subst}\left(\int \frac{\sin(2c+2dx)}{f+ex}\,dx, x, \frac{1}{x}\right)}{e} - \frac{(2abd\cdots}{\cdots}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} + \frac{2ab\sin\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{b^2\sin^2\left(c+\frac{d}{x}\right)}{e\left(f+\frac{e}{x}\right)} + \frac{2abd\sin\left(\cdots\right.}{\cdots}$$

$$= \frac{a^2}{e\left(f+\frac{e}{x}\right)} - \frac{2abd\cos\left(c-\frac{df}{e}\right)\operatorname{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^2} - \frac{b^2d\operatorname{Ci}\left(\frac{2d\left(f+\frac{e}{x}\right)}{e}\right)\sin\left(2c-\frac{2df}{e}\right)}{e^2} + \frac{2ab\sin\left(c+\cdots\right.}{e\left(f+\frac{e}{x}\right)}$$

Mathematica [A] time = 1.45129, size = 263, normalized size = 1.35

$$-\frac{2a^2e^2 + 4abdf(e+fx)\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right) - 4abdf^2x\sin\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right) - 4abdef\sin\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^2,x]
```

```
[Out] -(2*a^2*e^2 + b^2*e^2 + b^2*e*f*x*Cos[2*(c + d/x)] + 4*a*b*d*f*(e + f*x)*Co
s[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))] + 2*b^2*d*f*(e + f*x)*CosInteg
ral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e] - 4*a*b*e*f*x*Sin[c + d/x] - 4
*a*b*d*e*f*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] - 4*a*b*d*f^2*x*S
in[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 2*b^2*d*e*f*Cos[2*c - (2*d*
f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 2*b^2*d*f^2*x*Cos[2*c - (2*d*f)/e]*
SinIntegral[2*d*(f/e + x^(-1))])/(2*e^2*f*(e + f*x))
```

Maple [A] time = 0.029, size = 308, normalized size = 1.6

$$-d\left(-\frac{a^2}{e}\left(e\left(c+\frac{d}{x}\right)-ce+df\right)^{-1} + 2\,ab\left(-\frac{1}{e}\sin\left(c+\frac{d}{x}\right)\left(e\left(c+\frac{d}{x}\right)-ce+df\right)^{-1} + \frac{1}{e}\left(\frac{1}{e}\text{Si}\left(\frac{d}{x}+c+\frac{-ce+df}{e}\right)\sin\left(\frac{-c}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e)^2,x)
```

```
[Out] -d*(-a^2/(e*(c+d/x)-c*e+d*f)/e+2*a*b*(-sin(c+d/x)/(e*(c+d/x)-c*e+d*f)/e+(Si
(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d
*f)/e)/e)/e)-1/2*b^2/(e*(c+d/x)-c*e+d*f)/e-1/4*b^2*(-2*cos(2*d/x+2*c)/(e*(c
+d/x)-c*e+d*f)/e-2*(2*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e-2*
Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e)/e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69288, size = 805, normalized size = 4.13

$$-\frac{2\,b^2efx\cos\left(\frac{cx+d}{x}\right)^2 - 4\,abefx\sin\left(\frac{cx+d}{x}\right) - b^2efx + \left(2\,a^2+b^2\right)e^2 + 2\left(b^2df^2x + b^2def\right)\cos\left(-\frac{2\left(ce-df\right)}{e}\right)\text{Si}\left(\frac{2\left(dfx+de\right)}{ex}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*e*f*x*cos((c*x + d)/x)^2 - 4*a*b*e*f*x*sin((c*x + d)/x) - b^2*e
*f*x + (2*a^2 + b^2)*e^2 + 2*(b^2*d*f^2*x + b^2*d*e*f)*cos(-2*(c*e - d*f)/e
)*sin_integral(2*(d*f*x + d*e)/(e*x)) + 4*(a*b*d*f^2*x + a*b*d*e*f)*sin(-(c
*e - d*f)/e)*sin_integral((d*f*x + d*e)/(e*x)) + 2*((a*b*d*f^2*x + a*b*d*e*
f)*cos_integral((d*f*x + d*e)/(e*x)) + (a*b*d*f^2*x + a*b*d*e*f)*cos_integr
al(-(d*f*x + d*e)/(e*x)))*cos(-(c*e - d*f)/e) - ((b^2*d*f^2*x + b^2*d*e*f)*
cos_integral(2*(d*f*x + d*e)/(e*x)) + (b^2*d*f^2*x + b^2*d*e*f)*cos_integra
l(-2*(d*f*x + d*e)/(e*x)))*sin(-2*(c*e - d*f)/e))/(e^2*f^2*x + e^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(c + \frac{d}{x}\right) + a\right)^2}{\left(fx + e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)^2/(f*x + e)^2, x)
```

3.298 $\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx$

Optimal. Leaf size=470

$$\frac{a^2}{e^2\left(\frac{e}{x}+f\right)}-\frac{a^2f}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{abd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}$$

```
[Out] -(a^2*f)/(2*e^2*(f + e/x)^2) + a^2/(e^2*(f + e/x)) - (a*b*d*f*Cos[c + d/x])
/(e^3*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])
/e^3 + (b^2*d^2*f*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/e^4
 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^3 - (a*b*
d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/e^4 - (a*b*f*Sin[c +
d/x])/(e^2*(f + e/x)^2) + (2*a*b*Sin[c + d/x])/(e^2*(f + e/x)) - (b^2*d*f*C
os[c + d/x]*Sin[c + d/x])/(e^3*(f + e/x)) - (b^2*f*Sin[c + d/x]^2)/(2*e^2*(
f + e/x)^2) + (b^2*Sin[c + d/x]^2)/(e^2*(f + e/x)) - (a*b*d^2*f*Cos[c - (d*
f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^4 + (2*a*b*d*Sin[c - (d*f)/e]*SinInt
egral[d*(f/e + x^(-1))])/e^3 - (b^2*d*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*
(f/e + x^(-1))])/e^3 - (b^2*d^2*f*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e
 + x^(-1))])/e^4
```

Rubi [A] time = 0.957261, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3431, 3317, 3297, 3303, 3299, 3302, 3314, 31, 3312, 3313, 12}

$$\frac{a^2}{e^2\left(\frac{e}{x}+f\right)}-\frac{a^2f}{2e^2\left(\frac{e}{x}+f\right)^2}-\frac{abd^2f\sin\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^4}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```


```
[Out] -(a^2*f)/(2*e^2*(f + e/x)^2) + a^2/(e^2*(f + e/x)) - (a*b*d*f*Cos[c + d/x])
/(e^3*(f + e/x)) - (2*a*b*d*Cos[c - (d*f)/e]*CosIntegral[d*(f/e + x^(-1))])
/e^3 + (b^2*d^2*f*Cos[2*c - (2*d*f)/e]*CosIntegral[2*d*(f/e + x^(-1))])/e^4
 - (b^2*d*CosIntegral[2*d*(f/e + x^(-1))]*Sin[2*c - (2*d*f)/e])/e^3 - (a*b*
d^2*f*CosIntegral[d*(f/e + x^(-1))]*Sin[c - (d*f)/e])/e^4 - (a*b*f*Sin[c +
d/x])/(e^2*(f + e/x)^2) + (2*a*b*Sin[c + d/x])/(e^2*(f + e/x)) - (b^2*d*f*C
os[c + d/x]*Sin[c + d/x])/(e^3*(f + e/x)) - (b^2*f*Sin[c + d/x]^2)/(2*e^2*(
f + e/x)^2) + (b^2*Sin[c + d/x]^2)/(e^2*(f + e/x)) - (a*b*d^2*f*Cos[c - (d*
f)/e]*SinIntegral[d*(f/e + x^(-1))])/e^4 + (2*a*b*d*Sin[c - (d*f)/e]*SinInt
egral[d*(f/e + x^(-1))])/e^3 - (b^2*d*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*
(f/e + x^(-1))])/e^3 - (b^2*d^2*f*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e
 + x^(-1))])/e^4
```


Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}{(e+fx)^3}\,dx = -\text{Subst}\left(\int\left(-\frac{f(a+b\sin(c+dx))^2}{e(f+ex)^3}+\frac{(a+b\sin(c+dx))^2}{e(f+ex)^2}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{(f+ex)^2}dx,x,\frac{1}{x}\right)}{e}+\frac{f\,\text{Subst}\left(\int\frac{(a+b\sin(c+dx))^2}{(f+ex)^3}dx,x,\frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a^2}{(f+ex)^2}+\frac{2ab\sin(c+dx)}{(f+ex)^2}+\frac{b^2\sin^2(c+dx)}{(f+ex)^2}\right)dx,x,\frac{1}{x}\right)}{e}+\frac{f\,\text{Subst}\left(\int\left(\frac{a^2}{(f+ex)^3}+\frac{2ab\sin(\ldots}{(f+e\ldots}\right.\right.}{e}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{(2ab)\,\text{Subst}\left(\int\frac{\sin(c+dx)}{(f+ex)^2}dx,x,\frac{1}{x}\right)}{e}-\frac{b^2\,\text{Subst}\left(\int\frac{\sin^2(c+dx)}{(f+ex)^2}\ldots\right.}{e}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2}+\frac{2ab\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)}-\frac{b^2df\cos\left(c+\frac{d}{x}\right)\sin\left(\ldots\right.}{e^3\left(f+\frac{e}{x}\right)}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}+\frac{b^2d^2f\log\left(f+\frac{e}{x}\right)}{e^4}-\frac{abf\sin\left(c+\frac{d}{x}\right)}{e^2\left(f+\frac{e}{x}\right)^2}+\ldots$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-\frac{abf\,\text{si}\ldots}{e^2(\ldots}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}-\frac{b^2d\,\text{C}\ldots}{\ldots}$$

$$= -\frac{a^2f}{2e^2\left(f+\frac{e}{x}\right)^2}+\frac{a^2}{e^2\left(f+\frac{e}{x}\right)}-\frac{abdf\cos\left(c+\frac{d}{x}\right)}{e^3\left(f+\frac{e}{x}\right)}-\frac{2abd\cos\left(c-\frac{df}{e}\right)\text{Ci}\left(\frac{d\left(f+\frac{e}{x}\right)}{e}\right)}{e^3}+\frac{b^2d^2f\ldots}{\ldots}$$

Mathematica [A] time = 3.46648, size = 740, normalized size = 1.57

$$-\frac{2a^2e^4+4abdf(e+fx)^2\text{CosIntegral}\left(d\left(\frac{f}{e}+\frac{1}{x}\right)\right)\left(df\sin\left(c-\frac{df}{e}\right)+2e\cos\left(c-\frac{df}{e}\right)\right)+4abd^2e^2f^2\cos\left(c-\frac{df}{e}\right)\text{Si}\left(d\left(\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d/x])^2/(e + f*x)^3,x]
```

```
[Out] -(2*a^2*e^4 + b^2*e^4 + 4*a*b*d*e^2*f^2*x*Cos[c + d/x] + 4*a*b*d*e*f^3*x^2*
Cos[c + d/x] + 2*b^2*e^3*f*x*Cos[2*(c + d/x)] + b^2*e^2*f^2*x^2*Cos[2*(c +
d/x)] - 4*b^2*d*f*(e + f*x)^2*CosIntegral[2*d*(f/e + x^(-1))]*(d*f*Cos[2*c
- (2*d*f)/e] - e*Sin[2*c - (2*d*f)/e]) + 4*a*b*d*f*(e + f*x)^2*CosIntegral[
```

```
d*(f/e + x^(-1))]*(2*e*Cos[c - (d*f)/e] + d*f*Sin[c - (d*f)/e]) - 8*a*b*e^3
*f*x*Sin[c + d/x] - 4*a*b*e^2*f^2*x^2*Sin[c + d/x] + 2*b^2*d*e^2*f^2*x*Sin[
2*(c + d/x)] + 2*b^2*d*e*f^3*x^2*Sin[2*(c + d/x)] + 4*a*b*d^2*e^2*f^2*Cos[c
 - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] + 8*a*b*d^2*e*f^3*x*Cos[c - (d*f)
/e]*SinIntegral[d*(f/e + x^(-1))] + 4*a*b*d^2*f^4*x^2*Cos[c - (d*f)/e]*SinI
ntegral[d*(f/e + x^(-1))] - 8*a*b*d*e^3*f*Sin[c - (d*f)/e]*SinIntegral[d*(f
/e + x^(-1))] - 16*a*b*d*e^2*f^2*x*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^
(-1))] - 8*a*b*d*e*f^3*x^2*Sin[c - (d*f)/e]*SinIntegral[d*(f/e + x^(-1))] +
 4*b^2*d*e^3*f*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 8*b^2
*d*e^2*f^2*x*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2*d
*e*f^3*x^2*Cos[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2*d^2
*e^2*f^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 8*b^2*d^2*e
*f^3*x*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))] + 4*b^2*d^2*f^4
*x^2*Sin[2*c - (2*d*f)/e]*SinIntegral[2*d*(f/e + x^(-1))])/(4*e^4*f*(e + f*
x)^2)
```

Maple [B] time = 0.038, size = 1124, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(c+d/x))^2/(f*x+e)^3,x)
```

```
[Out] -d*(-a^2/e^2/(e*(c+d/x)-c*e+d*f)-1/2*a^2*(c*e-d*f)/e^2/(e*(c+d/x)-c*e+d*f)^
2+2*(c*e-d*f)/e*a*b*(-1/2*sin(c+d/x)/(e*(c+d/x)-c*e+d*f)^2/e+1/2*(-cos(c+d/
x)/(e*(c+d/x)-c*e+d*f)/e-(Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x
+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e)/e)/e)+2*a*b/e*(-sin(c+d/x)/(e*(c+d/x)
-c*e+d*f)/e+(Si(d/x+c+(-c*e+d*f)/e)*sin((-c*e+d*f)/e)/e+Ci(d/x+c+(-c*e+d*f)
/e)*cos((-c*e+d*f)/e)/e)/e)-1/2*b^2/e^2/(e*(c+d/x)-c*e+d*f)-1/4*b^2*(c*e-d*
f)/e^2/(e*(c+d/x)-c*e+d*f)^2-1/4*b^2*(c*e-d*f)/e*(-cos(2*d/x+2*c)/(e*(c+d/x
)-c*e+d*f)^2/e-(-2*sin(2*d/x+2*c)/(e*(c+d/x)-c*e+d*f)/e+2*(2*Si(2*d/x+2*c+2
*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e+2*Ci(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(
-c*e+d*f)/e)/e)/e)/e)-1/4*b^2/e*(-2*cos(2*d/x+2*c)/(e*(c+d/x)-c*e+d*f)/e-2*
(2*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e-2*Ci(2*d/x+2*c+2*(-c*
e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e)/e)+1/2*c*a^2/(e*(c+d/x)-c*e+d*f)^2/e-2*c*b
*a*(-1/2*sin(c+d/x)/(e*(c+d/x)-c*e+d*f)^2/e+1/2*(-cos(c+d/x)/(e*(c+d/x)-c*e
+d*f)/e-(Si(d/x+c+(-c*e+d*f)/e)*cos((-c*e+d*f)/e)/e-Ci(d/x+c+(-c*e+d*f)/e)*
sin((-c*e+d*f)/e)/e)/e)/e)+1/4*b^2*c/(e*(c+d/x)-c*e+d*f)^2/e+1/4*b^2*c*(-co
s(2*d/x+2*c)/(e*(c+d/x)-c*e+d*f)^2/e-(-2*sin(2*d/x+2*c)/(e*(c+d/x)-c*e+d*f)
/e+2*(2*Si(2*d/x+2*c+2*(-c*e+d*f)/e)*sin(2*(-c*e+d*f)/e)/e+2*Ci(2*d/x+2*c+2
*(-c*e+d*f)/e)*cos(2*(-c*e+d*f)/e)/e)/e)/e))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.06944, size = 2053, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*e^2*f^2*x^2 + 2*b^2*e^3*f*x - (2*a^2 + b^2)*e^4 - 2*(b^2*e^2*f^2*x
^2 + 2*b^2*e^3*f*x)*cos((c*x + d)/x)^2 - 4*((a*b*d*e*f^3*x^2 + 2*a*b*d*e^2*
f^2*x + a*b*d*e^3*f)*cos_integral((d*f*x + d*e)/(e*x)) + (a*b*d*e*f^3*x^2 +
 2*a*b*d*e^2*f^2*x + a*b*d*e^3*f)*cos_integral(-(d*f*x + d*e)/(e*x)) + (a*b
*d^2*f^4*x^2 + 2*a*b*d^2*e*f^3*x + a*b*d^2*e^2*f^2)*sin_integral((d*f*x + d
*e)/(e*x)))*cos(-(c*e - d*f)/e) + 2*((b^2*d^2*f^4*x^2 + 2*b^2*d^2*e*f^3*x +
 b^2*d^2*e^2*f^2)*cos_integral(2*(d*f*x + d*e)/(e*x)) + (b^2*d^2*f^4*x^2 +
2*b^2*d^2*e*f^3*x + b^2*d^2*e^2*f^2)*cos_integral(-2*(d*f*x + d*e)/(e*x)) -
 2*(b^2*d*e*f^3*x^2 + 2*b^2*d*e^2*f^2*x + b^2*d*e^3*f)*sin_integral(2*(d*f*
x + d*e)/(e*x)))*cos(-2*(c*e - d*f)/e) - 4*(a*b*d*e*f^3*x^2 + a*b*d*e^2*f^2
*x)*cos((c*x + d)/x) + 2*((a*b*d^2*f^4*x^2 + 2*a*b*d^2*e*f^3*x + a*b*d^2*e^
2*f^2)*cos_integral((d*f*x + d*e)/(e*x)) + (a*b*d^2*f^4*x^2 + 2*a*b*d^2*e*f
^3*x + a*b*d^2*e^2*f^2)*cos_integral(-(d*f*x + d*e)/(e*x)) - 4*(a*b*d*e*f^3
*x^2 + 2*a*b*d*e^2*f^2*x + a*b*d*e^3*f)*sin_integral((d*f*x + d*e)/(e*x)))*
sin(-(c*e - d*f)/e) + 2*((b^2*d*e*f^3*x^2 + 2*b^2*d*e^2*f^2*x + b^2*d*e^3*f
)*cos_integral(2*(d*f*x + d*e)/(e*x)) + (b^2*d*e*f^3*x^2 + 2*b^2*d*e^2*f^2*
x + b^2*d*e^3*f)*cos_integral(-2*(d*f*x + d*e)/(e*x)) + 2*(b^2*d^2*f^4*x^2
+ 2*b^2*d^2*e*f^3*x + b^2*d^2*e^2*f^2)*sin_integral(2*(d*f*x + d*e)/(e*x)))
*sin(-2*(c*e - d*f)/e) + 4*(a*b*e^2*f^2*x^2 + 2*a*b*e^3*f*x - (b^2*d*e*f^3*
x^2 + b^2*d*e^2*f^2*x)*cos((c*x + d)/x))*sin((c*x + d)/x))/(e^4*f^3*x^2 + 2
*e^5*f^2*x + e^6*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))**2/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(c + \frac{d}{x}\right) + a\right)^2}{\left(fx + e\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(c+d/x))^2/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)^2/(f*x + e)^3, x)
```

3.299 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi [A] time = 0.031669, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

```
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A] time = 1.0457, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

```
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Maple [A] time = 1.049, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{f^2x^2+2\,efx+e^2}{b\sin\left(\frac{cx+d}{x}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f^2*x^2 + 2*e*f*x + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((e + f*x)**2/(a + b*sin(c + d/x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

3.300 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi [A] time = 0.017325, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A] time = 0.569678, size = 0, normalized size = 0.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

```
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Maple [A] time = 0.94, size = 0, normalized size = 0.

$$\int (fx+e)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx+e}{b\sin\left(\frac{cx+d}{x}\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((e + f*x)/(a + b*sin(c + d/x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

3.301 $\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d/x])^(-1), x]
```

Rubi [A] time = 0.0051691, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d/x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d/x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A] time = 0.0367629, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d/x])^(-1),x]
```

```
[Out] Integrate[(a + b*Sin[c + d/x])^(-1), x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(c+d/x)),x)
```

```
[Out] int(1/(a+b*sin(c+d/x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(c + d/x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b\sin\left(\frac{cx+d}{x}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral(1/(a + b*sin(c + d/x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(c + d/x) + a), x)
```

3.302 $\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi [A] time = 0.0172068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx=\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Mathematica [A] time = 0.077505, size = 0, normalized size = 0.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x]),x]
```

```
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x]), x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx+e}{b\sin\left(\frac{cx+d}{x}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{e+fx}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((e + f*x)/(a + b*sin(c + d/x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a), x)
```

3.303 $\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)},x\right)$$

```
[Out] Unintegrable[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi [A] time = 0.0314051, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

```
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)} dx = \int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)} dx$$

Mathematica [A] time = 0.134575, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]),x]
```

```
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x]), x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{f^2x^2+2\,efx+e^2}{b\sin\left(\frac{cx+d}{x}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="fricas")
```

```
[Out] integral((f^2*x^2 + 2*e*f*x + e^2)/(b*sin((c*x + d)/x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2}{a+b\sin\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x)),x)
```

```
[Out] Integral((e + f*x)**2/(a + b*sin(c + d/x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2}{b\sin\left(c+\frac{d}{x}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x)),x, algorithm="giac")

[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a), x)
```

3.304 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi [A] time = 0.0295132, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx = \int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A] time = 112.861, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Maple [A] time = 3.425, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2 \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{f^2x^2 + 2\,efx + e^2}{b^2\cos\left(\frac{cx+d}{x}\right)^2 - 2\,ab\sin\left(\frac{cx+d}{x}\right) - a^2 - b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f^2*x^2 + 2*e*f*x + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*
x + d)/x) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2}{\left(b\sin\left(c + \frac{d}{x}\right) + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

3.305 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi [A] time = 0.0165308, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A] time = 19.4511, size = 0, normalized size = 0.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Maple [A] time = 2.381, size = 0, normalized size = 0.

$$\int (fx+e)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f*x^3 + a*b*e*x^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f*
x^3 + a*b*e*x^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x +
d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4
- a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a
^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)
*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c*x + d)
/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2 + 2*a*b*
e*x)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*cos(2*(c*x
+ d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*cos((c*x + d)/x) + (3*b^2*f*x^2 + 2*b^2
*e*x + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*s
in((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x
))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x +
 d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b
^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 +
 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a
*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(
b^2*f*x^3 + b^2*e*x^2 + (a*b*f*x^3 + a*b*e*x^2)*sin((c*x + d)/x))*sin(2*(c*
x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*co
s((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x)
+ (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a
^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx+e}{b^2\cos\left(\frac{cx+d}{x}\right)^2 - 2\,ab\sin\left(\frac{cx+d}{x}\right) - a^2 - b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
- b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

3.306 $\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d/x])^(-2), x]
```

Rubi [A] time = 0.0052627, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d/x])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d/x])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A] time = 3.2747, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d/x])^(-2),x]
```

```
[Out] Integrate[(a + b*Sin[c + d/x])^(-2), x]
```

Maple [A] time = 1.77, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int(1/(a+b*sin(c+d/x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a*b*x^2*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*a*b*x^2*cos((c*x + d)/x
) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x
+ d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*
b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2
+ 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b -
a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))*integrat
e(-2*(2*a^2*d*cos((c*x + d)/x)^2 + 2*a^2*d*sin((c*x + d)/x)^2 + 2*a*b*x*cos
((c*x + d)/x) + a*b*d*sin((c*x + d)/x) + (2*a*b*x*cos((c*x + d)/x) - a*b*d*
sin((c*x + d)/x))*cos(2*(c*x + d)/x) + (a*b*d*cos((c*x + d)/x) + 2*a*b*x*si
n((c*x + d)/x) + 2*b^2*x)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x
 + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*c
os((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x +
d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*
b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(a*b*x^2*sin((c*x + d)/x) + b^2*x
^2)*sin(2*(c*x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 -
a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*
(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d
*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4
)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c
*x + d)/x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{b^2\cos\left(\frac{cx+d}{x}\right)^2 - 2\,ab\sin\left(\frac{cx+d}{x}\right) - a^2 - b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-1/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2 - b^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(c + d/x) + a)^(-2), x)
```

3.307 $\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi [A] time = 0.0158103, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Defer[Int][(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A] time = 2.94656, size = 0, normalized size = 0.

$$\int \frac{e+fx}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Integrate[(e + f*x)/(a + b*Sin[c + d/x])^2, x]
```

Maple [A] time = 0.002, size = 0, normalized size = 0.

$$\int (fx+e)\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)/(a+b*sin(c+d/x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b*f*x^3 + a*b*e*x^2)*cos(2*(c*x + d)/x)*cos((c*x + d)/x) + 2*(a*b*f*
x^3 + a*b*e*x^2)*cos((c*x + d)/x) + ((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2
 + 4*(a^4 - a^2*b^2)*d*cos((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x +
d)/x)*sin(2*(c*x + d)/x) + (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4
- a^2*b^2)*d*sin((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a
^2*b^2 - b^4)*d - 2*(2*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)
*d)*cos(2*(c*x + d)/x))*integrate(-2*(2*(a^2*d*f*x + a^2*d*e)*cos((c*x + d)
/x)^2 + 2*(a^2*d*f*x + a^2*d*e)*sin((c*x + d)/x)^2 + ((3*a*b*f*x^2 + 2*a*b*
e*x)*cos((c*x + d)/x) - (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x))*cos(2*(c*x
+ d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*cos((c*x + d)/x) + (3*b^2*f*x^2 + 2*b^2
*e*x + (a*b*d*f*x + a*b*d*e)*cos((c*x + d)/x) + (3*a*b*f*x^2 + 2*a*b*e*x)*s
in((c*x + d)/x))*sin(2*(c*x + d)/x) + (a*b*d*f*x + a*b*d*e)*sin((c*x + d)/x
))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*cos((c*x +
 d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x) + (a^2*b
^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d)/x)^2 +
 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a^3*b - a
*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x)), x) + 2*(
b^2*f*x^3 + b^2*e*x^2 + (a*b*f*x^3 + a*b*e*x^2)*sin((c*x + d)/x))*sin(2*(c*
x + d)/x))/((a^2*b^2 - b^4)*d*cos(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*co
s((c*x + d)/x)^2 + 4*(a^3*b - a*b^3)*d*cos((c*x + d)/x)*sin(2*(c*x + d)/x)
+ (a^2*b^2 - b^4)*d*sin(2*(c*x + d)/x)^2 + 4*(a^4 - a^2*b^2)*d*sin((c*x + d
)/x)^2 + 4*(a^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d - 2*(2*(a
^3*b - a*b^3)*d*sin((c*x + d)/x) + (a^2*b^2 - b^4)*d)*cos(2*(c*x + d)/x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx+e}{b^2\cos\left(\frac{cx+d}{x}\right)^2-2\,ab\sin\left(\frac{cx+d}{x}\right)-a^2-b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f*x + e)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*x + d)/x) - a^2
- b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sin(c + d/x) + a)^2, x)
```

3.308 $\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi [A] time = 0.0302505, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Defer[Int][(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Rubi steps

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx=\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Mathematica [A] time = 99.5512, size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2}{\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2,x]
```

```
[Out] Integrate[(e + f*x)^2/(a + b*Sin[c + d/x])^2, x]
```

Maple [A] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

```
[Out] int((f*x+e)^2/(a+b*sin(c+d/x))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{f^2x^2 + 2\,efx + e^2}{b^2\cos\left(\frac{cx+d}{x}\right)^2 - 2\,ab\sin\left(\frac{cx+d}{x}\right) - a^2 - b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(f^2*x^2 + 2*e*f*x + e^2)/(b^2*cos((c*x + d)/x)^2 - 2*a*b*sin((c*
x + d)/x) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2/(a+b*sin(c+d/x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2}{\left(b\sin\left(c+\frac{d}{x}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2/(a+b*sin(c+d/x))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2/(b*sin(c + d/x) + a)^2, x)
```

3.309 $\int (e+fx)^m \left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left((e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p,x\right)$$

```
[Out] Unintegrable[(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]
```

Rubi [A] time = 0.0279417, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]
```

```
[Out] Defer[Int][(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]
```

Rubi steps

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx=\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Mathematica [A] time = 1.39924, size = 0, normalized size = 0.

$$\int (e+fx)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*Sin[c + d/x])^p, x]
```

Maple [A] time = 0.513, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^m\left(a+b\sin\left(c+\frac{d}{x}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

```
[Out] int((f*x+e)^m*(a+b*sin(c+d/x))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^m \left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((fx+e)^m \left(b\sin\left(\frac{cx+d}{x}\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^m*(b*sin((c*x + d)/x) + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*sin(c+d/x))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^m \left(b\sin\left(c+\frac{d}{x}\right)+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*sin(c+d/x))^p,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^m*(b*sin(c + d/x) + a)^p, x)
```

3.310 $\int x^m \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal. Leaf size=115

$$-\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\text{Gamma}(m+1,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\text{Gamma}(m+1,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

```
[Out] -(E^(I*a)*x^m*Csc[a + b*x]*Gamma[1 + m, (-I)*b*x]*(c*Sin[a + b*x]^3)^(1/3))
/(2*b*((-I)*b*x)^m) - (x^m*Csc[a + b*x]*Gamma[1 + m, I*b*x]*(c*Sin[a + b*x]
^3)^(1/3))/(2*b*E^(I*a)*(I*b*x)^m)
```

Rubi [A] time = 0.287303, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6720, 3308, 2181}

$$-\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\text{Gamma}(m+1,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}-\frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\text{Gamma}(m+1,ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -(E^(I*a)*x^m*Csc[a + b*x]*Gamma[1 + m, (-I)*b*x]*(c*Sin[a + b*x]^3)^(1/3))
/(2*b*((-I)*b*x)^m) - (x^m*Csc[a + b*x]*Gamma[1 + m, I*b*x]*(c*Sin[a + b*x]
^3)^(1/3))/(2*b*E^(I*a)*(I*b*x)^m)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\int x^m \sqrt[3]{c\sin^3(a+bx)}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^m \sin(a+bx)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int e^{-i(a+bx)}x^m\,dx - \frac{1}{2}\left(i\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int e^{i(a+b}$$
$$= -\frac{e^{ia}x^m(-ibx)^{-m}\csc(a+bx)\Gamma(1+m,-ibx)\sqrt[3]{c\sin^3(a+bx)}}{2b} - \frac{e^{-ia}x^m(ibx)^{-m}\csc(a+bx)\Gamma(1+}{2b}$$

Mathematica [A] time = 0.125477, size = 94, normalized size = 0.82

$$-\frac{e^{-ia}x^m\left(b^2x^2\right)^{-m}\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(e^{2ia}(ibx)^m\text{Gamma}(m+1,-ibx)+(-ibx)^m\text{Gamma}(m+1,ibx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -(x^m*Csc[a + b*x]*(E^((2*I)*a)*(I*b*x)^m*Gamma[1 + m, (-I)*b*x] + ((-I)*b*
x)^m*Gamma[1 + m, I*b*x])*(c*Sin[a + b*x]^3)^(1/3))/(2*b*E^(I*a)*(b^2*x^2)^
m)
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int x^m \sqrt[3]{c\left(\sin\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(b*x+a)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

Fricas [A] time = 1.74468, size = 213, normalized size = 1.85

$$-\frac{\left(e^{\left(-m\log(ib)-ia\right)}\Gamma\left(m+1,ibx\right)+e^{\left(-m\log(-ib)+ia\right)}\Gamma\left(m+1,-ibx\right)\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{2b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) + e^(-m*log(-I*b) + I*a)*ga
mma(m + 1, -I*b*x))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b*sin(b*x
 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt[3]{c \sin^3(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x)**3)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin(bx + a)^3\right)^{\frac{1}{3}} x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^m, x)
```

3.311 $\int x^3 \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal. Leaf size=96

$$\frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{6\sqrt[3]{c\sin^3(a+bx)}}{b^4}+\frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] (-6*(c*Sin[a + b*x]^3)^(1/3))/b^4 + (3*x^2*(c*Sin[a + b*x]^3)^(1/3))/b^2 +
(6*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b^3 - (x^3*Cot[a + b*x]*(c*Sin[
a + b*x]^3)^(1/3))/b
```

Rubi [A] time = 0.207996, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6720, 3296, 2637}

$$\frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{6\sqrt[3]{c\sin^3(a+bx)}}{b^4}+\frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (-6*(c*Sin[a + b*x]^3)^(1/3))/b^4 + (3*x^2*(c*Sin[a + b*x]^3)^(1/3))/b^2 +
(6*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b^3 - (x^3*Cot[a + b*x]*(c*Sin[
a + b*x]^3)^(1/3))/b
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3\sqrt[3]{c\sin^3(a+bx)}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^3\sin(a+bx)\,dx$$
$$= -\frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} + \frac{\left(3\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^2\cos(a+bx)\,dx}{b}$$
$$= \frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2} - \frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} - \frac{\left(6\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x}{b^2}$$
$$= \frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2} + \frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3} - \frac{x^3\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} - $$
$$= -\frac{6\sqrt[3]{c\sin^3(a+bx)}}{b^4} + \frac{3x^2\sqrt[3]{c\sin^3(a+bx)}}{b^2} + \frac{6x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3} - \frac{x^3\cot(a+b}{}$$

Mathematica [A] time = 0.197763, size = 47, normalized size = 0.49

$$-\frac{\left(bx\left(b^2x^2-6\right)\cot(a+bx)-3b^2x^2+6\right)\sqrt[3]{c\sin^3(a+bx)}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -(((6 - 3*b^2*x^2 + b*x*(-6 + b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/
3))/b^4)
```

Maple [C] time = 0.079, size = 151, normalized size = 1.6

$$\frac{-\frac{i}{2}\left(b^3x^3+3\,ib^2x^2-6\,bx-6\,i\right)e^{2\,i(bx+a)}}{\left(e^{2\,i(bx+a)}-1\right)b^4}\sqrt[3]{ic\left(e^{2\,i(bx+a)}-1\right)^3e^{-3\,i(bx+a)}} - \frac{\frac{i}{2}\left(b^3x^3-3\,ib^2x^2-6\,bx+6\,i\right)}{\left(e^{2\,i(bx+a)}-1\right)b^4}\sqrt[3]{ic\left(e^{2\,i(bx+a)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x+a)^3)^(1/3),x)
```

```
[Out] -1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a
))-1)*(b^3*x^3+3*I*b^2*x^2-6*b*x-6*I)/b^4*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(
2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*(b^3*x^3-3*
I*b^2*x^2-6*b*x+6*I)/b^4
```

Maxima [A] time = 1.60558, size = 197, normalized size = 2.05

$$\frac{3\left((bx+a)\cos(bx+a)-\sin(bx+a)\right)a^2c^{\frac{1}{3}} - 3\left(\left((bx+a)^2-2\right)\cos(bx+a)-2\,(bx+a)\sin(bx+a)\right)ac^{\frac{1}{3}} + \frac{4\,a^3c^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*(3*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a^2*c^(1/3) - 3*(((b*x + a)^
2 - 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*a*c^(1/3) + 4*a^3*c^(1/3)/(
sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1) + (((b*x + a)^3 - 6*b*x - 6*a)*cos
(b*x + a) - 3*((b*x + a)^2 - 2)*sin(b*x + a))*c^(1/3))/b^4
```

Fricas [A] time = 1.68003, size = 176, normalized size = 1.83

$$-\frac{\left(\left(b^3x^3 - 6\,bx\right)\cos\left(bx+a\right) - 3\left(b^2x^2 - 2\right)\sin\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2 - c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{b^4\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -((b^3*x^3 - 6*b*x)*cos(b*x + a) - 3*(b^2*x^2 - 2)*sin(b*x + a))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(1/3)/(b^4*sin(b*x + a))
```

Sympy [A] time = 29.8114, size = 143, normalized size = 1.49

$$\begin{cases} \frac{x^4\sqrt[3]{c\sin^3(a)}}{4} & \text{for } b = 0 \\ 0 & \text{for } a = -bx \vee a = -bx + \pi \\ -\frac{\sqrt[3]{c}x^3\sqrt[3]{\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)} + \frac{3\sqrt[3]{c}x^2\sqrt[3]{\sin^3(a+bx)}}{b^2} + \frac{6\sqrt[3]{c}x\sqrt[3]{\sin^3(a+bx)}\cos(a+bx)}{b^3\sin(a+bx)} - \frac{6\sqrt[3]{c}\sqrt[3]{\sin^3(a+bx)}}{b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**4*(c*sin(a)**3)**(1/3)/4, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-c**(1/3)*x**3*(sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a
 + b*x)) + 3*c**(1/3)*x**2*(sin(a + b*x)**3)**(1/3)/b**2 + 6*c**(1/3)*x*(si
n(a + b*x)**3)**(1/3)*cos(a + b*x)/(b**3*sin(a + b*x)) - 6*c**(1/3)*(sin(a
+ b*x)**3)**(1/3)/b**4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^3, x)
```

3.312 $\int x^2 \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal. Leaf size=74

$$\frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] (2*x*(c*Sin[a + b*x]^3)^(1/3))/b^2 + (2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/
3))/b^3 - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rubi [A] time = 0.182098, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6720, 3296, 2638}

$$\frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2}+\frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3}-\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (2*x*(c*Sin[a + b*x]^3)^(1/3))/b^2 + (2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/
3))/b^3 - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2\sqrt[3]{c\sin^3(a+bx)}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x^2\sin(a+bx)\,dx$$
$$= -\frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} + \frac{\left(2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\cos(a+bx)\,dx}{b}$$
$$= \frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2} - \frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b} - \frac{\left(2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \sin(a}{b^2}$$
$$= \frac{2x\sqrt[3]{c\sin^3(a+bx)}}{b^2} + \frac{2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b^3} - \frac{x^2\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Mathematica [A] time = 0.223712, size = 40, normalized size = 0.54

$$\frac{\left(\left(2-b^2x^2\right)\cot(a+bx)+2bx\right)\sqrt[3]{c\sin^3(a+bx)}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] ((2*b*x + (2 - b^2*x^2)*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^3
```

Maple [C] time = 0.075, size = 133, normalized size = 1.8

$$\frac{-\frac{i}{2}\left(x^2b^2+2\,ibx-2\right)e^{2\,i(bx+a)}}{\left(e^{2\,i(bx+a)}-1\right)b^3}\sqrt[3]{ic\left(e^{2\,i(bx+a)}-1\right)^3e^{-3\,i(bx+a)}} - \frac{\frac{i}{2}\left(x^2b^2-2\,ibx-2\right)}{\left(e^{2\,i(bx+a)}-1\right)b^3}\sqrt[3]{ic\left(e^{2\,i(bx+a)}-1\right)^3e^{-3\,i(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x+a)^3)^(1/3),x)
```

```
[Out] -1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a
))-1)*(x^2*b^2+2*I*b*x-2)/b^3*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(2*I*(b*x+a))
-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*(x^2*b^2-2*I*b*x-2)/b^3
```

Maxima [A] time = 1.54939, size = 134, normalized size = 1.81

$$-\frac{2\left((bx+a)\cos(bx+a)-\sin(bx+a)\right)ac^{\frac{1}{3}} - \left(\left((bx+a)^2-2\right)\cos(bx+a)-2\,(bx+a)\sin(bx+a)\right)c^{\frac{1}{3}} + \frac{4\,a^2c^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/2*(2*((b*x + a)*cos(b*x + a) - sin(b*x + a))*a*c^(1/3) - (((b*x + a)^2 -
 2)*cos(b*x + a) - 2*(b*x + a)*sin(b*x + a))*c^(1/3) + 4*a^2*c^(1/3)/(sin(b
*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^3
```

Fricas [A] time = 1.75473, size = 155, normalized size = 2.09

$$\frac{\left(2\,bx\sin\left(bx+a\right)-\left(b^2x^2-2\right)\cos\left(bx+a\right)\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}}{b^3\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] (2*b*x*sin(b*x + a) - (b^2*x^2 - 2)*cos(b*x + a))*(-(c*cos(b*x + a)^2 - c)*
sin(b*x + a))^(1/3)/(b^3*sin(b*x + a))
```

Sympy [A] time = 11.8304, size = 117, normalized size = 1.58

$$\begin{cases} \frac{x^3\sqrt[3]{c\sin^3(a)}}{3} & \text{for } b=0 \\ 0 & \text{for } a=-bx \vee a=-bx+\pi \\ -\frac{\sqrt[3]{c}x^2\sqrt[3]{\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)}+\frac{2\sqrt[3]{c}x\sqrt[3]{\sin^3(a+bx)}}{b^2}+\frac{2\sqrt[3]{c}\sqrt[3]{\sin^3(a+bx)}\cos(a+bx)}{b^3\sin(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**3*(c*sin(a)**3)**(1/3)/3, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-c**(1/3)*x**2*(sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a
 + b*x)) + 2*c**(1/3)*x*(sin(a + b*x)**3)**(1/3)/b**2 + 2*c**(1/3)*(sin(a +
 b*x)**3)**(1/3)*cos(a + b*x)/(b**3*sin(a + b*x)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx+a\right)^3\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x^2, x)
```

3.313 $\int x\sqrt[3]{c\sin^3(a+bx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] (c*Sin[a + b*x]^3)^(1/3)/b^2 - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rubi [A] time = 0.127389, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6720, 3296, 2637}

$$\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] (c*Sin[a + b*x]^3)^(1/3)/b^2 - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{c\sin^3(a+bx)}\,dx&=\left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int x\sin(a+bx)\,dx\\&=-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}+\frac{\left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int\cos(a+bx)\,dx}{b}\\&=\frac{\sqrt[3]{c\sin^3(a+bx)}}{b^2}-\frac{x\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.132428, size = 30, normalized size = 0.67

$$\frac{(1-bx\cot(a+bx))\sqrt[3]{c\sin^3(a+bx)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] ((1 - b*x*Cot[a + b*x])*(c*Sin[a + b*x]^3)^(1/3))/b^2
```

Maple [C] time = 0.072, size = 117, normalized size = 2.6

$$-\frac{\frac{i}{2}(bx+i)e^{2i(bx+a)}}{\left(e^{2i(bx+a)}-1\right)b^2}\sqrt[3]{ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}}-\frac{\frac{i}{2}(bx-i)}{\left(e^{2i(bx+a)}-1\right)b^2}\sqrt[3]{ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x+a)^3)^(1/3),x)
```

```
[Out] -1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a
))-1)*(b*x+I)/b^2*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3
*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*(b*x-I)/b^2
```

Maxima [A] time = 1.51733, size = 81, normalized size = 1.8

$$\frac{((bx+a)\cos(bx+a)-\sin(bx+a))c^{\frac{1}{3}}+\frac{4ac^{\frac{1}{3}}}{\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x + a)*cos(b*x + a) - sin(b*x + a))*c^(1/3) + 4*a*c^(1/3)/(sin(b*x
 + a)^2/(cos(b*x + a) + 1)^2 + 1))/b^2
```

Fricas [A] time = 1.69519, size = 135, normalized size = 3.

$$-\frac{(bx\cos(bx+a)-\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{1}{3}}}{b^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(b*x*cos(b*x + a) - sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(
1/3)/(b^2*sin(b*x + a))
```

Sympy [A] time = 5.14652, size = 76, normalized size = 1.69

$$\begin{cases} \frac{x^2\sqrt[3]{c\sin^3(a)}}{2} & \text{for } b = 0 \\ 0 & \text{for } a = -bx \lor a = -bx + \pi \\ -\frac{\sqrt[3]{c}x\sqrt[3]{\sin^3(a+bx)}\cos(a+bx)}{b\sin(a+bx)} + \frac{\sqrt[3]{c}\sqrt[3]{\sin^3(a+bx)}}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x**2*(c*sin(a)**3)**(1/3)/2, Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a,
-b*x + pi)), (-c**(1/3)*x*(sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a +
b*x)) + c**(1/3)*(sin(a + b*x)**3)**(1/3)/b**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx+a)^3\right)^{\frac{1}{3}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)*x, x)
```

3.314 $\int \sqrt[3]{c \sin^3(a+bx)}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Rubi [A] time = 0.0175031, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3207, 2638}

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c\sin^3(a+bx)}\,dx &= \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \sin(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0627671, size = 25, normalized size = 1.

$$-\frac{\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3))/b)
```

Maple [C] time = 0.111, size = 105, normalized size = 4.2

$$\frac{-\frac{i}{2}\mathrm{e}^{2i(bx+a)}}{\left(\mathrm{e}^{2i(bx+a)}-1\right)b}\sqrt[3]{ic\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\mathrm{e}^{-3i(bx+a)}}-\frac{\frac{i}{2}}{\left(\mathrm{e}^{2i(bx+a)}-1\right)b}\sqrt[3]{ic\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\mathrm{e}^{-3i(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3),x)
```

```
[Out] -1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a
))-1)/b*exp(2*I*(b*x+a))-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)
))^(1/3)/(exp(2*I*(b*x+a))-1)/b
```

Maxima [A] time = 1.45972, size = 42, normalized size = 1.68

$$-\frac{2\,c^{\frac{1}{3}}}{b\left(\frac{\sin(bx+a)^{2}}{(\cos(bx+a)+1)^{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -2*c^(1/3)/(b*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))
```

Fricas [A] time = 1.69497, size = 104, normalized size = 4.16

$$-\frac{\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}\cos\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [A] time = 1.94314, size = 53, normalized size = 2.12

$$\begin{cases} x\sqrt[3]{c\sin^{3}\left(a\right)} & \text{for}\: b=0 \\ 0 & \text{for}\: a=-bx\vee a=-bx+\pi \\ -\frac{\sqrt[3]{c}\sqrt[3]{\sin^{3}\left(a+bx\right)}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3),x)
```

```
[Out] Piecewise((x*(c*sin(a)**3)**(1/3), Eq(b, 0)), (0, Eq(a, -b*x) | Eq(a, -b*x
+ pi)), (-c**(1/3)*(sin(a + b*x)**3)**(1/3)*cos(a + b*x)/(b*sin(a + b*x)),
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^{3}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3), x)
```

3.315 $\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x} dx$

Optimal. Leaf size=55

$$\sin(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}+\cos(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}$$

```
[Out] CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3) + Cos[a]*Csc[
a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x]
```

Rubi [A] time = 0.165793, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6720, 3303, 3299, 3302}

$$\sin(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}+\cos(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

```
[Out] CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3) + Cos[a]*Csc[
a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x}\,dx$$
$$= \left(\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(bx)}{x}\,dx + \left(\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)}\right)\int$$
$$= \text{Ci}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a+bx)} + \cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\text{Si}(bx)$$

Mathematica [A] time = 0.0573214, size = 36, normalized size = 0.65

$$\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}(\sin(a)\text{CosIntegral}(bx)+\cos(a)\text{Si}(bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x,x]
```

```
[Out] Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(CosIntegral[b*x]*Sin[a] + Cos[a]*Sin
Integral[b*x])
```

Maple [C] time = 0.078, size = 228, normalized size = 4.2

$$-\frac{\text{Ei}(1,-ibx)\,e^{i(bx+2a)}}{2e^{2i(bx+a)}-2}\sqrt[3]{ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}} - \frac{\frac{i}{2}e^{ibx}\pi\,\text{csgn}(bx)}{e^{2i(bx+a)}-1}\sqrt[3]{ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}} + \frac{ie^{ibx}\text{Si}(bx)}{e^{2i(bx+a)}-1}\sqrt[3]{i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x,x)
```

```
[Out] -1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))
-1)*Ei(1,-I*b*x)*exp(I*(b*x+2*a))-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*
I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*exp(I*b*x)*Pi*csgn(b*x)+I*(I*c*(exp(
2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*exp(I*b*x)*
Si(b*x)+1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(
b*x+a))-1)*exp(I*b*x)*Ei(1,-I*b*x)
```

Maxima [C] time = 1.64273, size = 57, normalized size = 1.04

$$\frac{1}{4}\left((iE_1(ibx)-iE_1(-ibx))\cos(a)+(E_1(ibx)+E_1(-ibx))\sin(a)\right)c^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] 1/4*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x))*cos(a) + (e
xp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x))*sin(a))*c^(1/3)
```

Fricas [A] time = 1.69393, size = 265, normalized size = 4.82

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(a\right)\operatorname{Si}\left(bx\right)+\left(4^{\frac{2}{3}}\operatorname{Ci}\left(bx\right)+4^{\frac{2}{3}}\operatorname{Ci}\left(-bx\right)\right)\sin\left(a\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{1}{3}}\sin\left(bx+a\right)}{8\left(\cos\left(bx+a\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(2*4^(2/3)*cos(a)*sin_integral(b*x) + (4^(2/3)*cos_integral(b*
x) + 4^(2/3)*cos_integral(-b*x))*sin(a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x +
 a))^(1/3)*sin(b*x + a)/(cos(b*x + a)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt[3]{c\sin^{3}\left(a+bx\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\sin\left(bx+a\right)^{3}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x, x)
```

3.316 $\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=77

$$b\cos(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\sin(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}$$

```
[Out] -((c*Sin[a + b*x]^3)^(1/3)/x) + b*Cos[a]*CosIntegral[b*x]*Csc[a + b*x]*(c*S
in[a + b*x]^3)^(1/3) - b*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3)*SinIn
tegral[b*x]
```

Rubi [A] time = 0.176734, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 3297, 3303, 3299, 3302}

$$b\cos(a)\text{CosIntegral}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\sin(a)\text{Si}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - \frac{\sqrt[3]{c\sin^3(a+bx)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```


```
[Out] -((c*Sin[a + b*x]^3)^(1/3)/x) + b*Cos[a]*CosIntegral[b*x]*Csc[a + b*x]*(c*S
in[a + b*x]^3)^(1/3) - b*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/3)*SinIn
tegral[b*x]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^2}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x^2}\,dx$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + \left(b\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(a+bx)}{x}\,dx$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + \left(b\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(bx)}{x}\,dx - \left(b\csc(a+bx)\sin(a\right.$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{x} + b\cos(a)\text{Ci}(bx)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)} - b\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a}$$

Mathematica [A] time = 0.179418, size = 51, normalized size = 0.66

$$\frac{\sqrt[3]{c\sin^3(a+bx)}(bx\cos(a)\text{CosIntegral}(bx)\csc(a+bx) - bx\sin(a)\text{Si}(bx)\csc(a+bx) - 1)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^2,x]
```

```
[Out] ((c*Sin[a + b*x]^3)^(1/3)*(-1 + b*x*Cos[a]*CosIntegral[b*x]*Csc[a + b*x] -
b*x*Csc[a + b*x]*Sin[a]*SinIntegral[b*x]))/x
```

Maple [C] time = 0.083, size = 155, normalized size = 2.

$$\frac{\frac{i}{2}b}{e^{2i(bx+a)}-1}\sqrt[3]{ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}}\left(\frac{ie^{2i(bx+a)}}{bx} - \text{Ei}(1,-ibx)e^{i(bx+2a)}\right) - \frac{\frac{i}{2}b}{e^{2i(bx+a)}-1}\sqrt[3]{ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x^2,x)
```

```
[Out] 1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a)
)-1)*b*(I/x/b*exp(2*I*(b*x+a))-Ei(1,-I*b*x)*exp(I*(b*x+2*a)))-1/2*I*(I*c*(e
xp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))-1)*b*(I/x/b
+exp(I*b*x)*Ei(1,I*b*x))
```

Maxima [C] time = 1.66097, size = 328, normalized size = 4.26

$$\left(\left(\left(8\sqrt{3}-8i\right)E_2(ibx) + \left(8\sqrt{3}+8i\right)E_2(-ibx)\right)\cos(a)^3 + \left(\left(8\sqrt{3}-8i\right)E_2(ibx) + \left(8\sqrt{3}+8i\right)E_2(-ibx)\right)\cos(a)\sin(a)^2 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/64*(((8*sqrt(3) - 8*I)*exp_integral_e(2, I*b*x) + (8*sqrt(3) + 8*I)*exp_i
ntegral_e(2, -I*b*x))*cos(a)^3 + ((8*sqrt(3) - 8*I)*exp_integral_e(2, I*b*x
) + (8*sqrt(3) + 8*I)*exp_integral_e(2, -I*b*x))*cos(a)*sin(a)^2 + 8*((-I*s
qrt(3) - 1)*exp_integral_e(2, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(2, -I
*b*x))*sin(a)^3 - ((8*sqrt(3) + 8*I)*exp_integral_e(2, I*b*x) + (8*sqrt(3)
- 8*I)*exp_integral_e(2, -I*b*x))*cos(a) + 8*(((-I*sqrt(3) - 1)*exp_integra
l_e(2, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(2, -I*b*x))*cos(a)^2 + (I*sq
rt(3) - 1)*exp_integral_e(2, I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(2, -I
*b*x))*sin(a))*b*c^(1/3)/(a*cos(a)^2 + a*sin(a)^2 - (b*x + a)*(cos(a)^2 + s
in(a)^2))
```

Fricas [A] time = 1.79112, size = 339, normalized size = 4.4

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(bx+a\right)^{2}-\left(2\cdot 4^{\frac{2}{3}}bx\sin\left(a\right)\operatorname{Si}\left(bx\right)-\left(4^{\frac{2}{3}}bx\operatorname{Ci}\left(bx\right)+4^{\frac{2}{3}}bx\operatorname{Ci}\left(-bx\right)\right)\cos\left(a\right)\right)\sin\left(bx+a\right)-2\cdot 4^{\frac{2}{3}}\right)\left(-\right.}{8\left(x\cos\left(bx+a\right)^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(2*4^(2/3)*cos(b*x + a)^2 - (2*4^(2/3)*b*x*sin(a)*sin_integral
(b*x) - (4^(2/3)*b*x*cos_integral(b*x) + 4^(2/3)*b*x*cos_integral(-b*x))*co
s(a))*sin(b*x + a) - 2*4^(2/3))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3
)/(x*cos(b*x + a)^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx+a\right)^{3}\right)^{\frac{1}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x^2, x)
```

3.317 $\int \frac{\sqrt[3]{c \sin^3(a+bx)}}{x^3} \, dx$

Optimal. Leaf size=116

$$-\frac{1}{2}b^2 \sin(a)\text{CosIntegral}(bx) \csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} - \frac{1}{2}b^2 \cos(a)\text{Si}(bx) \csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} - \frac{\sqrt[3]{c \sin^3(a+b}}{2x^2}$$

```
[Out] -(c*Sin[a + b*x]^3)^(1/3)/(2*x^2) - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3
))/(2*x) - (b^2*CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/
3))/2 - (b^2*Cos[a]*Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x])
/2
```

Rubi [A] time = 0.205576, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}b^2 \sin(a)\text{CosIntegral}(bx) \csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} - \frac{1}{2}b^2 \cos(a)\text{Si}(bx) \csc(a+bx)\sqrt[3]{c \sin^3(a+bx)} - \frac{\sqrt[3]{c \sin^3(a+b}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

```
[Out] -(c*Sin[a + b*x]^3)^(1/3)/(2*x^2) - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(1/3
))/(2*x) - (b^2*CosIntegral[b*x]*Csc[a + b*x]*Sin[a]*(c*Sin[a + b*x]^3)^(1/
3))/2 - (b^2*Cos[a]*Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*SinIntegral[b*x])
/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx = \left(\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\sin(a+bx)}{x^3}\,dx$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} + \frac{1}{2}\left(b\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\cos(a+bx)}{x^2}\,dx$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}\left(b^2\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right)\int \frac{\mathrm{si}}{}$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}\left(b^2\cos(a)\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\right.$$
$$= -\frac{\sqrt[3]{c\sin^3(a+bx)}}{2x^2} - \frac{b\cot(a+bx)\sqrt[3]{c\sin^3(a+bx)}}{2x} - \frac{1}{2}b^2\mathrm{Ci}(bx)\csc(a+bx)\sin(a)\sqrt[3]{c\sin^3(a}$$

Mathematica [A] time = 0.145025, size = 69, normalized size = 0.59

$$-\frac{\csc(a+bx)\sqrt[3]{c\sin^3(a+bx)}\left(b^2x^2\sin(a)\mathrm{CosIntegral}(bx)+b^2x^2\cos(a)\mathrm{Si}(bx)+\sin(a+bx)+bx\cos(a+bx)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(1/3)/x^3,x]
```

```
[Out] -(Csc[a + b*x]*(c*Sin[a + b*x]^3)^(1/3)*(b*x*Cos[a + b*x] + b^2*x^2*CosInte
gral[b*x]*Sin[a] + Sin[a + b*x] + b^2*x^2*Cos[a]*SinIntegral[b*x]))/(2*x^2)
```

Maple [C] time = 0.082, size = 183, normalized size = 1.6

$$-\frac{b^2}{2\,e^{2\,i(bx+a)}-2}\sqrt[3]{ic\left(e^{2\,i(bx+a)}-1\right)^3e^{-3\,i(bx+a)}}\left(\frac{e^{2\,i(bx+a)}}{2\,x^2b^2}+\frac{\frac{i}{2}e^{2\,i(bx+a)}}{bx}-\frac{\mathrm{Ei}\,(1,-ibx)\,e^{i(bx+2\,a)}}{2}\right)+\frac{b^2}{2\,e^{2\,i(bx+a)}-2}\sqrt[3]{ic\left(e^{2\,i(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(1/3)/x^3,x)
```

```
[Out] -1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(1/3)/(exp(2*I*(b*x+a))
-1)*b^2*(1/2/x^2/b^2*exp(2*I*(b*x+a))+1/2*I/x/b*exp(2*I*(b*x+a))-1/2*Ei(1,-
I*b*x)*exp(I*(b*x+2*a)))+1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))
^(1/3)/(exp(2*I*(b*x+a))-1)*b^2*(1/2/x^2/b^2-1/2*I/x/b-1/2*exp(I*b*x)*Ei(1,
I*b*x))
```

Maxima [C] time = 1.67445, size = 365, normalized size = 3.15

$$-\frac{\left(\left(\left(8\sqrt{3}-8i\right)E_3(ibx)+\left(8\sqrt{3}+8i\right)E_3(-ibx)\right)\cos(a)^3+\left(\left(8\sqrt{3}-8i\right)E_3(ibx)+\left(8\sqrt{3}+8i\right)E_3(-ibx)\right)\cos(a)\sin(a)^2 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/64*(((8*sqrt(3) - 8*I)*exp_integral_e(3, I*b*x) + (8*sqrt(3) + 8*I)*exp_
integral_e(3, -I*b*x))*cos(a)^3 + ((8*sqrt(3) - 8*I)*exp_integral_e(3, I*b*
x) + (8*sqrt(3) + 8*I)*exp_integral_e(3, -I*b*x))*cos(a)*sin(a)^2 + 8*((-I*
sqrt(3) - 1)*exp_integral_e(3, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(3, -
I*b*x))*sin(a)^3 - ((8*sqrt(3) + 8*I)*exp_integral_e(3, I*b*x) + (8*sqrt(3)
 - 8*I)*exp_integral_e(3, -I*b*x))*cos(a) + 8*(((-I*sqrt(3) - 1)*exp_integr
al_e(3, I*b*x) + (I*sqrt(3) - 1)*exp_integral_e(3, -I*b*x))*cos(a)^2 + (I*s
qrt(3) - 1)*exp_integral_e(3, I*b*x) + (-I*sqrt(3) - 1)*exp_integral_e(3, -
I*b*x))*sin(a))*b^2*c^(1/3)/(a^2*cos(a)^2 + a^2*sin(a)^2 + (b*x + a)^2*(cos
(a)^2 + sin(a)^2) - 2*(a*cos(a)^2 + a*sin(a)^2)*(b*x + a))
```

Fricas [A] time = 1.69489, size = 401, normalized size = 3.46

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos(bx+a)^2-\left(2\cdot 4^{\frac{2}{3}}b^2x^2\cos(a)\operatorname{Si}(bx)+2\cdot 4^{\frac{2}{3}}bx\cos(bx+a)+\left(4^{\frac{2}{3}}b^2x^2\operatorname{Ci}(bx)+4^{\frac{2}{3}}b^2x^2\operatorname{Ci}(-bx)\right)\sin(a)\right.}{16\left(x^2\cos(bx+a)^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(2*4^(2/3)*cos(b*x + a)^2 - (2*4^(2/3)*b^2*x^2*cos(a)*sin_int
egral(b*x) + 2*4^(2/3)*b*x*cos(b*x + a) + (4^(2/3)*b^2*x^2*cos_integral(b*x
) + 4^(2/3)*b^2*x^2*cos_integral(-b*x))*sin(a))*sin(b*x + a) - 2*4^(2/3))*(
-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(1/3)/(x^2*cos(b*x + a)^2 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c\sin^3(a+bx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(1/3)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin(bx+a)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(1/3)/x^3, x)
```

3.318 $\int x^m \sqrt[3]{c \sin^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=153

$$\frac{1}{4}ie^{ia}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\csc\left(a+bx^2\right)\text{Gamma}\left(\frac{m+1}{2},-ibx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{4}ie^{-ia}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m-1)}\csc\left(a+bx^2\right.$$

```
[Out] (I/4)*E^(I*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 +
 m)/2, (-I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3) - ((I/4)*x^(1 + m)*(I*b*x^2)^
((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 + m)/2, I*b*x^2]*(c*Sin[a + b*x^2]^3)^
(1/3))/E^(I*a)
```

Rubi [A] time = 0.301327, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6720, 3389, 2218}

$$\frac{1}{4}ie^{ia}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\csc\left(a+bx^2\right)\text{Gamma}\left(\frac{m+1}{2},-ibx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{4}ie^{-ia}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m-1)}\csc\left(a+bx^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```


```
[Out] (I/4)*E^(I*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 +
 m)/2, (-I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3) - ((I/4)*x^(1 + m)*(I*b*x^2)^
((-1 - m)/2)*Csc[a + b*x^2]*Gamma[(1 + m)/2, I*b*x^2]*(c*Sin[a + b*x^2]^3)^
(1/3))/E^(I*a)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int x^m\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^m\sin\left(a+bx^2\right)\,dx\\
&= \frac{1}{2}\left(i\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int e^{-ia-ibx^2}x^m\,dx-\frac{1}{2}\left(i\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right.\\
&= \frac{1}{4}ie^{ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc\left(a+bx^2\right)\Gamma\left(\frac{1+m}{2},-ibx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{4}ie^{-ia}x^{1+m}\left(ibx\right.
\end{aligned}$$

Mathematica [A] time = 0.300039, size = 138, normalized size = 0.9

$$\frac{1}{4}ix^{m+1}\left(b^2x^4\right)^{\frac{1}{2}(-m-1)}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left((\cos(a)+i\sin(a))\left(ibx^2\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2},-ibx^2\right)-(\cos(a)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (I/4)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]*(-(((-I)*b*x^2)^((1 +
 m)/2)*Gamma[(1 + m)/2, I*b*x^2]*(Cos[a] - I*Sin[a])) + (I*b*x^2)^((1 + m)/
2)*Gamma[(1 + m)/2, (-I)*b*x^2]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^2]^3)^(
1/3)
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int x^m\sqrt[3]{c\left(\sin\left(bx^2+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(b*x^2+a)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

Fricas [A] time = 1.80492, size = 270, normalized size = 1.76

$$-\frac{\left(e^{\left(-\frac{1}{2}(m-1)\log(ib)-ia\right)}\Gamma\left(\frac{1}{2}m+\frac{1}{2},ibx^2\right)+e^{\left(-\frac{1}{2}(m-1)\log(-ib)+ia\right)}\Gamma\left(\frac{1}{2}m+\frac{1}{2},-ibx^2\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right.\right.}{4b\sin\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(e^(-1/2*(m - 1)*log(I*b) - I*a)*gamma(1/2*m + 1/2, I*b*x^2) + e^(-1/2
*(m - 1)*log(-I*b) + I*a)*gamma(1/2*m + 1/2, -I*b*x^2))*(-(c*cos(b*x^2 + a)
^2 - c)*sin(b*x^2 + a))^(1/3)/(b*sin(b*x^2 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt[3]{c \sin^3\left(a + bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral(x**m*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin\left(bx^2 + a\right)^3\right)^{\frac{1}{3}} x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^m, x)
```

3.319 $\int x^3 \sqrt[3]{c \sin^3(a+bx^2)}\, dx$

Optimal. Leaf size=58

$$\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^2}-\frac{x^2\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}$$

```
[Out] (c*Sin[a + b*x^2]^3)^(1/3)/(2*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(1/3))/(2*b)
```

Rubi [A] time = 0.181158, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3379, 3296, 2637}

$$\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^2}-\frac{x^2\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (c*Sin[a + b*x^2]^3)^(1/3)/(2*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(1/3))/(2*b)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^3\sin\left(a+bx^2\right)\,dx\\ &= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,x^2\right)\\ &= -\frac{x^2\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}+\frac{\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int\cos(a+b\right.}{2b}\\ &= \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^2}-\frac{x^2\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}\end{aligned}$$

Mathematica [A] time = 0.0909529, size = 38, normalized size = 0.66

$$-\frac{\left(bx^2\cot\left(a+bx^2\right)-1\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -((-1 + b*x^2*Cot[a + b*x^2])*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b^2)
```

Maple [C] time = 0.08, size = 135, normalized size = 2.3

$$\frac{-\frac{i}{4}\left(bx^2+i\right)e^{2i\left(bx^2+a\right)}}{\left(e^{2i\left(bx^2+a\right)}-1\right)b^2}\sqrt[3]{ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}}-\frac{\frac{i}{4}\left(bx^2-i\right)}{\left(e^{2i\left(bx^2+a\right)}-1\right)b^2}\sqrt[3]{ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] -1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b
*x^2+a))-1)*(b*x^2+I)/b^2*exp(2*I*(b*x^2+a))-1/4*I*(I*c*(exp(2*I*(b*x^2+a))
-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*(b*x^2-I)/b^2
```

Maxima [A] time = 1.54007, size = 43, normalized size = 0.74

$$\frac{\left(bx^2\cos\left(bx^2+a\right)-\sin\left(bx^2+a\right)\right)c^{\frac{1}{3}}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*c^(1/3)/b^2
```

Fricas [A] time = 1.60669, size = 157, normalized size = 2.71

$$-\frac{\left(bx^2\cos\left(bx^2+a\right)-\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}}{2\,b^2\sin\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*cos(b*x^2 + a) - sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin
(b*x^2 + a))^(1/3)/(b^2*sin(b*x^2 + a))
```

Sympy [A] time = 30.2861, size = 92, normalized size = 1.59

$$\begin{cases} 0 & \text{for } a = -bx^2 \vee a = -bx^2 + \pi \\ \frac{x^4 \sqrt[3]{c\sin^3(a)}}{4} & \text{for } b = 0 \\ -\frac{\sqrt[3]{c}x^2\sqrt[3]{\sin^3\left(a+bx^2\right)}\cos\left(a+bx^2\right)}{2b\sin\left(a+bx^2\right)} + \frac{\sqrt[3]{c}\sqrt[3]{\sin^3\left(a+bx^2\right)}}{2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Piecewise((0, Eq(a, -b*x**2) | Eq(a, -b*x**2 + pi)), (x**4*(c*sin(a)**3)**(
1/3)/4, Eq(b, 0)), (-c**(1/3)*x**2*(sin(a + b*x**2)**3)**(1/3)*cos(a + b*x*
*2)/(2*b*sin(a + b*x**2)) + c**(1/3)*(sin(a + b*x**2)**3)**(1/3)/(2*b**2),
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^3, x)
```

3.320 $\int x^2 \sqrt[3]{c \sin^3(a+bx^2)}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^{3/2}}$$

```
[Out] -(x*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b) + (Sqrt[Pi/2]*Cos[a]*C
sc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3))/(2
*b^(3/2)) - (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a
]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b^(3/2))
```

Rubi [A] time = 0.2145, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 3385, 3354, 3352, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -(x*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b) + (Sqrt[Pi/2]*Cos[a]*C
sc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3))/(2
*b^(3/2)) - (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a
]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b^(3/2))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^2\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int x^2\sin\left(a+bx^2\right)\,dx$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\cos\left(a+bx^2\right)\,d}{2b}$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int\cos\left(bx^2\right.}{2b}$$

$$= -\frac{x\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+\right.}}{2b^{3/2}}$$

Mathematica [A] time = 0.271106, size = 105, normalized size = 0.68

$$-\frac{\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(-\sqrt{2\pi}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)+\sqrt{2\pi}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+2\sqrt{b}x\cos\left(a+bx^2\right)\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -(Csc[a + b*x^2]*(2*Sqrt[b]*x*Cos[a + b*x^2] - Sqrt[2*Pi]*Cos[a]*FresnelC[S
qrt[b]*Sqrt[2/Pi]*x] + Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])*(c
*Sin[a + b*x^2]^3)^(1/3))/(4*b^(3/2))
```

Maple [C] time = 0.123, size = 240, normalized size = 1.6

$$\frac{1}{2\,e^{2\,i\left(bx^2+a\right)}-2}\sqrt[3]{ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3e^{-3\,i\left(bx^2+a\right)}}\left(\frac{-\frac{i}{2}xe^{2\,i\left(bx^2+a\right)}}{b}+\frac{\frac{i}{4}\sqrt{\pi}e^{i\left(bx^2+2\,a\right)}}{b}\text{Erf}\left(\sqrt{-ibx}\right)\frac{1}{\sqrt{-ib}}\right)-\frac{\frac{i}{4}x}{\left(e^{2\,i\left(bx^2+a\right)}-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] 1/2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^
2+a))-1)*(-1/2*I*x/b*exp(2*I*(b*x^2+a))+1/4*I/b*Pi^(1/2)/(-I*b)^(1/2)*erf((
-I*b)^(1/2)*x)*exp(I*(b*x^2+2*a)))-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(
-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)/b*x+1/8*I*(I*c*(exp(2*I*(b*x^
2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)/b
*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Maxima [C] time = 1.71994, size = 481, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/32*(8*x*abs(b)*cos(b*x^2 + a) - sqrt(pi)*((((-I*sqrt(3) + 1)*cos(1/4*pi +
 1/2*arctan2(0, b)) + (I*sqrt(3) + 1)*cos(-1/4*pi + 1/2*arctan2(0, b)) - (s
qrt(3) + I)*sin(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) - I)*sin(-1/4*pi + 1
/2*arctan2(0, b)))*cos(a) - ((sqrt(3) + I)*cos(1/4*pi + 1/2*arctan2(0, b))
- (sqrt(3) - I)*cos(-1/4*pi + 1/2*arctan2(0, b)) - (I*sqrt(3) - 1)*sin(1/4*
pi + 1/2*arctan2(0, b)) - (I*sqrt(3) + 1)*sin(-1/4*pi + 1/2*arctan2(0, b)))
*sin(a))*erf(sqrt(I*b)*x) + (((I*sqrt(3) + 1)*cos(1/4*pi + 1/2*arctan2(0, b
)) + (-I*sqrt(3) + 1)*cos(-1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) - I)*sin(
1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) + I)*sin(-1/4*pi + 1/2*arctan2(0, b)
))*cos(a) - ((sqrt(3) - I)*cos(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) + I)*
cos(-1/4*pi + 1/2*arctan2(0, b)) - (-I*sqrt(3) - 1)*sin(1/4*pi + 1/2*arctan
2(0, b)) - (-I*sqrt(3) + 1)*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf(s
qrt(-I*b)*x))*sqrt(abs(b)))*c^(1/3)/(b*abs(b))
```

Fricas [A] time = 1.7084, size = 425, normalized size = 2.74

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)-4^{\frac{2}{3}}\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)\sin(a)-2\cdot 4^{\frac{2}{3}}bx\cos\left(bx^2+a\right.}{16\left(b^2\cos\left(bx^2+a\right)^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*s
qrt(b/pi))*sin(b*x^2 + a) - 4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(
2)*x*sqrt(b/pi))*sin(b*x^2 + a)*sin(a) - 2*4^(2/3)*b*x*cos(b*x^2 + a)*sin(b
*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(b^2*cos(b*x^2
+ a)^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x**2)**3)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x^2, x)
```

3.321 $\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=31

$$-\frac{\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}$$

```
[Out] -(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b)
```

Rubi [A] time = 0.104194, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 3207, 2638}

$$-\frac{\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sqrt[3]{c\sin^3(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\operatorname{Subst}\left(\int \sin(a+bx)\,dx,x,x^2\right)\\
&= -\frac{\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0517917, size = 31, normalized size = 1.

$$-\frac{\cot\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] -(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/(2*b)
```

Maple [C] time = 0.071, size = 119, normalized size = 3.8

$$\frac{-\frac{i}{4}e^{2i\left(bx^2+a\right)}}{\left(e^{2i\left(bx^2+a\right)}-1\right)b}\sqrt[3]{ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}}-\frac{\frac{i}{4}}{\left(e^{2i\left(bx^2+a\right)}-1\right)b}\sqrt[3]{ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] -1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b
*x^2+a))-1)/b*exp(2*I*(b*x^2+a))-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3
*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)/b
```

Maxima [A] time = 1.53615, size = 22, normalized size = 0.71

$$\frac{c^{\frac{1}{3}}\cos\left(bx^2+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*c^(1/3)*cos(b*x^2 + a)/b
```

Fricas [A] time = 1.60993, size = 120, normalized size = 3.87

$$-\frac{\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}\cos\left(bx^2+a\right)}{2b\sin\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)*cos(b*x^2 + a)/(b*sin
(b*x^2 + a))
```

Sympy [A] time = 5.09722, size = 66, normalized size = 2.13

$$\begin{cases} 0 & \text{for } a = -bx^2 \vee a = -bx^2 + \pi \\ \frac{x^2 \sqrt[3]{c \sin^3(a)}}{2} & \text{for } b = 0 \\ -\frac{\sqrt[3]{c} \sqrt[3]{\sin^3(a+bx^2)} \cos(a+bx^2)}{2b \sin(a+bx^2)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Piecewise((0, Eq(a, -b*x**2) | Eq(a, -b*x**2 + pi)), (x**2*(c*sin(a)**3)**(
1/3)/2, Eq(b, 0)), (-c**(1/3)*(sin(a + b*x**2)**3)**(1/3)*cos(a + b*x**2)/(
2*b*sin(a + b*x**2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin\left(bx^2 + a\right)^3\right)^{\frac{1}{3}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)*x, x)
```

3.322 $\int \sqrt[3]{c\sin^3(a+bx^2)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+b\right.}}{\sqrt{b}}$$

```
[Out] (Sqrt[Pi/2]*Cos[a]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a +
 b*x^2]^3)^(1/3))/Sqrt[b] + (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqr
t[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Rubi [A] time = 0.0586227, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 3353, 3352, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+b\right.}}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a +
 b*x^2]^3)^(1/3))/Sqrt[b] + (Sqrt[Pi/2]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqr
t[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx = \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \sin\left(a+bx^2\right)\,dx$$
$$= \left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \sin\left(bx^2\right)\,dx + \left(\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx\right.}\right.$$
$$= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\csc\left(a+bx^2\right)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a}{\sqrt{b}}$$

Mathematica [A] time = 0.12134, size = 80, normalized size = 0.68

$$\frac{\sqrt{\frac{\pi}{2}}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)+\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3),x]
```

```
[Out] (Sqrt[Pi/2]*Csc[a + b*x^2]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Fresnel
C[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/Sqrt[b]
```

Maple [C] time = 0.083, size = 157, normalized size = 1.3

$$\frac{\sqrt{\pi}e^{i\left(bx^2+2a\right)}}{4e^{2i\left(bx^2+a\right)}-4}\sqrt[3]{ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}}\text{Erf}\left(\sqrt{-ib}x\right)\frac{1}{\sqrt{-ib}}-\frac{e^{ibx^2}\sqrt{\pi}}{4e^{2i\left(bx^2+a\right)}-4}\sqrt[3]{ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}}\text{Erf}\left(\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3),x)
```

```
[Out] 1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^
2+a))-1)*Pi^(1/2)/(-I*b)^(1/2)*erf((-I*b)^(1/2)*x)*exp(I*(b*x^2+2*a))-1/4*(
I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))
-1)*exp(I*b*x^2)*Pi^(1/2)/(I*b)^(1/2)*erf((I*b)^(1/2)*x)
```

Maxima [C] time = 1.71268, size = 444, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*((((sqrt(3) + I)*cos(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3)
- I)*cos(-1/4*pi + 1/2*arctan2(0, b)) - (I*sqrt(3) - 1)*sin(1/4*pi + 1/2*ar
ctan2(0, b)) - (I*sqrt(3) + 1)*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - (
(I*sqrt(3) - 1)*cos(1/4*pi + 1/2*arctan2(0, b)) + (-I*sqrt(3) - 1)*cos(-1/4
*pi + 1/2*arctan2(0, b)) + (sqrt(3) + I)*sin(1/4*pi + 1/2*arctan2(0, b)) +
(sqrt(3) - I)*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf(sqrt(I*b)*x) +
(((sqrt(3) - I)*cos(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) + I)*cos(-1/4*pi
 + 1/2*arctan2(0, b)) - (-I*sqrt(3) - 1)*sin(1/4*pi + 1/2*arctan2(0, b)) -
```

```
(-I*sqrt(3) + 1)*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(a) - ((-I*sqrt(3) -
1)*cos(1/4*pi + 1/2*arctan2(0, b)) + (I*sqrt(3) - 1)*cos(-1/4*pi + 1/2*arct
an2(0, b)) + (sqrt(3) - I)*sin(1/4*pi + 1/2*arctan2(0, b)) + (sqrt(3) + I)*
sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(a))*erf(sqrt(-I*b)*x))*c^(1/3)/sqrt(a
bs(b))
```

Fricas [A] time = 1.81931, size = 356, normalized size = 3.04

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)+4^{\frac{2}{3}}\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)\sin(a)\right)\left(-\left(c\cos\left(bx^2+a\right)^2 \cdots\right.\right.}{8\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sq
rt(b/pi))*sin(b*x^2 + a) + 4^(2/3)*sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2
)*x*sqrt(b/pi))*sin(b*x^2 + a)*sin(a))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2
 + a))^(1/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c\sin^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3), x)
```

3.323 $\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}\sin(a)\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}+\frac{1}{2}\cos(a)\text{Si}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

```
[Out] (CosIntegral[b*x^2]*Csc[a + b*x^2]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/2 + (
Cos[a]*Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2
```

Rubi [A] time = 0.120985, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3377, 3376, 3375}

$$\frac{1}{2}\sin(a)\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}+\frac{1}{2}\cos(a)\text{Si}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

```
[Out] (CosIntegral[b*x^2]*Csc[a + b*x^2]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3))/2 + (
Cos[a]*Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(a+bx^2\right)}{x}\,dx\\
&= \left(\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(bx^2\right)}{x}\,dx+\left(\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+b\right.\right.\\
&= \frac{1}{2}\text{Ci}\left(bx^2\right)\csc\left(a+bx^2\right)\sin(a)\sqrt[3]{c\sin^3\left(a+bx^2\right)}+\frac{1}{2}\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\text{Si}
\end{aligned}$$

Mathematica [A] time = 0.0584489, size = 47, normalized size = 0.64

$$\frac{1}{2}\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sin(a)\text{CosIntegral}\left(bx^2\right)+\cos(a)\text{Si}\left(bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x,x]
```

```
[Out] (Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(CosIntegral[b*x^2]*Sin[a] + Cos
[a]*SinIntegral[b*x^2]))/2
```

Maple [C] time = 0.083, size = 268, normalized size = 3.7

$$-\frac{\text{Ei}\left(1,-ibx^2\right)\mathrm{e}^{i\left(bx^2+2a\right)}}{4\,\mathrm{e}^{2i\left(bx^2+a\right)}-4}\sqrt[3]{ic\left(\mathrm{e}^{2i\left(bx^2+a\right)}-1\right)^3\mathrm{e}^{-3i\left(bx^2+a\right)}}-\frac{\frac{i}{4}\mathrm{e}^{ibx^2}\pi\,\text{csgn}\left(bx^2\right)}{\mathrm{e}^{2i\left(bx^2+a\right)}-1}\sqrt[3]{ic\left(\mathrm{e}^{2i\left(bx^2+a\right)}-1\right)^3\mathrm{e}^{-3i\left(bx^2+a\right)}}+\frac{\frac{i}{2}\mathrm{e}^{ibx^2}\text{S}}{\mathrm{e}^{2i\left(bx^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3)/x,x)
```

```
[Out] -1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x
^2+a))-1)*Ei(1,-I*b*x^2)*exp(I*(b*x^2+2*a))-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-
1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*Pi*csgn
(b*x^2)+1/2*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp
(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*Si(b*x^2)+1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3
*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*Ei(1,-I*b*x
^2)
```

Maxima [C] time = 1.69687, size = 63, normalized size = 0.86

$$\frac{1}{8}\left(\left(i\,\text{Ei}\left(i\,bx^2\right)-i\,\text{Ei}\left(-i\,bx^2\right)\right)\cos\left(a\right)-\left(\text{Ei}\left(i\,bx^2\right)+\text{Ei}\left(-i\,bx^2\right)\right)\sin\left(a\right)\right)c^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] 1/8*((I*Ei(I*b*x^2) - I*Ei(-I*b*x^2))*cos(a) - (Ei(I*b*x^2) + Ei(-I*b*x^2))
*sin(a))*c^(1/3)
```

Fricas [A] time = 1.74076, size = 285, normalized size = 3.9

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(a\right)\text{Si}\left(bx^2\right)+\left(4^{\frac{2}{3}}\,\text{Ci}\left(bx^2\right)+4^{\frac{2}{3}}\,\text{Ci}\left(-bx^2\right)\right)\sin\left(a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{1}{3}}\sin\left(bx^2+\right.}{16\left(\cos\left(bx^2+a\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(2*4^(2/3)*cos(a)*sin_integral(b*x^2) + (4^(2/3)*cos_integral
(b*x^2) + 4^(2/3)*cos_integral(-b*x^2))*sin(a))*(-(c*cos(b*x^2 + a)^2 - c)*
sin(b*x^2 + a))^(1/3)*sin(b*x^2 + a)/(cos(b*x^2 + a)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x, x)
```

3.324 $\int \frac{\sqrt[3]{c\sin^3(a+bx^2)}}{x^2}\,dx$

Optimal. Leaf size=135

$$\sqrt{2\pi}\sqrt{b}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\sqrt{2\pi}\sqrt{b}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

```
[Out] -((c*Sin[a + b*x^2]^3)^(1/3)/x) + Sqrt[b]*Sqrt[2*Pi]*Cos[a]*Csc[a + b*x^2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3) - Sqrt[b]*Sqrt[2*
Pi]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^
3)^(1/3)
```

Rubi [A] time = 0.15641, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 3387, 3354, 3352, 3351}

$$\sqrt{2\pi}\sqrt{b}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\sqrt{2\pi}\sqrt{b}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x^2]^3)^(1/3)/x) + Sqrt[b]*Sqrt[2*Pi]*Cos[a]*Csc[a + b*x^2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*(c*Sin[a + b*x^2]^3)^(1/3) - Sqrt[b]*Sqrt[2*
Pi]*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]*(c*Sin[a + b*x^2]^
3)^(1/3)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3387

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*x
)^(m + 1)*Sin[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(a+bx^2\right)}{x^2}\,dx\\ &= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}+\left(2b\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \cos\left(a+bx^2\right)\,dx\\ &= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}+\left(2b\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \cos\left(bx^2\right)\,dx-\left(2b\csc\left(a\right.\right.\\ &= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x}+\sqrt{b}\sqrt{2\pi}\cos(a)\csc\left(a+bx^2\right)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\sqrt{b}\sqrt{2\pi}\,c\end{aligned}$$

Mathematica [A] time = 0.276604, size = 105, normalized size = 0.78

$$\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(\sqrt{2\pi}\sqrt{b}x\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)\csc\left(a+bx^2\right)-\sqrt{2\pi}\sqrt{b}x\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\csc\left(a+bx^2\right)-1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^2,x]
```

```
[Out] ((-1 + Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]*Csc[a + b*x^2]*FresnelC[Sqrt[b]*Sqrt[2/P
i]*x] - Sqrt[b]*Sqrt[2*Pi]*x*Csc[a + b*x^2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*
Sin[a])*(c*Sin[a + b*x^2]^3)^(1/3))/x
```

Maple [C] time = 0.092, size = 232, normalized size = 1.7

$$\frac{1}{2\,e^{2\,i\left(bx^2+a\right)}-2}\sqrt[3]{ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3 e^{-3\,i\left(bx^2+a\right)}}\left(-\frac{e^{2\,i\left(bx^2+a\right)}}{x}+ib\sqrt{\pi}e^{i\left(bx^2+2\,a\right)}\text{Erf}\left(\sqrt{-ib}x\right)\frac{1}{\sqrt{-ib}}\right)+\frac{1}{\left(2\,e^{2\,i\left(bx^2+a\right)}-2\right)x}\sqrt[3]{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3)/x^2,x)
```

```
[Out] 1/2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^
2+a))-1)*(-1/x*exp(2*I*(b*x^2+a))+I*b*Pi^(1/2)/(-I*b)^(1/2)*erf((-I*b)^(1/2
)*x)*exp(I*(b*x^2+2*a)))+1/2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+
a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)/x+1/2*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp
(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*b*Pi^(1/2)/(I*b
)^(1/2)*erf((I*b)^(1/2)*x)
```

Maxima [C] time = 1.7393, size = 489, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(x^2*abs(b))*((((sqrt(3) + I)*gamma(-1/2, I*b*x^2) + (sqrt(3) - I)
*gamma(-1/2, -I*b*x^2))*cos(1/4*pi + 1/2*arctan2(0, b)) - ((sqrt(3) - I)*ga
mma(-1/2, I*b*x^2) + (sqrt(3) + I)*gamma(-1/2, -I*b*x^2))*cos(-1/4*pi + 1/2
*arctan2(0, b)) + ((I*sqrt(3) - 1)*gamma(-1/2, I*b*x^2) + (-I*sqrt(3) - 1)*
gamma(-1/2, -I*b*x^2))*sin(1/4*pi + 1/2*arctan2(0, b)) + ((I*sqrt(3) + 1)*g
amma(-1/2, I*b*x^2) + (-I*sqrt(3) + 1)*gamma(-1/2, -I*b*x^2))*sin(-1/4*pi +
 1/2*arctan2(0, b)))*cos(a) + (((-I*sqrt(3) + 1)*gamma(-1/2, I*b*x^2) + (I*
sqrt(3) + 1)*gamma(-1/2, -I*b*x^2))*cos(1/4*pi + 1/2*arctan2(0, b)) + ((I*s
qrt(3) + 1)*gamma(-1/2, I*b*x^2) + (-I*sqrt(3) + 1)*gamma(-1/2, -I*b*x^2))*
cos(-1/4*pi + 1/2*arctan2(0, b)) + ((sqrt(3) + I)*gamma(-1/2, I*b*x^2) + (s
qrt(3) - I)*gamma(-1/2, -I*b*x^2))*sin(1/4*pi + 1/2*arctan2(0, b)) + ((sqrt
(3) - I)*gamma(-1/2, I*b*x^2) + (sqrt(3) + I)*gamma(-1/2, -I*b*x^2))*sin(-1
/4*pi + 1/2*arctan2(0, b)))*sin(a))*c^(1/3)/x
```

Fricas [A] time = 1.58083, size = 412, normalized size = 3.05

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)-4^{\frac{2}{3}}\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(bx^2+a\right)\sin\left(a\right)+4^{\frac{2}{3}}\cos\left(bx^2+a\right.\right.}{4\left(x\cos\left(bx^2+a\right)^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*4^(1/3)*(4^(2/3)*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*
sqrt(b/pi))*sin(b*x^2 + a) - 4^(2/3)*sqrt(2)*pi*x*sqrt(b/pi)*fresnel_sin(sq
rt(2)*x*sqrt(b/pi))*sin(b*x^2 + a)*sin(a) + 4^(2/3)*cos(b*x^2 + a)^2 - 4^(2
/3))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(1/3)/(x*cos(b*x^2 + a)^2 -
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^2, x)
```

3.325 $\int \frac{\sqrt[3]{c \sin^3\left(a+bx^2\right)}}{x^3} \, dx$

Optimal. Leaf size=98

$$\frac{1}{2}b\cos(a)\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{2}b\sin(a)\text{Si}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-$$

[Out] -(c*Sin[a + b*x^2]^3)^(1/3)/(2*x^2) + (b*Cos[a]*CosIntegral[b*x^2]*Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/2 - (b*Csc[a + b*x^2]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2

Rubi [A] time = 0.20267, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6720, 3379, 3297, 3303, 3299, 3302}

$$\frac{1}{2}b\cos(a)\text{CosIntegral}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{2}b\sin(a)\text{Si}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-$$

Antiderivative was successfully verified.

[In] Int[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]

[Out] -(c*Sin[a + b*x^2]^3)^(1/3)/(2*x^2) + (b*Cos[a]*CosIntegral[b*x^2]*Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3))/2 - (b*Csc[a + b*x^2]*Sin[a]*(c*Sin[a + b*x^2]^3)^(1/3)*SinIntegral[b*x^2])/2

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx &= \left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\int \frac{\sin\left(a+bx^2\right)}{x^3}\,dx\\ &= \frac{1}{2}\left(\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}\left(b\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,x^2\right)\\ &= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}\left(b\cos(a)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,x^2\right.\\ &= -\frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{2x^2}+\frac{1}{2}b\cos(a)\text{Ci}\left(bx^2\right)\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}-\frac{1}{2}b\csc\left(a+bx^2\right)\text{si}\end{aligned}$$

Mathematica [A] time = 0.127633, size = 67, normalized size = 0.68

$$-\frac{\csc\left(a+bx^2\right)\sqrt[3]{c\sin^3\left(a+bx^2\right)}\left(-bx^2\cos(a)\text{CosIntegral}\left(bx^2\right)+bx^2\sin(a)\text{Si}\left(bx^2\right)+\sin\left(a+bx^2\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(1/3)/x^3,x]
```

```
[Out] -(Csc[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(1/3)*(-(b*x^2*Cos[a]*CosIntegral[b*x
^2]) + Sin[a + b*x^2] + b*x^2*Sin[a]*SinIntegral[b*x^2]))/(2*x^2)
```

Maple [C] time = 0.087, size = 214, normalized size = 2.2

$$\frac{1}{2\,e^{2\,i\left(bx^2+a\right)}-2}\sqrt[3]{ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3 e^{-3\,i\left(bx^2+a\right)}}\left(-\frac{e^{2\,i\left(bx^2+a\right)}}{2\,x^2}-\frac{i}{2}b\text{Ei}\left(1,-ibx^2\right)e^{i\left(bx^2+2\,a\right)}\right)+\frac{1}{\left(4\,e^{2\,i\left(bx^2+a\right)}-4\right)x^2}\sqrt[3]{ic\left(e^{2\,i(b\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(1/3)/x^3,x)
```

```
[Out] 1/2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I*(b*x^
2+a))-1)*(-1/2/x^2*exp(2*I*(b*x^2+a))-1/2*I*b*Ei(1,-I*b*x^2)*exp(I*(b*x^2+2
*a)))+1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(1/3)/(exp(2*I
*(b*x^2+a))-1)/x^2-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))
```

```
^(1/3)/(exp(2*I*(b*x^2+a))-1)*exp(I*b*x^2)*b*Ei(1,I*b*x^2)
```

Maxima [C] time = 1.69097, size = 70, normalized size = 0.71

$$-\frac{1}{8}\left(\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\cos\left(a\right)-\left(i\,\Gamma\left(-1,i\,bx^2\right)-i\,\Gamma\left(-1,-i\,bx^2\right)\right)\sin\left(a\right)\right)bc^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*((gamma(-1, I*b*x^2) + gamma(-1, -I*b*x^2))*cos(a) - (I*gamma(-1, I*b*
x^2) - I*gamma(-1, -I*b*x^2))*sin(a))*b*c^(1/3)
```

Fricas [A] time = 1.66378, size = 375, normalized size = 3.83

$$-\frac{4^{\frac{1}{3}}\left(2\cdot 4^{\frac{2}{3}}\cos\left(bx^2+a\right)^2-\left(2\cdot 4^{\frac{2}{3}}bx^2\sin\left(a\right)\operatorname{Si}\left(bx^2\right)-\left(4^{\frac{2}{3}}bx^2\operatorname{Ci}\left(bx^2\right)+4^{\frac{2}{3}}bx^2\operatorname{Ci}\left(-bx^2\right)\right)\cos\left(a\right)\right)\sin\left(bx^2+a\right)-2\cdots}{16\left(x^2\cos\left(bx^2+a\right)^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*4^(1/3)*(2*4^(2/3)*cos(b*x^2 + a)^2 - (2*4^(2/3)*b*x^2*sin(a)*sin_int
egral(b*x^2) - (4^(2/3)*b*x^2*cos_integral(b*x^2) + 4^(2/3)*b*x^2*cos_integ
ral(-b*x^2))*cos(a))*sin(b*x^2 + a) - 2*4^(2/3))*(-(c*cos(b*x^2 + a)^2 - c)
*sin(b*x^2 + a))^(1/3)/(x^2*cos(b*x^2 + a)^2 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c\sin^3\left(a+bx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(1/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(1/3)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(1/3)/x^3, x)
```

3.326 $\int x^m \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal. Leaf size=157

$$\frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\text{Gamma}}{2n}$$

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Csc[a + b*x^n]*Gamma[(1 + m)/n, (-I)*b*x^n]*(c*Sin
[a + b*x^n]^3)^(1/3))/(n*((-I)*b*x^n)^((1 + m)/n)) - ((I/2)*x^(1 + m)*Csc[a
 + b*x^n]*Gamma[(1 + m)/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*
(I*b*x^n)^((1 + m)/n))
```

Rubi [A] time = 0.380559, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6720, 3423, 2218}

$$\frac{ie^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\csc(a+bx^n)\,\text{Gamma}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^(1 + m)*Csc[a + b*x^n]*Gamma[(1 + m)/n, (-I)*b*x^n]*(c*Sin
[a + b*x^n]^3)^(1/3))/(n*((-I)*b*x^n)^((1 + m)/n)) - ((I/2)*x^(1 + m)*Csc[a
 + b*x^n]*Gamma[(1 + m)/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*
(I*b*x^n)^((1 + m)/n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^m \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x^m\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x^m\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx\right.$$
$$= \frac{ie^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc(a+bx^n)\Gamma\left(\frac{1+m}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^{1+m}(ibx^n)^{-\frac{1+}{}}}{}$$

Mathematica [A] time = 0.324668, size = 142, normalized size = 0.9

$$\frac{ix^{m+1}\left(b^2x^{2n}\right)^{-\frac{m+1}{n}}\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left((\cos(a)+i\sin(a))(ibx^n)^{\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)-(\cos(a)-i\sin\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^(1 + m)*Csc[a + b*x^n]*(-(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n
, I*b*x^n]*(Cos[a] - I*Sin[a])) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-
I)*b*x^n]*(Cos[a] + I*Sin[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))
^((1 + m)/n))
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int x^m \sqrt[3]{c(\sin(a+bx^n))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int(x^m*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{1}{3}}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx^n + a\right)^3\right)^{\frac{1}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^m, x)
```

3.327 $\int x^3 \sqrt[3]{c \sin^3(a + bx^n)}\, dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}x^4(-ibx^n)^{-4/n}\operatorname{Gamma}\left(\frac{4}{n}, -ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\operatorname{Gamma}\left(\frac{4}{n}, ibx^n\right)\csc(a+b$$

```
[Out] ((I/2)*E^(I*a)*x^4*Csc[a + b*x^n]*Gamma[4/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(4/n)) - ((I/2)*x^4*Csc[a + b*x^n]*Gamma[4/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(4/n))
```

Rubi [A] time = 0.277831, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6720, 3423, 2218}

$$\frac{ie^{ia}x^4(-ibx^n)^{-4/n}\operatorname{Gamma}\left(\frac{4}{n}, -ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\operatorname{Gamma}\left(\frac{4}{n}, ibx^n\right)\csc(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^4*Csc[a + b*x^n]*Gamma[4/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(4/n)) - ((I/2)*x^4*Csc[a + b*x^n]*Gamma[4/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(4/n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^3\sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x^3\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x^3\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)$$
$$= \frac{ie^{ia}x^4(-ibx^n)^{-4/n}\csc(a+bx^n)\Gamma\left(\frac{4}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^4(ibx^n)^{-4/n}\csc(a+bx}{}$$

Mathematica [A] time = 0.188758, size = 129, normalized size = 0.9

$$\frac{ix^4\left(b^2x^{2n}\right)^{-4/n}\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left((\cos(a)+i\sin(a))(ibx^n)^{4/n}\text{Gamma}\left(\frac{4}{n},-ibx^n\right)-(\cos(a)-i\sin(a))(-ibx\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^4*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(4/n)*Gamma[4/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(4/n)*Gamma[4/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(4/n))
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int x^3\sqrt[3]{c(\sin(a+bx^n))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int(x^3*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{1}{3}}x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx^n + a\right)^3\right)^{\frac{1}{3}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^3, x)
```

3.328 $\int x^2 \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}x^3(-ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},ibx^n\right)\csc(a+bx^n)}{2n}$$

```
[Out] ((I/2)*E^(I*a)*x^3*Csc[a + b*x^n]*Gamma[3/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(3/n)) - ((I/2)*x^3*Csc[a + b*x^n]*Gamma[3/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(3/n))
```

Rubi [A] time = 0.266028, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6720, 3423, 2218}

$$\frac{ie^{ia}x^3(-ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x^3(ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},ibx^n\right)\csc(a+bx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^3*Csc[a + b*x^n]*Gamma[3/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(3/n)) - ((I/2)*x^3*Csc[a + b*x^n]*Gamma[3/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(3/n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^2 \sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x^2\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x^2\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n}\right.$$
$$= \frac{ie^{ia}x^3(-ibx^n)^{-3/n}\csc(a+bx^n)\Gamma\left(\frac{3}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^3(ibx^n)^{-3/n}\csc(a+}{}$$

Mathematica [A] time = 0.187471, size = 129, normalized size = 0.9

$$\frac{ix^3\left(b^2x^{2n}\right)^{-3/n}\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left((\cos(a)+i\sin(a))(ibx^n)^{3/n}\operatorname{Gamma}\left(\frac{3}{n},-ibx^n\right)-(\cos(a)-i\sin(a))(-\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^3*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(3/n)*Gamma[3/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(3/n)*Gamma[3/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(3/n))
```

Maple [F] time = 0.182, size = 0, normalized size = 0.

$$\int x^2\sqrt[3]{c(\sin(a+bx^n))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int(x^2*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{1}{3}}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x^2, x)
```

3.329 $\int x\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}x^2\left(-ibx^n\right)^{-2/n}\operatorname{Gamma}\left(\frac{2}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}-\frac{ie^{-ia}x^2\left(ibx^n\right)^{-2/n}\operatorname{Gamma}\left(\frac{2}{n},ibx^n\right)\csc\left(a+b\right.}{2n}$$

```
[Out] ((I/2)*E^(I*a)*x^2*Csc[a + b*x^n]*Gamma[2/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(2/n)) - ((I/2)*x^2*Csc[a + b*x^n]*Gamma[2/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(2/n))
```

Rubi [A] time = 0.186104, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6720, 3423, 2218}

$$\frac{ie^{ia}x^2\left(-ibx^n\right)^{-2/n}\operatorname{Gamma}\left(\frac{2}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2n}-\frac{ie^{-ia}x^2\left(ibx^n\right)^{-2/n}\operatorname{Gamma}\left(\frac{2}{n},ibx^n\right)\csc\left(a+b\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*E^(I*a)*x^2*Csc[a + b*x^n]*Gamma[2/n, (-I)*b*x^n]*(c*Sin[a + b*x^n]^
3)^(1/3))/(n*((-I)*b*x^n)^(2/n)) - ((I/2)*x^2*Csc[a + b*x^n]*Gamma[2/n, I*b
*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^(2/n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x\sqrt[3]{c\sin^3(a+bx^n)}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int x\sin(a+bx^n)\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}x\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int$$
$$= \frac{ie^{ia}x^2(-ibx^n)^{-2/n}\csc(a+bx^n)\Gamma\left(\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n} - \frac{ie^{-ia}x^2(ibx^n)^{-2/n}\csc(a+bx^n)}{2}$$

Mathematica [A] time = 0.184956, size = 129, normalized size = 0.9

$$\frac{ix^2\left(b^2x^{2n}\right)^{-2/n}\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left((\cos(a)+i\sin(a))(ibx^n)^{2/n}\text{Gamma}\left(\frac{2}{n},-ibx^n\right)-(\cos(a)-i\sin(a))(-ibx\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x^2*Csc[a + b*x^n]*(-(((-I)*b*x^n)^(2/n)*Gamma[2/n, I*b*x^n]*(Cos[a]
 - I*Sin[a])) + (I*b*x^n)^(2/n)*Gamma[2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))
*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^(2/n))
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int x\sqrt[3]{c(\sin(a+bx^n))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int(x*(c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{1}{3}}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx^n + a\right)^3\right)^{\frac{1}{3}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)*x, x)
```

3.330 $\int \sqrt[3]{c \sin^3(a+bx^n)}\,dx$

Optimal. Leaf size=135

$$\frac{ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{}}{2n}$$

```
[Out] ((I/2)*E^(I*a)*x*Csc[a + b*x^n]*Gamma[n^(-1), (-I)*b*x^n]*(c*Sin[a + b*x^n]
^3)^(1/3))/(n*((-I)*b*x^n)^n^(-1)) - ((I/2)*x*Csc[a + b*x^n]*Gamma[n^(-1),
I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rubi [A] time = 0.0471616, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6720, 3365, 2208}

$$\frac{ie^{ia}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-ibx^n\right)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},ibx^n\right)\csc(a+bx^n)\sqrt[3]{}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3),x]
```


```
[Out] ((I/2)*E^(I*a)*x*Csc[a + b*x^n]*Gamma[n^(-1), (-I)*b*x^n]*(c*Sin[a + b*x^n]
^3)^(1/3))/(n*((-I)*b*x^n)^n^(-1)) - ((I/2)*x*Csc[a + b*x^n]*Gamma[n^(-1),
I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*(I*b*x^n)^n^(-1))
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3365

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c\sin^3(a+bx^n)}\,dx &= \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \sin(a+bx^n)\,dx\\ &= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{-ia-ibx^n}\,dx-\frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int e^{ia}\\ &= \frac{ie^{ia}x(-ibx^n)^{-1/n}\csc(a+bx^n)\Gamma\left(\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2n}-\frac{ie^{-ia}x(ibx^n)^{-1/n}\csc(a+bx^n)\Gamma\left(\right.}{2n}\end{aligned}$$

Mathematica [A] time = 0.137111, size = 119, normalized size = 0.88

$$\frac{ix\left(b^2x^{2n}\right)^{-1/n}\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}\left(\left(\cos(a)+i\sin(a)\right)\left(ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(\frac{1}{n},-ibx^n\right)-\left(\cos(a)-i\sin(a)\right)\left(-ib\right.\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3),x]
```

```
[Out] ((I/2)*x*Csc[a + b*x^n]*(-(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[
a] - I*Sin[a])) + (I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Si
n[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*(b^2*x^(2*n))^n^(-1))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int\sqrt[3]{c\left(\sin\left(a+bx^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3),x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[3]{c\sin^3\left(a+bx^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx^n+a\right)^3\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3), x)
```

3.331 $\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x} dx$

Optimal. Leaf size=73

$$\frac{\sin(a)\text{CosIntegral}\left(bx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{n}+\frac{\cos(a)\text{Si}\left(bx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{n}$$

```
[Out] (CosIntegral[b*x^n]*Csc[a + b*x^n]*Sin[a]*(c*Sin[a + b*x^n]^3)^(1/3))/n + (
Cos[a]*Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*SinIntegral[b*x^n])/n
```

Rubi [A] time = 0.150866, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3377, 3376, 3375}

$$\frac{\sin(a)\text{CosIntegral}\left(bx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{n}+\frac{\cos(a)\text{Si}\left(bx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

```
[Out] (CosIntegral[b*x^n]*Csc[a + b*x^n]*Sin[a]*(c*Sin[a + b*x^n]^3)^(1/3))/n + (
Cos[a]*Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*SinIntegral[b*x^n])/n
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3377

```
Int[Sin[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sin[c], Int[Cos[d*x
^n]/x, x], x] + Dist[Cos[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(a+bx^n)}{x}\,dx$$
$$= \left(\cos(a)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(bx^n)}{x}\,dx + \left(\csc(a+bx^n)\sin(a)\sqrt[3]{c\sin^3(a+bx\right.$$
$$= \frac{\mathrm{Ci}(bx^n)\csc(a+bx^n)\sin(a)\sqrt[3]{c\sin^3(a+bx^n)}}{n} + \frac{\cos(a)\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\mathrm{Si}(bx^n)}{n}$$

Mathematica [A] time = 0.0744186, size = 47, normalized size = 0.64

$$\frac{\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left(\sin(a)\mathrm{CosIntegral}(bx^n)+\cos(a)\mathrm{Si}(bx^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x,x]
```

```
[Out] (Csc[a + b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3)*(CosIntegral[b*x^n]*Sin[a] + Cos
[a]*SinIntegral[b*x^n]))/n
```

Maple [C] time = 0.144, size = 280, normalized size = 3.8

$$-\frac{\mathrm{Ei}(1,-ibx^n)\,e^{i(bx^n+2a)}}{\left(2e^{2i(a+bx^n)}-2\right)n}\sqrt[3]{ic\left(e^{2i(a+bx^n)}-1\right)^3e^{-3i(a+bx^n)}} - \frac{\frac{i}{2}e^{ibx^n}\pi\,\mathrm{csgn}(bx^n)}{\left(e^{2i(a+bx^n)}-1\right)n}\sqrt[3]{ic\left(e^{2i(a+bx^n)}-1\right)^3e^{-3i(a+bx^n)}} + \frac{ie^{ibx^n}\mathrm{Si}(}{\left(e^{2i(a+bx^n)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x,x)
```

```
[Out] -1/2*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp(2*I*(a+b
*x^n))-1)/n*Ei(1,-I*b*x^n)*exp(I*(b*x^n+2*a))-1/2*I*(I*c*(exp(2*I*(a+b*x^n)
)-1)^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp(2*I*(a+b*x^n))-1)*exp(I*b*x^n)/n*Pi*
csgn(b*x^n)+I*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp
(2*I*(a+b*x^n))-1)*exp(I*b*x^n)/n*Si(b*x^n)+1/2*(I*c*(exp(2*I*(a+b*x^n))-1)
^3*exp(-3*I*(a+b*x^n)))^(1/3)/(exp(2*I*(a+b*x^n))-1)*exp(I*b*x^n)/n*Ei(1,-I
*b*x^n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 1.77753, size = 293, normalized size = 4.01

$$-\frac{4^{\frac{1}{3}}\left(4^{\frac{2}{3}}\,\mathrm{Ci}\left(bx^{n}\right)\sin\left(a\right)+4^{\frac{2}{3}}\,\mathrm{Ci}\left(-bx^{n}\right)\sin\left(a\right)+2\cdot4^{\frac{2}{3}}\cos\left(a\right)\mathrm{Si}\left(bx^{n}\right)\right)\left(-\left(c\cos\left(bx^{n}+a\right)^{2}-c\right)\sin\left(bx^{n}+a\right)\right)^{\frac{1}{3}}\sin\left(bx\right.}{8\left(n\cos\left(bx^{n}+a\right)^{2}-n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/8*4^(1/3)*(4^(2/3)*cos_integral(b*x^n)*sin(a) + 4^(2/3)*cos_integral(-b*
x^n)*sin(a) + 2*4^(2/3)*cos(a)*sin_integral(b*x^n))*(-(c*cos(b*x^n + a)^2 -
 c)*sin(b*x^n + a))^(1/3)*sin(b*x^n + a)/(n*cos(b*x^n + a)^2 - n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{c\sin^{3}\left(a+bx^{n}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**n)**3)**(1/3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x, x)
```

3.332 $\int \frac{\sqrt[3]{c \sin^3(a+bx^n)}}{x^2} dx$

Optimal. Leaf size=139

$$\frac{ie^{ia}\left(-ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx}-\frac{ie^{-ia}\left(ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx}$$

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]*Gamma[-n^(-1), (-I)*b*x^n
]*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x) - ((I/2)*(I*b*x^n)^n^(-1)*Csc[a + b*x^n
]*Gamma[-n^(-1), I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x)
```

Rubi [A] time = 0.204583, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6720, 3423, 2218}

$$\frac{ie^{ia}\left(-ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx}-\frac{ie^{-ia}\left(ibx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]*Gamma[-n^(-1), (-I)*b*x^n
]*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x) - ((I/2)*(I*b*x^n)^n^(-1)*Csc[a + b*x^n
]*Gamma[-n^(-1), I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^2}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(a+bx^n)}{x^2}\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{e^{-ia-ibx^n}}{x^2}\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)$$
$$= \frac{ie^{ia}(-ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\Gamma\left(-\frac{1}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx} - \frac{ie^{-ia}(ibx^n)^{\frac{1}{n}}\csc(a+bx^n)\Gamma\left(-\right.}{2nx}$$

Mathematica [A] time = 0.159705, size = 110, normalized size = 0.79

$$\frac{i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left((\cos(a)+i\sin(a))(-ibx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},-ibx^n\right)-(\cos(a)-i\sin(a))(ibx^n)^{\frac{1}{n}}\,\text{Gamm}\right.}{2nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^2,x]
```

```
[Out] ((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^n^(-1)*Gamma[-n^(-1), I*b*x^n]*(Cos[a] -
 I*Sin[a])) + ((-I)*b*x^n)^n^(-1)*Gamma[-n^(-1), (-I)*b*x^n]*(Cos[a] + I*Si
n[a]))*(c*Sin[a + b*x^n]^3)^(1/3))/(n*x)
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[3]{c(\sin(a+bx^n))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{1}{3}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c \sin \left(bx^n + a\right)^3\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^2, x)
```

3.333 $\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx$

Optimal. Leaf size=143

$$\frac{ie^{ia}\left(-ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx^2}-\frac{ie^{-ia}\left(ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},ibx^n\right)\csc\left(a+bx^n\right)}{2nx^2}$$

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]*Gamma[-2/n, (-I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(1/3))/(n*x^2) - ((I/2)*(I*b*x^n)^(2/n)*Csc[a + b*x^n]*G
amma[-2/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x^2)
```

Rubi [A] time = 0.21195, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6720, 3423, 2218}

$$\frac{ie^{ia}\left(-ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},-ibx^n\right)\csc\left(a+bx^n\right)\sqrt[3]{c\sin^3\left(a+bx^n\right)}}{2nx^2}-\frac{ie^{-ia}\left(ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},ibx^n\right)\csc\left(a+bx^n\right)}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

```
[Out] ((I/2)*E^(I*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]*Gamma[-2/n, (-I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(1/3))/(n*x^2) - ((I/2)*(I*b*x^n)^(2/n)*Csc[a + b*x^n]*G
amma[-2/n, I*b*x^n]*(c*Sin[a + b*x^n]^3)^(1/3))/(E^(I*a)*n*x^2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{c\sin^3(a+bx^n)}}{x^3}\,dx = \left(\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{\sin(a+bx^n)}{x^3}\,dx$$
$$= \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{e^{-ia-ibx^n}}{x^3}\,dx - \frac{1}{2}\left(i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\right)\int \frac{e}{}$$
$$= \frac{ie^{ia}(-ibx^n)^{2/n}\csc(a+bx^n)\Gamma\left(-\frac{2}{n},-ibx^n\right)\sqrt[3]{c\sin^3(a+bx^n)}}{2nx^2} - \frac{ie^{-ia}(ibx^n)^{2/n}\csc(a+bx^n)\Gamma\left(-\frac{2}{}\right.}{2nx^2}$$

Mathematica [A] time = 0.173913, size = 114, normalized size = 0.8

$$\frac{i\csc(a+bx^n)\sqrt[3]{c\sin^3(a+bx^n)}\left((\cos(a)+i\sin(a))(-ibx^n)^{2/n}\,\text{Gamma}\left(-\frac{2}{n},-ibx^n\right)-(\cos(a)-i\sin(a))(ibx^n)^{2/n}\,\text{Gamm}\right.}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(1/3)/x^3,x]
```

```
[Out] ((I/2)*Csc[a + b*x^n]*(-((I*b*x^n)^(2/n)*Gamma[-2/n, I*b*x^n]*(Cos[a] - I*S
in[a])) + ((-I)*b*x^n)^(2/n)*Gamma[-2/n, (-I)*b*x^n]*(Cos[a] + I*Sin[a]))*(
c*Sin[a + b*x^n]^3)^(1/3))/(n*x^2)
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt[3]{c(\sin(a+bx^n))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(1/3)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin(bx^n+a)^3\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\left(c\cos(bx^n+a)^2-c\right)\sin(bx^n+a)\right)^{\frac{1}{3}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(1/3)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(1/3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c \sin\left(bx^n + a\right)^3\right)^{\frac{1}{3}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(1/3)/x^3, x)
```

3.334 $\int x^m \left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal. Leaf size=169

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\csc^2(a+bx)\text{Gamma}(m+1,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\csc^2(a+bx)\text{Gamm}}{b}$$

```
[Out] (x^(1 + m)*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(2*(1 + m)) + (I*2^(-3
- m)*E^((2*I)*a)*x^m*Csc[a + b*x]^2*Gamma[1 + m, (-2*I)*b*x]*(c*Sin[a + b*x
]^3)^(2/3))/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Csc[a + b*x]^2*Gamma[1 + m
, (2*I)*b*x]*(c*Sin[a + b*x]^3)^(2/3))/(b*E^((2*I)*a)*(I*b*x)^m)
```

Rubi [A] time = 0.301069, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6720, 3312, 3307, 2181}

$$\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\csc^2(a+bx)\text{Gamma}(m+1,-2ibx)\left(c\sin^3(a+bx)\right)^{2/3}}{b}-\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\csc^2(a+bx)\text{Gamm}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(2*(1 + m)) + (I*2^(-3
- m)*E^((2*I)*a)*x^m*Csc[a + b*x]^2*Gamma[1 + m, (-2*I)*b*x]*(c*Sin[a + b*x
]^3)^(2/3))/(b*((-I)*b*x)^m) - (I*2^(-3 - m)*x^m*Csc[a + b*x]^2*Gamma[1 + m
, (2*I)*b*x]*(c*Sin[a + b*x]^3)^(2/3))/(b*E^((2*I)*a)*(I*b*x)^m)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^m \left(c\sin^3(a+bx)\right)^{2/3} dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^m \sin^2(a+bx)\,dx \\
&= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx)\right)dx \\
&= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} - \frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^m \cos(2a \\
&= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} - \frac{1}{4}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int e^{-i(2a+2bx)} \\
&= \frac{x^{1+m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2(1+m)} + \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\csc^2(a+bx)\Gamma(1+m,-2i}{b}
\end{aligned}$$

Mathematica [A] time = 0.536991, size = 142, normalized size = 0.84

$$\frac{2^{-m-3}x^m\left(b^2x^2\right)^{-m}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(-i(m+1)(\cos(a)-i\sin(a))^2(-ibx)^m\text{Gamma}(m+1,2ibx)+i(m+\right.}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (2^(-3 - m)*x^m*Csc[a + b*x]^2*(2^(2 + m)*b*x*(b^2*x^2)^m - I*(1 + m)*((-I)
*b*x)^m*Gamma[1 + m, (2*I)*b*x]*(Cos[a] - I*Sin[a])^2 + I*(1 + m)*(I*b*x)^m
*Gamma[1 + m, (-2*I)*b*x]*(Cos[a] + I*Sin[a])^2)*(c*Sin[a + b*x]^3)^(2/3))/
(b*(1 + m)*(b^2*x^2)^m)
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int x^m \left(c\left(\sin(bx+a)\right)^3\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(b*x+a)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left((m+1)\int x^m\cos(2\,bx+2\,a)\,dx - e^{(m\log(x)+\log(x))}\right)c^{\frac{2}{3}}}{4\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/4*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) - e^(m*log(x) + log(x)))*c^
(2/3)/(m + 1)
```

Fricas [A] time = 1.86369, size = 301, normalized size = 1.78

$$-\frac{\left(4\,bxx^{m}-(i\,m+i)e^{(-m\log(2i\,b)-2i\,a)}\Gamma\left(m+1,2i\,bx\right)-(-i\,m-i)e^{(-m\log(-2i\,b)+2i\,a)}\Gamma\left(m+1,-2i\,bx\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{8\left(\left(bm+b\right)\cos\left(bx+a\right)^{2}-bm-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*x^m - (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
 - (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))*(-(c*cos(b
*x + a)^2 - c)*sin(b*x + a))^(2/3)/((b*m + b)*cos(b*x + a)^2 - b*m - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^{3}\right)^{\frac{2}{3}}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^m, x)
```

3.335 $\int x^3 \left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal. Leaf size=165

$$\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{3x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{8b^2}-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}$$

```
[Out] (-3*(c*Sin[a + b*x]^3)^(2/3))/(8*b^4) + (3*x^2*(c*Sin[a + b*x]^3)^(2/3))/(4
*b^2) + (3*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(4*b^3) - (x^3*Cot[a +
b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (3*x^2*Csc[a + b*x]^2*(c*Sin[a + b*x
]^3)^(2/3))/(8*b^2) + (x^4*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/8
```

Rubi [A] time = 0.188101, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6720, 3311, 30, 3310}

$$\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{3x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{8b^2}-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (-3*(c*Sin[a + b*x]^3)^(2/3))/(8*b^4) + (3*x^2*(c*Sin[a + b*x]^3)^(2/3))/(4
*b^2) + (3*x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(4*b^3) - (x^3*Cot[a +
b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (3*x^2*Csc[a + b*x]^2*(c*Sin[a + b*x
]^3)^(2/3))/(8*b^2) + (x^4*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/8
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\int x^3\left(c\sin^3(a+bx)\right)^{2/3}dx=\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^3\sin^2(a+bx)\,dx$$
$$=\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x^3\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx\right.\right.$$
$$=-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^3\,\mathrm{co}}{}$$
$$=-\frac{3\left(c\sin^3(a+bx)\right)^{2/3}}{8b^4}+\frac{3x^2\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{3x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x^3\,\mathrm{co}}{}$$

Mathematica [A] time = 0.296209, size = 79, normalized size = 0.48

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\left(6bx-4b^3x^3\right)\sin(2(a+bx))+\left(3-6b^2x^2\right)\cos(2(a+bx))+2b^4x^4\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*b^4*x^4 + (3 - 6*b^2*x^2)*Cos[2
*(a + b*x)] + (6*b*x - 4*b^3*x^3)*Sin[2*(a + b*x)]))/(16*b^4)
```

Maple [C] time = 0.08, size = 208, normalized size = 1.3

$$-\frac{x^4e^{2i(bx+a)}}{8\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}\right)^{\frac{2}{3}}-\frac{\frac{i}{32}\left(4b^3x^3+6ib^2x^2-6bx-3i\right)e^{4i(bx+a)}}{\left(e^{2i(bx+a)}-1\right)^2b^4}\left(ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] -1/8*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))
-1)^2*x^4*exp(2*I*(b*x+a))-1/32*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x
+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*(4*b^3*x^3+6*I*b^2*x^2-6*b*x-3*I)/b^4*ex
p(4*I*(b*x+a))+1/32*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/
(exp(2*I*(b*x+a))-1)^2*(4*b^3*x^3-6*I*b^2*x^2-6*b*x+3*I)/b^4
```

Maxima [B] time = 1.63359, size = 386, normalized size = 2.34

$$-\frac{32\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a^3+6\left(2(bx+a)^2-2(bx+a)\sin(2bx+2a)-\cos(2bx+2a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/32*(32*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(
2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
 + 1))*a^3 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
2*a))*a^2*c^(2/3) - 2*(4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*
(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*c^(2/3) + (2*(b*x + a)^4 - 3*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2
*a))*c^(2/3))/b^4
```

Fricas [A] time = 1.63508, size = 259, normalized size = 1.57

$$-\frac{\left(2\,b^4x^4+6\,b^2x^2-6\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)^2-4\left(2\,b^3x^3-3\,bx\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-3\right)\left(-\left(c\cos\left(bx+a\right)^2-c\right.\right.}{16\left(b^4\cos\left(bx+a\right)^2-b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4*x^4 + 6*b^2*x^2 - 6*(2*b^2*x^2 - 1)*cos(b*x + a)^2 - 4*(2*b^3*
x^3 - 3*b*x)*cos(b*x + a)*sin(b*x + a) - 3)*(-(c*cos(b*x + a)^2 - c)*sin(b*
x + a))^(2/3)/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx+a\right)^3\right)^{\frac{2}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^3, x)
```

3.336 $\int x^2 \left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal. Leaf size=139

$$\frac{x\left(c\sin^3(a+bx)\right)^{2/3}}{2b^2}+\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx\right.}{2b}$$

```
[Out] (x*(c*Sin[a + b*x]^3)^(2/3))/(2*b^2) + (Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/
3))/(4*b^3) - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (x*Csc[a
+ b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(4*b^2) + (x^3*Csc[a + b*x]^2*(c*Sin[a +
 b*x]^3)^(2/3))/6
```

Rubi [A] time = 0.163858, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 3311, 30, 2635, 8}

$$\frac{x\left(c\sin^3(a+bx)\right)^{2/3}}{2b^2}+\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3}-\frac{x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x^2\cot(a+bx)\left(c\sin^3(a+bx\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (x*(c*Sin[a + b*x]^3)^(2/3))/(2*b^2) + (Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/
3))/(4*b^3) - (x^2*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) - (x*Csc[a
+ b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/(4*b^2) + (x^3*Csc[a + b*x]^2*(c*Sin[a +
 b*x]^3)^(2/3))/6
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^2 \left(c\sin^3(a+bx)\right)^{2/3} dx = \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x^2\sin^2(a+bx)\,dx$$
$$= \frac{x\left(c\sin^3(a+bx)\right)^{2/3}}{2b^2} - \frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b} + \frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+b\right.\right.$$
$$= \frac{x\left(c\sin^3(a+bx)\right)^{2/3}}{2b^2} + \frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3} - \frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^2}{2b}$$
$$= \frac{x\left(c\sin^3(a+bx)\right)^{2/3}}{2b^2} + \frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b^3} - \frac{x^2\cot(a+bx)\left(c\sin^3(a+bx)\right)^2}{2b}$$

Mathematica [A] time = 0.281151, size = 69, normalized size = 0.5

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(\left(3-6b^2x^2\right)\sin(2(a+bx))-6bx\cos(2(a+bx))+4b^3x^3\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)
] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)]))/(24*b^3)
```

Maple [C] time = 0.078, size = 190, normalized size = 1.4

$$-\frac{x^3 e^{2i(bx+a)}}{6\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} - \frac{\frac{i}{16}\left(2x^2b^2+2ibx-1\right)e^{4i(bx+a)}}{\left(e^{2i(bx+a)}-1\right)^2 b^3}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}} + \frac{i}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] -1/6*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))
-1)^2*x^3*exp(2*I*(b*x+a))-1/16*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x
+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*(2*x^2*b^2+2*I*b*x-1)/b^3*exp(4*I*(b*x+a
))+1/16*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*
x+a))-1)^2*(2*x^2*b^2-2*I*b*x-1)/b^3
```

Maxima [A] time = 1.58114, size = 296, normalized size = 2.13

$$\frac{48\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right) - \frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1} - \frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a^2 + 6\left(2(bx+a)^2 - 2(bx+a)\sin(2bx+2a) - \cos(2bx+2\right.}{48b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/48*(48*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*
x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2
*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
+ 1))*a^2 + 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a))*a*c^(2/3) - (4*(b*x + a)^3 - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*c^(2/3))/b^3
```

Fricas [A] time = 1.71005, size = 227, normalized size = 1.63

$$-\frac{\left(2\,b^{3}x^{3}-6\,bx\cos\left(bx+a\right)^{2}-3\left(2\,b^{2}x^{2}-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+3\,bx\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{\frac{2}{3}}}{12\left(b^{3}\cos\left(bx+a\right)^{2}-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*si
n(b*x + a) + 3*b*x)*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^3*cos(b
*x + a)^2 - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx+a\right)^{3}\right)^{\frac{2}{3}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x^2, x)
```

3.337 $\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

```
[Out] (c*Sin[a + b*x]^3)^(2/3)/(4*b^2) - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3)
)/(2*b) + (x^2*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/4
```

Rubi [A] time = 0.102483, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6720, 3310, 30}

$$\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (c*Sin[a + b*x]^3)^(2/3)/(4*b^2) - (x*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3)
)/(2*b) + (x^2*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/4
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(c\sin^3(a+bx)\right)^{2/3}\,dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int x\sin^2(a+bx)\,dx\\
&=\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{\dots}\right.\\
&=\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{4b^2}-\frac{x\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{4}x^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right.\dots
\end{aligned}$$

Mathematica [A] time = 0.173772, size = 55, normalized size = 0.7

$$-\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(2bx(\sin(2(a+bx))-bx)+\cos(2(a+bx)))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] -(Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(Cos[2*(a + b*x)] + 2*b*x*(-(b*x)
 + Sin[2*(a + b*x)])))/(8*b^2)
```

Maple [C] time = 0.073, size = 174, normalized size = 2.2

$$-\frac{x^2 e^{2i(bx+a)}}{4\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}-\frac{\frac{i}{16}(2bx+i)e^{4i(bx+a)}}{\left(e^{2i(bx+a)}-1\right)^2 b^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}+\frac{\frac{i}{16}(2bx-i)}{\left(e^{2i(bx+a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] -1/4*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))
-1)^2*x^2*exp(2*I*(b*x+a))-1/16*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x
+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*(2*b*x+I)/b^2*exp(4*I*(b*x+a))+1/16*I*(I
*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*(
2*b*x-I)/b^2
```

Maxima [B] time = 1.51562, size = 219, normalized size = 2.77

$$-\frac{16\left(c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}\right)a+\left(2(bx+a)^2-2(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\right)c}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/16*(16*(c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b
*x + a)/(cos(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(
2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4
 + 1))*a + (2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
)*c^(2/3))/b^2
```

Fricas [A] time = 1.78849, size = 198, normalized size = 2.51

$$-\frac{\left(2b^2x^2-4bx\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2+1\right)\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{2}{3}}}{8\left(b^2\cos(bx+a)^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^2*x^2 - 4*b*x*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 1)*(
-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(c\sin^3(a+bx)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral(x*(c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx+a)^3\right)^{\frac{2}{3}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)*x, x)
```

3.338 $\int \left(c \sin^3(a+bx)\right)^{2/3} dx$

Optimal. Leaf size=55

$$\frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

```
[Out] -(Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) + (x*Csc[a + b*x]^2*(c*Sin[a
 + b*x]^3)^(2/3))/2
```

Rubi [A] time = 0.023013, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3207, 2635, 8}

$$\frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] -(Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3))/(2*b) + (x*Csc[a + b*x]^2*(c*Sin[a
 + b*x]^3)^(2/3))/2
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^3(a+bx)\right)^{2/3} dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \sin^2(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int 1\,dx\\ &= -\frac{\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{2b}+\frac{1}{2}x\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\end{aligned}$$

Mathematica [A] time = 0.0939423, size = 47, normalized size = 0.85

$$\frac{(2(a+bx)-\sin(2(a+bx)))\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(2*(a + b*x) - Sin[2*(a + b*x)]))/
(4*b)
```

Maple [C] time = 0.113, size = 158, normalized size = 2.9

$$-\frac{x e^{2i(bx+a)}}{2\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}-\frac{\frac{i}{8}e^{4i(bx+a)}}{\left(e^{2i(bx+a)}-1\right)^2 b}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}+\frac{\frac{i}{8}}{\left(e^{2i(bx+a)}-1\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3),x)
```

```
[Out] -1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))
-1)^2*x*exp(2*I*(b*x+a))-1/8*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)
))^(2/3)/(exp(2*I*(b*x+a))-1)^2/b*exp(4*I*(b*x+a))+1/8*I*(I*c*(exp(2*I*(b*x
+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2/b
```

Maxima [B] time = 1.47115, size = 157, normalized size = 2.85

$$\frac{c^{\frac{2}{3}}\arctan\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)-\frac{\frac{c^{\frac{2}{3}}\sin(bx+a)}{\cos(bx+a)+1}-\frac{c^{\frac{2}{3}}\sin(bx+a)^3}{(\cos(bx+a)+1)^3}}{\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (c^(2/3)*arctan(sin(b*x + a)/(cos(b*x + a) + 1)) - (c^(2/3)*sin(b*x + a)/(c
os(b*x + a) + 1) - c^(2/3)*sin(b*x + a)^3/(cos(b*x + a) + 1)^3)/(2*sin(b*x
+ a)^2/(cos(b*x + a) + 1)^2 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4 + 1))/b
```

Fricas [A] time = 1.62901, size = 146, normalized size = 2.65

$$-\frac{(bx-\cos(bx+a)\sin(bx+a))\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^{\frac{2}{3}}}{2\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - cos(b*x + a)*sin(b*x + a))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a
))^(2/3)/(b*cos(b*x + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin^3(a+bx)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin(bx+a)^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3), x)
```

3.339 $\int \frac{\left(c \sin^3(a+bx)\right)^{2/3}}{x} dx$

Optimal. Leaf size=99

$$-\frac{1}{2} \cos(2a)\text{CosIntegral}(2bx) \csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} + \frac{1}{2} \sin(2a)\text{Si}(2bx) \csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} + \frac{1}{2}$$

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/2 +
(Csc[a + b*x]^2*Log[x]*(c*Sin[a + b*x]^3)^(2/3))/2 + (Csc[a + b*x]^2*Sin[2*
a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*x])/2
```

Rubi [A] time = 0.208733, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 3312, 3303, 3299, 3302}

$$-\frac{1}{2} \cos(2a)\text{CosIntegral}(2bx) \csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} + \frac{1}{2} \sin(2a)\text{Si}(2bx) \csc^2(a+bx) \left(c \sin^3(a+bx)\right)^{2/3} + \frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3))/2 +
(Csc[a + b*x]^2*Log[x]*(c*Sin[a + b*x]^3)^(2/3))/2 + (Csc[a + b*x]^2*Sin[2*
a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*x])/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}\,dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x}\,dx\\ &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \left(\frac{1}{2x}-\frac{\cos(2a+2bx)}{2x}\right)dx\\ &= \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{1}{2}\left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\cos(2a+2b}{x}\\ &= \frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{1}{2}\left(\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\cos(}{}\\ &= -\frac{1}{2}\cos(2a)\mathrm{Ci}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+\frac{1}{2}\csc^2(a+bx)\log(x)\left(c\sin^3(a+bx)\right)^{2/3}\end{aligned}$$

Mathematica [A] time = 0.0870483, size = 50, normalized size = 0.51

$$\frac{1}{2}\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(-\cos(2a)\text{CosIntegral}(2bx)+\sin(2a)\text{Si}(2bx)+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x]) +
Log[x] + Sin[2*a]*SinIntegral[2*b*x]))/2
```

Maple [C] time = 0.079, size = 283, normalized size = 2.9

$$\frac{\frac{i}{4}e^{2\,ibx}\pi\,\mathrm{csgn}\left(bx\right)}{\left(e^{2\,i(bx+a)}-1\right)^2}\left(ic\left(e^{2\,i(bx+a)}-1\right)^3e^{-3\,i(bx+a)}\right)^{\frac{2}{3}}-\frac{\frac{i}{2}e^{2\,ibx}\mathrm{Si}\left(2\,bx\right)}{\left(e^{2\,i(bx+a)}-1\right)^2}\left(ic\left(e^{2\,i(bx+a)}-1\right)^3e^{-3\,i(bx+a)}\right)^{\frac{2}{3}}-\frac{e^{2\,ibx}\mathrm{Ei}\left(1,-2\,ibx\right)}{4\left(e^{2\,i(bx+a)}-1\right)^2}\left(ic\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x,x)
```

```
[Out] 1/4*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a)
)-1)^2*exp(2*I*b*x)*Pi*csgn(b*x)-1/2*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I
*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*exp(2*I*b*x)*Si(2*b*x)-1/4*(I*c*(ex
p(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*exp(2*I
*b*x)*Ei(1,-2*I*b*x)-1/4*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/
3)/(exp(2*I*(b*x+a))-1)^2*Ei(1,-2*I*b*x)*exp(2*I*(b*x+2*a))-1/2*(I*c*(exp(2
*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*ln(x)*exp(
2*I*(b*x+a))
```

Maxima [C] time = 1.57603, size = 70, normalized size = 0.71

$$-\frac{1}{8}\left(\left(E_1\left(2i\,bx\right)+E_1\left(-2i\,bx\right)\right)\cos\left(2\,a\right)+\left(-i\,E_1\left(2i\,bx\right)+i\,E_1\left(-2i\,bx\right)\right)\sin\left(2\,a\right)+2\,\log\left(bx\right)\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/8*((exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a) +
 (-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))*sin(2*a) +
 2*log(b*x))*c^(2/3)
```

Fricas [A] time = 1.74951, size = 288, normalized size = 2.91

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx\right)-\left(4^{\frac{1}{3}}\,\mathrm{Ci}\left(2\,bx\right)+4^{\frac{1}{3}}\,\mathrm{Ci}\left(-2\,bx\right)\right)\cos\left(2\,a\right)+2\cdot 4^{\frac{1}{3}}\log\left(x\right)\right)\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx\right.\right.}{16\left(\cos\left(bx+a\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/16*4^(2/3)*(2*4^(1/3)*sin(2*a)*sin_integral(2*b*x) - (4^(1/3)*cos_integr
al(2*b*x) + 4^(1/3)*cos_integral(-2*b*x))*cos(2*a) + 2*4^(1/3)*log(x))*(-(c
*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(cos(b*x + a)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx+a\right)^{3}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x, x)
```

3.340 $\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=86

$$b\sin(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+b\cos(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{\left(c\sin^3(a\right.}{}$$

```
[Out] -((c*Sin[a + b*x]^3)^(2/3)/x) + b*CosIntegral[2*b*x]*Csc[a + b*x]^2*Sin[2*a
]*(c*Sin[a + b*x]^3)^(2/3) + b*Cos[2*a]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(
2/3)*SinIntegral[2*b*x]
```

Rubi [A] time = 0.183908, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 3313, 12, 3303, 3299, 3302}

$$b\sin(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}+b\cos(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}-\frac{\left(c\sin^3(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x]^3)^(2/3)/x) + b*CosIntegral[2*b*x]*Csc[a + b*x]^2*Sin[2*a
]*(c*Sin[a + b*x]^3)^(2/3) + b*Cos[2*a]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(
2/3)*SinIntegral[2*b*x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^2}\,dx &= \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x^2}\,dx\\ &= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+\left(2b\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2a+2bx)}{2x}\,dx\\ &= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+\left(b\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2a+2bx)}{x}\,dx\\ &= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+\left(b\cos(2a)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin(2bx)}{x}\,dx+\left(b\csc^2\right.\\ &= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x}+b\mathrm{Ci}(2bx)\csc^2(a+bx)\sin(2a)\left(c\sin^3(a+bx)\right)^{2/3}+b\cos(2a)\csc^2(a\end{aligned}$$

Mathematica [A] time = 0.148407, size = 65, normalized size = 0.76

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}(2bx\sin(2a)\mathrm{CosIntegral}(2bx)+2bx\cos(2a)\mathrm{Si}(2bx)+\cos(2(a+bx))-1)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 2*b*x*Cos
Integral[2*b*x]*Sin[2*a] + 2*b*x*Cos[2*a]*SinIntegral[2*b*x]))/(2*x)
```

Maple [C] time = 0.089, size = 211, normalized size = 2.5

$$\frac{\frac{i}{4}b}{\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}\right)^{\frac{2}{3}}\left(\frac{i}{bx}+2e^{2ibx}\mathrm{Ei}(1,2ibx)\right)+\frac{\frac{i}{4}b}{\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3e^{-3i(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x^2,x)
```

```
[Out] 1/4*I*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a)
)-1)^2*b*(I/x/b+2*exp(2*I*b*x)*Ei(1,2*I*b*x))+1/4*I*(I*c*(exp(2*I*(b*x+a))-
1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*b*(I/x/b*exp(4*I*(b*x+
a))-2*Ei(1,-2*I*b*x)*exp(2*I*(b*x+2*a)))+1/2*(I*c*(exp(2*I*(b*x+a))-1)^3*ex
p(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2/x*exp(2*I*(b*x+a))
```

Maxima [C] time = 1.64815, size = 378, normalized size = 4.4

$$\frac{\left(64\left(\left(-i\sqrt{3}+1\right)E_{2}\left(2i\,bx\right)+\left(i\sqrt{3}+1\right)E_{2}\left(-2i\,bx\right)\right)\cos\left(2\,a\right)^{3}-\left(\left(64\sqrt{3}+64i\right)E_{2}\left(2i\,bx\right)+\left(64\sqrt{3}-64i\right)E_{2}\left(-2i\,bx\right)\right)\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*((-I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*e
xp_integral_e(2, -2*I*b*x))*cos(2*a)^3 - ((64*sqrt(3) + 64*I)*exp_integral_
e(2, 2*I*b*x) + (64*sqrt(3) - 64*I)*exp_integral_e(2, -2*I*b*x))*sin(2*a)^3
 + 64*(((-I*sqrt(3) + 1)*exp_integral_e(2, 2*I*b*x) + (I*sqrt(3) + 1)*exp_i
ntegral_e(2, -2*I*b*x))*cos(2*a) - 4)*sin(2*a)^2 + 64*((I*sqrt(3) + 1)*exp_
integral_e(2, 2*I*b*x) + (-I*sqrt(3) + 1)*exp_integral_e(2, -2*I*b*x))*cos(
2*a) - 256*cos(2*a)^2 - (((64*sqrt(3) + 64*I)*exp_integral_e(2, 2*I*b*x) +
(64*sqrt(3) - 64*I)*exp_integral_e(2, -2*I*b*x))*cos(2*a)^2 - (64*sqrt(3) -
 64*I)*exp_integral_e(2, 2*I*b*x) - (64*sqrt(3) + 64*I)*exp_integral_e(2, -
2*I*b*x))*sin(2*a))*b*c^(2/3)/(a*cos(2*a)^2 + a*sin(2*a)^2 - (b*x + a)*(cos
(2*a)^2 + sin(2*a)^2))
```

Fricas [A] time = 1.79522, size = 332, normalized size = 3.86

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}bx\cos\left(2\,a\right)\mathrm{Si}\left(2\,bx\right)+2\cdot 4^{\frac{1}{3}}\cos\left(bx+a\right)^{2}+\left(4^{\frac{1}{3}}bx\,\mathrm{Ci}\left(2\,bx\right)+4^{\frac{1}{3}}bx\,\mathrm{Ci}\left(-2\,bx\right)\right)\sin\left(2\,a\right)-2\cdot 4^{\frac{1}{3}}\right)\left(-\left(c\cos\left(bx\right.\right.\right.}{8\left(x\cos\left(bx+a\right)^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*4^(2/3)*(2*4^(1/3)*b*x*cos(2*a)*sin_integral(2*b*x) + 2*4^(1/3)*cos(b*
x + a)^2 + (4^(1/3)*b*x*cos_integral(2*b*x) + 4^(1/3)*b*x*cos_integral(-2*b
*x))*sin(2*a) - 2*4^(1/3))*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(x*
cos(b*x + a)^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\sin\left(bx+a\right)^{3}\right)^{\frac{2}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x^2, x)
```

3.341 $\int \frac{(c\sin^3(a+bx))^{2/3}}{x^3}\,dx$

Optimal. Leaf size=119

$$b^2\cos(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - b^2\sin(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - \frac{\left(c\sin^3\right.}{}$$

```
[Out] -(c*Sin[a + b*x]^3)^(2/3)/(2*x^2) - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3
))/x + b^2*Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2
/3) - b^2*Csc[a + b*x]^2*Sin[2*a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*
x]
```

Rubi [A] time = 0.230095, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6720, 3314, 29, 3312, 3303, 3299, 3302}

$$b^2\cos(2a)\text{CosIntegral}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - b^2\sin(2a)\text{Si}(2bx)\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3} - \frac{\left(c\sin^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```


```
[Out] -(c*Sin[a + b*x]^3)^(2/3)/(2*x^2) - (b*Cot[a + b*x]*(c*Sin[a + b*x]^3)^(2/3
))/x + b^2*Cos[2*a]*CosIntegral[2*b*x]*Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2
/3) - b^2*Csc[a + b*x]^2*Sin[2*a]*(c*Sin[a + b*x]^3)^(2/3)*SinIntegral[2*b*
x]
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\left(c\sin^3(a+bx)\right)^{2/3}}{x^3}\,dx = \left(\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\right)\int \frac{\sin^2(a+bx)}{x^3}\,dx$$
$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)\right.$$
$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b^2\csc^2(a+bx)\log(x)\left(c\sin^3(a+\right.$$
$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\csc^2(a+bx)\left(c\sin^3(a+bx)\right)\right.$$
$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + \left(b^2\cos(2a)\csc^2(a+bx)\left(c\sin^3(\right.\right.$$
$$= -\frac{\left(c\sin^3(a+bx)\right)^{2/3}}{2x^2} - \frac{b\cot(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}}{x} + b^2\cos(2a)\text{Ci}(2bx)\csc^2(a+bx)\left(\right.$$

Mathematica [A] time = 0.203367, size = 85, normalized size = 0.71

$$\frac{\csc^2(a+bx)\left(c\sin^3(a+bx)\right)^{2/3}\left(4b^2x^2\cos(2a)\text{CosIntegral}(2bx) - 4b^2x^2\sin(2a)\text{Si}(2bx) - 2bx\sin(2(a+bx)) + \cos(2\right.}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x]^2*(c*Sin[a + b*x]^3)^(2/3)*(-1 + Cos[2*(a + b*x)] + 4*b^2*x^2
*Cos[2*a]*CosIntegral[2*b*x] - 2*b*x*Sin[2*(a + b*x)] - 4*b^2*x^2*Sin[2*a]*
SinIntegral[2*b*x]))/(4*x^2)
```

Maple [C] time = 0.088, size = 238, normalized size = 2.

$$-\frac{b^2}{4\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}-1\right)^3 e^{-3i(bx+a)}\right)^{\frac{2}{3}}\left(\frac{1}{2x^2b^2} - \frac{i}{bx} - 2e^{2ibx}\text{Ei}\left(1,2ibx\right)\right) - \frac{b^2}{4\left(e^{2i(bx+a)}-1\right)^2}\left(ic\left(e^{2i(bx+a)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^(2/3)/x^3,x)
```

```
[Out] -1/4*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))
-1)^2*b^2*(1/2/x^2/b^2-I/x/b-2*exp(2*I*b*x)*Ei(1,2*I*b*x))-1/4*(I*c*(exp(2*
I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(b*x+a))-1)^2*b^2*(1/2/x^
2/b^2*exp(4*I*(b*x+a))+I/x/b*exp(4*I*(b*x+a))-2*Ei(1,-2*I*b*x)*exp(2*I*(b*x
+2*a)))+1/4*(I*c*(exp(2*I*(b*x+a))-1)^3*exp(-3*I*(b*x+a)))^(2/3)/(exp(2*I*(
b*x+a))-1)^2/x^2*exp(2*I*(b*x+a))
```

Maxima [C] time = 1.70297, size = 420, normalized size = 3.53

$$-\frac{\left(128\left(\left(-i\sqrt{3}+1\right)E_{3}\left(2i\,bx\right)+\left(i\sqrt{3}+1\right)E_{3}\left(-2i\,bx\right)\right)\cos\left(2\,a\right)^{3}-\left(\left(128\sqrt{3}+128i\right)E_{3}\left(2i\,bx\right)+\left(128\sqrt{3}-128i\right)E_{3}\left(-2i\,bx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2048*(128*((-I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1)
*exp_integral_e(3, -2*I*b*x))*cos(2*a)^3 - ((128*sqrt(3) + 128*I)*exp_integ
ral_e(3, 2*I*b*x) + (128*sqrt(3) - 128*I)*exp_integral_e(3, -2*I*b*x))*sin(
2*a)^3 + 128*(((-I*sqrt(3) + 1)*exp_integral_e(3, 2*I*b*x) + (I*sqrt(3) + 1
)*exp_integral_e(3, -2*I*b*x))*cos(2*a) - 2)*sin(2*a)^2 + 128*((I*sqrt(3) +
 1)*exp_integral_e(3, 2*I*b*x) + (-I*sqrt(3) + 1)*exp_integral_e(3, -2*I*b*
x))*cos(2*a) - 256*cos(2*a)^2 - (((128*sqrt(3) + 128*I)*exp_integral_e(3, 2
*I*b*x) + (128*sqrt(3) - 128*I)*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 - (
128*sqrt(3) - 128*I)*exp_integral_e(3, 2*I*b*x) - (128*sqrt(3) + 128*I)*exp
_integral_e(3, -2*I*b*x))*sin(2*a))*b^2*c^(2/3)/(a^2*cos(2*a)^2 + a^2*sin(2
*a)^2 + (b*x + a)^2*(cos(2*a)^2 + sin(2*a)^2) - 2*(a*cos(2*a)^2 + a*sin(2*a
)^2)*(b*x + a))
```

Fricas [A] time = 1.84185, size = 404, normalized size = 3.39

$$\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}b^{2}x^{2}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx\right)+2\cdot 4^{\frac{1}{3}}bx\cos\left(bx+a\right)\sin\left(bx+a\right)-4^{\frac{1}{3}}\cos\left(bx+a\right)^{2}-\left(4^{\frac{1}{3}}b^{2}x^{2}\operatorname{Ci}\left(2\,bx\right)+4^{\frac{1}{3}}b^{2}x^{2}\operatorname{Ci}\left(-\right.\right.\right.}{8\left(x^{2}\cos\left(bx+a\right)^{2}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*4^(2/3)*(2*4^(1/3)*b^2*x^2*sin(2*a)*sin_integral(2*b*x) + 2*4^(1/3)*b*x
*cos(b*x + a)*sin(b*x + a) - 4^(1/3)*cos(b*x + a)^2 - (4^(1/3)*b^2*x^2*cos_
integral(2*b*x) + 4^(1/3)*b^2*x^2*cos_integral(-2*b*x))*cos(2*a) + 4^(1/3))
*(-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^(2/3)/(x^2*cos(b*x + a)^2 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin^{3}\left(a+bx\right)\right)^{\frac{2}{3}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**(2/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**(2/3)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx+a\right)^{3}\right)^{\frac{2}{3}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^(2/3)/x^3, x)
```

3.342 $\int x^m \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx$

Optimal. Leaf size=209

$$e^{2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\text{Gamma}\left(\frac{m+1}{2},-2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+e^{-2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m}$$

```
[Out] (x^(1 + m)*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(2*(1 + m)) + 2^(-7
/2 - m/2)*E^((2*I)*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*
Gamma[(1 + m)/2, (-2*I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3) + (2^(-7/2 - m/2)
*x^(1 + m)*(I*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*Gamma[(1 + m)/2, (2*I)*b
*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/E^((2*I)*a)
```

Rubi [A] time = 0.277051, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3403, 3390, 2218}

$$e^{2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(-ibx^2\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\text{Gamma}\left(\frac{m+1}{2},-2ibx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+e^{-2ia}2^{-\frac{m}{2}-\frac{7}{2}}x^{m+1}\left(ibx^2\right)^{\frac{1}{2}(-m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(2*(1 + m)) + 2^(-7
/2 - m/2)*E^((2*I)*a)*x^(1 + m)*((-I)*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*
Gamma[(1 + m)/2, (-2*I)*b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3) + (2^(-7/2 - m/2)
*x^(1 + m)*(I*b*x^2)^((-1 - m)/2)*Csc[a + b*x^2]^2*Gamma[(1 + m)/2, (2*I)*b
*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/E^((2*I)*a)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^m \left(c\sin^3\left(a+bx^2\right)\right)^{2/3} dx = \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^m \sin^2\left(a+bx^2\right) dx$$
$$= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cos\left(2a+2bx^2\right)\right) dx$$
$$= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)} - \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int$$
$$= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)} - \frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int$$
$$= \frac{x^{1+m}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{2(1+m)} + 2^{-\frac{7}{2}-\frac{m}{2}}e^{2ia}x^{1+m}\left(-ibx^2\right)^{\frac{1}{2}(-1-m)}\csc^2\left(a+bx\right.$$

Mathematica [A] time = 0.895489, size = 189, normalized size = 0.9

$$\frac{2^{\frac{1}{2}(-m-7)}x^{m+1}\left(b^2x^4\right)^{\frac{1}{2}(-m-1)}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left((m+1)(\cos(2a)-i\sin(2a))\left(-ibx^2\right)^{\frac{m+1}{2}}\text{Gamma}\left(\frac{m+1}{2}\right.\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (2^((-7 - m)/2)*x^(1 + m)*(b^2*x^4)^((-1 - m)/2)*Csc[a + b*x^2]^2*(2^((5 +
m)/2)*(b^2*x^4)^((1 + m)/2) + (1 + m)*((-I)*b*x^2)^((1 + m)/2)*Gamma[(1 + m
)/2, (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) + (1 + m)*(I*b*x^2)^((1 + m)/2)*G
amma[(1 + m)/2, (-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a]))*(c*Sin[a + b*x^2]^3)
^(2/3))/(1 + m)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int x^m \left(c\left(\sin\left(bx^2+a\right)\right)^3\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(b*x^2+a)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(xx^m-(m+1)\int x^m\cos\left(2\,bx^2+2\,a\right)\,dx\right)c^{\frac{2}{3}}}{4\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^2 + 2*a), x))*c^(2/3)/(m + 1)
```

Fricas [A] time = 1.80256, size = 359, normalized size = 1.72

$$-\frac{\left(8\,bxx^{m}-(im+i)e^{\left(-\frac{1}{2}\,(m-1)\log(2i\,b)-2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2i\,bx^{2}\right)-(-im-i)e^{\left(-\frac{1}{2}\,(m-1)\log(-2i\,b)+2i\,a\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-2i\,bx^{2}\right)\right)\left(-\left(c\,c\right.\right.}{16\left((bm+b)\cos\left(bx^{2}+a\right)^{2}-bm-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(8*b*x*x^m - (I*m + I)*e^(-1/2*(m - 1)*log(2*I*b) - 2*I*a)*gamma(1/2*
m + 1/2, 2*I*b*x^2) - (-I*m - I)*e^(-1/2*(m - 1)*log(-2*I*b) + 2*I*a)*gamma
(1/2*m + 1/2, -2*I*b*x^2))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)
/((b*m + b)*cos(b*x^2 + a)^2 - b*m - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx^{2}+a\right)^{3}\right)^{\frac{2}{3}} x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^m, x)
```

3.343 $\int x^3 \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx$

Optimal. Leaf size=91

$$\frac{\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2 \cot\left(a+bx^2\right)\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{8}x^4 \csc^2\left(a+bx^2\right)\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}$$

```
[Out] (c*Sin[a + b*x^2]^3)^(2/3)/(8*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(2/3))/(4*b) + (x^4*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/8
```

Rubi [A] time = 0.183493, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3379, 3310, 30}

$$\frac{\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2 \cot\left(a+bx^2\right)\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{8}x^4 \csc^2\left(a+bx^2\right)\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (c*Sin[a + b*x^2]^3)^(2/3)/(8*b^2) - (x^2*Cot[a + b*x^2]*(c*Sin[a + b*x^2]^
3)^(2/3))/(4*b) + (x^4*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/8
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx=\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^3\sin^2\left(a+bx^2\right)dx$$
$$=\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\text{Subst}\left(\int x\sin^2(a+bx)\,dx,x,x^2\right)$$
$$=\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a\right.\right.\right.$$
$$=\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b^2}-\frac{x^2\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{8}x^4\csc^2\left(a+bx^2\right)\left(c\sin^3\right.$$

Mathematica [A] time = 0.233949, size = 67, normalized size = 0.74

$$-\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2bx^2\left(\sin\left(2\left(a+bx^2\right)\right)-bx^2\right)+\cos\left(2\left(a+bx^2\right)\right)\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] -(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(Cos[2*(a + b*x^2)] + 2*b*x^2
*(-(b*x^2) + Sin[2*(a + b*x^2)])))/(16*b^2)
```

Maple [C] time = 0.086, size = 200, normalized size = 2.2

$$-\frac{x^4e^{2i(bx^2+a)}}{8\left(e^{2i(bx^2+a)}-1\right)^2}\left(ic\left(e^{2i(bx^2+a)}-1\right)^3e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}-\frac{\frac{i}{32}\left(2bx^2+i\right)e^{4i(bx^2+a)}}{\left(e^{2i(bx^2+a)}-1\right)^2b^2}\left(ic\left(e^{2i(bx^2+a)}-1\right)^3e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}+\frac{\frac{i}{32}\left(\right.}{\left(e^{2i(bx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] -1/8*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2*x^4*exp(2*I*(b*x^2+a))-1/32*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp
(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*(2*b*x^2+I)/b^2*exp(4*I*(b
*x^2+a))+1/32*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(e
xp(2*I*(b*x^2+a))-1)^2*(2*b*x^2-I)/b^2
```

Maxima [A] time = 1.55119, size = 63, normalized size = 0.69

$$-\frac{\left(2b^2x^4-2bx^2\sin\left(2bx^2+2a\right)-\cos\left(2bx^2+2a\right)\right)c^{\frac{2}{3}}}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b^2*x^4 - 2*b*x^2*sin(2*b*x^2 + 2*a) - cos(2*b*x^2 + 2*a))*c^(2/3)
/b^2
```

Fricas [A] time = 1.59906, size = 219, normalized size = 2.41

$$-\frac{\left(2 b^{2} x^{4}-4 b x^{2} \cos \left(b x^{2}+a\right) \sin \left(b x^{2}+a\right)-2 \cos \left(b x^{2}+a\right)^{2}+1\right)\left(-\left(c \cos \left(b x^{2}+a\right)^{2}-c\right) \sin \left(b x^{2}+a\right)\right)^{\frac{2}{3}}}{16\left(b^{2} \cos \left(b x^{2}+a\right)^{2}-b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^2*x^4 - 4*b*x^2*cos(b*x^2 + a)*sin(b*x^2 + a) - 2*cos(b*x^2 + a)
^2 + 1)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a
)^2 - b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(b x^{2}+a\right)^{3}\right)^{\frac{2}{3}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^3, x)
```

3.344 $\int x^2 \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\pi}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}+\frac{\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}$$

```
[Out] (x^3*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/6 + (Sqrt[Pi]*Cos[2*a]*Cs
c[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))
/(16*b^(3/2)) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]
*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(16*b^(3/2)) - (x*Csc[a + b*x^2]^2*(c
*Sin[a + b*x^2]^3)^(2/3)*Sin[2*a + 2*b*x^2])/(8*b)
```

Rubi [A] time = 0.230888, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6720, 3403, 3386, 3353, 3352, 3351}

$$\frac{\sqrt{\pi}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}+\frac{\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```


```
[Out] (x^3*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/6 + (Sqrt[Pi]*Cos[2*a]*Cs
c[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))
/(16*b^(3/2)) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]
*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(16*b^(3/2)) - (x*Csc[a + b*x^2]^2*(c
*Sin[a + b*x^2]^3)^(2/3)*Sin[2*a + 2*b*x^2])/(8*b)
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^2\sin^2\left(a+bx^2\right)dx\\ &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos\left(2a+2bx^2\right)\right)dx\\ &= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int x^2\\ &= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\sin\left(2a\right.}{8b}\\ &= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\sin\left(2a\right.}{8b}\\ &= \frac{1}{6}x^3\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(\right.\right.}{16b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.287334, size = 113, normalized size = 0.58

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(3\sqrt{\pi}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+3\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+2\sqrt{b}x\left(4bx^2-3\sin\left(2\left(a\right.\right.\right.\right.}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(3*Sqrt[Pi]*Cos[2*a]*FresnelS[
(2*Sqrt[b]*x)/Sqrt[Pi]] + 3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2
*a] + 2*Sqrt[b]*x*(4*b*x^2 - 3*Sin[2*(a + b*x^2)])))/(48*b^(3/2))
```

Maple [C] time = 0.112, size = 309, normalized size = 1.6

$$\frac{\frac{i}{16}x}{\left(e^{2i\left(bx^2+a\right)}-1\right)^2 b}\left(ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}\right)^{\frac{2}{3}}-\frac{\frac{i}{64}e^{2ibx^2}\sqrt{\pi}\sqrt{2}}{\left(e^{2i\left(bx^2+a\right)}-1\right)^2 b}\left(ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}\right)^{\frac{2}{3}}\text{Erf}\left(\sqrt{2}\sqrt{ibx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] 1/16*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b
*x^2+a))-1)^2/b*x-1/64*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))
^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)/b*Pi^(1/2)*2^(1/2)/(I*b)^(1/
2)*erf(2^(1/2)*(I*b)^(1/2)*x)+1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b
*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*(-1/4*I/b*x*exp(4*I*(b*x^2+a))+1/8
*I/b*Pi^(1/2)/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))-1/
6*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+
a))-1)^2*x^3*exp(2*I*(b*x^2+a))
```

Maxima [C] time = 1.76104, size = 539, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/768*(sqrt(2)*sqrt(pi)*((((3*sqrt(3) - 3*I)*cos(1/4*pi + 1/2*arctan2(0, b)
) - (3*sqrt(3) + 3*I)*cos(-1/4*pi + 1/2*arctan2(0, b)) - 3*(I*sqrt(3) + 1)*
sin(1/4*pi + 1/2*arctan2(0, b)) - 3*(I*sqrt(3) - 1)*sin(-1/4*pi + 1/2*arcta
n2(0, b)))*cos(2*a) - (3*(I*sqrt(3) + 1)*cos(1/4*pi + 1/2*arctan2(0, b)) +
3*(-I*sqrt(3) + 1)*cos(-1/4*pi + 1/2*arctan2(0, b)) + (3*sqrt(3) - 3*I)*sin
(1/4*pi + 1/2*arctan2(0, b)) + (3*sqrt(3) + 3*I)*sin(-1/4*pi + 1/2*arctan2(
0, b)))*sin(2*a))*erf(sqrt(2*I*b)*x) + (((3*sqrt(3) + 3*I)*cos(1/4*pi + 1/2
*arctan2(0, b)) - (3*sqrt(3) - 3*I)*cos(-1/4*pi + 1/2*arctan2(0, b)) - 3*(-
I*sqrt(3) + 1)*sin(1/4*pi + 1/2*arctan2(0, b)) - 3*(-I*sqrt(3) - 1)*sin(-1/
4*pi + 1/2*arctan2(0, b)))*cos(2*a) - (3*(-I*sqrt(3) + 1)*cos(1/4*pi + 1/2*
arctan2(0, b)) + 3*(I*sqrt(3) + 1)*cos(-1/4*pi + 1/2*arctan2(0, b)) + (3*sq
rt(3) + 3*I)*sin(1/4*pi + 1/2*arctan2(0, b)) + (3*sqrt(3) - 3*I)*sin(-1/4*p
i + 1/2*arctan2(0, b)))*sin(2*a))*erf(sqrt(-2*I*b)*x))*c^(2/3)*sqrt(abs(b))
 - 16*(4*b*x^3*abs(b) - 3*x*abs(b)*sin(2*b*x^2 + 2*a))*c^(2/3))/(b*abs(b))
```

Fricas [A] time = 1.85172, size = 387, normalized size = 1.98

$$-\frac{4^{\frac{2}{3}}\left(8\cdot 4^{\frac{1}{3}}b^2x^3-12\cdot 4^{\frac{1}{3}}bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)+3\cdot 4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+3\cdot 4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right.\right.}{192\left(b^2\cos\left(bx^2+a\right)^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/192*4^(2/3)*(8*4^(1/3)*b^2*x^3 - 12*4^(1/3)*b*x*cos(b*x^2 + a)*sin(b*x^2
 + a) + 3*4^(1/3)*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi)) + 3*4^
(1/3)*pi*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))*sin(2*a))*(-(c*cos(b*x^2 +
a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b^2*cos(b*x^2 + a)^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x**2*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx^2 + a\right)^3\right)^{\frac{2}{3}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x^2, x)
```

3.345 $\int x \left(c \sin^3\left(a+bx^2\right)\right)^{2/3} dx$

Optimal. Leaf size=65

$$\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}$$

```
[Out] -(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/(4*b) + (x^2*Csc[a + b*x^2]^2*
(c*Sin[a + b*x^2]^3)^(2/3))/4
```

Rubi [A] time = 0.0997761, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6715, 3207, 2635, 8}

$$\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] -(Cot[a + b*x^2]*(c*Sin[a + b*x^2]^3)^(2/3))/(4*b) + (x^2*Csc[a + b*x^2]^2*
(c*Sin[a + b*x^2]^3)^(2/3))/4
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx &= \frac{1}{2}\operatorname{Subst}\left(\int\left(c\sin^3(a+bx)\right)^{2/3}dx,x,x^2\right)\\ &= \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\operatorname{Subst}\left(\int\sin^2(a+bx)\,dx,x,x^2\right)\\ &= -\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\operatorname{Subst}\left(\int\right.\\ &= -\frac{\cot\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4b}+\frac{1}{4}x^2\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\end{aligned}$$

Mathematica [A] time = 0.118257, size = 55, normalized size = 0.85

$$\frac{\left(2\left(a+bx^2\right)-\sin\left(2\left(a+bx^2\right)\right)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(2*(a + b*x^2) - Sin[2*(a + b*
x^2)]))/(8*b)
```

Maple [C] time = 0.075, size = 182, normalized size = 2.8

$$-\frac{x^2e^{2i(bx^2+a)}}{4\left(e^{2i(bx^2+a)}-1\right)^2}\left(ic\left(e^{2i(bx^2+a)}-1\right)^3e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}-\frac{\frac{i}{16}e^{4i(bx^2+a)}}{\left(e^{2i(bx^2+a)}-1\right)^2b}\left(ic\left(e^{2i(bx^2+a)}-1\right)^3e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}+\frac{}{\left(e^{2i(bx^2+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] -1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2*x^2*exp(2*I*(b*x^2+a))-1/16*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp
(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2/b*exp(4*I*(b*x^2+a))+1/16*
I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+
a))-1)^2/b
```

Maxima [A] time = 1.54614, size = 38, normalized size = 0.58

$$-\frac{\left(2\,bx^2-\sin\left(2\,bx^2+2\,a\right)\right)c^{\frac{2}{3}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/16*(2*b*x^2 - sin(2*b*x^2 + 2*a))*c^(2/3)/b
```

Fricas [A] time = 1.58751, size = 162, normalized size = 2.49

$$-\frac{\left(bx^2-\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{4\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^2 - cos(b*x^2 + a)*sin(b*x^2 + a))*(-(c*cos(b*x^2 + a)^2 - c)*sin
(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral(x*(c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)*x, x)
```

3.346 $\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\,dx$

Optimal. Leaf size=148

$$-\frac{\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx\right.\right.}{4\sqrt{b}}$$

```
[Out] (x*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/2 - (Sqrt[Pi]*Cos[2*a]*Csc[
a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))/(
4*Sqrt[b]) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Si
n[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Rubi [A] time = 0.0583402, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6720, 3357, 3354, 3352, 3351}

$$-\frac{\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4\sqrt{b}}+\frac{\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx\right.\right.}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (x*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/2 - (Sqrt[Pi]*Cos[2*a]*Csc[
a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3))/(
4*Sqrt[b]) + (Sqrt[Pi]*Csc[a + b*x^2]^2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Si
n[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3357

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^3\left(a+bx^2\right)\right)^{2/3}dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin^2\left(a+bx^2\right)dx\\ &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\left(\frac{1}{2}-\frac{1}{2}\cos\left(2a+2bx^2\right)\right)dx\\ &= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int\cos\left(2a+\right.\\ &= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int c\\ &= \frac{1}{2}x\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}-\frac{\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right.\right.}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.102306, size = 93, normalized size = 0.63

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+2\sqrt{b}x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3),x]
```

```
[Out] (Csc[a + b*x^2]^2*(2*Sqrt[b]*x - Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/S
qrt[Pi]] + Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b
*x^2]^3)^(2/3))/(4*Sqrt[b])
```

Maple [C] time = 0.087, size = 224, normalized size = 1.5

$$\frac{e^{2\,ibx^2}\sqrt{\pi}\sqrt{2}}{16\left(e^{2\,i\left(bx^2+a\right)}-1\right)^2}\left(ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3e^{-3\,i\left(bx^2+a\right)}\right)^{\frac{2}{3}}\text{Erf}\left(\sqrt{2}\sqrt{ib}x\right)\frac{1}{\sqrt{ib}}+\frac{\sqrt{\pi}e^{2\,i\left(bx^2+2\,a\right)}}{8\left(e^{2\,i\left(bx^2+a\right)}-1\right)^2}\left(ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3e^{-3\,i\left(bx^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3),x)
```

```
[Out] 1/16*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2*exp(2*I*b*x^2)*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*erf(2^(1/2)*(I*b)^(
1/2)*x)+1/8*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2
*I*(b*x^2+a))-1)^2*Pi^(1/2)/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b
*x^2+2*a))-1/2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(ex
p(2*I*(b*x^2+a))-1)^2*x*exp(2*I*(b*x^2+a))
```

Maxima [C] time = 1.72626, size = 483, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/64*(sqrt(2)*sqrt(pi)*((((-I*sqrt(3) - 1)*cos(1/4*pi + 1/2*arctan2(0, b))
 + (I*sqrt(3) - 1)*cos(-1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) - I)*sin(1/4
*pi + 1/2*arctan2(0, b)) - (sqrt(3) + I)*sin(-1/4*pi + 1/2*arctan2(0, b)))*
cos(2*a) - ((sqrt(3) - I)*cos(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) + I)*c
os(-1/4*pi + 1/2*arctan2(0, b)) - (I*sqrt(3) + 1)*sin(1/4*pi + 1/2*arctan2(
0, b)) - (I*sqrt(3) - 1)*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(2*a))*erf(sq
rt(2*I*b)*x) + (((I*sqrt(3) - 1)*cos(1/4*pi + 1/2*arctan2(0, b)) + (-I*sqrt
(3) - 1)*cos(-1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) + I)*sin(1/4*pi + 1/2*
arctan2(0, b)) - (sqrt(3) - I)*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(2*a) -
 ((sqrt(3) + I)*cos(1/4*pi + 1/2*arctan2(0, b)) - (sqrt(3) - I)*cos(-1/4*pi
 + 1/2*arctan2(0, b)) - (-I*sqrt(3) + 1)*sin(1/4*pi + 1/2*arctan2(0, b)) -
(-I*sqrt(3) - 1)*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin(2*a))*erf(sqrt(-2*I*
b)*x))*c^(2/3)*sqrt(abs(b)) + 16*c^(2/3)*x*abs(b))/abs(b)
```

Fricas [A] time = 1.70179, size = 305, normalized size = 2.06

$$\frac{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}\cos\left(2a\right)C\left(2x\sqrt{\frac{b}{\pi}}\right)-4^{\frac{1}{3}}\pi\sqrt{\frac{b}{\pi}}S\left(2x\sqrt{\frac{b}{\pi}}\right)\sin\left(2a\right)-2\cdot 4^{\frac{1}{3}}bx\right)\left(-\left(c\cos\left(bx^2+a\right)^2-c\right)\sin\left(bx^2+a\right)\right)^{\frac{2}{3}}}{16\left(b\cos\left(bx^2+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/16*4^(2/3)*(4^(1/3)*pi*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x*sqrt(b/pi)) -
4^(1/3)*pi*sqrt(b/pi)*fresnel_sin(2*x*sqrt(b/pi))*sin(2*a) - 2*4^(1/3)*b*x)
*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(b*cos(b*x^2 + a)^2 - b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3), x)
```

3.347 $\int \frac{\left(c \sin^3\left(a+bx^2\right)\right)^{2/3}}{x} \, dx$

Optimal. Leaf size=115

$$-\frac{1}{4} \cos(2a)\text{CosIntegral}\left(2bx^2\right) \csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx\right.\right.$$

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^2]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)
)/4 + (Csc[a + b*x^2]^2*Log[x]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (Csc[a + b*x
^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/4
```

Rubi [A] time = 0.133211, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 3403, 3378, 3376, 3375}

$$-\frac{1}{4} \cos(2a)\text{CosIntegral}\left(2bx^2\right) \csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^2]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)
)/4 + (Csc[a + b*x^2]^2*Log[x]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (Csc[a + b*x
^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/4
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x}\,dx = \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x}\,dx$$
$$= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2x}-\frac{\cos\left(2a+2bx^2\right)}{2x}\right)dx$$
$$= \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} - \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\cos}{}$$
$$= \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} - \frac{1}{2}\left(\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right.$$
$$= -\frac{1}{4}\cos(2a)\text{Ci}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} + \frac{1}{2}\csc^2\left(a+bx^2\right)\log(x)\left(c\sin^3\left(a\right.\right.$$

Mathematica [A] time = 0.096372, size = 60, normalized size = 0.52

$$\frac{1}{4}\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(-\cos(2a)\text{CosIntegral}\left(2bx^2\right)+\sin(2a)\text{Si}\left(2bx^2\right)+2\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x^
2]) + 2*Log[x] + Sin[2*a]*SinIntegral[2*b*x^2]))/4
```

Maple [C] time = 0.085, size = 331, normalized size = 2.9

$$\frac{\frac{i}{8}e^{2\,ibx^2}\pi\,\text{csgn}\left(bx^2\right)}{\left(e^{2\,i\left(bx^2+a\right)}-1\right)^2}\left(ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3e^{-3\,i\left(bx^2+a\right)}\right)^{\frac{2}{3}} - \frac{\frac{i}{4}e^{2\,ibx^2}\text{Si}\left(2\,bx^2\right)}{\left(e^{2\,i\left(bx^2+a\right)}-1\right)^2}\left(ic\left(e^{2\,i\left(bx^2+a\right)}-1\right)^3e^{-3\,i\left(bx^2+a\right)}\right)^{\frac{2}{3}} - \frac{e^{2\,ibx^2}\text{Ei}\left(1,-\right.}{8\left(e^{2\,i\left(bx^2+a\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x,x)
```

```
[Out] 1/8*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*
x^2+a))-1)^2*exp(2*I*b*x^2)*Pi*csgn(b*x^2)-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1
)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*Si(2
*b*x^2)-1/8*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2
*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*Ei(1,-2*I*b*x^2)-1/8*(I*c*(exp(2*I*(b*x^2
+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*Ei(1,-2*I*b*x
^2)*exp(2*I*(b*x^2+2*a))-1/2*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+
a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*ln(x)*exp(2*I*(b*x^2+a))
```

Maxima [C] time = 1.68878, size = 74, normalized size = 0.64

$$\frac{1}{16}\left(\left(\text{Ei}\left(2i\,bx^2\right)+\text{Ei}\left(-2i\,bx^2\right)\right)\cos\left(2\,a\right)-\left(-i\,\text{Ei}\left(2i\,bx^2\right)+i\,\text{Ei}\left(-2i\,bx^2\right)\right)\sin\left(2\,a\right)-4\,\log\left(x\right)\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/16*((Ei(2*I*b*x^2) + Ei(-2*I*b*x^2))*cos(2*a) - (-I*Ei(2*I*b*x^2) + I*Ei(
-2*I*b*x^2))*sin(2*a) - 4*log(x))*c^(2/3)
```

Fricas [A] time = 1.61394, size = 304, normalized size = 2.64

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx^{2}\right)-\left(4^{\frac{1}{3}}\,\mathrm{Ci}\left(2\,bx^{2}\right)+4^{\frac{1}{3}}\,\mathrm{Ci}\left(-2\,bx^{2}\right)\right)\cos\left(2\,a\right)+4\cdot 4^{\frac{1}{3}}\log\left(x\right)\right)\left(-\left(c\cos\left(bx^{2}+a\right)^{2}-c\right)\sin\right.}{32\left(\cos\left(bx^{2}+a\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/32*4^(2/3)*(2*4^(1/3)*sin(2*a)*sin_integral(2*b*x^2) - (4^(1/3)*cos_inte
gral(2*b*x^2) + 4^(1/3)*cos_integral(-2*b*x^2))*cos(2*a) + 4*4^(1/3)*log(x)
)*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/(cos(b*x^2 + a)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin^{3}\left(a+bx^{2}\right)\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^{2}+a\right)^{3}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x, x)
```

3.348 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=132

$$\sqrt{\pi}\sqrt{b}\sin(2a)\mathrm{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\sqrt{\pi}\sqrt{b}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+\right.\right.$$

```
[Out] -((c*Sin[a + b*x^2]^3)^(2/3)/x) + Sqrt[b]*Sqrt[Pi]*Cos[2*a]*Csc[a + b*x^2]^
2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3) + Sqrt[b]*Sqr
t[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a]*(c*Sin[a +
 b*x^2]^3)^(2/3)
```

Rubi [A] time = 0.148321, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6720, 3393, 4573, 3373, 3353, 3352, 3351}

$$\sqrt{\pi}\sqrt{b}\sin(2a)\mathrm{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\sqrt{\pi}\sqrt{b}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

```
[Out] -((c*Sin[a + b*x^2]^3)^(2/3)/x) + Sqrt[b]*Sqrt[Pi]*Cos[2*a]*Csc[a + b*x^2]^
2*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*(c*Sin[a + b*x^2]^3)^(2/3) + Sqrt[b]*Sqr
t[Pi]*Csc[a + b*x^2]^2*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a]*(c*Sin[a +
 b*x^2]^3)^(2/3)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3393

```
Int[(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[(x^(m +
 1)*Sin[a + b*x^n]^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[Sin[a + b*x^n
]^(p - 1)*Cos[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4573

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rule 3373

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] && !BinomialMatchQ
[u, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^2}\,dx &= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x^2}\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} + \left(4b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \cos\left(a+bx^2\right)\sin\left(a+\right.\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} + \left(2b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(2\left(a+bx^2\right)\right)\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} + \left(2b\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(2a+2bx^2\right)\,dx\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} + \left(2b\cos(2a)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \sin\left(2bx^2\right)\,d\\
&= -\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x} + \sqrt{b}\sqrt{\pi}\cos(2a)\csc^2\left(a+bx^2\right)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3} +
\end{aligned}$$

Mathematica [A] time = 0.202151, size = 107, normalized size = 0.81

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2\sqrt{\pi}\sqrt{b}x\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+2\sqrt{\pi}\sqrt{b}x\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+\cos\left(2\left(a+bx^2\right)\right.\right.}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x^2]^2*(-1 + Cos[2*(a + b*x^2)] + 2*Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]*
FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] + 2*Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[b]
*x)/Sqrt[Pi]]*Sin[2*a])*(c*Sin[a + b*x^2]^3)^(2/3))/(2*x)
```

Maple [C] time = 0.096, size = 301, normalized size = 2.3

$$-\frac{1}{4\left(e^{2i\left(bx^2+a\right)}-1\right)^2x}\left(ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}\right)^{\frac{2}{3}}-\frac{\frac{i}{4}e^{2ibx^2}b\sqrt{\pi}\sqrt{2}}{\left(e^{2i\left(bx^2+a\right)}-1\right)^2}\left(ic\left(e^{2i\left(bx^2+a\right)}-1\right)^3e^{-3i\left(bx^2+a\right)}\right)^{\frac{2}{3}}\text{Erf}\left(\sqrt{2}\sqrt{i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x^2,x)
```

```
[Out] -1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2/x-1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3
)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*b*Pi^(1/2)*2^(1/2)/(I*b)^(1/2)*er
f(2^(1/2)*(I*b)^(1/2)*x)+1/4*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+
a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*(-1/x*exp(4*I*(b*x^2+a))+2*I*b*Pi^(1/2)
/(-2*I*b)^(1/2)*erf((-2*I*b)^(1/2)*x)*exp(2*I*(b*x^2+2*a)))+1/2*(I*c*(exp(2
*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2/x*ex
p(2*I*(b*x^2+a))
```

Maxima [C] time = 1.75053, size = 510, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(2)*sqrt(x^2*abs(b))*((((I*sqrt(3) + 1)*gamma(-1/2, 2*I*b*x^2) +
 (-I*sqrt(3) + 1)*gamma(-1/2, -2*I*b*x^2))*cos(1/4*pi + 1/2*arctan2(0, b))
+ ((-I*sqrt(3) + 1)*gamma(-1/2, 2*I*b*x^2) + (I*sqrt(3) + 1)*gamma(-1/2, -2
*I*b*x^2))*cos(-1/4*pi + 1/2*arctan2(0, b)) - ((sqrt(3) - I)*gamma(-1/2, 2*
I*b*x^2) + (sqrt(3) + I)*gamma(-1/2, -2*I*b*x^2))*sin(1/4*pi + 1/2*arctan2(
0, b)) - ((sqrt(3) + I)*gamma(-1/2, 2*I*b*x^2) + (sqrt(3) - I)*gamma(-1/2,
-2*I*b*x^2))*sin(-1/4*pi + 1/2*arctan2(0, b)))*cos(2*a) + (((sqrt(3) - I)*g
amma(-1/2, 2*I*b*x^2) + (sqrt(3) + I)*gamma(-1/2, -2*I*b*x^2))*cos(1/4*pi +
 1/2*arctan2(0, b)) - ((sqrt(3) + I)*gamma(-1/2, 2*I*b*x^2) + (sqrt(3) - I)
*gamma(-1/2, -2*I*b*x^2))*cos(-1/4*pi + 1/2*arctan2(0, b)) + ((I*sqrt(3) +
1)*gamma(-1/2, 2*I*b*x^2) + (-I*sqrt(3) + 1)*gamma(-1/2, -2*I*b*x^2))*sin(1
/4*pi + 1/2*arctan2(0, b)) + ((I*sqrt(3) - 1)*gamma(-1/2, 2*I*b*x^2) + (-I*
sqrt(3) - 1)*gamma(-1/2, -2*I*b*x^2))*sin(-1/4*pi + 1/2*arctan2(0, b)))*sin
(2*a))*c^(2/3) - 8*c^(2/3))/x
```

Fricas [A] time = 1.7511, size = 339, normalized size = 2.57

$$-\frac{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\pi x\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+4^{\frac{1}{3}}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)+4^{\frac{1}{3}}\cos\left(bx^{2}+a\right)^{2}-4^{\frac{1}{3}}\right)\left(-\left(c\cos\left(bx^{2}+a\right)^{2}-c\right)\mathrm{si}\right.}{4\left(x\cos\left(bx^{2}+a\right)^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*4^(2/3)*(4^(1/3)*pi*x*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi))
+ 4^(1/3)*pi*x*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b/pi))*sin(2*a) + 4^(1/3)*co
s(b*x^2 + a)^2 - 4^(1/3))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*x^2 + a))^(2/3)/
(x*cos(b*x^2 + a)^2 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x**2,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^2, x)
```

3.349 $\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=161

$$\frac{1}{2}b\sin(2a)\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)$$

```
[Out] -(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(4*x^2) + (Cos[2*(a + b*x^2)
]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(4*x^2) + (b*CosIntegral[2*b
*x^2]*Csc[a + b*x^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (b*Cos[2*a]
*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/2
```

Rubi [A] time = 0.214949, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6720, 3403, 3380, 3297, 3303, 3299, 3302}

$$\frac{1}{2}b\sin(2a)\text{CosIntegral}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}+\frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

```
[Out] -(Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(4*x^2) + (Cos[2*(a + b*x^2)
]*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3))/(4*x^2) + (b*CosIntegral[2*b
*x^2]*Csc[a + b*x^2]^2*Sin[2*a]*(c*Sin[a + b*x^2]^3)^(2/3))/2 + (b*Cos[2*a]
*Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*SinIntegral[2*b*x^2])/2
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3403

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{x^3}\,dx = \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\sin^2\left(a+bx^2\right)}{x^3}\,dx$$

$$= \left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \left(\frac{1}{2x^3}-\frac{\cos\left(2a+2bx^2\right)}{2x^3}\right)dx$$

$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} - \frac{1}{2}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\int \frac{\cos\left(2a\right.}{}$$

$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} - \frac{1}{4}\left(\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\right)\mathrm{Subst}\left(\int\right.$$

$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} + \frac{\cos\left(2\left(a+bx^2\right)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right.}{4x^2}$$

$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} + \frac{\cos\left(2\left(a+bx^2\right)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right.}{4x^2}$$

$$= -\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}}{4x^2} + \frac{\cos\left(2\left(a+bx^2\right)\right)\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right.}{4x^2}$$

Mathematica [A] time = 0.145008, size = 79, normalized size = 0.49

$$\frac{\csc^2\left(a+bx^2\right)\left(c\sin^3\left(a+bx^2\right)\right)^{2/3}\left(2bx^2\sin(2a)\mathrm{CosIntegral}\left(2bx^2\right)+2bx^2\cos(2a)\mathrm{Si}\left(2bx^2\right)+\cos\left(2\left(a+bx^2\right)\right)-1\right.}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^2]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x^2]^2*(c*Sin[a + b*x^2]^3)^(2/3)*(-1 + Cos[2*(a + b*x^2)] + 2*b
*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + 2*b*x^2*Cos[2*a]*SinIntegral[2*b*x^2])
)/(4*x^2)
```

Maple [C] time = 0.095, size = 277, normalized size = 1.7

$$-\frac{1}{8\left(e^{2i(bx^2+a)}-1\right)^2 x^2}\left(ic\left(e^{2i(bx^2+a)}-1\right)^3 e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}+\frac{\frac{i}{4}e^{2ibx^2}b\,\mathrm{Ei}\left(1,2ibx^2\right)}{\left(e^{2i(bx^2+a)}-1\right)^2}\left(ic\left(e^{2i(bx^2+a)}-1\right)^3 e^{-3i(bx^2+a)}\right)^{\frac{2}{3}}+\frac{}{4\left(e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x^2+a)^3)^(2/3)/x^3,x)
```

```
[Out] -1/8*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x
^2+a))-1)^2/x^2+1/4*I*(I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2
/3)/(exp(2*I*(b*x^2+a))-1)^2*exp(2*I*b*x^2)*b*Ei(1,2*I*b*x^2)+1/4*(I*c*(exp
(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))-1)^2*(-
1/2/x^2*exp(4*I*(b*x^2+a))-I*b*Ei(1,-2*I*b*x^2)*exp(2*I*(b*x^2+2*a)))+1/4*(
I*c*(exp(2*I*(b*x^2+a))-1)^3*exp(-3*I*(b*x^2+a)))^(2/3)/(exp(2*I*(b*x^2+a))
-1)^2/x^2*exp(2*I*(b*x^2+a))
```

Maxima [C] time = 1.69055, size = 86, normalized size = 0.53

$$-\frac{\left(\left(\left(i\Gamma\left(-1,2i\,bx^2\right)-i\Gamma\left(-1,-2i\,bx^2\right)\right)\cos\left(2a\right)+\left(\Gamma\left(-1,2i\,bx^2\right)+\Gamma\left(-1,-2i\,bx^2\right)\right)\sin\left(2a\right)\right)bx^2-1\right)c^{\frac{2}{3}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(((I*gamma(-1, 2*I*b*x^2) - I*gamma(-1, -2*I*b*x^2))*cos(2*a) + (gamma
(-1, 2*I*b*x^2) + gamma(-1, -2*I*b*x^2))*sin(2*a))*b*x^2 - 1)*c^(2/3)/x^2
```

Fricas [A] time = 1.69334, size = 366, normalized size = 2.27

$$-\frac{4^{\frac{2}{3}}\left(2\cdot 4^{\frac{1}{3}}bx^2\cos\left(2a\right)\mathrm{Si}\left(2bx^2\right)+2\cdot 4^{\frac{1}{3}}\cos\left(bx^2+a\right)^2+\left(4^{\frac{1}{3}}bx^2\,\mathrm{Ci}\left(2bx^2\right)+4^{\frac{1}{3}}bx^2\,\mathrm{Ci}\left(-2bx^2\right)\right)\sin\left(2a\right)-2\cdot 4^{\frac{1}{3}}\right)\left(-\left(\right.\right.}{16\left(x^2\cos\left(bx^2+a\right)^2-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*4^(2/3)*(2*4^(1/3)*b*x^2*cos(2*a)*sin_integral(2*b*x^2) + 2*4^(1/3)*c
os(b*x^2 + a)^2 + (4^(1/3)*b*x^2*cos_integral(2*b*x^2) + 4^(1/3)*b*x^2*cos_
integral(-2*b*x^2))*sin(2*a) - 2*4^(1/3))*(-(c*cos(b*x^2 + a)^2 - c)*sin(b*
x^2 + a))^(2/3)/(x^2*cos(b*x^2 + a)^2 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\sin^3\left(a+bx^2\right)\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x**2+a)**3)**(2/3)/x**3,x)
```

```
[Out] Integral((c*sin(a + b*x**2)**3)**(2/3)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^2+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x^2+a)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^2 + a)^3)^(2/3)/x^3, x)
```

3.350 $\int x^m \left(c \sin^3(a+bx^n)\right)^{2/3}\,dx$

Optimal. Leaf size=217

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(-ibx^n\right)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(ibx^n\right)^{-\frac{m+1}{n}}}{}$$

```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(2*(1 + m)) + (E^((
2*I)*a)*x^(1 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (-2*I)*b*x^n]*(c*Sin[a
+ b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1
 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)/n))
```

Rubi [A] time = 0.364997, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3425, 3424, 2218}

$$\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(-ibx^n\right)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\,\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(ibx^n\right)^{-\frac{m+1}{n}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```


```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(2*(1 + m)) + (E^((
2*I)*a)*x^(1 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (-2*I)*b*x^n]*(c*Sin[a
+ b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) + (x^(1
 + m)*Csc[a + b*x^n]^2*Gamma[(1 + m)/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)/n))
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^m \left(c\sin^3(a+bx^n)\right)^{2/3} dx = \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x^m \sin^2(a+bx^n)\,dx$$
$$= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\left(\frac{x^m}{2}-\frac{1}{2}x^m\cos(2a+2bx^n)\right)dx$$
$$= \frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x$$
$$= \frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e$$
$$= \frac{x^{1+m}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2(1+m)}+\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\csc^2(a+bx^n}{n}$$

Mathematica [A] time = 0.831941, size = 194, normalized size = 0.89

$$\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}\left(b^2x^{2n}\right)^{-\frac{m+1}{n}}\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\left(e^{4ia}(m+1)(ibx^n)^{\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)+(m\right.}{(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^(1 + m)*Csc[a + b*x^n]^2*(2^((1 + m + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n))^
((1 + m)/n) + E^((4*I)*a)*(1 + m)*(I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-
2*I)*b*x^n] + (1 + m)*((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (2*I)*b*x^n
])*(c*Sin[a + b*x^n]^3)^(2/3))/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*(1 + m)*n*(
b^2*x^(2*n))^((1 + m)/n))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int x^m\left(c\left(\sin(a+bx^n)\right)^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^m*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(xx^m-(m+1)\int x^m\cos\left(2\,bx^n+2\,a\right)\,dx\right)c^{\frac{2}{3}}}{4\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(x*x^m - (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))*c^(2/3)/(m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{2}{3}}x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*sin(a+b*x**n)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^m, x)
```

3.351 $\int x^3 \left(c\sin^3(a+bx^n)\right)^{2/3}\,dx$

Optimal. Leaf size=188

$$\frac{e^{2ia}4^{-\frac{2}{n}-1}x^4(-ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{2}{n}-1}x^4(ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n}\right.}{}$$

```
[Out] (x^4*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/8 + (4^(-1 - 2/n)*E^((2*I
)*a)*x^4*Csc[a + b*x^n]^2*Gamma[4/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(4/n)) + (4^(-1 - 2/n)*x^4*Csc[a + b*x^n]^2*Gamma[4/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(4/n))
```

Rubi [A] time = 0.341263, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3425, 3424, 2218}

$$\frac{e^{2ia}4^{-\frac{2}{n}-1}x^4(-ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{2}{n}-1}x^4(ibx^n)^{-4/n}\,\text{Gamma}\left(\frac{4}{n}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^4*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/8 + (4^(-1 - 2/n)*E^((2*I
)*a)*x^4*Csc[a + b*x^n]^2*Gamma[4/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(4/n)) + (4^(-1 - 2/n)*x^4*Csc[a + b*x^n]^2*Gamma[4/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(4/n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^3\left(c\sin^3(a+bx^n)\right)^{2/3}dx = \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x^3\sin^2(a+bx^n)\,dx$$
$$= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\left(\frac{x^3}{2}-\frac{1}{2}x^3\cos(2a+2bx^n)\right)dx$$
$$= \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x^3\cos$$
$$= \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{-2ia-2}$$
$$= \frac{1}{8}x^4\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{4^{-1-\frac{2}{n}}e^{2ia}x^4(-ibx^n)^{-4/n}\csc^2(a+bx^n)\Gamma\left(\frac{4}{n},-2ib\right.}{n}$$

Mathematica [A] time = 0.543004, size = 161, normalized size = 0.86

$$\frac{e^{-2ia}2^{-\frac{4}{n}-3}x^4\left(b^2x^{2n}\right)^{-4/n}\csc^2(a+bx^n)\left(2e^{4ia}(ibx^n)^{4/n}\operatorname{Gamma}\left(\frac{4}{n},-2ibx^n\right)+2(-ibx^n)^{4/n}\operatorname{Gamma}\left(\frac{4}{n},2ibx^n\right)+e^{2ia}16^{\frac{1}{n}}n\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-3 - 4/n)*x^4*Csc[a + b*x^n]^2*(16^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(
4/n) + 2*E^((4*I)*a)*(I*b*x^n)^(4/n)*Gamma[4/n, (-2*I)*b*x^n] + 2*((-I)*b*x
^n)^(4/n)*Gamma[4/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)
*n*(b^2*x^(2*n))^(4/n))
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int x^3\left(c(\sin(a+bx^n))^3\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^3*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16}\left(x^4-4\int x^3\cos(2\,bx^n+2\,a)\,dx\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/16*(x^4 - 4*integrate(x^3*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos\left(bx^{n}+a\right)^{2}-c\right)\sin\left(bx^{n}+a\right)\right)^{\frac{2}{3}}x^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*sin(a+b*x**n)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{2}{3}}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^3, x)
```

3.352 $\int x^2 \left(c\sin^3(a+bx^n)\right)^{2/3} dx$

Optimal. Leaf size=188

$$\frac{e^{2ia}2^{-\frac{3}{n}-2}x^3(-ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{3}{n}-2}x^3(ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},2ib\right.}{n}$$

```
[Out] (x^3*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/6 + (2^(-2 - 3/n)*E^((2*I
)*a)*x^3*Csc[a + b*x^n]^2*Gamma[3/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(3/n)) + (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*Gamma[3/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(3/n))
```

Rubi [A] time = 0.314549, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3425, 3424, 2218}

$$\frac{e^{2ia}2^{-\frac{3}{n}-2}x^3(-ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{3}{n}-2}x^3(ibx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n},2ib\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^3*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/6 + (2^(-2 - 3/n)*E^((2*I
)*a)*x^3*Csc[a + b*x^n]^2*Gamma[3/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/
3))/(n*((-I)*b*x^n)^(3/n)) + (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*Gamma[3/n,
(2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(3/n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule steps

$$\begin{aligned}\int x^2\left(c\sin^3(a+bx^n)\right)^{2/3}dx &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x^2\sin^2(a+bx^n)\,dx\\ &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\left(\frac{x^2}{2}-\frac{1}{2}x^2\cos(2a+2bx^n)\right)dx\\ &= \frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x^2 c\\ &= \frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{-2i}\\ &= \frac{1}{6}x^3\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{2^{-2-\frac{3}{n}}e^{2ia}x^3(-ibx^n)^{-3/n}\csc^2(a+bx^n)\Gamma\left(\frac{3}{n},-\right.}{n}\end{aligned}$$

Mathematica [A] time = 0.55804, size = 168, normalized size = 0.89

$$\frac{e^{-2ia}2^{-\frac{3}{n}-2}x^3\left(b^2x^{2n}\right)^{-3/n}\csc^2(a+bx^n)\left(3e^{4ia}(ibx^n)^{3/n}\text{Gamma}\left(\frac{3}{n},-2ibx^n\right)+3(-ibx^n)^{3/n}\text{Gamma}\left(\frac{3}{n},2ibx^n\right)+e^{2ia}2\right.}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-2 - 3/n)*x^3*Csc[a + b*x^n]^2*(2^((3 + n)/n)*E^((2*I)*a)*n*(b^2*x^(2*n
))^(3/n) + 3*E^((4*I)*a)*(I*b*x^n)^(3/n)*Gamma[3/n, (-2*I)*b*x^n] + 3*((-I)
*b*x^n)^(3/n)*Gamma[3/n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(3*E^((2
*I)*a)*n*(b^2*x^(2*n))^(3/n))
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int x^2\left(c\left(\sin(a+bx^n)\right)^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x^2*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\left(x^3-3\int x^2\cos(2\,bx^n+2\,a)\,dx\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/12*(x^3 - 3*integrate(x^2*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{2}{3}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*sin(a+b*x**n)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x^2, x)
```

3.353 $\int x \left(c \sin^3(a+bx^n)\right)^{2/3} dx$

Optimal. Leaf size=188

$$\frac{e^{2ia}4^{-\frac{1}{n}-1}x^2(-ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{1}{n}-1}x^2(ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},\right.}{}$$

```
[Out] (x^2*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/4 + (4^(-1 - n^(-1))*E^((
2*I)*a)*x^2*Csc[a + b*x^n]^2*Gamma[2/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^
(2/3))/(n*((-I)*b*x^n)^(2/n)) + (4^(-1 - n^(-1))*x^2*Csc[a + b*x^n]^2*Gamma
[2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(2/
n))
```

Rubi [A] time = 0.240431, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6720, 3425, 3424, 2218}

$$\frac{e^{2ia}4^{-\frac{1}{n}-1}x^2(-ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-2ibx^n\right)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{n}+\frac{e^{-2ia}4^{-\frac{1}{n}-1}x^2(ibx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^2*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/4 + (4^(-1 - n^(-1))*E^((
2*I)*a)*x^2*Csc[a + b*x^n]^2*Gamma[2/n, (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^
(2/3))/(n*((-I)*b*x^n)^(2/n)) + (4^(-1 - n^(-1))*x^2*Csc[a + b*x^n]^2*Gamma
[2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^(2/
n))
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x\left(c\sin^3(a+bx^n)\right)^{2/3}dx=\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x\sin^2(a+bx^n)\,dx$$
$$=\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\left(\frac{x}{2}-\frac{1}{2}x\cos(2a+2bx^n)\right)dx$$
$$=\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int x\cos(2$$
$$=\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{-2ia-2ib}$$
$$=\frac{1}{4}x^2\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{4^{-1-\frac{1}{n}}e^{2ia}x^2(-ibx^n)^{-2/n}\csc^2(a+bx^n)\Gamma\left(\frac{2}{n},-2ibx\right.}{n}$$

Mathematica [A] time = 0.548596, size = 160, normalized size = 0.85

$$\frac{e^{-2ia}4^{-\frac{n+1}{n}}x^2\left(b^2x^{2n}\right)^{-2/n}\csc^2(a+bx^n)\left(e^{4ia}(ibx^n)^{2/n}\text{Gamma}\left(\frac{2}{n},-2ibx^n\right)+(-ibx^n)^{2/n}\text{Gamma}\left(\frac{2}{n},2ibx^n\right)+e^{2ia}4^{\frac{1}{n}}n\left(b^2x\right.\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (x^2*Csc[a + b*x^n]^2*(4^n^(-1)*E^((2*I)*a)*n*(b^2*x^(2*n))^(2/n) + E^((4*I
)*a)*(I*b*x^n)^(2/n)*Gamma[2/n, (-2*I)*b*x^n] + ((-I)*b*x^n)^(2/n)*Gamma[2/
n, (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4^((1 + n)/n)*E^((2*I)*a)*n*(
b^2*x^(2*n))^(2/n))
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int x\left(c(\sin(a+bx^n))^3\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int(x*(c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\left(x^2-2\int x\cos(2\,bx^n+2\,a)\,dx\right)c^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/8*(x^2 - 2*integrate(x*cos(2*b*x^n + 2*a), x))*c^(2/3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{2}{3}}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x**n)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)*x, x)
```

3.354 $\int \left(c\sin^3(a+bx^n)\right)^{2/3}\,dx$

Optimal. Leaf size=178

$$\frac{e^{2ia}2^{-\frac{1}{n}-2}x\left(-ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x\left(ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2ibx\right.}{n}$$

```
[Out] (x*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (2^(-2 - n^(-1))*E^((2*
I)*a)*x*Csc[a + b*x^n]^2*Gamma[n^(-1), (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(n*((-I)*b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*Gamma[n
^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^n^
(-1))
```

Rubi [A] time = 0.0888038, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 3367, 3366, 2208}

$$\frac{e^{2ia}2^{-\frac{1}{n}-2}x\left(-ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{n}+\frac{e^{-2ia}2^{-\frac{1}{n}-2}x\left(ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},2ibx\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3),x]
```


```
[Out] (x*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (2^(-2 - n^(-1))*E^((2*
I)*a)*x*Csc[a + b*x^n]^2*Gamma[n^(-1), (-2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(
2/3))/(n*((-I)*b*x^n)^n^(-1)) + (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*Gamma[n
^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a)*n*(I*b*x^n)^n^
(-1))
```


Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3367

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rubi steps

$$\int \left(c\sin^3(a+bx^n)\right)^{2/3}dx = \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \sin^2(a+bx^n)\,dx$$
$$= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \left(\frac{1}{2}-\frac{1}{2}\cos(2a+2bx^n)\right)dx$$
$$= \frac{1}{2}x\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \cos(2a$$
$$= \frac{1}{2}x\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int e^{-2ia-2i}$$
$$= \frac{1}{2}x\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}+\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\csc^2(a+bx^n)\Gamma\left(\frac{1}{n},-2ibx\right.}{n}$$

Mathematica [A] time = 0.266812, size = 149, normalized size = 0.84

$$\frac{e^{-2ia}2^{-\frac{1}{n}-2}x\left(b^2x^{2n}\right)^{-1/n}\csc^2(a+bx^n)\left(e^{4ia}(ibx^n)^{\frac{1}{n}}\operatorname{Gamma}\left(\frac{1}{n},-2ibx^n\right)+(-ibx^n)^{\frac{1}{n}}\operatorname{Gamma}\left(\frac{1}{n},2ibx^n\right)+e^{2ia}2^{\frac{1}{n}+1}n\right.(}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3),x]
```

```
[Out] (2^(-2 - n^(-1))*x*Csc[a + b*x^n]^2*(2^(1 + n^(-1))*E^((2*I)*a)*n*(b^2*x^(2
*n))^n^(-1) + E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-2*I)*b*x^n] + ((
-I)*b*x^n)^n^(-1)*Gamma[n^(-1), (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(
E^((2*I)*a)*n*(b^2*x^(2*n))^n^(-1))
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \left(c\left(\sin(a+bx^n)\right)^3\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3),x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}c^{\frac{2}{3}}\left(x-\int \cos(2bx^n+2a)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*c^(2/3)*(x - integrate(cos(2*b*x^n + 2*a), x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3), x)
```

3.355 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx$

Optimal. Leaf size=121

$$-\frac{\cos(2a)\text{CosIntegral}\left(2bx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}+\frac{\sin(2a)\text{Si}\left(2bx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}$$

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^n]*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)
)/(2*n) + (Csc[a + b*x^n]^2*Log[x]*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (Csc[a +
 b*x^n]^2*Sin[2*a]*(c*Sin[a + b*x^n]^3)^(2/3)*SinIntegral[2*b*x^n])/(2*n)
```

Rubi [A] time = 0.178981, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6720, 3425, 3378, 3376, 3375}

$$-\frac{\cos(2a)\text{CosIntegral}\left(2bx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}+\frac{\sin(2a)\text{Si}\left(2bx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

```
[Out] -(Cos[2*a]*CosIntegral[2*b*x^n]*Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)
)/(2*n) + (Csc[a + b*x^n]^2*Log[x]*(c*Sin[a + b*x^n]^3)^(2/3))/2 + (Csc[a +
 b*x^n]^2*Sin[2*a]*(c*Sin[a + b*x^n]^3)^(2/3)*SinIntegral[2*b*x^n])/(2*n)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x}\,dx = \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\sin^2(a+bx^n)}{x}\,dx$$
$$= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int\left(\frac{1}{2x}-\frac{\cos(2a+2bx^n)}{2x}\right)dx$$
$$= \frac{1}{2}\csc^2(a+bx^n)\log(x)\left(c\sin^3(a+bx^n)\right)^{2/3} - \frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\cos(}{}$$
$$= \frac{1}{2}\csc^2(a+bx^n)\log(x)\left(c\sin^3(a+bx^n)\right)^{2/3} - \frac{1}{2}\left(\cos(2a)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)$$
$$= -\frac{\cos(2a)\mathrm{Ci}(2bx^n)\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2n} + \frac{1}{2}\csc^2(a+bx^n)\log(x)\left(c\sin^3(a+b\right.$$

Mathematica [A] time = 0.1383, size = 63, normalized size = 0.52

$$\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\left(-\cos(2a)\mathrm{CosIntegral}(2bx^n)+\sin(2a)\mathrm{Si}(2bx^n)+n\log(x)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x,x]
```

```
[Out] (Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3)*(-(Cos[2*a]*CosIntegral[2*b*x^
n]) + n*Log[x] + Sin[2*a]*SinIntegral[2*b*x^n]))/(2*n)
```

Maple [C] time = 0.144, size = 343, normalized size = 2.8

$$\frac{\frac{i}{4}e^{2\,ibx^n}\pi\,\mathrm{csgn}(bx^n)}{\left(e^{2\,i(a+bx^n)}-1\right)^2 n}\left(ic\left(e^{2\,i(a+bx^n)}-1\right)^3 e^{-3\,i(a+bx^n)}\right)^{\frac{2}{3}} - \frac{\frac{i}{2}e^{2\,ibx^n}\mathrm{Si}(2\,bx^n)}{\left(e^{2\,i(a+bx^n)}-1\right)^2 n}\left(ic\left(e^{2\,i(a+bx^n)}-1\right)^3 e^{-3\,i(a+bx^n)}\right)^{\frac{2}{3}} - \frac{e^{2\,ibx^n}\mathrm{Ei}(1,-}{4\left(e^{2\,i(a+bx^n)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x,x)
```

```
[Out] 1/4*I*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(exp(2*I*(a+
b*x^n))-1)^2*exp(2*I*b*x^n)/n*Pi*csgn(b*x^n)-1/2*I*(I*c*(exp(2*I*(a+b*x^n))
-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(exp(2*I*(a+b*x^n))-1)^2*exp(2*I*b*x^n)/n*
Si(2*b*x^n)-1/4*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(e
xp(2*I*(a+b*x^n))-1)^2*exp(2*I*b*x^n)/n*Ei(1,-2*I*b*x^n)-1/4*(I*c*(exp(2*I*
(a+b*x^n))-1)^3*exp(-3*I*(a+b*x^n)))^(2/3)/(exp(2*I*(a+b*x^n))-1)^2/n*Ei(1,
-2*I*b*x^n)*exp(2*I*(b*x^n+2*a))-1/2*(I*c*(exp(2*I*(a+b*x^n))-1)^3*exp(-3*I
*(a+b*x^n)))^(2/3)/(exp(2*I*(a+b*x^n))-1)^2*ln(x)*exp(2*I*(a+b*x^n))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 1.86783, size = 317, normalized size = 2.62

$$\frac{4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^{n}\right)+4^{\frac{1}{3}}\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,bx^{n}\right)-2\cdot4^{\frac{1}{3}}n\log\left(x\right)-2\cdot4^{\frac{1}{3}}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^{n}\right)\right)\left(-\left(c\cos\left(bx^{n}+a\right)^{2}-\right.\right.}{16\left(n\cos\left(bx^{n}+a\right)^{2}-n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/16*4^(2/3)*(4^(1/3)*cos(2*a)*cos_integral(2*b*x^n) + 4^(1/3)*cos(2*a)*cos
_integral(-2*b*x^n) - 2*4^(1/3)*n*log(x) - 2*4^(1/3)*sin(2*a)*sin_integral(
2*b*x^n))*(-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/(n*cos(b*x^n + a
)^2 - n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x, x)
```

3.356 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=180

$$\frac{e^{2ia}2^{\frac{1}{n}-2}\left(-ibx^n\right)^{\frac{1}{n}}\operatorname{Gamma}\left(-\frac{1}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx}+\frac{e^{-2ia}2^{\frac{1}{n}-2}\left(ibx^n\right)^{\frac{1}{n}}\operatorname{Gamma}\left(-\frac{1}{n},2ibx^n\right)\csc}{nx}$$

```
[Out] -(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(2*x) + (2^(-2 + n^(-1))*E^(
(2*I)*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]^2*Gamma[-n^(-1), (-2*I)*b*x^n]*
(c*Sin[a + b*x^n]^3)^(2/3))/(n*x) + (2^(-2 + n^(-1))*(I*b*x^n)^n^(-1)*Csc[a
 + b*x^n]^2*Gamma[-n^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*
I)*a)*n*x)
```

Rubi [A] time = 0.275436, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3425, 3424, 2218}

$$\frac{e^{2ia}2^{\frac{1}{n}-2}\left(-ibx^n\right)^{\frac{1}{n}}\operatorname{Gamma}\left(-\frac{1}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx}+\frac{e^{-2ia}2^{\frac{1}{n}-2}\left(ibx^n\right)^{\frac{1}{n}}\operatorname{Gamma}\left(-\frac{1}{n},2ibx^n\right)\csc}{nx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

```
[Out] -(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(2*x) + (2^(-2 + n^(-1))*E^(
(2*I)*a)*((-I)*b*x^n)^n^(-1)*Csc[a + b*x^n]^2*Gamma[-n^(-1), (-2*I)*b*x^n]*
(c*Sin[a + b*x^n]^3)^(2/3))/(n*x) + (2^(-2 + n^(-1))*(I*b*x^n)^n^(-1)*Csc[a
 + b*x^n]^2*Gamma[-n^(-1), (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*
I)*a)*n*x)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^2}dx = \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\sin^2(a+bx^n)}{x^2}dx$$
$$= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \left(\frac{1}{2x^2}-\frac{\cos(2a+2bx^n)}{2x^2}\right)dx$$
$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x} - \frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\cos(2a}{x}$$
$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x} - \frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{e^{-2ia-2ib}}{x^2}$$
$$= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{2x} + \frac{2^{-2+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\csc^2(a+bx^n)\Gamma\left(-\frac{1}{n},-2ibx^n\right)(}{nx}$$

Mathematica [A] time = 0.344419, size = 125, normalized size = 0.69

$$\frac{e^{-2ia}\csc^2(a+bx^n)\left(e^{4ia}2^{\frac{1}{n}}(-ibx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-2ibx^n\right)+2^{\frac{1}{n}}(ibx^n)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},2ibx^n\right)-2e^{2ia}n\right)\left(c\sin^3(a+bx\right.}{4nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^2,x]
```

```
[Out] (Csc[a + b*x^n]^2*(-2*E^((2*I)*a)*n + 2^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^n^(
-1)*Gamma[-n^(-1), (-2*I)*b*x^n] + 2^n^(-1)*(I*b*x^n)^n^(-1)*Gamma[-n^(-1),
 (2*I)*b*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(c(\sin(a+bx^n))^3\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(x\int \frac{\cos(2bx^n+2a)}{x^2}dx+1\right)c^{\frac{2}{3}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x*integrate(cos(2*b*x^n + 2*a)/x^2, x) + 1)*c^(2/3)/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\left(c\cos\left(bx^{n}+a\right)^{2}-c\right)\sin\left(bx^{n}+a\right)\right)^{\frac{2}{3}}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^{n}+a\right)^{3}\right)^{\frac{2}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x^2, x)
```

3.357 $\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=184

$$\frac{e^{2ia}4^{\frac{1}{n}-1}\left(-ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx^2}+\frac{e^{-2ia}4^{\frac{1}{n}-1}\left(ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},2ibx^n\right.}{nx^2}$$

```
[Out] -(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(4*x^2) + (4^(-1 + n^(-1))*E
^((2*I)*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]^2*Gamma[-2/n, (-2*I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(2/3))/(n*x^2) + (4^(-1 + n^(-1))*(I*b*x^n)^(2/n)*Csc[a
+ b*x^n]^2*Gamma[-2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a
)*n*x^2)
```

Rubi [A] time = 0.268642, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6720, 3425, 3424, 2218}

$$\frac{e^{2ia}4^{\frac{1}{n}-1}\left(-ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},-2ibx^n\right)\csc^2\left(a+bx^n\right)\left(c\sin^3\left(a+bx^n\right)\right)^{2/3}}{nx^2}+\frac{e^{-2ia}4^{\frac{1}{n}-1}\left(ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},2ibx^n\right.}{nx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

```
[Out] -(Csc[a + b*x^n]^2*(c*Sin[a + b*x^n]^3)^(2/3))/(4*x^2) + (4^(-1 + n^(-1))*E
^((2*I)*a)*((-I)*b*x^n)^(2/n)*Csc[a + b*x^n]^2*Gamma[-2/n, (-2*I)*b*x^n]*(c
*Sin[a + b*x^n]^3)^(2/3))/(n*x^2) + (4^(-1 + n^(-1))*(I*b*x^n)^(2/n)*Csc[a
+ b*x^n]^2*Gamma[-2/n, (2*I)*b*x^n]*(c*Sin[a + b*x^n]^3)^(2/3))/(E^((2*I)*a
)*n*x^2)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 3425

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sin[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
```

```
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c\sin^3(a+bx^n)\right)^{2/3}}{x^3}\,dx &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\sin^2(a+bx^n)}{x^3}\,dx\\ &= \left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \left(\frac{1}{2x^3}-\frac{\cos(2a+2bx^n)}{2x^3}\right)dx\\ &= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4x^2}-\frac{1}{2}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{\cos(2a+2\ldots}{x^3}\\ &= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4x^2}-\frac{1}{4}\left(\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}\right)\int \frac{e^{-2ia-2ibx^n}}{x^3}\,d\ldots\\ &= -\frac{\csc^2(a+bx^n)\left(c\sin^3(a+bx^n)\right)^{2/3}}{4x^2}+\frac{4^{-1+\frac{1}{n}}e^{2ia}\left(-ibx^n\right)^{2/n}\csc^2(a+bx^n)\,\Gamma\left(-\frac{2}{n},-2ibx^n\right)\left(c\ldots\right.}{nx^2}\end{aligned}$$

Mathematica [A] time = 0.352419, size = 129, normalized size = 0.7

$$\frac{e^{-2ia}\csc^2(a+bx^n)\left(e^{4ia}4^{\frac{1}{n}}\left(-ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},-2ibx^n\right)+4^{\frac{1}{n}}\left(ibx^n\right)^{2/n}\text{Gamma}\left(-\frac{2}{n},2ibx^n\right)-e^{2ia}n\right)\left(c\sin^3(a+bx^n)\ldots\right.}{4nx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x^n]^3)^(2/3)/x^3,x]
```

```
[Out] (Csc[a + b*x^n]^2*(-(E^((2*I)*a)*n) + 4^n^(-1)*E^((4*I)*a)*((-I)*b*x^n)^(2/
n)*Gamma[-2/n, (-2*I)*b*x^n] + 4^n^(-1)*(I*b*x^n)^(2/n)*Gamma[-2/n, (2*I)*b
*x^n])*(c*Sin[a + b*x^n]^3)^(2/3))/(4*E^((2*I)*a)*n*x^2)
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\left(c\left(\sin(a+bx^n)\right)^3\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

```
[Out] int((c*sin(a+b*x^n)^3)^(2/3)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,x^2\int \frac{\cos(2\,bx^n+2\,a)}{x^3}\,dx+1\right)c^{\frac{2}{3}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*x^2*integrate(cos(2*b*x^n + 2*a)/x^3, x) + 1)*c^(2/3)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\left(c\cos\left(bx^n+a\right)^2-c\right)\sin\left(bx^n+a\right)\right)^{\frac{2}{3}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x^n + a)^2 - c)*sin(b*x^n + a))^(2/3)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x**n)**3)**(2/3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sin\left(bx^n+a\right)^3\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(a+b*x^n)^3)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x^n + a)^3)^(2/3)/x^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```